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QUANTIFYING THE SUPPLY-SIDE BENEFITS FROM FORWARD CONTRACTING IN WHOLESALE ELECTRICITY MARKETS

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SUMMARY

The assumption of expected profit-maximizing bidding behavior in a multi-unit, multi-period auction with step-function supply curves is used to estimate cost functions for electricity generation units and derive tests of expected profit-maximizing behavior. Applying these techniques to data from the National Electricity Market in Australia reveals statistically significant evidence of output-dependent marginal costs within and across half-hours of the day, but no evidence against the hypothesis of expected profit-maximizing behavior. These cost function estimates quantify the economic significance of output-varying costs and how forward financial contract obligations impact the amount of these costs the generation unit owner incurs. This supplier's existing obligations imply average daily production costs that are 8% lower than the profit-maximizing pattern of output with no forward contract obligations. Copyright © 2007 John Wiley & Sons, Ltd.

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1. INTRODUCTION

This paper uses the assumption of expected profit-maximizing bidding behavior in a multi-unit, multi-period auction where bidders submit non-decreasing step-function supply curves to derive a procedure that estimates cost functions for individual electricity generation units. This framework extends the results in Wolak (2003) to the case of multi-output cost functions and incorporates significantly more of the implications of expected profit-maximizing bidding behavior contained in the step-function bid supply curves to derive more efficient cost function estimates and a number of tests of the hypothesis of expected profit-maximizing bidding behavior.

Applications of this procedure are not limited to wholesale electricity markets. This procedure can be used to recover bidder cost or valuation functions from participants in any multi-unit auction that uses non-decreasing step-function supply curves or non-increasing step-function demand curves. Because this estimation procedure does not require solving for an auction market equilibrium, it can be applied to uniform-price, second-price, pay-as-bid, or any other auction mechanism that specifies precisely how a player's action impacts its payoff. Potential applications outside of bid-based wholesale electricity markets include treasury bill, eBay, and spectrum auctions.

This paper uses data from the Australian National Electricity Market 1 (NEM1) to recover multi-output generation unit-level behavioral cost function estimates and to test the assumption

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of expected profit-maximizing bidding behavior. I refer to these cost functions as 'behavioral' because they are derived from the generation unit-owner's bids using the assumption of expected profit-maximizing behavior, rather than from explicitly minimizing the cost of producing a vector of outputs given the technological characteristics of the generation unit and fuel and other input prices. Nevertheless, the marginal costs obtained from these behavioral cost function estimates are broadly consistent with marginal costs computed from the technological characteristics of the generation units and fuel and other input prices.

My primary motivation for estimating this multi-output behavioral cost function is to quantify the extent to which generation unit-level marginal costs are non-constant within a given half-hour period and across half-hour periods of the day. Generation unit owners argue that they face 'hard' technological operating constraints that limit their ability to increase and decrease the electricity produced by their generation units within a given time interval—what is typically termed a generation unit's ramp rate. For example, a generation unit may have a ramp rate of 10 MW per minute, meaning that the generation unit can increase or decrease the amount of its capacity producing electricity by 10 MW in 1 minute.

Although it is difficult to deny the existence of hard technological operating constraints, generation units are often observed increasing or decreasing their output by more than this stated 10 MW per minute 'hard' technological ramp rate. This observation suggests that these 'hard' technological constraints can be violated at a cost. The purpose of my empirical work is to quantify both the statistical and economic significance of these ramping costs using the estimated generation-unit-level behavioral cost functions.

Economically significant ramping costs can alter the size and temporal distribution of benefits a supplier might obtain from entering into forward financial contracts. Substantial ramping costs can make it unilaterally profitable for a supplier to take actions to limit the sensitivity of its expected profit-maximizing pattern of electricity production throughout the day to the behavior of spot prices by signing a fixed-price long-term contract. As shown in Wolak (2000), a supplier's fixed-price forward financial contract obligations alter its expected profit-maximizing bidding behavior, and through the market-clearing mechanism, the supplier's expected profit-maximizing pattern of electricity production throughout the day. Wolak (2000) demonstrates that the larger a supplier's forward contract holdings, the less incentive it has to bid to raise the spot market price by varying its output in response to changes in the residual demand curves it faces across half-hours of the day.

This logic implies that, for the same distribution of residual demand curves, by signing forward financial contracts an expected profit-maximizing supplier can commit to a less volatile and lower average cost pattern of production throughout the day. If ramping costs are economically significant, then certain patterns of electricity production throughout the day across all of a supplier's generation units achieve a significantly lower daily average cost. For example, a supplier may find that a constant level of output from each of its generation units throughout the day achieves significantly lower daily average operating costs than producing less in certain half-hours of the day and more in others in response to changes in the residual demand curves it faces each half-hour, even if this pattern of output yields a higher average price for the supplier. Consequently, a generation unit owner facing significant ramping costs may find it expected profit maximizing to sign a fixed-price forward contract even if the contract price is less than the supplier's expectations of short-term market prices if it does not sign the forward contract, because signing a forward contract typically commits the supplier to a smooth daily pattern of output that has a lower average cost of production.

My generation unit-level behavioral cost function estimates are used to quantify the daily operating cost reductions that are possible from forward contracting. These behavioral cost functions are first estimated by imposing the moment restrictions implied by the supplier choosing the vector of daily price bid steps to maximize expected profits. To investigate the impact of the additional information embodied in the half-hourly quantity bids, I then estimate the cost function using both the daily price and half-hourly quantity bids. The assumption of expected profit-maximizing bidding behavior implies that a set of over-identifying moment restrictions hold for the behavioral cost function estimates. I find little evidence against the validity of these over-identifying moment restrictions implied by expected profit-maximizing behavior for both behavioral cost function estimates. As a further test of the validity of the assumption of expected profit-maximizing bidding behavior, I derive a test statistic for the null hypothesis that the probability limit of parameters of the behavioral cost function estimated using just the daily price bids equals the probability limit of the parameters of the behavioral cost function estimated using both the daily price and half-hourly quantity bids. I find little evidence against this implication of the assumption of expected profit-maximizing bidding behavior. A final implication of the assumption of expected profit-maximizing bidding behavior is that a set of moment inequalities should hold for a subset of the half-hourly bid quantities. Using the multivariate inequality constraints hypothesis testing framework from Wolak (1989), I find little evidence against the null hypothesis that these moment inequalities implied by expected profit-maximizing bidding behavior are valid.

Having found no statistically significant evidence against the assumption of expected profit-maximizing bidding behavior, I then use the two behavioral cost function estimates derived from this assumption to determine the extent of ramping costs. Both multi-output, generation unit-level cost function estimates find statistically significant ramping costs. The joint hypothesis test that all of the coefficients in the behavioral cost function associated with ramping costs are equal to zero is overwhelmingly rejected for both behavioral cost function estimates. The parameters of the generation unit-level cost functions that capture ramping costs are more precisely estimated for the case in which both the price and quantity bid moment restrictions are used versus just the case of price bid moment restrictions.

To quantify the magnitude of ramping costs and the implied potential benefit to this supplier from signing forward contracts, I perform two sets of two counterfactual calculations using the behavioral cost function estimate derived from the price and quantity bid moment restrictions. For each day of the sample period, first I compute both the minimum and maximum daily operating cost, according to this behavioral cost function, of replicating the actual half-hourly pattern of output produced by the supplier across all generation units that it owns. I also compute both the minimum and maximum daily cost of replicating the total daily output from all of the generation units owned by that supplier. Both of these optimization problems impose the restriction that no generation unit can produce less than its minimum operating level or more than its maximum operating level during a half-hour period. If there were no ramping costs and the marginal costs of all generation units owned by the supplier were equal, then the minimum and maximum daily cost for both scenarios would equal the actual daily variable operating cost, because the marginal cost of producing electricity is invariant to which half-hour of the day the energy is produced in and which generation unit produces it.

Comparing the average daily maximum cost to replicate the supplier's actual pattern of half-hourly production (across all of the generation units it owns) for the day to the average daily minimum cost that replicates the supplier's actual pattern of half-hourly production

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for the day provides one measure of the economic significance of ramping costs. This average daily maximum cost is 7% higher than this average daily minimum cost. For the case of just replicating the total daily output of the suppler's generation units (without regard to which half-hours of that day this output is produced in), the sample average daily maximum cost is 9% higher than the sample average daily minimum cost. I find that the average actual daily cost (according to this behavioral cost function) is very close to the average minimum daily cost that replicates the 48 half-hourly values of total output across all of the supplier's generation units. This result demonstrates that by signing a forward contract for a substantial fraction of its capacity, a supplier avoids ramping costs that can average as high as 9% of the average minimum cost of producing its daily pattern of output.

To quantify the extent to which a reduction in forward contracting would result in a more costly pattern of production throughout the day, I use this estimated cost function and the 48 half-hourly residual demand curves actually faced by that supplier each day of the sample to compute the profit-maximizing daily pattern of output for each day during the sample period, assuming the firm has zero forward contract obligations in all half-hours. This zero-forward-contract daily profit-maximizing pattern of output respects the constraints that each unit cannot produce less than its minimum operating level or more than its maximum operating level during each half-hour of the day.

For this zero-contract, profit-maximizing half-hourly pattern of output for each day of the sample period, I compute the two minimum and maximum daily costs described above. The average daily maximum cost that replicates the 48 half-hourly values of the supplier's profit-maximizing pattern of output with zero forward contracts is 9% higher than the average daily minimum cost that replicates the 48 half-hourly values of the supplier's profit-maximizing pattern of total output with zero forward contracts. The profit-maximizing pattern of daily production with zero forward contracts is close to the maximum daily cost pattern of production. This result is evidence that profit-maximizing suppliers without significant forward contract holdings choose more costly patterns of daily production.

Comparing the average cost of daily production under the zero-forward-contracts counterfactual pattern of production to the average cost of daily production with the actual level of forward contracts quantifies the average cost savings from forward contracting. The sample average of the average cost of the actual pattern of daily output (at the actual level of forward contracts) is 92% of the sample average of the daily average cost of the counterfactual profit-maximizing pattern of daily production with no forward contracts. This implies that for an expected profit-maximizing supplier facing the same distribution of residual demand curves, forward contracting can cause the supplier to realize up to an 8% reduction in the daily average cost of production relative to a pattern of output that is expected profit maximizing for the case of no forward contracts.

The remainder of the paper proceeds as follows. In the next section, I present the theoretical model of expected profit-maximizing bidding behavior for step-function bid supply curves and derive the full set of moment restrictions that can be used to estimate the behavioral cost function. Section 3 discusses the actual estimation procedure and derives a number of tests of the hypothesis of expected profit-maximizing behavior that I implement in the empirical section. Section 4 briefly summarizes the NEM1 electricity market and the data necessary for the analysis and presents the cost function estimation and testing results. This section then describes the various counterfactual scenarios I use to quantify the economic significance of ramping costs and the benefits from forward contracting in terms of its ability to limit these costs.

2. MOMENT RESTRICTIONS IMPLIED BY EXPECTED PROFIT-MAXIMIZING BIDDING BEHAVIOR

This section derives the full set of population moment restrictions implied by expected profit-maximizing bidding behavior in a multi-unit auction where firms bid step-function willingness-to-supply curves. I then describe how these moment restrictions are used to estimate the parameters of the supplier's multi-unit behavioral cost function and derive tests of the null hypothesis of expected profit-maximizing bidding behavior.

In the NEM1, a supplier submits 10 daily price bids and 10 half-hour quantity bids to construct the 48 half-hourly bid supply curves for the day for each generation unit, or genset. These price bids are required to be greater than or equal to an administratively-set minimum price bid and less than or equal to an administratively-set maximum price bid. Each quantity bid is required to be greater than or equal to zero, and less than the capacity of the generation unit, and the sum of the 10 half-hourly quantity bids must be less than or equal to the capacity of the genset. These market rules imply that a NEM1 supplier has a daily strategy set for each genset that it owns that is a compact subset of 490-dimensional Euclidian space—10 daily price bids and 480 half-hourly quantity bids. Electricity generation plants are typically composed of multiple gensets or units at a given location. Moreover, suppliers in wholesale electricity markets typically own multiple plants, so the dimension of a supplier's daily strategy set can easily exceed 2000–3000.

