Optimal Offer-Bid Strategy of an Energy Storage Portfolio: A Linear Quasi-Relaxation Approach

Journal:	*IEEE Transactions on Smart Grid*
Manuscript ID	TSG-00263-2019
Manuscript Type:	Transactions
Date Submitted by the Author:	18-Feb-2019
Complete List of Authors:	Tómasson, Egill; Kungliga Tekniska Hogskolan Skolan for teknikvetenskap, EPE Hesamzadeh, Mohammad Reza; KTH Royal Institute of Technology, Electric Power Systems Wolak, Frank; Stanford University, Economics
Technical Topic Area :	Analytic Methods for Smart Grid (optimization, stochastic, hierarchical control) < Transactions on Smart Grid, Demand-Side Management and Scheduling (appliances, electric vehicles, aggregation, demand response) < Transactions on Smart Grid
Key Words:	Merchant storage, Offer-bid strategy, Bilinear program, Disjunctive program, Linear quasi-relaxation

Optimal Offer-Bid Strategy of an Energy Storage Portfolio: A Linear Quasi-Relaxation Approach

Egill Tómasson, *Student Member, IEEE*, Mohammad Reza Hesamzadeh, *Senior Member, IEEE* and Frank A. Wolak

***Abstract*—This paper proposes a model of the behavior of an expected profit-maximizing merchant storage owner with the ability to exercise unilateral market power. The resulting non-linear bilevel optimization problem is transformed into a single-level stochastic bilinear program using the KKT conditions of the lower-level Independent System Operator (ISO) dispatch problem. By discretizing the offers and bids of the merchant storage owner, the problem is formulated as a stochastic disjunctive program. Using the disjunctive nature of the derived program, a specialized branch-and-bound algorithm that applies a linear quasi-relaxation of the merchant storage problem is proposed. Our solution algorithm is able to solve the problem in an efficient manner; returning the charge and discharge strategies for the merchant storage owner that yield the highest expected profits. Simulations of test systems reveal the various abilities of the merchant storage owner to exercise unilateral market power. Those include *demand withholding*, *generation withholding* and *under-use* which result in an increased congestion in both space and time when compared to the welfare-maximizing use of storage. Moreover, numerical results demonstrate the superior computational performance of the proposed solution algorithm when benchmarked against current practices in the literature.**

***Index Terms*—Merchant storage, Offer-bid strategy, Bilinear program, Disjunctive program, Linear quasi-relaxation**

NOMENCLATURE

Indices and sets

i	Index of units.
n, m	Indices of buses.
t	Index of time periods.
l	Index of discrete offer-bid values.
w	Index of stochastic scenarios.
$\mathcal{I}^S$	Set of strategic storage units.
$\mathcal{I}^{NS}$	Set of units of non-strategic players.

Variables

$\hat{p}_{it}$	Generation offer of players.
$\hat{d}_{it}$	Demand bid of players.
$\hat{c}_{it}$	Offered price of players.
f_{nmtw}	Real power transmission line flow.
p_{itw}	Real power generation of players.
d_{itw}	Real power demand of players.
s_{itw}	State of charge of storage unit.
θ_{ntw}	Voltage angle at bus n.
λ, μ	Dual variables.
$\lambda^{(2)}_{ntw}$	Electricity price at bus n.
$x^P_{itlw}, x^D_{itlw}, x^C_{itlw}$	Binary variables in MILP.
$z^P_{itlw}, z^D_{itlw}, z^C_{itlw}$	Bilinear term intermediate values.
$\pi^P_{itw}, \pi^D_{itw}, \pi^C_{itw}$	Bilinear term intermediate values.
$y^P_{it}, y^D_{it}, y^C_{it}$	Spanning variables.

Parameters

b_{nm}	Susceptance of a transmission line.
D_{ntw}	Net real power demand at bus n.
c_i	Variable cost of units.
ρ_w	Probability of scenario w.
$\underline{S}_i$, $\overline{S}_i$	Charge limits of merchant storage.
S^0_i	Initial state of charge.
$\underline{F}_{nm}$, $\overline{F}_{nm}$	Real flow limits of a transmission line.
$\underline{\Theta}_n$, $\overline{\Theta}_n$	Voltage angle limits on bus n.
$\underline{P}_i$, $\overline{P}_i$	Real power generation limits of units.
$\underline{D}_i$, $\overline{D}_i$	Real power demand limits of units.
C_{ni}	Node-unit connection matrix.
$\hat{P}_l, \hat{D}_l, \hat{C}_l$	Discrete offer-bid values.
n_P, n_D, n_C	Number of possible discrete values.
M^P, M^D, M^C	Sufficiently large constants.

Operator

$\mathbf{I}(condition)$	If operator.

I. INTRODUCTION

ENERGY storage systems have the potential to significantly improve the operation of the power system of today, especially because of the ever increasing generation from intermittent renewable resources. The applications of energy storage systems are diverse and include voltage support [1], frequency regulation, synchronous/non-synchronous reserve [2] as well as spatio-temporal energy arbitrage [3]–[7]. Furthermore, applications of storage systems for micro grids and demand response [8], [9] as well as grid-tied applications of electric vehicle storage [10]–[12] are gaining ground.

It is expected that a decrease in the capital cost of energy storage systems will eventually spur the deployment of large amounts of energy storage [13]. This raises the issue of market power. Exercise of unilateral market power[1] is a concern in today's electricity markets and numerous mathematical models have been developed to study such behavior [14]. The author of [15] quantifies the impact of the exercise of unilateral market power by a large hydroelectric generation facility in the western U.S. The author develops a model which solves for a sub-game perfect equilibrium of a multi-period Cournot game between strategic producers, one of which owns hydroelectric capacity with the ability to store water. Reference [16] models the offer behavior of a plant owner maximizing its expected profit and with the ability to exercise unilateral market power. In [17], the framework from [16] is used to test the assumption of expected profit-maximizing offer behavior in a short-term electricity market and the results show no evidence against it.

The issue of unilateral market power is especially interesting in the case of energy storage because storage units can act both

[1]This term is generally used in the economics literature while the terms *strategic* or *strategic behavior* are more common in the engineering literature.

as generators and loads. A portfolio of storage units is able to influence the market price in multiple ways; by creating congestion in both space and time, by withholding generation as well as by withholding demand. Looking at the literature, relatively little focus has been put on studying large, price-maker storage portfolios. However, recent work such as [18] evaluates the impact of strategic behavior of an independent trader operating energy storage systems while the authors of [19] assess the various consequences, including those of market power, of storage via a complementarity model of a stylized Western European power system. Reference [20] studies how storage, operating as a price maker, may be optimally operated over an extended period of time. In [21], the author proposes an optimization framework to coordinate the operation of large, price-maker, geographically dispersed storage systems in a nodal transmission-constrained market.