Fixing the 10 price bids for a genset for all of the half-hours of the day limits a supplier's flexibility to alter its unit-level willingness-to-supply curve across half-hours of the day. Figure 1 provides an example of how this market rule limits changes in a genset's willingness-to-supply curve across two half-hours of the day for the case of three bid increments. The prices, P_1, P_2, and P_3, are the same for both the off-peak and peak period bid curves. The quantities associated with each of these price bids can differ across the two periods. The assumption used to recover the behavioral cost function is that the supplier chooses the daily price bids and half-hourly quantity bids to maximize its expected profits from participating in the wholesale electricity market subject to the constraints on the price and quantity bids described above.

I now present the bid price and bid quantity moment restrictions used to estimate the multi-output, unit-level behavioral cost functions for a NEM1 supplier. In the NEM1, each day of the market, d, is divided into half-hour load periods denoted by the subscript i beginning with 4:00–4:30 a.m. and ending with 3:30–4:00 a.m. the following day. Suppliers are required to submit their price and quantity bids for the entire day by 11 a.m. of the day before the energy is to be produced.

Let firm A denote the supplier that owns multiple gensets whose expected profit-maximizing bidding strategy is being computed. Define the following variables for each day of the sample period:

Q_{id}: market demand in load period i of day d

$SO_{id}(p)$: amount of capacity bid by all other firms besides firm A into the market in load period i of day d at price p

$DR_{id}(p) = Q_{id} - SO_{id}(p)$: residual demand curve faced by firm A in load period i of day d, at price p

QC_{id}: contract quantity for load period i of day d for firm A

PC_{id}: quantity-weighted average (over all hedge contracts signed for that load period and day) contract price for load period i of day d for firm A

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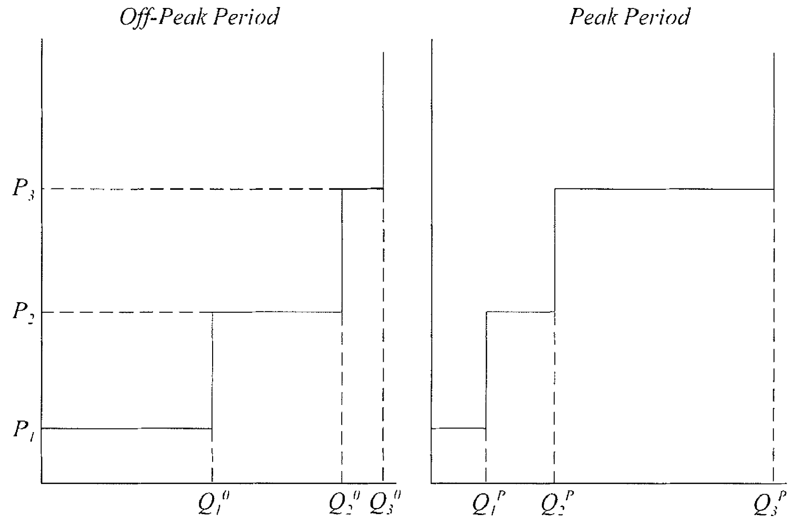

Figure 1. Sample bid functions for Australian electricity market

$\pi_{id}(p)$: variable profits to firm A at price p, in load period i of day d
Q_{ijd}: output in load period i from genset j owned by firm A for day d
Q_{jd}: vector of half-hourly outputs of genset j, $Q_{jd} = (Q_{1jd}, Q_{2jd}, \ldots, Q_{48jd})$, for day d
$C_j(Q_{jd}, \beta_j)$: daily operating cost of producing the output vector Q_{jd} for genset j
β_j: parameters of daily operating cost function for genset j
$SA_{id}(p, \Theta)$: bid function of firm A for load period i of day d giving the amount it is willing to supply as a function of the price p and bid parameter vector Θ defined below

The market clearing price p is determined by solving for the smallest price such that the equation $SA_{id}(p, \Theta) = DR_{id}(p)$ holds.

The forward contract variables, QC_{id} and PC_{id}, are set in advance of the day-ahead bidding process. Suppliers sign hedge contracts with large consumers or electricity retailers for a predetermined pattern of fixed prices or a single fixed price throughout the day, week, or month and for a predetermined quantity each half-hour of the day or pattern of quantities throughout the day, week, or month for an entire year or number of years. There is a small amount of short-term activity in the forward contract market for electricity retailers requiring price certainty for a larger or smaller-than-planned quantity of electricity at some point during the year, but the vast majority of a supplier's contract position is known far in advance of the settlement dates of the contracts. Consequently, from the perspective of formulating its day-ahead expected profit-maximizing bidding strategy, the values of QC_{id} and PC_{id} for all half-hours of the day are known to the supplier at the time it submits its vector of price and quantity bids for the following day.

A supplier bids to maximize its expected profits, where this expectation is taken with respect to two sources of uncertainty in the residual demand function the supplier faces each half-hour period of the following trading day. The first is due to the fact that firm A does not exactly know the form of $SO_{id}(p)$, the aggregate willingness-to-supply curve submitted by all other market participants during load period i of day d. The second accounts for the fact that the firm does not know the value of market demand that will set the market price when it submits its bid. Because I am not

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solving for equilibrium outcomes in the multi-unit auction, I do not need to be specific about the causes of these two sources of uncertainty in the residual demand function that firm A faces. I only need to assume that firm A knows the joint distribution of the uncertainty in the 48 half-hourly residual demand curves it is bidding against. Joint tests of this assumption and the assumption that firm A maximizes expected profits given the distribution of these residual demand curves are proposed in Section 3.

Let ε_i equal a continuously distributed random vector parameterizing the uncertainty in firm A's residual demand function in load period $i\,(i = 1, \ldots, 48)$. Rewrite firm A's residual demand in load period i of day d accounting for this demand shock as $DR_{id}(p, \varepsilon_i)$. Define $\Theta = (p_{11}, \ldots, p_{JK}, q_{1,11}, \ldots, q_{1,JK}, q_{2,11}, \ldots, q_{2,JK}, \ldots, q_{48,11}, \ldots, q_{48,JK})$ as the vector of daily bid prices and quantities submitted by firm A. There are K increments for each of the J gensets owned by firm A. As noted earlier, NEM1 rules require that the price bid for increment k of unit j, p_{jk}, is set for each of the $k = 1, \ldots, K$ bid increments for each of the $j = 1, \ldots, J$ gensets owned by firm A for the entire day. The quantity bid in load period i, for increment k of unit j, q_{ijk}, made available to produce electricity in load period i from each of the $k = 1, \ldots, K$ bid increments for the $j = 1, \ldots, J$ gensets owned by firm A can vary across the $i = 1, \ldots, 48$ load periods throughout the day. The value of K is 10, so the dimension of Θ is $10J + 48 \times 10J$. Firm A operates seven gensets during our sample period, so the dimension of Θ is more than several thousand. Let $SA_{id}(p, \Theta)$ equal firm A's bid function in half-hour period i of day d as parameterized by Θ. The NEM1 rules require that bid increments are dispatched based on the order of their bid prices, from lowest to highest, which implies that $SA_{id}(p, \Theta)$ is non-decreasing in p.

Deriving the moment conditions necessary to estimate generation unit-level behavioral cost functions requires additional notation to represent $SA_{id}(p, \Theta)$ in terms of the genset-level bid supply functions. Let

$SA_{ijd}(p, \Theta) =$ the amount bid by genset j at price p during load period i of day d;

$SA_{id}(p, \Theta) = \sum_{j=1}^{J} SA_{ijd}(p, \Theta) =$ total amount supplied by firm A at price p during load period i of day d.

In terms of this notation, write the realized variable profit for firm A during day d as

$$\Pi_d(\Theta, \varepsilon) = \sum_{i=1}^{48} [DR_{id}(p_i(\varepsilon_i, \Theta))p_i(\varepsilon_i, \Theta) - (p_i(\varepsilon_i, \Theta) - PC_{id})QC_{id}] - \sum_{j=1}^{J} C_j(Q_{jd}, \beta_j) \qquad (1)$$

where ε is the vector of realizations of ε_i, $\varepsilon = ({\varepsilon_1}', {\varepsilon_2}', \ldots, {\varepsilon_{48}}')'$. As discussed above, $p_i(\varepsilon_i, \Theta)$, the market-clearing price for load period i for the residual demand shock realization, ε_i, and daily bid vector, Θ, is the solution in p to the equation $DR_{id}(p, \varepsilon_i) = SA_{id}(p, \Theta)$. Because $SA_{id}(p)$ is non-decreasing and $DR_{id}(p, \varepsilon_i)$ is non-increasing, the equilibrium price is unique with probability one if the joint distribution function of ε_i is continuous. To economize on notation, in the discussion that follows I abbreviate $p_i(\varepsilon_i, \Theta)$ as p_i. The ith element of the vector Q_{jd} is equal to $SA_{ijd}(p_i, \Theta)$, the bid supply curve for genset j in half-hour period i of day d evaluated at the market-clearing price in this half-hour period, p_i.

The first term in (1) is the daily total revenue received by firm A for selling the energy it produces in the spot market. The last term is the total daily operating cost to produce the electricity sold. The middle term is the payment made by firm A if the spot price exceeds the contract price, or received by firm A if the contract price exceeds the spot price. Forward financial contracts typically settle through these so-called 'difference payments' between the buyer and seller of the contract. Under this forward contract settlement scheme, the spot market operator simply pays for all energy produced at the spot market price and charges for all energy withdrawn from the network at the spot market price. The market/system operator does not need to know the forward financial contract arrangements between market participants.

The best-reply bidding strategy maximizes the expected value of $\Pi_d(\Theta, \varepsilon)$ with respect to Θ, subject to the constraints that all bid quantity increments, q_{ijk}, must be greater than or equal to zero and less than the capacity of the unit for all load periods, i, bid increments, k, and gensets, j, and that for each genset the sum of bid quantity increments during each half-hour period is less than the capacity, $\text{CAP}_j^{\max}$, of genset j. All daily price increments must be greater than −\$9,999.99/MWh and less than \$5000/MWh, where all dollar magnitudes are in Australian dollars. All of these constraints can be written as a linear combination of the elements of Θ. Define R as the matrix of these linear combinations.

In terms of the above notation, the firm's expected profit-maximizing bidding strategy can be written as

$$\max_{\Theta} E_\varepsilon(\Pi_d(\Theta, \varepsilon)) \text{ subject to } b_u \geq R\Theta \geq b_l \tag{2}$$

where $E_\varepsilon(.)$ is the expectation with respect to the joint distribution of ε. The first-order conditions for this optimization problem are

$$\begin{gathered} \frac{\partial E_\varepsilon(\Pi_d(\Theta, \varepsilon))}{\partial \Theta} = R'\lambda - R'\mu, \\ R\Theta \geq b_l, \quad b_u \geq R\Theta \\ \text{if } (R\Theta - b_l)_k > 0, \text{ then } \mu_k = 0 \text{ and if } (R\Theta - b_u)_k < 0, \text{ then } \lambda_k = 0 \end{gathered} \tag{3}$$

where $(X)_k$ is the kth element of vector X and μ_k and λ_k are the kth elements of the vectors of Kuhn-Tucker multipliers, μ and λ.

If all the inequality constraints associated with an element of Θ, say p_{jk}, are slack, then the first-order condition for this price increment reduces to

$$\frac{\partial E_\varepsilon(\Pi_d(\Theta_d, \varepsilon))}{\partial_{p_{jk}}} = 0, \tag{4}$$

where Θ_d is the value of Θ for day d.

All the daily price bids associated with firm A's gensets over the sample period lie in the interior of the interval (−9999.99, 5000), which implies that all price bids satisfy the first-order conditions given in (4) for all days, d; gensets, j; and daily bid increments, k. Because firm A operates 7 units during the sample period and each of them has 10 bid increments, this implies 70 daily price moment should be restrictions. These first-order conditions for daily expected profit maximization with respect to firm A's choice of the vector of daily price increments are used to estimate the parameters of the genset-level cost functions.

The first-order conditions with respect to bid quantity increments can also be used to estimate the behavioral cost function. There are two conditions that must hold for the quantity bid associated with bid increment k from unit j in load period i, q_{ijk}, to yield a first-order condition of the form

$$\frac{\partial E_{\varepsilon}(\Pi_d(\Theta_d, \varepsilon))}{\partial q_{ijk}} = 0 \tag{5}$$

These conditions are: (1) the value of q_{ijk} is strictly greater than zero; and (2) the sum of the q_{ijk} over all bid increments k for unit j in load period i is less than the capacity of the unit. The constraint that the bid increment is less than the capacity of the genset is not binding for any half-hourly quantity bid increment for any genset owned by firm A, so this constraint can be ignored for the purposes of our estimation procedure.