In line with the work in [21], this paper studies the operation of a profit-maximizing merchant storage owner with the ability to exercise unilateral market power. The paper is motivated by the recent FERC order to allow medium to large scale storage resources to directly participate in the wholesale market (either through direct bidding or self-scheduling). Worldwide there has also been a growing trend for more storage resources to participate in wholesale markets. The point of the model is to understand what could happen in the future when storage capacity is expected to increase and the potential of one firm to own a significant amount of storage is likely. The problem formulation is a bilevel one where in the upper-level problem, the merchant storage owner makes offers and bids to the market in order to maximize its expected profit, subject to the lower-level optimal dispatch problem of the independent system operator (ISO) for a variety of possible scenarios. The main contributions of this paper are:

1) A derivation of a stochastic disjunctive program model for finding the optimal offer-bid strategy of a merchant storage portfolio which maximizes the expected profit over several possible market scenarios.
2) A Specialized Branch-and-Bound (SBB) solution algorithm that applies a linear quasi-relaxation which significantly reduces the computational requirements when solving the merchant storage problem.
3) Both the proposed stochastic disjunctive programming model and the SBB solution algorithm are benchmarked against current practices in the literature for modeling and solving these types of problems.

The rest of the paper is organized as follows. Section II covers the mathematical model where the physical constraints of the system are listed and the market players of the system are introduced. A stochastic bilevel merchant storage problem is then derived and using the Karush-Kuhn-Tucker (KKT) conditions as well as bid discretization, it is reformulated as a single-level stochastic disjunctive program. Section III derives a specialized solution algorithm that applies a linear quasi-relaxation of the merchant storage problem and solves the problem in a computationally efficient way. Sections IV and V show an illustrative example and numerical simulations, respectively, of test systems that confirm the performance of the proposed method. Section VI concludes the paper.

II. Mathematical Model

The market is composed of two types of market players; a single large merchant storage owner whose units are in the set $\mathcal{I}^S$ and traditional generators whose units are in the set $\mathcal{I}^{NS}$. Index i represents all units in the market. The production of each player at time t, scenario w is represented by p_{itw} and the demand of each player is represented by d_{itw}. The production and demand are limited by the lower and upper limits $(\underline{P}_i, \overline{P}_i)$ and $(\underline{D}_i, \overline{D}_i)$, respectively. The charging power of a storage unit is represented by d_{itw} (unit acts as a load) and the discharging power of a storage unit is represented by p_{itw} (unit acts as a generator). Without loss of generality, the storage units are assumed to have ideal round-trip efficiencies. The storage level of each storage unit is represented by s_{itw} and the storage capacity is limited by the lower and upper limits $(\underline{S}_i, \overline{S}_i)$. The initial state of charge is represented by S_i^0. The storage owner chooses its offer price and quantity pairs to maximize its expected profits. The storage owner computes the expected profits associated with a combination of offer price and quantity pairs by solving a lower-level ISO market equilibrium several times for a variety of possible scenarios. The expected profits are the probability weighted sum of these realized profit outcomes. The ISO is assumed to solve an optimization problem minimizing the as offered cost of generation and therefore the complete optimization problem that the merchant storage owner needs to solve is a bilevel one.

A. Stochastic Bilevel Program

The bilevel merchant storage problem is given in (1).

$$\underset{\hat{p}_{it},\hat{d}_{it},\hat{c}_{it}}{\text{maximize}} \sum_{i\in\mathcal{I}^S,t,w} \rho_w \sum_n C_{ni}\lambda^{(2)}_{ntw}(p_{itw} - d_{itw}) \tag{1a}$$

subject to:

$$\hat{p}_{it}\hat{d}_{it} = 0,\ \forall(i \in \mathcal{I}^S)t, \tag{1b}$$

$$\underline{P}_i \le \hat{p}_{it} \le \overline{P}_i,\ \underline{D}_i \le \hat{d}_{it} \le \overline{D}_i,\ \forall(i\in\mathcal{I}^S)t, \tag{1c}$$

$$\hat{p}_{it} = \overline{P}_i,\ \hat{d}_{it} = 0,\ \hat{c}_{it} = c_i,\ \forall(i\in\mathcal{I}^{NS})t, \tag{1d}$$

$$s_{itw} = s_{i,t-1,w} + S_i^0\mathbf{I}(t=1) + d_{itw} - p_{itw},\ \forall(i\in\mathcal{I}^S)tw, \tag{1e}$$

$$\underline{S}_i \le s_{itw} \le \overline{S}_i,\ \forall(i\in\mathcal{I}^S)tw, \tag{1f}$$

where $\{\lambda^{(2)}_{ntw},\, p_{itw},\, d_{itw}\} \in$

$$\arg\left\{\underset{p_{itw},d_{itw},f_{nmtw},\theta_{ntw}}{\text{minimize}} \sum_{i,t}(p_{itw} - d_{itw})\hat{c}_{it}\right. \tag{1g}$$

subject to:

$$f_{nmtw} = -b_{nm}(\theta_{ntw} - \theta_{mtw}) : \lambda^{(1)}_{nmtw},\ \forall nmtw, \tag{1h}$$

$$\sum_i (p_{itw} - d_{itw})C_{ni} - \sum_m^{n\neq m} f_{nmtw} = D_{ntw} : \lambda^{(2)}_{ntw},\ \forall ntw, \tag{1i}$$

$$\underline{F}_{nm} \le f_{nmtw} \le \overline{F}_{nm} : \mu^{(1)}_{nmtw}, \mu^{(2)}_{nmtw},\ \forall nmtw, \tag{1j}$$

$$\underline{\Theta}_n \le \theta_{ntw} \le \overline{\Theta}_n : \mu^{(3)}_{ntw}, \mu^{(4)}_{ntw},\ \forall ntw, \tag{1k}$$

$$\left.\begin{aligned}&\underline{P}_i \le p_{itw} \le \hat{p}_{it} : \mu^{(5)}_{itw}, \mu^{(6)}_{itw},\ \forall itw, &(1l)\\ &\underline{D}_i \le d_{itw} \le \hat{d}_{it} : \mu^{(7)}_{itw}, \mu^{(8)}_{itw},\ \forall itw\end{aligned}\right\}. \quad (1m)$$

Uncertainties in the market are captured by probabilistic scenarios, indexed by w with probabilities ρ_w. The main source of uncertainty is the net demand D_{ntw}, which depends on the intermittent generation. Scenarios are constructed based on forecasts of the demand and intermittent generation. In the upper-level problem, the merchant storage owner maximizes the expected profit over the time horizon and the different scenarios. The resulting solution is the generation offer quantity $\hat{p}_{it}$, the demand bid quantity $\hat{d}_{it}$ and the price $\hat{c}_{it}$. Note that they are independent of the scenarios. The term $\sum_n C_{ni}\lambda^{(2)}_{ntw}$ in the upper-level objective function represents the nodal price that player i is exposed to. $\lambda^{(2)}_{ntw}$ is the nodal price and C_{ni} is a binary node-unit connection matrix. Constraint (1b) makes sure that in each time period, the storage units submit offers or bids to the market either as a generator or as a load [21]. The offer-bid values $\hat{p}_{it}$ and $\hat{d}_{it}$ must confine to the physical limits of each unit (1c). Constraints (1d) ensure that the non-strategic players offer their true capacities $\overline{P}_i$ and bids c_i and that their demand is zero. One can also treat the bids of the non-strategic generators as uncertain by using several scenarios for them because they may have private information. The energy balance of the storage units is captured by (1e) and the energy limits by (1f). The storage owner is considered responsible for the energy limits so those constraints appear in the upper-level problem [21]. The lower-level ISO dispatch problem minimizes the as offered generation cost in the system while taking into account the power flow constraints (1h) and the energy balance on each bus (1i). There are also lower and upper limits on the real power flows f_{nmtw} (1j), the voltage angles θ_{ntw} (1k) and the dispatch values p_{itw} and d_{itw} (1l)–(1m). The lower-level dual variables are given after the colon.

The formulation above assumes that the merchant storage owner submits *price-quantity pairs* to the market, that is offers or bids that contain both price and quantity. The model in (1) can also capture *self-scheduling*, e.g. offers or bids without a price component, if minor changes are made to the formulation. Specifically, the lower-level objective function becomes $\sum_{i,t}(p_{itw}\hat{c}_{it} - d_{itw}\hat{u}_{it})$ where for all storage units the submitted generation offer price $\hat{c}_{it}$ is zero, and the submitted demand bid price $\hat{u}_{it}$ is a sufficiently high constant [21].

The above model is in general very hard to solve because: i) it is bilevel, ii) it is non-linear and iii) transmission congestion implies that small changes in offer behavior can create large changes in realized market outcomes.

B. The proposed stochastic disjunctive program

In order to compose a single-level stochastic optimization problem from the stochastic bilevel program given in (1), the KKT conditions are derived for the lower-level problem. The stationary conditions are given in (2).

$$\frac{\partial L}{\partial f_{nmtw}} = \lambda^{(1)}_{nmtw} - \lambda^{(2)}_{ntw}\mathbf{I}(n \neq m) - \mu^{(1)}_{nmtw} + \mu^{(2)}_{nmtw} = 0,\ \forall nmtw, \quad (2a)$$

$$\frac{\partial L}{\partial \theta_{nmtw}} = \sum_m [b_{nm}\lambda^{(1)}_{nmtw} - b_{mn}\lambda^{(1)}_{mntw}] - \mu^{(3)}_{nw} + \mu^{(4)}_{nw} = 0,\ \forall ntw, \quad (2b)$$

$$\frac{\partial L}{\partial p_{itw}} = \sum_n [C_{ni}\lambda^{(2)}_{ntw}] - \mu^{(5)}_{itw} + \mu^{(6)}_{itw} = \hat{c}_{it},\ \forall itw, \quad (2c)$$

$$\frac{\partial L}{\partial d_{itw}} = \sum_n [C_{ni}\lambda^{(2)}_{ntw}] + \mu^{(7)}_{itw} - \mu^{(8)}_{itw} = \hat{c}_{it},\ \forall itw. \quad (2d)$$

All complementary slackness conditions are collected in the strong duality condition given in (3).

$$\begin{aligned}\sum_{i,t}(p_{itw} - d_{itw})\hat{c}_i &= \sum_{n,t}\lambda^{(2)}_{ntw}D_{ntw} + \sum_{n,m,t}[\mu^{(2)}_{nmtw}\overline{F}_{nm} - \mu^{(1)}_{nmtw}\underline{F}_{nm}] \\ &+ \sum_{n,t}[\mu^{(4)}_{ntw}\overline{\Theta}_n - \mu^{(3)}_{ntw}\underline{\Theta}_n] + \sum_{i,t}[\mu^{(6)}_{itw}\hat{p}_{it} - \mu^{(5)}_{itw}\underline{P}_i] \\ &+ \sum_{i,t}[\mu^{(8)}_{itw}\hat{d}_{it} - \mu^{(7)}_{itw}\underline{D}_i],\ \forall w.\end{aligned} \quad (3)$$

There are bilinear terms that appear in the objective function (1a) as well as the strong duality constraint (3). These terms are $\sum_n C_{ni}\lambda^{(2)}_{ntw}(p_{itw} - d_{itw})$, $\mu^{(6)}_{itw}\hat{p}_{it}$ and $\mu^{(8)}_{itw}\hat{d}_{it}$. Using (2c), (2d) and the complementary slackness (CS) conditions for constraints (1l) and (1m), the objective function can be rewritten as can be seen in (4).

$$\begin{aligned}&\sum_{i\in\mathcal{I}^S,t,w}\rho_w\sum_n C_{ni}\lambda^{(2)}_{ntw}(p_{itw} - d_{itw}) \overset{(2c),(2d)}{=} \\ &\sum_{i\in\mathcal{I}^S,t,w}\rho_w\Big[(p_{itw} - d_{itw})\hat{c}_{it} + \mu^{(5)}_{itw}p_{itw} - \mu^{(6)}_{itw}p_{itw} \\ &+ \mu^{(7)}_{itw}d_{itw} - \mu^{(8)}_{itw}d_{itw}\Big] \overset{(CS)}{=} \sum_{i\in\mathcal{I}^S,t,w}\rho_w\Big[\underbrace{(p_{itw} - d_{itw})\hat{c}_{it}}_{\pi^C_{itw}} \\ &+ \mu^{(5)}_{itw}\underline{P}_i - \underbrace{\mu^{(6)}_{itw}\hat{p}_{it}}_{\pi^P_{itw}} + \mu^{(7)}_{itw}\underline{D}_i - \underbrace{\mu^{(8)}_{itw}\hat{d}_{it}}_{\pi^D_{itw}}\Big]\end{aligned} \quad (4)$$

The bilinear terms have therefore been reduced to $(p_{itw} - d_{itw})\hat{c}_{it}$, $\mu^{(6)}_{itw}\hat{p}_{it}$ and $\mu^{(8)}_{itw}\hat{d}_{it}$ where each term is a continuous variable multiplied by an offer-bid value. We assume discrete offer-bid values which can take values from a possible pool of ordered values $\hat{p}_{it} \in \{\hat{P}_1, \hat{P}_2, ..., \hat{P}_{n_P}\}$, $\hat{d}_{it} \in \{\hat{D}_1, \hat{D}_2, ..., \hat{D}_{n_D}\}$ and $\hat{c}_{it} \in \{\hat{C}_1, \hat{C}_2, ..., \hat{C}_{n_C}\}$. Therefore, one can rewrite the bilinear terms in the following disjunctive manner:

$$\begin{aligned}&\mu^{(6)}_{itw}\hat{p}_{it} = \bigvee_{l=1}^{n_P}\mu^{(6)}_{itw}\hat{P}_l, \quad \mu^{(8)}_{itw}\hat{d}_{it} = \bigvee_{l=1}^{n_D}\mu^{(8)}_{itw}\hat{D}_l, \\ &(p_{itw} - d_{itw})\hat{c}_{it} = \bigvee_{l=1}^{n_C}(p_{itw} - d_{itw})\hat{C}_l,\end{aligned} \quad (5)$$

where the disjunction is represented by the disjunction (OR) operator $\bigvee$. One can then rewrite the whole stochastic bilevel program as the stochastic disjunctive program (6).

$$\underset{\Omega}{\text{maximize}} \sum_{i\in\mathcal{I}^S,t,w}\rho_w\Big[\bigvee_{l=1}^{n_C}(p_{itw} - d_{itw})\hat{C}_l +$$

$$\mu^{(5)}_{itw}\underline{P}_i - \bigvee_{l=1}^{n_P} \mu^{(6)}_{itw}\hat{P}_l + \mu^{(7)}_{itw}\underline{D}_i - \bigvee_{l=1}^{n_D} \mu^{(8)}_{itw}\hat{D}_l\Bigg]$$

subject to: (6)

$(1b)-(1f),\ (1h)-(1m),\ (2a)-(2d),$

(3) rewritten with (5),

$\mu^{(1)}_{nmtw},\ \mu^{(2)}_{nmtw},\ \mu^{(3)}_{ntw},\ \mu^{(4)}_{ntw} \leq 0\ ,$