To implement this approach, define the following two indicator variables:

$$y_{ijk} = 1 \text{ if } q_{ijk} > 0 \text{ and zero otherwise} \tag{6}$$

$$z_{ij} = 1 \text{ if } \sum_{k=1}^{10} q_{ijk} < \text{CAP}_j^{\max} \text{ and zero otherwise} \tag{7}$$

Because suppliers can and often do produce more than the nameplate capacity of their generation unit, I assume that $\text{CAP}_j^{\max}$ of the generation unit is the sample maximum half-hourly amount of energy produced from that unit over all half-hours during the sample period. This definition of the indicator variables y_{rst} and z_{rs} implies the following quantity bid increment moment restrictions for all JK quantity bid increments:

$$\frac{1}{48}\sum_{r=1}^{48} y_{rst} z_{rs} \frac{\partial E_{\varepsilon}(\Pi_d(\Theta_d, \varepsilon))}{\partial q_{rst}} = 0 \tag{8}$$

Consequently, I have 70 additional moment restrictions from the first-order conditions for the 70 quantity increments, as long as the population means of y_{rst} and z_{rs} are non-zero.

There are also moment inequality constraints implied by expected profit-maximizing bidding behavior. These will form the basis for a specification test of the assumption of expected profit-maximizing bidding behavior given our unit-level behavioral cost function estimates. Recall the definitions of y_{ijk} and z_{ij}. If q_{ijk} is such that $y_{ijk} = 0$ and $z_{ij} = 1$, then the following moment equality restriction holds:

$$\frac{\partial E_{\varepsilon}(\Pi_d(\Theta_d, \varepsilon))}{\partial q_{ijk}} \leq 0 \tag{9}$$

If q_{ijk} is such that $y_{ijk} = 1$ and $z_{ij} = 0$, then the following moment equality restriction holds:

$$\frac{\partial E_{\varepsilon}(\Pi_d(\Theta_d, \varepsilon))}{\partial q_{ijk}} \geq 0 \tag{10}$$

This implies the following inequality constraints moment restriction for all 70 quantity bid increments on a daily basis:

$$\frac{1}{48}\sum_{r=1}^{48} (y_{rst} - z_{rs}) \frac{\partial E_{\varepsilon}(\Pi_d(\Theta_d, \varepsilon))}{\partial q_{rst}}, \tag{11}$$

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which should be greater than or equal to zero if the quantity increments are chosen to maximize expected daily profits. Given a consistent estimate of the behavioral cost function, I will construct a test of the null hypothesis that the population value of this moment restriction is greater than or equal to zero.

3. IMPLEMENTATION OF ESTIMATION AND TESTING PROCEDURES

Deriving a procedure to recover the genset-level daily operating cost function from the first-order conditions for expected profit-maximizing bidding behavior is complicated by the fact that even though $E_\varepsilon(\Pi_d(\Theta, \varepsilon))$ is differentiable with respect to Θ, the daily realized profit function, $\Pi_d(\Theta, \varepsilon)$, is not. The bid functions in the NEM1 are step functions, which implies that the residual demand curve each supplier faces is non-differentiable, because $DR_{id}(p) = Q_{id} - SO_{id}(p)$.

I use a flexible smoothing procedure to construct a differentiable approximation to $\Pi_d(\Theta, \varepsilon)$, which is indexed by a smoothing parameter, h. Let $\Pi_d^h(\Theta, \varepsilon)$ equal the differentiable version of firm A's daily variable profit function. When $h = 0$, there is no approximation, because $\Pi_d^h(\Theta, \varepsilon) = \Pi_d(\Theta, \varepsilon)$. Using this smooth, differentiable approximation to $\Pi_d(\Theta, \varepsilon)$, the order of integration and differentiation can be switched in the first-order conditions for expected profit-maximizing bidding behavior to produce the equality

$$\left.\frac{\partial E_\varepsilon(\Pi_d^h(\Theta, \varepsilon))}{\partial \Theta}\right|_{h=0} = E_\varepsilon\left[\frac{\partial \Pi_d^h(\Theta, \varepsilon))}{\partial \Theta}\right]_{h=0} \tag{12}$$

For the elements of Θ that are unconstrained, the corresponding element of the vector on the right-hand side of (12) is equal to zero, so the value of $\partial \Pi_d^h(\Theta, \varepsilon))/\partial \Theta$ for each day in the sample period can be used to form a sample moment condition. Solving for the cost function parameters that make these sample moment restrictions as close to zero as possible yields a consistent estimate of these parameters if h tends to zero as the sample size grows.

This smooth, differentiable version of $\Pi_d^h(\Theta, \varepsilon)$ takes the following form. A differentiable residual demand function facing firm A that allows for both sources of residual demand uncertainty is constructed as follows:

$$DR_{id}^h(p, \varepsilon_i) = Q_{id}(\varepsilon_i) - SO_{id}^h(p, \varepsilon_i) \tag{13}$$

where the smoothed aggregated bid supply function of all other market participants besides firm A in load period i is equal to

$$SO_i^h(p, \varepsilon) = \sum_{n=1}^{N} \sum_{k=1}^{10} qo_{ink} \Phi((p - po_{nk})/h) \tag{14}$$

qo_{ink} is the kth bid increment of genset n in load period i and po_{nk} is bid price for increment k of genset n, where N is the total number of gensets in the market excluding those owned by firm A. Although the qo_{ink} and po_{nk} change on a daily basis, I suppress the subscript d from both of these variables to reduce notational clutter. $\Phi(t)$ is the standard normal cumulative distribution function and h is the smoothing parameter. This parameterization of $SO_i^h(p)$ smooths the corners on the step-function bid curves of all other market participants besides firm A to create a supply function

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that is differentiable in p. The degree of smoothing depends on the value of h. Smaller values of h introduce less smoothing. For $h = 0$, the procedure reproduces the original step function residual demand curve as is required for $\Pi_d^h(\Theta, \varepsilon)|_{h=0} = \Pi_d(\Theta, \varepsilon)$ to hold.

This smoothing procedure results in the following expression for derivatives of firm A's residual demand function with respect to the market price in load period i:

$$\partial DR_i^h(p, \varepsilon)/\partial p = -\frac{1}{h}\sum_{n=1}^{N}\sum_{k=1}^{10} qo_{ink}\varphi((p - po_{nk})/h) \tag{15}$$

where $\varphi(t)$ is the standard normal density function.

This same procedure is followed to make $SA_{ij}(p, \Theta)$ differentiable with respect to both the market price, p, and Θ, the price and quantity bid parameters that make up this willingness-to-supply function. Define $SA_{ij}^h(p, \Theta)$ as

$$SA_{ij}^h(p, \Theta) = \sum_{k=1}^{10} q_{ijk}\Phi((p - p_{jk})/h) \tag{16}$$

which implies

$$SA_i^h(p, \Theta) = \sum_{j=1}^{J}\sum_{k=1}^{10} q_{ijk}\Phi((p - p_{jk})/h) \tag{17}$$

where it is understood that q_{ijk} and p_{jk} change on a daily basis. This definition of $SA_{ij}(p, \Theta)$ yields the following three partial derivatives:

$$\frac{\partial SA_{ij}}{\partial q_{ijk}} = \Phi((p - p_{jk})/h),\ \frac{\partial SA_{ij}}{\partial p} = \frac{1}{h}\sum_{k=1}^{10} q_{ijk}\varphi((p - p_{jk})/h) \text{ and } \frac{\partial SA_{ij}}{\partial p_{jk}} = -\frac{1}{h} q_{ijk}\varphi((p - p_{jk})/h) \tag{18}$$

The final partial derivatives required to construct the elements of $\frac{\partial \Pi_d^h(\Theta, \varepsilon)}{\partial \Theta}$ can be computed by applying the implicit function theorem to the equation used to determine the market-clearing price $DR_i^h(p) = SA_i^h(p, \Theta)$. This yields the expression

$$\frac{\partial p_i^h(\varepsilon_i, \Theta)}{\partial p_{jk}} = \frac{\dfrac{\partial {SA_i}^h(p_i(\varepsilon_i, \Theta), \Theta)}{\partial p_{jk}}}{\partial {DR_i}^h(p_i(\varepsilon_i, \Theta), \varepsilon_i)/\partial p - \partial {SA_i}^h(p_i(\varepsilon_i, \Theta), \Theta)/\partial p} \tag{19}$$

where the partial derivative of the residual demand curve with respect to price used in this expression is given in equation (15) and the other partial derivatives are given in (18). For the case of quantity bids, the analogous expression is

$$\frac{\partial {p_i}^h(\varepsilon_i, \Theta)}{\partial q_{ijk}} = \frac{\dfrac{\partial {SA_i}^h(p_i(\varepsilon_i, \Theta), \Theta)}{\partial q_{ijk}}}{\partial {DR_i}^h(p_i(\varepsilon_i, \Theta), \varepsilon_i)/\partial p - \partial {SA_i}^h(p_i(\varepsilon_i, \Theta), \Theta)/\partial p} \tag{20}$$

These expressions quantify, respectively, how the market-clearing price changes in response to changes in the supplier's daily price bids and half-hourly quantity bids. By inspection, the sign of the right-hand side (19) is positive and the sign of the right-hand side of (20) is negative.

Note that expressions (19) and (20) depend on the uniform price auction mechanism used to determine the prices paid to the supplier. If a pay-as-bid or second-price mechanism is used to set the prices paid to the supplier, there will be a different relationship between the market participant's bid parameters and the price it is paid.

Given data on market-clearing prices and the bids for all market participants, I can compute all of the inputs into equations (13) through (20). I only need to choose a value for h, the smoothing parameter that enters the smoothed residual demand function and firm A's smoothed bid functions. Once this smoothing parameter has been selected, the magnitudes given in equations (13) through (20) remain constant for the entire estimation procedure.

The sample moment restriction for bid price increment t of genset s is

$$\frac{\partial \Pi_d^h(\Theta_d, \varepsilon)}{\partial p_{st}} = \sum_{i=1}^{48} \Bigg[(\partial DR_i{}^h(p_i(\varepsilon_i, \Theta), \varepsilon_i)/\partial p) p_i(\varepsilon_i, \Theta) + (DR_i{}^h(p_i(\varepsilon_i, \Theta), \varepsilon_i) - QC_i) - \sum_{j=1}^{J} \frac{\partial C_j(Q_j, \beta_j)}{\partial Q_{ij}} \left(\frac{\partial SA_{ij}{}^h}{\partial p_i} \right) \frac{\partial p_i{}^h}{\partial p_{st}} - \sum_{j=1}^{J} \frac{\partial C_j(Q_j, \beta_j)}{\partial Q_{ij}} \frac{\partial SA_{ij}{}^h}{\partial p_{st}} \Bigg] \tag{21}$$

where p_i is shorthand for the market-clearing price in load period i. By the assumption of expected profit-maximizing choice of the price bid increments:

$$\lim_{h \to 0} E_\varepsilon \left[\frac{\partial \Pi_d^h(\Theta_d, \varepsilon)}{\partial p_{st}} \right] = 0 \tag{22}$$

Let $\ell_d(\beta, h)$ denote the 70-dimensional vector of partial derivatives—10 bid price increments for seven gensets—given in (21), where β is the vector composed of β_j for $j = 1, \ldots, J$. Assuming that the functional form for $C_j(Q_j, \beta_j)$ is correct for all gensets, the first-order conditions for expected profit maximization with respect to the 70 daily bid prices imply that $E(\ell_d(\beta^0, h))_{h=0} = 0$, where β^0 is the true value of β. Consequently, solving for the value of b that minimizes

$$\left[\frac{1}{D} \sum_{d=1}^{D} \ell_d(b, h) \right]' \left[\frac{1}{D} \sum_{d=1}^{D} \ell_d(b, h) \right] \tag{23}$$

will yield a consistent estimate of β as both D tends to infinity and h tends to zero. This result follows directly from Theorem 1 of Horowitz (1992), which proves the consistency of an estimator defined as the optimum of a smoothed version of a discontinuous objective function where the smoothing parameter converges to zero as the sample size tends to infinity. Wolak (2001) discusses the technical conditions necessary for the consistency of this smoothed estimation problem for the case of multi-unit auctions with step function bids.

Let $b(I)$ denote this consistent estimate of β, where I denotes the fact that the identity matrix is used as the generalized method of moments (GMM) weighting matrix. I can construct a consistent estimate of the optimal GMM weighting matrix that accounts for potential heteroscedasticity in

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the covariance matrices of $\ell_d(\beta^0, h)$ across observations using this consistent estimate of β as follows:

$$V_D(b(I), h) = \frac{1}{D}\sum_{d=1}^{D} \ell_d(b(I), h)\ell_d(b(I), h)' \tag{24}$$

The optimal GMM estimator finds the value of b that minimizes

$$\left[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b, h)\right]' V_D(b(I), h)^{-1} \left[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b, h)\right] \tag{25}$$

as h tends to zero. Let $b(O)$ denote this estimator, where O denotes the fact this estimator is based on a consistent estimate of the optimal weighting matrix.

To incorporate the quantity bid moment restrictions, take the partial derivative of the smoothed realized profit function, $\Pi_d^h(\Theta, \varepsilon)$, with respect to the quantity increment t for unit s in half-hour r, q_{rst}:

$$\begin{aligned}\frac{\partial \Pi_d^h(\Theta_d, \varepsilon)}{\partial q_{rst}} &= [(\partial DR_r(p_r(\varepsilon_r, \Theta), \varepsilon_r)/\partial p) p_r(\varepsilon_r, \Theta) \\ &+ (DR_r{}^h(p_r(\varepsilon_r, \Theta), \varepsilon_r) - QC_r)]\frac{\partial p_r{}^h}{\partial q_{rst}} - \sum_{j=1}^{J}\sum_{i=1}^{48} \frac{\partial C_{ij}}{\partial Q_{ij}}\left[\frac{\partial SA_{ij}{}^h}{\partial P_r{}^h}\frac{\partial p_r^h}{\partial q_{rst}} + \frac{\partial SA_{ij}{}^h}{\partial q_{rst}}\right]\end{aligned} \tag{26}$$

where p_r is the market-clearing price in load period r. Compute the following daily observation for each quantity increment:

$$M_{dst}^h(\Theta_d, \varepsilon) = \frac{1}{48}\sum_{r=1}^{48} y_{rst} z_{rs} \frac{\partial \Pi_d^h(\Theta_d, \varepsilon)}{\partial q_{rst}} \tag{27}$$

By definition of the indicator variables y_{rst} and z_{rs}, the following result holds:

$$\lim_{h\to 0} E_\varepsilon[M_{dst}^h(\Theta_d, \varepsilon)] = 0 \tag{28}$$

Consequently, I have 70 additional moment restrictions from the first-order conditions for the 70 quantity increments, as long as population means of y_{rst} and z_{rt} are non-zero. Define $\mathcal{L}_d(\beta, h)$ as the 140-dimensional vector of daily price and quantity moments. Equations (22) and (28) imply that $E\big(\mathcal{L}_d(\beta^0, h)\big)\big|_{h=0} = 0$. Repeating the logic given in equations (23) through (25), the optimal GMM estimator of the parameters of the behavioral cost functions that incorporates both the price and quantity moment conditions can be derived.

To examine the empirical validity of the inequality constraint moment restrictions implied by the expected profit-maximizing choice of the 70 quantity bids, compute the following magnitude for each quantity increment:

$$MI_{dst}^h(\Theta_d, \varepsilon) = \frac{1}{48}\sum_{r=1}^{48} (y_{rst} - z_{rs}) \frac{\partial \Pi_d^h(\Theta_d, \varepsilon)}{\partial q_{rst}} \tag{29}$$

Given a consistent estimate of the elements of the cost function I can construct a test of the null hypothesis that the population value of this moment restriction is greater than or equal to zero. Define $\mathbb{I}_d(\beta, h)$ as the 70-dimensional vector composed of the values of $MI^h_{dst}(\Theta_d, \varepsilon)$ for genset s ($s = 1, \ldots, 7$) and bid increment t ($t = 1, \ldots, 10$). The expected profit-maximizing choice of the quantity bids implies $E(\mathbb{I}_d(\beta^0, h))_{h=0} \geq 0$.

Assuming β is known, compute

$$\begin{aligned} Z_D(\beta, h) &= D^{-1/2} \sum_{d=1}^{D} \mathbb{I}_d(\beta, h) \\ \text{and } V_D(\beta, h) &= \frac{1}{D} \sum_{d=1}^{D} (\mathbb{I}_d(\beta, h) - \bar{\mathbb{I}}_D(\beta, h))(\mathbb{I}_d(\beta, h) - \bar{\mathbb{I}}_D(\beta, h))' \\ \text{where } \bar{\mathbb{I}}_D(\beta, h) &= \frac{1}{D} \sum_{d=1}^{D} \mathbb{I}_d(\beta, h) \end{aligned} \tag{30}$$

Using $Z_D(\beta, h)$ and $V_D(\beta, h)$, the multivariate inequality constraints testing results from Wolak (1989) can be used to derive a test that $\lim_{h \to 0} E(Z_D(\beta, h)) \geq 0$ (which is implied by the hypothesis of expected profit-maximizing bidding behavior) against an unrestricted alternative.

4. ESTIMATION AND HYPOTHESIS-TESTING RESULTS

This section first describes the details of the estimation procedure. Then I present the results of the various tests of the implications of the hypothesis of expected profit-maximizing bidding behavior. Finally, I describe the counterfactual scenarios used to quantify the economic significance of ramping costs and the results of applying my behavioral cost function estimates to these scenarios.

4.1. Cost Function Estimation

The final step necessary to implement this estimation technique is the choice of functional form for the marginal cost function for each genset. Firm A owns two power plants. One power plant has four identical gensets that the firm operates during the sample period. I refer to this facility as plant 1. Two of the gensets at plant 1 have a $\text{CAP}_j^{\max}$ of 680 MW and the other two have a value 690 MW. Recall that $\text{CAP}_j^{\max}$ is the sample maximum half-hourly output from that generation unit. The lower operating limit of all of these units is 200 MW. The other power plant has three identical gensets that the firm operates during the sample period. I will refer to this facility as plant 2. All three gensets at plant 2 have a $\text{CAP}_j^{\max}$ of 500 MW and a lower operating limit of 180 MW.

There are an enormous number of parameters that could be estimated for the 48-dimensional daily cost function for unit j. Because I would like to capture ramping constraints across and within half hours of the day, I specify unit-level variable costs as a cubic function of the vector of 48 half-hourly outputs for the day. For plant 1, define

$$Q_{ij}{}^* = \max(0, Q_{ij} - 200) \tag{31}$$

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where Q_{ij} is the output in half-hour i from unit j and $Q_j{}^*$ is the 48-dimensional vector of the $Q_{ij}{}^*$ for all 48 half-hours of the day. The behavioral cost function for the four units associated with plant 1 is assumed to take the form

$$C_1(Q, a_1, A_1, B_1) = a_1(\iota' Q^*) + \frac{1}{2}Q^{*'}A_1Q^* + \frac{1}{6}\mathrm{vec}(Q^*Q^{*'})B_1Q^* \tag{32}$$

where a_1 is a scalar to denote the fact that without ramping costs the marginal cost would be the same for all hours of the day. ι is a 48-dimensional vector of 1's, A_1 is a (48×48) symmetric matrix, and B_1 is a $48^2 \times 48$ matrix that is composed of 48 symmetric (48×48) matrices, E_1, stacked as

$$B_1 = [E_1 E_1 \ldots E_1]' \tag{33}$$

To make the estimation manageable, I restrict A_1 to be a band symmetric matrix that depends on three parameters: A_1(diag), A_1(one) and A_1(two). A_1(diag) is the value of the diagonal elements, A_1(one) is the value of all of the elements above and below the diagonal elements, and A_1(two) is the value of all of the elements two above and two below the diagonal elements. The matrix E_1 is a diagonal matrix with the single element B_1(diag) along the diagonal. These restrictions reduce the number of parameters in the daily operating cost function from $\frac{1}{2}(48)(49)^2 + 1$ to 5. The parameters, A_1(diag), A_1(one), A_1(two), and B_1(diag) quantify the extent to which marginal costs are not constant across half-hours of the day because of the level of production in previous half-hours and in the present half-hour. Consequently, a joint test that these four coefficients are zero is a test of the null hypothesis of constant marginal cost. The parameter vector for the behavioral cost function for the four units at plant 1 is $\beta_1 = (a_1, A_1(\text{diag}), A_1(\text{one}), A_1(\text{two}), B_1(\text{diag}))$. All remaining elements of A_1 and B_1 are zero.

The unit-level cost function for the three units at plant 2 are defined in a similar manner. In this case

$$Q_{ij}{}^* = \max(0, Q_{ij} - 180) \tag{34}$$

The cost function takes the form

$$C_2(Q, a_2, A_2, B_2) = a_2(\iota' Q^*) + \frac{1}{2}Q^{*'}A_2Q^* + \frac{1}{6}\mathrm{vec}(Q^*Q^{*'})B_2Q^* \tag{35}$$

where a_2, A_2, B_2 and E_2 are defined analogously to a_1, A_1, B_1 and E_2. A_2(diag), A_2(one), A_2(two) and B_2(diag) are defined in the same manner as A_1(diag), A_1(one), A_1(two) and B_1(diag), respectively. The parameter vector associated with these three units is $\beta_2 = (a_2, A_2(\text{diag}), A_2(\text{one}), A_2(\text{two}), \beta_2(\text{diag}))'$. All remaining elements of A_2 and B_2 are zero.

These functional forms are substituted into the moment restrictions for price bids in equation (21) and quantity bid moment restrictions in equation (26) to construct the sample moment restrictions necessary to compute the objective function. The parameter vector is $(\beta_1', \beta_2')'$, where β_1 is the parameter vector associated with the behavioral cost function for the four units at plant 1 and β_2 is the parameter vector associated with the behavioral cost function for the three units at plant 2. Wolak (2001) derives the conditions necessary for nonparametric identification of these generation unit-level cost functions. Because the cost functions depend on the 48-dimensional daily output vector, obtaining precise nonparametric estimates would require an enormous number of observations to overcome the curse of dimensionality.

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Section 6 of Wolak (2003) summarizes the relevant features of the NEM1 market in Australia. Our sample period is from 15 May 1997 to 24 August 1997, so that $D = 102$. All half-hourly unit-level bid data and output data can be downloaded from the web site of the NEM1 operator. Forward contract data, the half-hourly values of PC and QC, is confidential. Consequently, the constraint on my sample size is that I only have the half-hourly forward contract values for firm A for the period 15 May 1997 to 24 August 1997. This sample size restriction necessitates our flexible parametric approach to estimating these generation unit-level cost functions.