$\mu^{(5)}_{itw},\ \mu^{(6)}_{itw},\ \mu^{(7)}_{itw},\ \mu^{(8)}_{itw} \leq 0,$

where the set of decision variables is $\Omega = \{\hat{p}_{it},\ \hat{d}_{it},\ \hat{c}_{it},\ p_{itw},\ d_{itw},\ s_{itw},\ \theta_{ntw},\ f_{nmtw},\ \lambda^{(1)}_{nmtw},\ \lambda^{(2)}_{ntw},\ \mu^{(1)}_{nmtw},\ \mu^{(2)}_{nmtw},\ \mu^{(3)}_{ntw},\ \mu^{(4)}_{ntw},\ \mu^{(5)}_{itw},\ \mu^{(6)}_{itw},\ \mu^{(7)}_{itw},\ \mu^{(8)}_{itw}\}$. Program (6) can be solved by the binary expansion approach [22]. Taking the disjunctive term $\bigvee_{l=1}^{n_P} \mu^{(6)}_{itw}\hat{P}_l$ as an example, one can introduce binary variables x^P_{itl} where $\sum_{l=1}^{n_P} x^P_{itl} = 1$ and write the disjunction as:

$$-M^P x^P_{itl} \leq z^P_{itlw} \leq M^P x^P_{itl},\ \forall itlw,$$
$$-M^P(1-x^P_{itl}) \leq z^P_{itlw} - \mu^{(6)}_{itw}\hat{P}_l \leq M^P(1-x^P_{itl}),\ \forall itlw,$$

where M^P is a sufficiently large constant, and z^P_{itlw} are continuous variables that are enforced to take the value of the bilinear term for a single index l. The disjunctive term can then be written as $\bigvee_{l=1}^{n_P} \mu^{(6)}_{itw}\hat{P}_l = \sum_{l=1}^{n_P} z^P_{itlw}$ and the offer value as $\hat{p}_{it} = \sum_{l=1}^{n_P} \hat{P}_l x^P_{itl}$. The same approach can be used to rewrite the other disjunctive terms. Additional constraints $\hat{p}_{it} \leq \overline{P}_i a_{it}$ and $\hat{d}_{it} \leq \overline{D}_i(1-a_{it})$ where $a_{it} \in \{0,1\}$ are introduced to ensure that in each period, each storage unit participates in the market either as a generator or as a load. This formulation allows the merchant storage problem to be written as a mixed-integer linear program (MILP) and the standard branch-and-bound algorithm can be used to solve it. A continuous linear relaxed optimization problem is then formed by allowing the binary variables to be continuous in the range from zero to one. Branches are created by finding a non-binary solution variable and setting it to zero at one node and to one at the other. The problem with the MILP formulation is however that it contains a large number of binary variables and the choice of M^P, M^D and M^C affects the performance of the solver. To tackle these shortcomings, following [22] we propose an alternative way to deal with the bilinear terms.

In the mixed integer linear formulation, each of the offer-bid values is expressed as a convex combination of discrete values. Instead of such a formulation, we propose that only two discrete values are used for the offer-bid values; a lower bound and an upper bound. Taking $\hat{p}_{it}$ as an example, $\underline{p}_{it}$ and $\overline{p}_{it}$ represent the lower and upper bounds, respectively. A continuous variable y^P_{it} is introduced to span the range of $\hat{p}_{it}$:

$$\hat{p}_{it} = \underline{p}_{it} y^P_{it} + \overline{p}_{it}(1-y^P_{it}),\ 0 \leq y^P_{it} \leq 1, \tag{7}$$

and the disjunction is enforced by the constraint

$$\bigvee_{l=1}^{n_P} [\underline{p}_{it} y^P_{it} + \overline{p}_{it}(1-y^P_{it}) = \hat{P}_l]. \tag{8}$$

The continuous variable $\mu^{(6)}_{itw}$ that appears in the disjunctive term is then represented by the sum of two values

$$\mu^{(6)}_{itw} = \mu^{(6)-}_{itw} + \mu^{(6)+}_{itw}, \tag{9}$$

and the bilinear term $\mu^{(6)}_{itw}\hat{p}_{it}$ can be written as

$$\pi^P_{itw} = (\mu^{(6)-}_{itw} + \mu^{(6)+}_{itw})[\underline{p}_{it} y^P_{it} + \overline{p}_{it}(1-y^P_{it})]. \tag{10}$$

The reformulation is equivalent for $\mu^{(8)}_{itw}\hat{d}_{it}$ and $(p_{itw}-d_{itw})\hat{c}_{it}$ resulting in π^D_{itw} and π^C_{itw}, respectively. Subsequently, one can write the stochastic disjunctive program as given in (11).

$$\underset{\Omega}{\text{maximize}} \sum_{i\in\mathcal{I}^S,t,w} \rho_w \left[\pi^C_{itw} + \mu^{(5)}_{itw}\underline{P}_i - \pi^P_{itw} + \mu^{(7)}_{itw}\underline{D}_i - \pi^D_{itw}\right]$$

subject to:

$(1b)-(1f),\ (1h)-(1m),\ (2a)-(2d),$

$(7)-(10)$ [Equiv. for $\mu^{(8)}_{itw}\hat{d}_{it}$ and $(p_{itw}-d_{itw})\hat{c}_{it}$], (11)

(3) rewritten with $\pi^P_{itw},\ \pi^D_{itw},\ \pi^C_{itw},$

$\mu^{(1)}_{nmtw},\ \mu^{(2)}_{nmtw},\ \mu^{(3)}_{ntw},\ \mu^{(4)}_{ntw} \leq 0,$

$\mu^{(5)}_{itw},\ \mu^{(6)}_{itw},\ \mu^{(7)}_{itw},\ \mu^{(8)}_{itw} \leq 0,$

where the set of decision variables is $\Omega = \{\hat{p}_{it},\ \hat{d}_{it},\ \hat{c}_{it},\ p_{itw},\ d_{itw},\ s_{itw},\ \theta_{ntw},\ f_{nmtw},\ \lambda^{(1)}_{nmtw},\ \lambda^{(2)}_{ntw},\ \mu^{(1)}_{nmtw},\ \mu^{(2)}_{nmtw},\ \mu^{(3)}_{ntw},\ \mu^{(4)}_{ntw},\ \mu^{(5)}_{itw},\ \mu^{(6)}_{itw},\ \mu^{(7)}_{itw},\ \mu^{(8)}_{itw}\}$.

The stochastic disjunctive program (11) can be solved directly by applying a specialized branch-and-bound solution approach that was proposed in [22]. The disjunctive formulation allows one to branch on the range of offer-bid values instead of binary variables. As an example, if an offer $\hat{p}_{it}$ can take values in the set $\{0, 10, 20, 30, 40, 50\}$ MW, one can enforce the constraint $0 \leq \hat{p}_{it} \leq 20$ in one branch and the constraint $30 \leq \hat{p}_{it} \leq 50$ in the other. In order for such a branch-and-bound approach to work properly, one needs a relaxed optimization problem to give a upper bound on the objective value. Unfortunately, the straightforward continuous relaxation of (11) is both non-linear and non-convex and does therefore not provide a suitable way of obtaining an upper bound. It is however possible to obtain an upper bound by applying *quasi-relaxation*.

Definition. *For a given constrained maximization problem P, a problem Q is a quasi-relaxation of P if for every feasible solution of P with objective value equal to v, there is a feasible solution of Q having an objective function value greater than or equal to v. The optimal value of Q is an upper bound on the optimal value of P.*

One can derive a quasi-relaxation as follows. Looking at the stochastic disjunctive program given in (11), everything is linear except for (1b) and the constraints given in (8) and (10) as well as their equivalents for $\mu^{(8)}_{itw}\hat{d}_{it}$ and $(p_{itw}-d_{itw})\hat{c}_{it}$. First, we drop constraint (1b) which will be dealt with directly in the solution algorithm in Section III. Then consider the following linear constraints in (12) which are obtained by dropping the disjunctive constraints in (8) and rewriting constraints (10). M^P represents a sufficiently large constant. The constraints for $\mu^{(8)}_{itw}\hat{d}_{it}$ and $(p_{itw}-d_{itw})\hat{c}_{it}$ are equivalent.

$$\pi^P_{itw} = \underline{p}_{it}\mu^{(6)-}_{itw} + \overline{p}_{it}\mu^{(6)+}_{itw}, \tag{12a}$$
$$-M^P y^P_{it} \leq \mu^{(6)-}_{itw} \leq 0, \tag{12b}$$
$$-M^P(1-y^P_{it}) \leq \mu^{(6)+}_{itw} \leq 0, \tag{12c}$$

$$\text{[Equivalent for } \mu^{(8)}_{itw}\hat{d}_{it} \text{ and } (p_{itw} - d_{itw})\hat{c}_{it}]. \quad (12d)$$

Remark 1. *Let (11Q) denote the optimization problem that results from taking problem (11), dropping constraint (1b) and replacing constraints (8) and (10) as well as the equivalent constraints for the other bilinear terms $\mu^{(8)}_{itw}\hat{d}_{it}$ and $(p_{itw} - d_{itw})\hat{c}_{it}$ with constraints (12) .*

Lemma 1. *Problem (11Q) is a linear program and a quasi-relaxation of problem (11). See Appendix for proof.*

Lemma 2. *If $\hat{p}_{it}\hat{d}_{it} = 0,\ \forall(i \in \mathcal{I}^S)t$, and each y^P_{it}, y^D_{it}, y^C_{it} is binary in a solution of (11Q), that solution is feasible in (11). See Appendix for proof.*

III. Solution Algorithm

Lemmas 1 and 2 can be used to derive a Specialized Branch-and-Bound (SBB) solution algorithm that solves the merchant storage problem (11). The SBB algorithm differs from the standard branch-and-bound algorithm (BB) used to solve the binary expansion MILP model in the following aspects:

1) The SBB algorithm branches on the lower and upper bounds of offer-bid values $(\underline{p}_{it}, \overline{p}_{it})$, $(\underline{d}_{it}, \overline{d}_{it})$, and $(\underline{c}_{it}, \overline{c}_{it})$ instead of branching on binary values in the BB algorithm.
2) The SBB algorithm uses the quasi-relaxation model in (11Q) to find valid upper bounds, while the BB algorithm uses a linear continuous relaxation by allowing the binary variables to be continuous in the range $[0, 1]$.

The solution algorithm is shown in Algorithm 1. In the algorithm, variable $\hat{x}_{it}$ represents any of the offer-bid values $\hat{p}_{it}$, $\hat{d}_{it}$ or $\hat{c}_{it}$ in order to simplify the algorithm and it is accompanied by the corresponding spanning variable y^X_{it}. For clarity, sets $\underline{\mathcal{X}}$ and $\overline{\mathcal{X}}$ include all lower and upper bounds for all of the offer-bid values for all i and t. The algorithm can branch on any single offer-bid value.[2] In order to make sure that the merchant storage owner participates in the market either as a generator or a demand, the following feasibility cut is applied when a branch is created. Whenever the algorithm branches on a single offer quantity $\hat{p}_{it}$ and a branch is created where the lower bound $\underline{p}_{it}$ is greater than zero, the corresponding demand bid $\hat{d}_{it}$ is set to zero in that branch. Similarly, whenever the algorithm branches on a single bid quantity $\hat{d}_{it}$ and a branch is created where the lower bound $\underline{d}_{it}$ is greater that zero, the corresponding generation offer quantity $\hat{p}_{it}$ is set to zero in that branch. This enforces constraint (1b).

IV. Illustrative Example

1) Price-quantity bidding and offering: For the illustrative example, a 5-node system is used. The system is based on the PJM 5-bus system of the MATPOWER package [23]. The following changes have been made to the system in order to make it suitable for illustration:

1) A generator on bus 1 has been replaced by a wind farm on bus 3 with an installed capacity of 300 MW.

[2]Since branching terminates when the y's are binary, convergence is ensured just as is the case with the standard branch-and-bound algorithm.

Algorithm 1: The SBB algorithm with linear quasi-relaxation.

Input : Linear quasi-relaxed problem (11Q) and discrete values $\{\hat{P}_1, \hat{P}_2, ..., \hat{P}_{n_P}\}$, $\{\hat{D}_1, \hat{D}_2, ..., \hat{D}_{n_D}\}$ and $\{\hat{C}_1, \hat{C}_2, ..., \hat{C}_{n_C}\}$.

Set lower and upper bounds $(\underline{p}_{it}, \overline{p}_{it}) = (\hat{P}_1, \hat{P}_{n_P})$, $(\underline{d}_{it}, \overline{d}_{it}) = (\hat{D}_1, \hat{D}_{n_D})$ and $(\underline{c}_{it}, \overline{c}_{it}) = (\hat{C}_1, \hat{C}_{n_C})$.
Set $LB = -\infty$.
Branch$(\underline{\mathcal{X}}, \overline{\mathcal{X}})$.
if $LB = -\infty$ **then**
  Problem is infeasible.
else
  sol^* is optimal for (11).

Function *Branch*$(\underline{\mathcal{X}}, \overline{\mathcal{X}})$
  if *(11Q) has a feasible solution sol with objective* $z > LB$ **then**
    if *some* $y^X_{it} \notin \{0, 1\}$ *and* $\underline{x}_{it} \neq \overline{x}_{it}$ **then**
      Let $\hat{X}_l$ be the largest value in the set $\{\hat{X}_1, \hat{X}_2, ..., \hat{X}_{n_X}\}$ that is smaller than $\underline{x}_{it}y^X_{it} + \overline{x}_{it}(1 - y^X_{it})$.
      Branch$(\underline{\mathcal{X}}', \overline{\mathcal{X}})$, where $\underline{\mathcal{X}}'$ is identical to $\underline{\mathcal{X}}$ apart from that $\underline{x}_{it} = \hat{X}_{l+1}$
      Branch$(\underline{\mathcal{X}}, \overline{\mathcal{X}}')$, where $\overline{\mathcal{X}}'$ is identical to $\overline{\mathcal{X}}$ apart from that $\overline{x}_{it} = \hat{X}_l$
    else
      Let $LB = z$ and $sol^* = sol$ with $\hat{x}_{it} = \underline{x}_{it}y^X_{it} + \overline{x}_{it}(1 - y^X_{it})$.

Output: Optimal solution sol^*.