Figure 2 presents smoothed values of residual demand curves for firm A during a low aggregate demand and a high aggregate demand half-hour period on 28 July 1997 for a value of $h = 1$. At

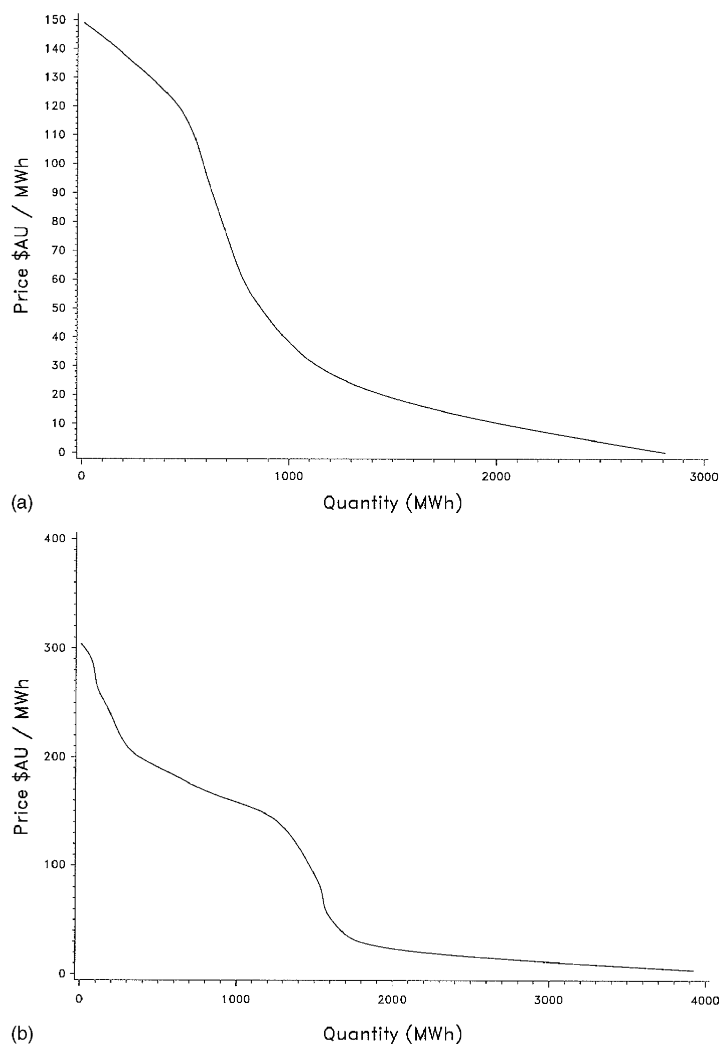

Figure 2. (a) Residual demand curve for 28 July 1997 low demand. (b) Residual demand curve for 28 July 1997 high demand

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all prices firm A has a higher demand for its output in the high system-demand period versus the low system-demand period. If the remaining NEM1 suppliers bid their entire capacity into the spot market during all half-hours with roughly the same quantity bids at the same bid prices, I would expect to observe this relationship between the half-hourly aggregate demand and the half-hourly residual demand that firm A faces.

Table I contains the parameter estimates for the two genset-level behavioral cost functions for the two sets of moment conditions used to estimate these parameters. All parameter estimates use a value $h = 0.01$, where the prices are reported in units of \$/MWh. The standard error estimates for the elements of β are computed using the GMM covariance matrix estimate given in Hansen (1982) using a consistent estimate of the optimal weighting matrix. The estimated asymptotic covariance matrix of the coefficient estimates for the case of the price moment restrictions only is equal to

$$V_\beta(b(O), h) = D\left[\left[\frac{1}{D}\sum_{d=1}^{D}\frac{\partial \ell_d(b(O), h)}{\partial b}\right]' V_D(b(I), h)^{-1}\left[\frac{1}{D}\sum_{d=1}^{D}\frac{\partial \ell_d(b(O), h)}{\partial b}\right]\right] \tag{36}$$

where $b(I)$ is the initial consistent estimate of β and $b(O)$ is the estimate of β using a consistent estimate of the optimal weighting matrix. Wolak (2001) outlines the conditions necessary for $\sqrt{D}(b(O) - \beta)$ to have an asymptotic normal distribution and for $V_\beta(b(O), h)$ to converge in probability to the asymptotic covariance matrix of $\sqrt{D}(b(O) - \beta)$ as D tends to infinity, h tends to zero and $D^{1/2}h$ tends to zero.

For both sets of moment restrictions, a Wald test of the null hypothesis that all elements of β, besides a_1 and a_2, are jointly zero is overwhelmingly rejected at an $\alpha = 0.01$ level of significance. This hypothesis testing result is consistent with the existence of genset-level ramping costs for both estimation methods. However, focusing on each genset-level cost function individually, the null hypothesis that all of the genset-level parameters, besides a_i, are zero is rejected at an

Table I. Parameter estimates

	Price and quantity moments		Price moments only	
	Coefficient	Standard error	Coefficient	Standard error
	Units from plant 1 (Nameplate capacity = 660)			
a_1	10.706	0.354	10.677	0.555
A_1(diag)	0.01139	0.00181	0.01197	0.00294
A_1(one)	−0.005232	0.00041	−0.00553	0.000835
A_1(two)	0.001709	0.00056	−0.001268	0.001072
B_1(diag)	−0.00000517	0.0000019	−0.00000575	0.00000308
	Units from plant 2 (Nameplate capacity = 500)			
a_2	14.456	0.3859	13.961	0.6555
A_2(diag)	0.0160	0.00796	0.01039	0.04515
A_2(one)	−0.005762	0.00158	−0.00692	0.02923
A_2(two)	0.04266	0.00341	0.01031	0.01678
B_2(diag)	−0.000056	0.000025	−0.000109	0.000108

Note: Standard errors of coefficient estimates computed using asymptotic covariance matrix estimate given in Hansen (1982).

$\alpha = 0.01$ level of significance for both gensets individually for the price and quantity bid moments restrictions, but for the gensets at plant 1 only for the case of the price moment restrictions.

Figure 3 presents plots of the estimated genset-level marginal cost function for units at plant 1 for various values of output in the previous half-hour period and the half-hour period immediately following the current period, given the value of output two half-hour periods ago and two half-hourly periods into the future. The differences in marginal costs due to differences in the level of production in previous periods are economically significant in the sense that the marginal cost is approximately \$0.50/MWh higher if the unit is producing 100 MWh less in period $t-1$ and the marginal cost is approximately \$0.20/MWh higher if the unit's output in period $t-2$ is 100 MWh higher.

Comparing the marginal costs in Figure 3 with those estimated by firm A using the technological relationship between the amount of input fuel burned and the megawatt-hours of electricity produced by the generation unit (called the heat rate), input fuel costs and other input prices, provides a rough check of the validity of these estimation results. The heat rate of a generation unit is analogous to the miles per gallon of an automobile in the sense that the miles per gallon a car achieves depends on how it is operated. These issues are discussed in Chapter 2 of Wood and Wollenberg (1996). Differences in the heat rate of the generation unit depending on how it is operated throughout the day are the technological source of ramping costs. Using the range of feasible marginal heat rates for the four large units owned by firm A, the input fuel prices and other variable operating and maintenance costs yield variable cost estimates in the range \$10/MWh to \$15/MWh, which contains virtually all of the marginal cost values given in Figure 3.

4.2. Testing the Hypothesis of Expected Profit Maximization

These behavioral cost function estimates can be used to examine the validity of the assumption of expected profit-maximizing bidding behavior. Because 70 moment restrictions are used to estimate 10 parameters in the case of the price bid moments, this leaves 60 over-identifying moment restrictions implied by the assumption of expected profit-maximizing bidding behavior. For the case of the price and quantity bid moments, 140 moment restrictions are used to estimate 10 parameters, which leaves 130 over-identifying moment restrictions implied by the assumption of expected profit-maximizing bidding. The test statistic for both of these null hypotheses is the optimized value of the GMM objective function times the number of observations, D. For the case of the price bid moment restrictions, this statistic takes the following form:

$$S_{\beta}(b(O), h) = D\left[\left[\frac{1}{D}\sum_{d=1}^{D}\ell_d(b(O), h)\right]' V_D(b(I), h)^{-1}\left[\frac{1}{D}\sum_{d=1}^{D}\ell_d(b(O), h)\right]\right] \tag{37}$$

Under the null hypothesis that the over-identifying moment restrictions implied by expected profit-maximizing bidding behavior hold, $S_{\beta}(b(O), h)$ is asymptotically distributed as a chi-squared random variable with degrees of freedom equal to the number of over-identifying restrictions. The value of this test statistic for the case of the price bid moment restrictions is equal to 47.62, which is less than the $\alpha = 0.05$ critical value from a chi-squared distribution with 60 degrees of freedom. The value of this test statistic for the case of the price and quantity bid moments is 84.11, which is less than the $\alpha = 0.05$ critical value from a chi-squared distribution with 130 degrees of freedom.

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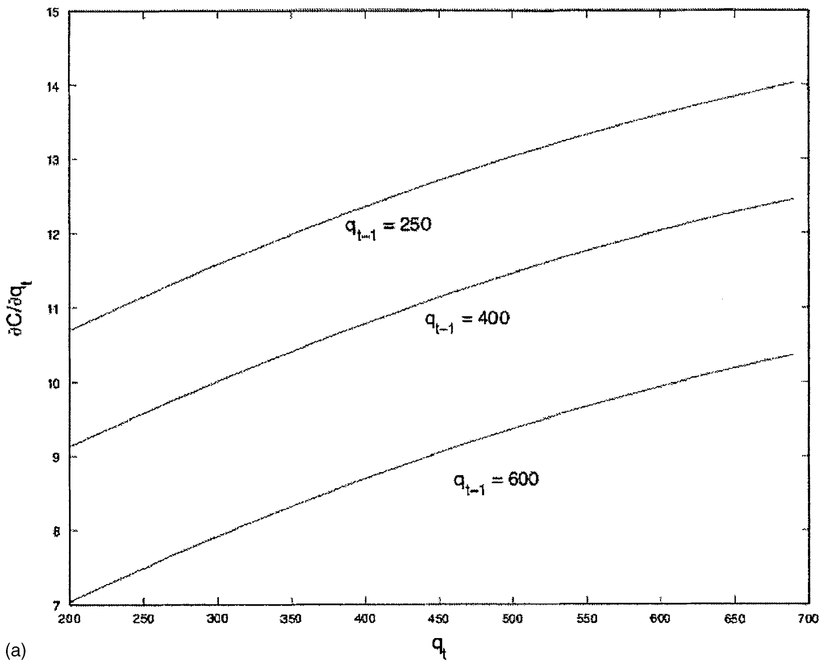
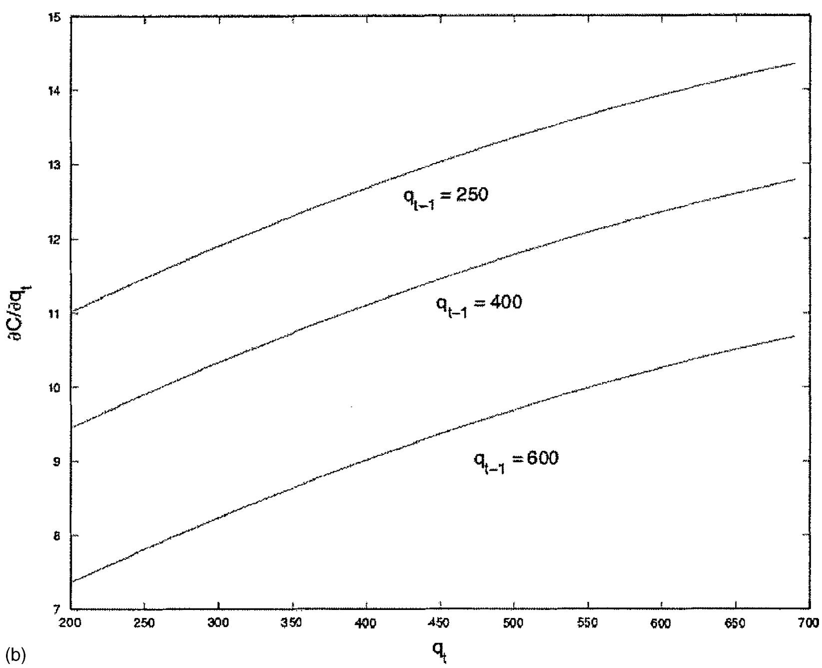


(a)



(b)

Figure 3. (a) Marginal cost for unit 1 (price and quantity moments), $q_{t-2} = 250$. (b) Marginal cost for unit 1 (price and quantity moments), $q_{t-2} = 400$. (c) Marginal cost for unit 1 (price and quantity moments), $q_{t-2} = 600$

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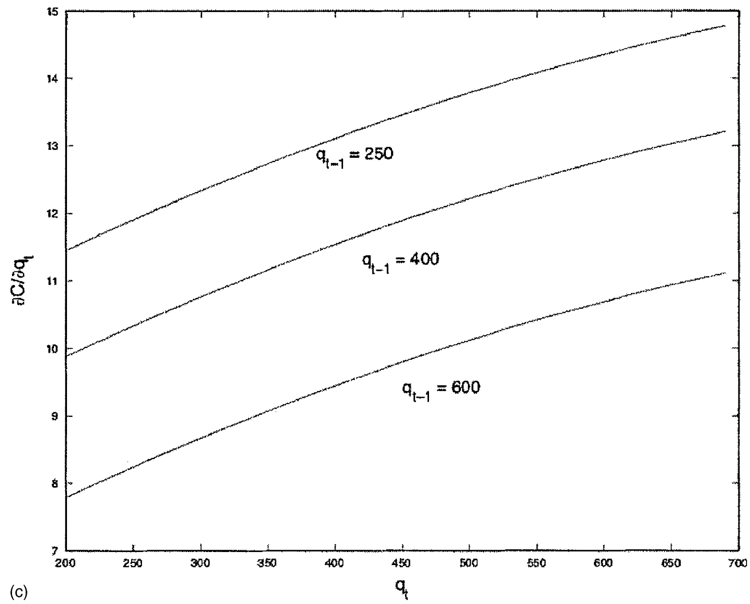

(c)

Figure 3. (*Continued*)

Both of these hypothesis-testing results provide no evidence against the null hypothesis that firm A chooses its price and quantity bids to maximize expected profits.

Another way to examine the validity of the moment restrictions implied by expected profit-maximizing bidding behavior is to test the null hypothesis that the probability limit of the difference between the GMM estimate of β that imposes the price bid moment restrictions only, $b^p(O)$, and the GMM estimate of β that imposes both the price and quantity bid moment restrictions, $b^{pq}(O)$, is equal to zero. It is possible to show that under the null hypothesis the plim of this difference is zero and the following asymptotic distribution result holds:

$$D^{1/2}(b^p(O) - b^{pq}(O)) \xrightarrow{d} N(0, V(\text{diff})) \tag{38}$$

where $V(\text{diff}, h) = V_\beta(b^p(O), h) - V_\beta(b^{pq}(O), h)$ converges in probability to $V(\text{diff})$ as D tends to infinity and h tends to zero. The matrix $V_\beta(b^p(O), h)$ is defined in equation (36) and $V_\beta(b^{pq}(O), h)$ is defined in an analogous manner with $\ell_d(b, h)$ replaced by $\mathcal{L}_d(b, h)$. The test statistic of the null hypothesis plim $(b^p(O) - b^{pq}(O)) = 0$ is

$$J = D(b^p(O) - b^{pq}(O))[V(\text{diff}, h)]^{-1}(b^p(O) - b^{pq}(O)) \tag{39}$$

The asymptotic distribution of this statistic under the null hypothesis is a chi-squared random variable with degrees of freedom equal to the dimension of β, which is equal to 10. The value of this test statistic is $J = 1.045$, which is smaller than the $\alpha = 0.05$ critical value of a chi-squared random variable with 10 degrees of freedom, indicating that the data provide no evidence against the null hypothesis that the price bid moment restrictions and price and quantity bid moment restrictions yield the same estimates of the parameters of the two unit-level behavioral cost functions.

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The 70-dimensional population moment inequality constraints associated with the quantity bids, $\lim_{h\to 0} E(Z_D(\beta, h)) \geq 0$, where $Z_D(\beta, h)$ is defined in equation (30), are an additional implication of expected profit-maximizing bidding behavior. Using the optimal GMM estimator of β that uses both the price and quantity bid moments, I compute

$$Z_D(b(O), h) = D^{-1/2} \sum_{d=1}^{D} \mathbb{I}_d(b(O), h)$$

$$\text{and } V_D(b(O), h) = \frac{1}{D} \sum_{d=1}^{D} (\mathbb{I}_d(b(O), h) - \bar{\mathbb{I}}_D(b(O), h))(\mathbb{I}_d(b(O), h) - \bar{\mathbb{I}}_D(b(O), h)' \quad (40)$$

$$\text{where } \bar{\mathbb{I}}_D(b(O), h) = \frac{1}{D} \sum_{d=1}^{D} \mathbb{I}_d(b(O), h)$$

Wolak (2004) derives the asymptotic distribution of $Z_D(b(O), h)$ for the least favorable value of moment restriction under the null hypothesis $\lim_{h\to 0} E(Z_D(\beta, h)) \geq 0$. Let $VI_D(b(O), h)$ denote a consistent estimate of the asymptotic covariance matrix of $Z_D(b(O), h)$. This asymptotic covariance estimate depends on $V_D(b(O), h)$, the estimate of the asymptotic covariance matrix of the GMM estimate of β, and the asymptotic covariance of the inequality moment restrictions with the equality moment restrictions. Applying results from Wolak (1989), the multivariate inequality constraints distance test for this null hypothesis against an unrestricted alternative is the solution to the following quadratic programming problem:

$$W = \min_{Z \geq 0} (Z_D(b(O), h) - Z)[VI_D(b(O), h)]^{-1}(Z_D(b(O), h) - Z) \quad (41)$$

I find that $W = 39.49$. Using the expression for the least-favorable distribution of the test statistic under the null hypothesis $\lim_{h\to 0} E(Z_D(\beta, h)) \geq 0$, the p-value for this hypothesis test can be computed using the expression

$$\sum_{k=0}^{M} Pr(\chi_k^2 > W) w(M, M-k, VI_D(b(O), h)) \quad (42)$$

where $Pr(\chi_k^2 > c)$ is the probability that a chi-square random variable with k degrees of freedom is greater than c, the weights $w(M, M-k, VI_D(b(O), h)$ are computed as described in Wolak (1989), and M is equal to number of inequality constraints being tested, which is equal to 70 in this case. Equation (42) gives the probability of rejecting the null hypothesis given the realized value of the test statistic, W. If this p-value is less than the size of the test, α, then the data provide evidence against the validity of the null hypothesis. For $W = 39.49$, equation (42) yields a probability value for this test statistic equal to 0.736. This result implies that the probability of obtaining a value from the least favorable the null asymptotic distribution of the test statistic greater than 39.49, the value of W computed using $b(O)$, is 0.736. This hypothesis testing result implies that the data provide little evidence against the validity of the moment inequality restrictions implied by expected profit-maximizing bidding behavior. Performing this same test for the estimate of β that only imposes the moment restrictions implied by the price bids yields the same conclusion of no evidence against the validity of the null hypothesis.

These specification testing results find no evidence against the null hypothesis of expected profit-maximizing bidding behavior, no evidence against the consistency of the unit-level cost function estimates using the price and quantity bid moment restrictions and those obtained using only the price bid moment restrictions, and overwhelming statistical evidence against the null hypothesis of no ramping costs for both genset unit-level cost functions for the case of price and quantity bid moment restrictions.

4.3. The Economic Significance of Ramping Costs

I now proceed to investigate the economic significance of ramping costs and quantify the magnitude of potential average daily production cost reductions that firm A achieves as a result of signing forward financial contracts to commit itself to a smoother pattern of output throughout the day. The following notation is required to define a number of counterfactual patterns of output throughout the day that quantify the magnitude of ramping costs that firm A faces:

Q_{ij}^{act} = actual output of genset j in half hour i

$Q_i{}^{\text{act}} = \sum_{j=1}^{J} Q_{ij}{}^{\text{act}}$ = the actual output from all gensets in half-hour i

$Q^{\text{act}} = \sum_{i=1}^{48} \sum_{j=1}^{J} Q_{ij}{}^{\text{act}}$ = actual output from all gensets owned by firm A during all half-hour periods of the day

$CAP_j^{\min}$ = minimum level of output that can be produced by unit j in a half-hour

As noted earlier, there are four identical units at Plant 1 and three identical units at Plant 2 operating during my sample period, so that $J = 7$. The value of $\text{CAP}_j^{\min}$ is 200 MW for the units at plant 1 and 180 MW for the units at Plant 2. $\text{CAP}_j^{\max}$, defined earlier, is the maximum half-hourly output observed from that unit during the sample period.

For each day during the sample I solve the following two optimization problems using the unit-level behavioral cost function estimates based on the price and quantity bid moments:

$$\min_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j, b_j^{pq}(O)) \text{ subject to } \begin{array}{c} \sum_{j=1}^{J} Q_{ij} = Q_i{}^{\text{act}} \text{ for } i = 1, \ldots, 48 \\ \text{CAP}_j^{\min} \leq Q_{ij} \leq \text{CAP}_j^{\max} \text{ for } j = 1, \ldots, J \\ \text{and } i = 1, \ldots, 48 \end{array} \tag{43}$$

and

$$\max_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j, b_j^{pq}(O)) \text{ subject to } \begin{array}{c} \sum_{j=1}^{J} Q_{ij} = Q_i{}^{\text{act}} \text{ for } i = 1, \ldots, 48 \\ \text{CAP}_j^{\min} \leq Q_{ij} \leq \text{CAP}_j^{\max} \text{ for } j = 1, \ldots, J \\ \text{and } i = 1, \ldots, 48 \end{array} \tag{44}$$

where $b_j^{pq}(O)$ is the vector of elements of $b^{pq}(O)$ corresponding to genset j. The first optimization problem solves for the daily pattern of output throughout the day that minimizes daily operating costs subject to the constraints that it replicates the half-hourly actual pattern of total output (across all J units) owned by firm A, and that no unit can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective

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function from (43) for day d, Min_Cost_Half_Hour(d). The second optimization problem solves for the daily pattern of output throughout the day that maximizes daily operating costs subject to the constraints that it replicates the half-hourly actual pattern of total output (across all J units) owned by firm A and that no unit can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (44) for day d, Max_Cost_Half_Hour(d). Let Actual_Cost(d) equal the value of operating costs during day d, based on the estimated genset-level cost functions, using the values of ${Q_{ij}}^{\text{act}}$ for all half-hours of the day. If there were no ramping costs and all units had the same marginal cost of production then

$$\text{Min\_Cost\_Half\_Hour}(d) = \text{Actual\_Cost}(d) = \text{Max\_Cost\_Half\_Hour}(d) \text{ for } d = 1, \ldots, D$$

With ramping costs

$$\text{Min\_Cost\_Half\_Hour}(d) \leq \text{Actual\_Cost}(d) \leq \text{Max\_Cost\_Half\_Hour}(d)$$

Consequently, one measure of the magnitude of ramping costs is the ratio of the sample mean of Max_Cost_Half_Hour(d) to the sample mean of Min_Cost_Half_Hour(d).

To allow greater flexibility to substitute across gensets within the day, I also compute the following two counterfactual daily minimum and maximum patterns of output using the behavioral cost function estimates obtained from the price and quantity bid moment restrictions:

$$\min_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j, b_j^{pq}(O)) \text{ subject to } \begin{array}{c} \sum_{i=1}^{48} \sum_{j=1}^{J} Q_{ij} = Q^{\text{act}} \\ \text{CAP}_j^{\min} \leq Q_{ij} \leq \text{CAP}_j^{\max} \text{ for } j = 1, \ldots, J \\ \text{and } i = 1, \ldots, 48 \end{array} \tag{45}$$

and

$$\max_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j, b_j^{pq}(O)) \text{ subject to } \begin{array}{c} \sum_{i=1}^{48} \sum_{j=1}^{J} Q_{ij} = Q^{\text{act}} \\ \text{CAP}_j^{\min} \leq Q_{ij} \leq \text{CAP}_j^{\max} \text{ for } j = 1, \ldots, J \\ \text{and } i = 1, \ldots, 48 \end{array} \tag{46}$$

Optimization problem (45) solves for the daily pattern of output throughout the day that minimizes daily operating costs subject to the constraints that it replicates the total daily output (across all J units and 48 half-hours) produced by firm A and that none of firm A's units can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (45) for day d, Min_Cost_Day(d). The second optimization problem solves for the daily pattern of output throughout the day that maximizes daily operating costs subject to the constraints that it replicates the total daily output (across all J units and 48 half-hours) produced by firm A and that none of firm A's units can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (46) for day d, Max_Cost_Day(d).

Because (45) and (46) impose one constraint on the pattern of output throughout the day and (43) and (44) impose 48 constraints, the following inequalities hold:

$$\text{Min\_Cost\_Day}(d) \leq \text{Min\_Cost\_Half\_Hour}(d)$$

$$\text{Max\_Cost\_Half\_Hour}(d) \leq \text{Max\_Cost\_Day}(d)$$

The sample mean of Max_Cost_Day(d) divided by the sample mean of Min_Cost_Day(d) is equal to 1.09 with a standard error of 0.025. The sample mean of Max_Cost_Half_Hour(d) divided by the sample mean of Min_Cost_Half_Hour(d) is equal 1.07 with a standard error of 0.021. These results indicate that ramping costs can increase the average daily cost necessary to meet firm A's actual daily output obligations by as much as 7–9%. The sample mean of Actual_Cost(d) divided by the sample mean of Min_Cost_Half_Hour(d) is 1.02 with a standard error of 0.014, and the ratio of the sample mean of Actual_Cost(d) divided by the sample mean of Min_Cost_Day(d) is 1.03 with a standard error of 0.017. These calculations indicate that firm A's present level of forward contracting implies that firm A finds it expected profit-maximizing to produce a pattern of output that is not statistically significantly different from the minimum daily cost pattern of output.