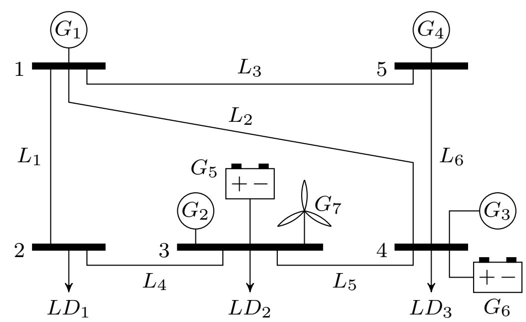


Fig. 1. The single-line diagram of the illustrative example.

2) Two merchant storage units with an installed generation/demand capacity of 100 MW and a storage capacity of 400 MWh each have been connected to buses 3 and 4, respectively.
3) All transmission lines with unlimited capacity in the original system have a capacity of 400 MW.
4) The marginal cost of generator 3 has been increased from $ 40/MWh to $ 60/MWh.

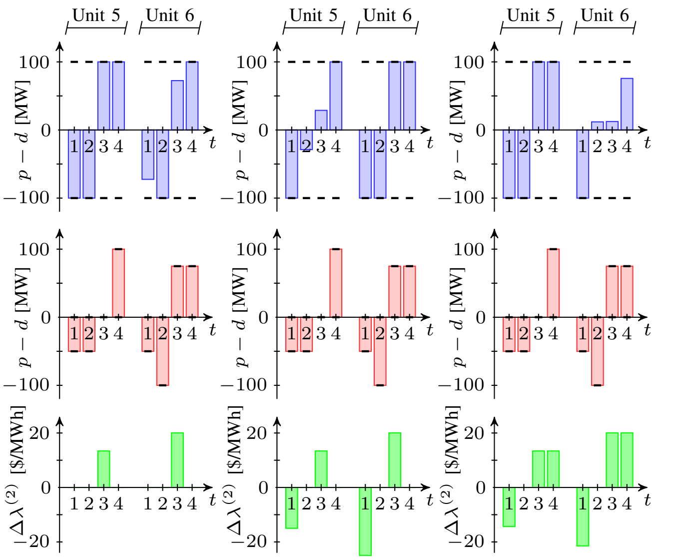
The single-line diagram of the illustrative example is shown in Fig. 1. Generator data are given in Table I.

The merchant storage owner optimizes its operation over a horizon of 4 time periods. The variable cost of the merchant storage units is considered to be negligible and their round-trip efficiency is considered to be 100 %. The merchant storage owner submits price-quantity pairs to the market with a price of either 0 $/MWh or 50 $/MWh as well as quantity of either 0 %, 50 %, 75 %, or 100 % of the 100 MW installed capacity. The bus loads are increasing over the horizon and are assumed to be known deterministically. In order to add stochasticity to the system, there are 3 equiprobable scenarios possible for the wind farm connected to bus 3 (1: low, 2: medium, 3: high). In

TABLE I

Generator data for the illustrative example.

Unit #	Type	$\overline{P}_i$ / $\overline{D}_i$ [MW]	Cost, c_i [$/MWh]	Bus
1	Dispatchable	40 / 0	15	1
2	Dispatchable	520 / 0	30	3
3	Dispatchable	200 / 0	60	4
4	Dispatchable	600 / 0	10	5
5	Storage	100 / 100	0	3
6	Storage	100 / 100	0	4
7	Wind	300 / 0	0	3



(a) Scenario 1 (b) Scenario 2 (c) Scenario 3

Fig. 2. Illustrative example results for the two storage units. Benchmark case (blue), strategic case (red) and price deviation at the buses corresponding to the two units (green). Thick black lines represent offer-bid values and colored bars represent dispatch or price deviation.

all scenarios the wind power production is decreasing over the horizon. The total load is considered to be the residual load after the wind power has been deducted from the bus loads.

Fig. 2 shows the charging and discharging (red) of the merchant storage units as well as their submitted offer-bid values to the market (black). For comparison, the figure also shows a *benchmark case* where the ISO controls the dispatch of the storage units completely (blue). Lastly, the figure shows the price deviation at the corresponding buses between the *benchmark case* and the *strategic case* (green). The results of the illustrative example show various types of strategic behavior from the merchant storage owner when compared to the benchmark case where the ISO completely controls the dispatch of the storage units. Some empirically relevant takeaways from the illustrative example are the following:

Demand withholding: The units withhold demand in periods 1 and 2 by bidding a fraction of their demand capacity. This results in a decrease in the price compared to the benchmark case on both buses in period 1 (for scenarios 2 and 3) which means that the storage units can charge at a lower price. This behavior also results in less stored energy in periods 3 and 4, which helps to drive up the price during those periods.

Generation withholding: In period 3, both units withhold their generation capacity. Furthermore in period 4, unit 6 withholds its generation capacity. This behavior results in a price increase compared to the benchmark case and therefore a higher expected profit for both units.

TABLE II

Total profit by scenario as well as expected profit of the generating units over the horizon.

Unit #	Profit [$]				
	$w=1$	$w=2$	$w=3$	E[Profit]	ΔE[Profit]
1	2933.7	2597.5	2597.5	2709.6	947.3
2	13932.9	13932.9	13932.9	13932.9	9288.6
3	0	0	0	0	0
4	0	3000.0	0	1000.0	1000.0
5	1339.7	2089.7	2339.7	1923.0	841.0
6	3008.6	4255.7	4505.7	3923.4	2303.3
7	5169.9	8539.7	15879.4	9863.0	-451.5

Portfolio effect: The abovementioned generation withholding of the units also shows a portfolio effect where the actions of one unit benefit the storage portfolio as a whole.

Increased profit: Table II shows the profit of the generating units over the horizon. The expected profit of the storage portfolio over the three scenarios and four periods is $ 5846.4. The last column shows how the expected profit of the generators compares with their expected profit from the benchmark case. The strategic actions of the two storage units result in an expected profit that is more than double the expected profit of the benchmark case ($ 2702.1). Moreover, the strategic actions of the storage units increase the expected profit of all of the other generators apart from generator 3 and the wind farm. This is because the storage owner manages to decrease the prices somewhat when there is high wind power production and increases the prices when there is lower wind production.

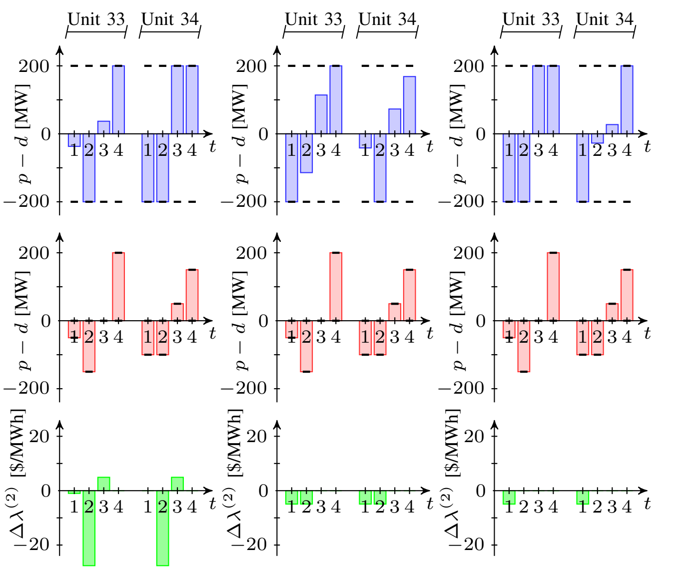
Under-use: While the expected profit is increased, the expected amount of energy sold by storage is around 25 % less in the strategic case than in the benchmark case; storage is under-used compared to the welfare-maximising use. In the benchmark case, the ISO flattens out the prices to minimize the generation costs. When the storage portfolio is controlled strategically, the owner tries to maintain the price difference while finding a trade-off between sold energy and price.

2) Self-scheduling: In this case the storage owner submits self-schedule bids and offers, that is without a price component, to the market instead of price-quantity pairs. The SBB algorithm is run again for the illustrative example where the merchant storage owner is allowed to submit *self-schedule* offer-bid values of 0 %, 20 %, 40 %, 60 %, 80 % or 100 % of the installed capacity which for both units is 100 MW. For this case, the expected profit of the storage portfolio is $ 5815.0.

V. Numerical Simulation

This section shows numerical simulations of larger test systems along with a comparison of the generation cost for i) the case without storage, ii) the case with competitive storage as well as iii) the case with strategic storage. Lastly, computational comparison demonstrates the superior performance of the proposed solution algorithm when benchmarked against current practices in the literature.

1) IEEE 24-bus system with transmission constraints – self-scheduling: In this case, we run the SBB algorithm on the IEEE 24-bus, 32-unit system where the two 400 MW nuclear units in the system are assumed to be off-line. The simulation



(a) Scenario 1 (b) Scenario 2 (c) Scenario 3

Fig. 3. IEEE 24-bus system results for the two storage units. Benchmark case (blue), strategic case (red) and price deviation at the buses corresponding to the two units (green). Thick black lines represent offer-bid values and colored bars represent dispatch or price deviation.

is carried out for 4 periods, 3 scenarios and there are two merchant storage units in the market (units 33 and 34); both of which have a storage capacity of 1000 MWh. They are connected to buses 13 and 15. There are 3 equiprobable scenarios possible for a 400 MW wind farm connected to bus 17 (1: high, 2: medium, 3: low). The units are allowed to submit self-schedule offer-bid values of 0 %, 25 %, 50 %, 75 % or 100 % of their 200 MW installed capacity to the market. The results are represented in Fig. 3 where one can see how the units are able to strategically decrease the system price substantially during the first 2 periods as well as increase the price in the third period. These strategic actions increase their expected profit over the 3 scenarios and 4 periods from \$ 655.9 in the benchmark case to \$ 3856.0 in the strategic case.

2) IEEE 24-bus system without transmission constraints – self-scheduling: The SBB algorithm is run on the same IEEE 24-bus system without considering transmission constraints. The simulation is carried out for 12 periods and there is a single merchant storage unit in the market which has a storage capacity of 1000 MWh. The unit is allowed to submit self-schedule offer-bid values of either 0 %, 25 %, 50 %, 75 %, or 100 % of its 300 MW installed capacity to the market. The unit is able to strategically apply generation withholding in order to increase the system price substantially and increase its expected profit from \$ 7983.6 in the benchmark case to \$ 40049.9 in the strategic case.

3) IEEE 118-bus without transmission constraints – price-quantity bidding: Finally, the SBB algorithm is run on the IEEE 118-bus system without considering transmission constraints for a horizon of 8 periods. A 2000 MWh storage unit submits price-quantity pairs with a price of either 0 \$/MWh or 40 \$/MWh as well as quantity of either 0 %, 50 % or 100 % of its installed capacity of 600 MW. The strategic actions of the player increase its expected profit over the eight periods from zero in the benchmark case to \$ 12000 in the strategic case.

TABLE III

COMPARISON OF EXPECTED GENERATION COST FOR THE SIMULATIONS.

	Case Without Storage	Case With Competitive Storage (Benchmark)	Case With Strategic Storage (Strategic)
Illustr. ex. I	\$ 80580	\$ 72367	\$ 74246
Illustr. ex. II	\$ 80580	\$ 72367	\$ 74277
24-bus I	\$ 125170	\$ 118896	\$ 119524
24-bus II	\$ 691567	\$ 641036	\$ 645797
118-bus	\$ 852852	\$ 839325	\$ 840852

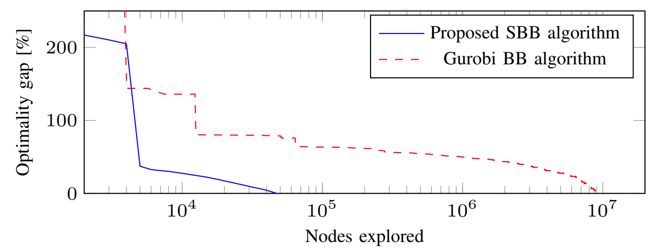


Fig. 4. Convergence of the optimality gap for the two solution algorithms for the 24-bus I case.

4) Generation cost comparison: Table III shows a comparison of the expected generation cost for the benchmark case and the strategic case. The generation cost is decreased when storage is introduced for all simulations, both in the benchmark case and in the strategic case. The generation cost is however lower in the benchmark case than in the strategic case.

5) Computational Comparison: Table IV shows the computational requirements of the Gurobi BB algorithm for solving the binary expansion MILP model and the SBB algorithm for solving the proposed disjunctive program (11). The SBB algorithm is implemented by the authors using a Python interface. In order to get a fair comparison, the Gurobi BB does not apply pre-solve algorithms or other heuristics. For the simulations performed, the proposed SBB algorithm is orders of magnitude more efficient than Gurobi BB in terms of nodes explored. For the two price-quantity bidding cases (the illustrative example I and the 118-bus), as well as the self-schedule 24-bus II case, SBB finds the optimal solution while Gurobi BB fails to find a proven optimal solution. Fig. 4 shows a graphical comparison of the optimality gap convergence for the two algorithms (proposed SBB and Gurobi BB) for the 24-bus I case.

VI. CONCLUSION

This paper proposes a stochastic disjunctive programming model for finding the optimal offer-bid strategy of a merchant

TABLE IV

COMPARISON OF THE COMPUTATIONAL REQUIREMENTS OF THE DIFFERENT SIMULATIONS AND ALGORITHMS. THE COMPLEXITY OF THE MILP MODEL IS ALSO REPORTED IN TERMS OF THE NUMBER OF CONTINUOUS VARIABLES, BINARY VARIABLES AND CONSTRAINTS.

Simulation	Nodes Explored		MILP complexity		
	SBB	Gurobi BB	cont.	bin.	constr.
Illustr. Ex. I	1.31×10^6	*	1584	88	2395
Illustr. Ex. II	59623	1.96×10^6	1608	104	2555
24-bus I	47155	8.90×10^6	7400	88	8203
24-bus II	69103	*	3348	132	3781
118-bus	1.11×10^6	*	4064	72	4313

* No proven optimal solution found after 10 million nodes explored

storage portfolio. The interaction between the merchant storage owner and the ISO is modeled as a stochastic bilevel optimization model and then reformulated as a stochastic disjunctive program. Employing the disjunctive nature of the optimization model, a specialized branch-and-bound algorithm is proposed. The proposed SBB solution algorithm branches on the ranges of discrete variables (rather than binary variables in the standard BB algorithm). To find a relaxed solution of the proposed stochastic disjunctive program, first the concept of quasi-relaxation is defined. Then a linear quasi-relaxation is derived for the case of the merchant storage model. Both the proposed disjunctive programming model and the SBB solution algorithm are benchmarked against current practices in literature for modeling and solving these types of problems (binary expansion MILP model and Gurobi BB). The numerical results confirm the performance of the modeling approach and its solution algorithm for dealing with the optimal offer-bid strategy of an energy storage portfolio. Although the modeling approach and the SBB algorithm are derived in the context of a merchant storage offer-bid model, the same modeling approach and solution algorithm might be applicable for other stochastic bilevel optimization problems.