The standard errors reported in the previous paragraph account for the fact that $b_j^{pq}(O)$, the GMM estimator using the price and quantity moment conditions, not β_j, is used in $C_j(Q_j, \beta_j)$ to compute the minimum, maximum and actual daily variable cost figures. These standard errors are computed by applying the delta method to the sample daily average variable cost ratio. Each sample average daily variable cost ratio can be written as

$$f_{AB}(b, D) = \frac{\sum_{d=1}^{D} \mathrm{Cost}^B(b, d)}{\sum_{d=1}^{D} \mathrm{Cost}^A(b, d)}$$

where $\mathrm{Cost}^J(b, d)$ is the value of the daily variable cost function for day d, with counterfactual scenario J ($J = A, B$), and cost function parameter vector equal to b. For example, for the first ratio reported in the previous paragraph, $\mathrm{Cost}^B(b, d)$ is Max_Cost_Day(d) and $\mathrm{Cost}^A(b, d)$ is Min_Cost_Day(d).

The function $f_{AB}(b, D)$ is continuously differentiable in b given D, the number of observations in the sample. Given the asymptotic distribution of the GMM estimate of β, applying the delta method yields the following estimate for the variance of $f_{AB}(b^{pq}(O), D)$:

$$\mathrm{Var}^{\mathrm{Est}}(f_{AB}(b^{pq}(O), D)) = \frac{\partial f_{AB}(b^{pq}(O), D)'}{\partial b}[D^* V_\beta(b^{pq}(O))]\frac{\partial f_{AB}(b^{pq}(O), D)}{\partial b}$$

where $V_\beta(b^{pq}(O))$ is a consistent estimate of the covariance matrix of the asymptotic distribution of $D^{1/2}(b^{pq}(O) - \beta)$. Note that this estimated variance only controls for the estimation error associated with using $\mathrm{Cost}^J(b^{pq}(O), d)$ instead of $\mathrm{Cost}^J(\beta, d)$ ($J = A, B$) in the computation of the ratio of the sample means of $\mathrm{Cost}^B(b^{pq}(O), d)$ and $\mathrm{Cost}^A(b^{pq}(O), d)$. Therefore, the square root of this variance estimate quantifies the imprecision in the ratio of the sample means caused by the need to use $b^{pq}(O)$ instead of β to compute the value of the variable cost function. All the standard errors reported above use this procedure.

The ratios of the sample means reported above are informative about the extent of potential benefits, in terms of reduced daily average production costs, accruing to firm A from its current level of forward contracting. Figure 4 plots the sample average (over all 102 days during the sample period) of Q_i^{act}, the actual total output of firm A during half-hour i for $i = 1, \ldots, 48$, and the sample average of QC_i, the forward contract quantity obligation of firm A during half-hour period $i = 1, \ldots, 48$. For all but the early-morning half-hour periods of the trading day (period

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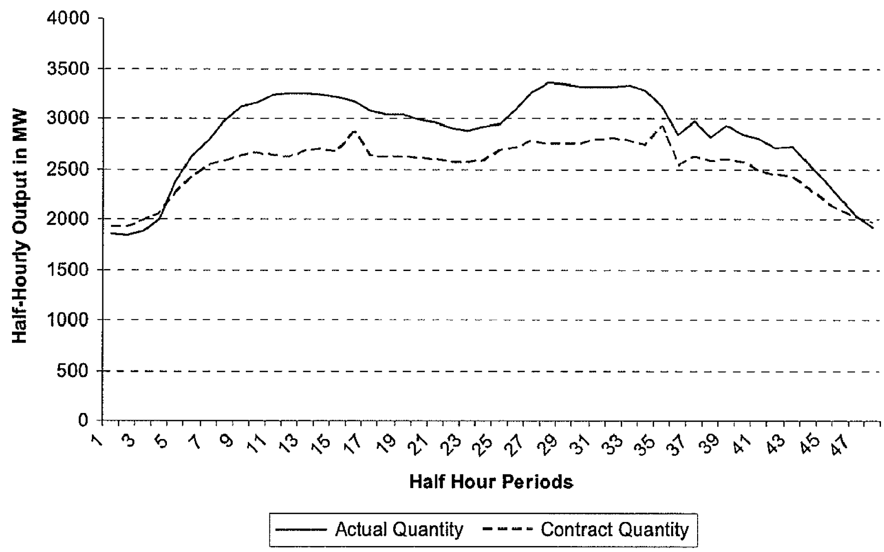

Figure 4. Average half-hourly output and half-hourly forward contract quantity

1 of the trading day is the half-hour beginning 4 a.m. and period 48 is the half-hour ending at 4 a.m. of the following day), the sample mean of Q_i^{act} is larger than the sample mean of QC_i. The ratio of the sample mean of QC_i over all days and half-hour periods divided by the sample mean of Q_i^{act} over all days and half-hour periods is equal to 0.881, indicating that during my sample period 88.1% of the MWh produced by firm A was hedged by a forward financial contract.

At a reduced level of forward contracts, it is unlikely that firm A would produce the same total daily output and it is very unlikely to produce the same total amount of output in each half-hour of the day because of the increased incentives it would have to exploit the distribution of residual demand curves that it faces in constructing its expected profit-maximizing bids. Thus, comparing the sample means of the two Max_Cost and Min_Cost figures does not account for the fact that the profit-maximizing pattern of daily output would change as a result of firm A having significantly less or no forward contract obligations. In particular, firm A would produce less output on average.

To address the impact of a reduced level of forward contracting on the profit-maximizing daily pattern of production, I compute the solution to the following optimization problem for each day of the sample period:

$$\max_{\{Q_j\}_{j=1}^{J}} \sum_{i=1}^{48} DR_i(p_i)p_i - \sum_{j=1}^{J} C_j(Q_j, b_j^{pq}(O)) - \sum_{i=1}^{48}(p_i - PC_i{}^*)QC_i{}^*$$

$$\text{subject to } DR_i(p_i) = \sum_{j=1}^{J} Q_{ij} \quad \text{for } i = 1, \ldots, 48 \tag{47}$$

$$\text{CAP}_j{}^{\min} \le Q_{ij} \le \text{CAP}_j{}^{\max} \text{ for } j = 1, \ldots, J \text{ and } i = 1, \ldots, 48$$

$DR_i(p)$ is the actual residual demand curve faced by firm A during load period i. $QC_i{}^*$ is the assumed counterfactual value of forward contracts held by firm A during load period i. $PC_i{}^*$ is

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the assumed forward contract price in period i. The solution to (47) yields the profit-maximizing pattern of output during the day for firm A given the actual residual demand curves that it faced during each half-hour of the day for a half-hourly level of forward contracts equal to $QC_i{}^*$.

To determine the profit-maximizing pattern of daily production with no forward contract holdings by firm A, I solve (47) assuming that $QC_i{}^* = 0$ for all half-hours i and for all days during the sample period. Let $Q_{ij}{}^{QC=0}$ equal the optimized value of Q_{ij} from solving (47). I then compute the value Max_Cost_Half_Hour(d) and Min_Cost_Half_Hour(d) by solving (43) and (44), replacing the actual value of Q_{ij} with $Q_{ij}{}^{QC=0}$. I also compute Max_Cost_Day(d) and Min_Cost_Day(d) by solving (45) and (46), replacing the actual value of Q_{ij} with $Q_{ij}{}^{QC=0}$. Denote this Min_Cost and Max_Cost with a '*'.

Dividing the sample mean of Max_Cost_Half_Hour(d)* by the sample mean of Min_Cost_Half_Hour(d)* yields 1.09, with a standard error of 0.031. The sample mean of Max_Cost_Day(d)* divided by the sample mean of Min_Cost_Day(d)* yields 1.12, with a standard error of 0.038. These ratios are both higher than the same ratios computed using the actual (rather than the zero-forward-contract profit-maximizing pattern of output by firm A). This result demonstrates that the profit-maximizing pattern of output within the day with no forward contracts is significantly more variable than the actual pattern of output within the day with a substantial amount of forward contracts.

Define Actual_Cost(d)* as the daily operating cost computed using $Q_{ij}{}^{QC=0}$ instead of the actual output of firm A. Dividing the sample mean of Max_Cost_Half_Hour(d)* by the sample mean of Actual_Cost(d)* yields a value of 1.02, with a standard error of 0.015. The sample mean of Max_Cost_Day(d)* divided by the sample mean of Actual_Cost(d)* is 1.03, with a standard error of 0.017. These results imply that the profit-maximizing pattern of production with zero forward contracts is not statistically different from the maximum daily cost pattern of production, which is exactly the opposite result from what I obtained for the case of firm A's actual level of forward contracts.

To further quantify the benefits of forward contracting, I perform the following computation for $AC(d) = \text{Actual\_Cost}(d)/Q^{act}$ and $AC(d)^* = \text{Actual\_Cost}(d)^*/Q^{QC=0}$, where $Q^{QC=0} = \sum_{i=1}^{48}\sum_{j=1}^{J} Q_{ij}{}^{QC=0}$, for each day of the sample period. Because suppliers have a unilateral incentive to produce more output with forward contract obligations, the sample mean of Actual_Cost(d) is greater than the sample mean of Actual_Cost(d)*. However, the sample mean of $AC(d)^*$ is greater than $AC(d)$ because forward contract obligations provide incentives for a smooth pattern of output throughout the day. The ratio of the sample mean of $AC(d)^*$ divided by the sample mean of $AC(d)$ is 1.08, with a standard error of 0.021, implying that the sample mean daily average cost of producing power according to the profit-maximizing pattern of daily output for $QC = 0$ for all half-hours of the sample is statistically significantly higher than the sample mean daily average cost of producing power according to the profit-maximizing pattern of daily output at the actual level of forward contracting. This computation implies significant benefits from forward contracting in terms of committing suppliers to a lower daily average cost pattern of production.

The standard errors reported in the above three paragraphs were computed in the same manner as described above using $Q_{ij}{}^{QC=0}$ in place of $Q_{ij}{}^{\text{act}}$. Consequently, these standard errors control for the uncertainty in the minimum, maximum and actual costs due to the use of $b^{pq}(O)$ in place of β, but not the uncertainty in the $Q_{ij}{}^{QC=0}$ due to the use of $b^{pq}(O)$ in place of β.

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Figure 5 reproduces the half-hourly mean values of Q_i^{act} over the sample period from Figure 5 and the hourly mean values of $Q_i^{QC=0} = \sum_{j=1}^{J} Q_{ij}^{QC=0}$ over the sample period. With the exception of the early-morning half-hour periods, the mean value of $Q_i^{QC=0}$ is significantly less than the half-hourly mean values Q_i^{act}. The mean of $Q_i^{QC=0}$ over all days and hours in the sample period divided by the mean of Q_i^{act} over all days and hours in the sample period is 0.851. This result implies that if firm A had no forward contract obligations, it would find it unilaterally profit-maximizing to produce an average of 85.1% less output given the half-hourly residual demand curves it faces over the sample period. This result is consistent with the substantially greater incentive a supplier with no forward contracts has to withhold output to raise the price it is paid for all of the output it sells.

The flat pattern of the sample means of $Q_i^{QC=0}$ for $i = 1, \ldots, 48$ throughout the day relative to the pattern of the sample means of Q_i^{act} for $i = 1, \ldots, 48$ throughout the day masks a substantially more volatile pattern of generation unit-level output with no forward contracts relative to firm A's actual pattern of output with its actual level of forward contracts. Figure 6 plots kernel density estimates of the half-hourly values of $Q_{ij}^{QC=0}$ and Q_{ij}^{act} for $j = 1$ and $j = 2$ over the sample period. Consistent with the incentive for a stable pattern of output throughout the day caused by a high level of forward contracts, the kernel densities for Q_{ij}^{act} for gensets 1 and 2 show substantial peaks around 660 MW, indicating that for the vast majority of half-hours during the sample period both gensets operated in the range of 660 MW. In contrast, the kernel densities for $Q_{ij}^{QC=0}$ have no significant peaks, indicating close to a uniform distribution of half-hourly output levels over the sample period under the counterfactual scenario that firm A has no forward contract obligations. The kernel densities of $Q_{ij}^{QC=0}$ and Q_{ij}^{act} for the remaining five gensets, $j = 3, \ldots, 7$, are qualitatively similar to those given in Figure 6.