Appendix

Proof of Lemma 1. The same approach as in [22] is used for proof of this lemma. Consider any feasible solution ($\hat{p}_{it}$, $\hat{d}_{it}$, $\hat{c}_{it}$, $\mu^{(6)}_{itw}$, $\mu^{(8)}_{itw}$, p_{itw}, d_{itw}) to (11). It suffices to construct a feasible solution ($\hat{p}_{it}$, $\hat{d}_{it}$, $\hat{c}_{it}$, $\mu^{(6)'}_{itw}$, $\mu^{(8)'}_{itw}$, p'_{itw}, d'_{itw}) of (11Q) since the latter has the same objective value in (11Q) as the former does in (11). Let

$$\mu^{(6)-'}_{itw} = y^{P}_{it}(\mu^{(6)-}_{itw} + \mu^{(6)+}_{itw}) \tag{13a}$$
$$\mu^{(6)+'}_{itw} = (1 - y^{P}_{it})(\mu^{(6)-}_{itw} + \mu^{(6)+}_{itw}) \tag{13b}$$
$$\mu^{(8)-'}_{itw} = y^{D}_{it}(\mu^{(8)-}_{itw} + \mu^{(8)+}_{itw}) \tag{13c}$$
$$\mu^{(8)+'}_{itw} = (1 - y^{D}_{it})(\mu^{(8)-}_{itw} + \mu^{(8)+}_{itw}) \tag{13d}$$
$$p^{-'}_{itw} = y^{C}_{it}(p^{-}_{itw} + p^{+}_{itw}) \tag{13e}$$
$$p^{+'}_{itw} = (1 - y^{C}_{it})(p^{-}_{itw} + p^{+}_{itw}) \tag{13f}$$
$$d^{-'}_{itw} = y^{C}_{it}(d^{-}_{itw} + d^{+}_{itw}) \tag{13g}$$
$$d^{+'}_{itw} = (1 - y^{C}_{it})(d^{-}_{itw} + d^{+}_{itw})\ . \tag{13h}$$

which is clearly feasible in (11Q). Furthermore, substituting (13a)–(13h) into (11Q) results in the same constraints as in (11), apart from the disjunctive constraints. All other constraints in (11Q) are identical to their counterparts in (11). □

Proof of Lemma 2. The same approach as in [22] is used for proof of this lemma. When a variable y_{it} is either 0 or 1, the offer-bid value is either at the lower bound or the upper bound of that branch, both of which are valid discrete values and therefore fulfill the disjunctive constraints. □

References

[1] X. Liu, A. Aichhorn, L. Liu, and H. Li, "Coordinated control of distributed energy storage system with tap changer transformers for voltage rise mitigation under high photovoltaic penetration," *IEEE Transactions on Smart Grid*, vol. 3, no. 2, pp. 897–906, 2012.

[2] P. Mercier, R. Cherkaoui, and A. Oudalov, "Optimizing a battery energy storage system for frequency control application in an isolated power system," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1469–1477, 2009.

[3] K. Divya and J. Østergaard, "Battery energy storage technology for power systemsan overview," *Electric Power Systems Research*, vol. 79, no. 4, pp. 511–520, 2009.

[4] B. P. Roberts and C. Sandberg, "The role of energy storage in development of smart grids," *Proceedings of the IEEE*, vol. 99, no. 6, pp. 1139–1144, 2011.

[5] K. Rahbar, J. Xu, and R. Zhang, "Real-time energy storage management for renewable integration in microgrid: An off-line optimization approach," *IEEE Transactions on Smart Grid*, vol. 6, no. 1, pp. 124–134, 2015.

[6] C. A. Hill, M. C. Such, D. Chen, J. Gonzalez, and W. M. Grady, "Battery energy storage for enabling integration of distributed solar power generation," *IEEE Transactions on Smart Grid*, vol. 3, no. 2, pp. 850–857, 2012.

[7] H. Akhavan-Hejazi and H. Mohsenian-Rad, "Optimal operation of independent storage systems in energy and reserve markets with high wind penetration," *IEEE Transactions on Smart Grid*, vol. 5, no. 2, pp. 1088–1097, 2014.

[8] S. Chen, H. B. Gooi, and M. Wang, "Sizing of energy storage for microgrids," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 142–151, 2012.

[9] I. Atzeni, L. G. Ordóñez, G. Scutari, D. P. Palomar, and J. R. Fonollosa, "Demand-side management via distributed energy generation and storage optimization," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 866–876, 2013.

[10] M. E. Khodayar, L. Wu, and M. Shahidehpour, "Hourly coordination of electric vehicle operation and volatile wind power generation in scuc," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1271–1279, 2012.

[11] Y. Ota, H. Taniguchi, T. Nakajima, K. M. Liyanage, J. Baba, and A. Yokoyama, "Autonomous distributed V2G (vehicle-to-grid) satisfying scheduled charging," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 559–564, 2012.

[12] C. Wu, H. Mohsenian-Rad, and J. Huang, "Vehicle-to-aggregator interaction game," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 434–442, 2012.

[13] Y. Dvorkin, R. Fernandez-Blanco, D. S. Kirschen, H. Pandžić, J.-P. Watson, and C. A. Silva-Monroy, "Ensuring profitability of energy storage," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 611–623, 2017.

[14] G. Bautista, M. F. Anjos, and A. Vannelli, "Modeling market power in electricity markets: Is the devil only in the details?" *The Electricity Journal*, vol. 20, no. 1, pp. 82–92, 2007.

[15] J. Bushnell, "A mixed complementarity model of hydrothermal electricity competition in the western United States," *Operations research*, vol. 51, no. 1, pp. 80–93, 2003.

[16] F. A. Wolak, "An empirical analysis of the impact of hedge contracts on bidding behavior in a competitive electricity market," *International Economic Journal*, vol. 14, no. 2, pp. 1–39, 2000.

[17] ——, "Quantifying the supply-side benefits from forward contracting in wholesale electricity markets," *Journal of Applied Econometrics*, vol. 22, no. 7, pp. 1179–1209, 2007.

[18] K. Hartwig and I. Kockar, "Impact of strategic behavior and ownership of energy storage on provision of flexibility," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 2, pp. 744–754, 2016.

[19] V. Virasjoki, P. Rocha, A. S. Siddiqui, and A. Salo, "Market impacts of energy storage in a transmission-constrained power system," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 4108–4117, 2016.

[20] J. R. Cruise, L. Flatley, and S. Zachary, "Impact of storage competition on energy markets," *European Journal of Operational Research*, vol. 269, no. 3, pp. 998–1012, 2018.

[21] H. Mohsenian-Rad, "Coordinated price-maker operation of large energy storage units in nodal energy markets," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 786–797, 2016.

[22] S. Bollapragada, O. Ghattas, and J. N. Hooker, "Optimal design of truss structures by logic-based branch and cut," *Operations Research*, vol. 49, no. 1, pp. 42–51, 2001.

[23] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.