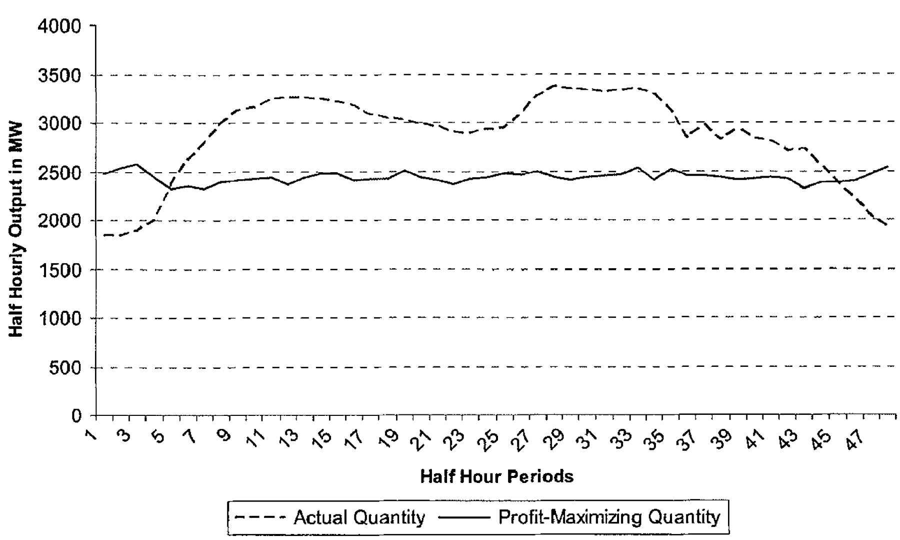

Figure 5. Average half-hourly actual-output and half-hourly profit-maximizing output with $QC = 0$

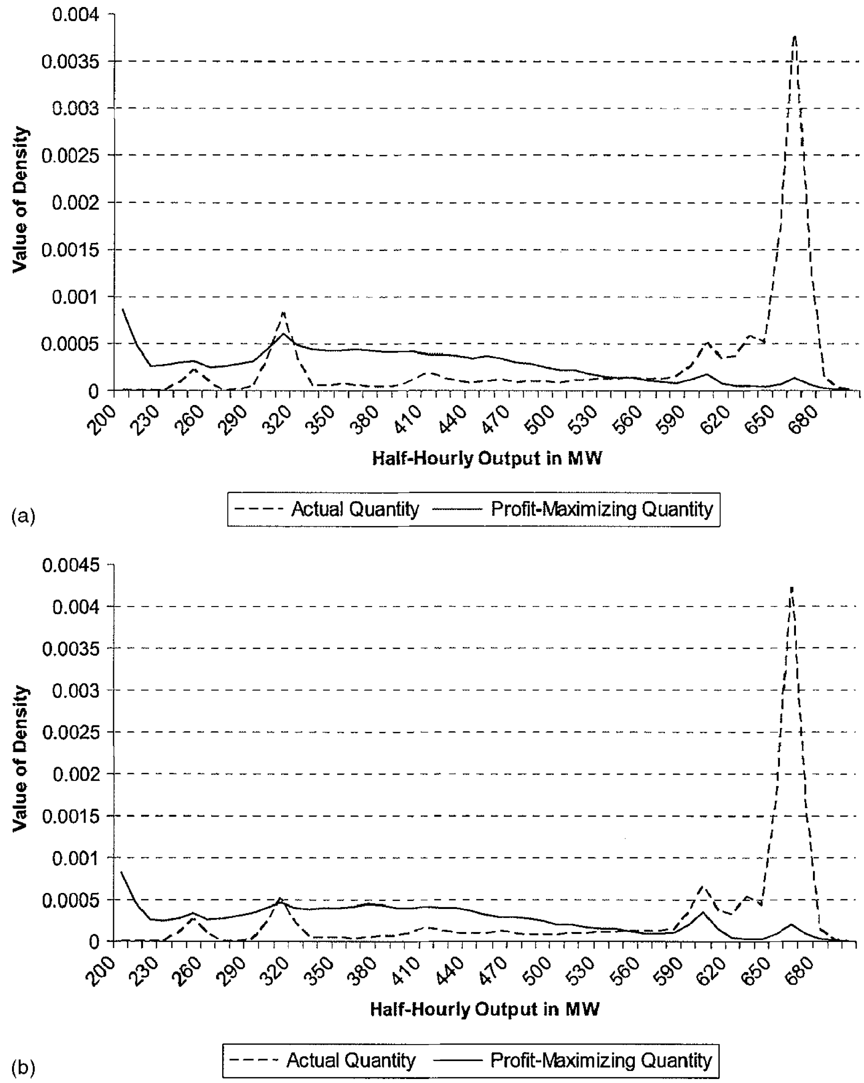

Figure 6. (a) Kernel densities of half-hourly output of generation unit 1. (b) Kernel densities of half-hourly output of generation unit 2

5. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

The estimation framework presented in this paper for recovering multi-output, genset-level behavioral cost functions from the assumption of expected profit-maximizing bidding behavior in a multi-unit auction where bidders submit non-decreasing step-function bid supply curves can be applied to any multi-unit auction that uses step-function bid curves, such as auctions for treasury bills, spectrum, and multiple items on eBay. Because this estimation procedure only relies on the assumption of best-reply behavior on the part of an individual market participant, it is unnecessary to solve for the market equilibrium to apply it, so the procedure can be applied to a wide range of auction mechanisms. Because step-function bid curves, rather than continuous bid functions, appear to be the rule rather than the exception in actual multi-unit auctions, this estimation framework should have wide empirical application.

Several tests of the null hypothesis of expected profit-maximizing bidding behavior are proposed and implemented. There is little empirical evidence against the validity of the over-identifying moment restrictions implied by the null hypothesis of expected profit-maximizing bidding behavior for both the price bid moments alone and the combination of price and quantity bid moments estimators. I also propose and implement a test of the null hypothesis that the parameters of the behavioral cost function implied by the expected profit-maximizing choice of the price and quantity bids of the step function supply curves are the same as the parameters of the behavioral cost function implied by the expected profit-maximizing choice of the price bids only. I find little evidence against this null hypothesis. Finally, expected profit-maximizing choice of the quantity bids also implies that certain moment inequalities hold for the true behavioral cost function. Using results from Wolak (1989) and Wolak (2004), I test and fail to reject these population moment inequality constraints.

These genset-level behavioral cost function estimates provide statistically significant evidence consistent with the existence of ramping costs. These ramping costs are quantified by comparing the minimum and maximum daily cost of producing the supplier's actual half-hourly pattern of daily output across all gensets it owns using these cost function estimates and the minimum and maximum daily cost of producing the supplier's actual daily output. These computations reveal that ramping costs can raise the average daily cost of producing the supplier's actual pattern of output by as much 9%.

To quantify the impact of forward contracting on the supplier's daily cost of producing its output, I use the actual bids submitted by the supplier's competitors and these behavioral cost function estimates to compute a counterfactual profit-maximizing pattern of production throughout the day for the supplier, assuming it holds no forward contracts for each day in the sample. I then repeat the two daily cost minimum and maximum computations for this pattern of output.

I find that the profit-maximizing pattern of output throughout the day assuming the supplier has zero forward contracts is very close to the maximum-cost method to supply this pattern of daily production. In contrast, the supplier's actual pattern of daily production with its current forward contract holdings is very close to the minimum-cost method to supply this pattern of daily production. Comparing the actual daily average cost and the daily average cost of producing the zero-contract profit-maximizing pattern of production, I find that the average cost to firm A of producing in a profit-maximizing manner with no forward contracts is approximately 8% higher than its actual average cost at its current level of forward contracts, which provides strong evidence for the operating cost reduction benefits to suppliers from signing forward financial contracts.

These results can be used to quantify the benefits to firm A from its current fixed-price forward contract obligations. Relative to the case of zero forward contract obligations, firm A's average cost of production is roughly 8% lower and it sells on average approximately 15% more energy. These two numbers can be used to provide an upper bound on the ratio of spot prices relative to forward contract prices that would cause firm A to sign its current level of fixed-price forward contracts. Assume that p_{id}^{C} is the average price that firm A receives for the energy it sells during period i of day d at its current level of fixed-price forward contract obligations. Using this notation,

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write firm A's actual profits over all days in the sample as

$$\Pi^C = \sum_{d=1}^{D} \left[\sum_{i=1}^{48} p_{id}^C \left[\sum_{j=1}^{J} Q_{ijd}^{\text{act}} \right] - \sum_{j=1}^{J} C_j(Q_{1jd}^{\text{act}}, Q_{2jd}^{\text{act}}, \ldots, Q_{48jd}^{\text{act}}) \right] \tag{48}$$

where Q_{ijd}^{act} is the period i output from unit j during day d of the sample period. Dividing the right-hand side of (48) through by $Q_{\text{tot}}^{\text{act}} = \sum_{d=1}^{D} \sum_{i=1}^{48} \sum_{j=1}^{J} Q_{ijd}^{\text{act}}$ and then multiplying by $Q_{\text{tot}}^{\text{act}}$ yields

$$\Pi^C = [p_{\text{avg}}^C - AVC_{\text{avg}}^C] Q_{\text{tot}}^{\text{act}} \tag{49}$$

where $p_{\text{avg}}^C = \dfrac{\sum_{d=1}^{D} \sum_{i=1}^{48} p_{id}^C \left[\sum_{j=1}^{J} Q_{ijd}^{\text{act}} \right]}{Q_{\text{tot}}^{\text{act}}}$ and $AVC_{\text{avg}}^C = \dfrac{\sum_{d=1}^{D} \sum_{j=1}^{J} C_j(Q_{1jd}^{\text{act}}, Q_{2jd}^{\text{act}}, \ldots, Q_{48jd}^{\text{act}})}{Q_{tot}^{act}}$ are the sample average price and sample average variable cost for firm A's current forward contract obligations. Sample average profits without forward contracts can be written as

$$\Pi^{NC} = \left[p_{\text{avg}}^{NC} - \left(\frac{1}{0.92} \right) AVC_{\text{avg}}^C \right] Q_{\text{tot}}^{\text{act}} (0.85) \tag{50}$$

because the sample average cost with contracts is 8% less than sample average cost without contracts and the total output over the sample period without contracts is 85% of the total output with the current level of forward contract obligations. The inequality that $\Pi^C \geq \Pi^{NC}$, firm A's profits with its current level of forward contracts is greater than or equal to its maximum profits without contracts, implies

$$\frac{p_{\text{avg}}^{NC}}{p_{\text{avg}}^C} \leq \frac{1}{0.85} * \left[1 - \left(\frac{0.07}{0.92} \right) \frac{AVC_{\text{avg}}^C}{p_{\text{avg}}^C} \right] \tag{51}$$

Firm A's average variable cost is roughly equal to 50% of the average price it received for energy over the sample period, which implies that p_{avg}^{NC} can be up to 1.13 times p_{avg}^C and firm A would still earn higher profits from its existing level of contracts relative to no forward contracts. Equation (51) implies that unless firm A expects to exercise enough unilateral market power with no fixed-price forward contract obligations to raise average short-term prices by more than 13%, it would prefer its existing level of forward contracts to no forward contract obligations. The ability to raise annual average prices by 13% would require that the firm possess a substantial amount of unilateral market power. Moreover, in most markets it is very difficult for suppliers to raise prices during the off-peak period of the day—roughly one-third of the hours in the day. This would imply that the firm would need to raise average prices by close to 20% (= 1.5 × 13%) in the remaining hours of the day, which would be very likely to trigger a regulatory intervention. This logic may explain why suppliers in all wholesale electricity markets currently operating around the world have found it unilaterally expected profit-maximizing to enter into fixed-price forward contract obligations for the vast majority of their expected annual output.

There are a number of other potential uses for these behavioral cost function estimates. Realistic genset-level marginal cost estimates are a key ingredient for computing competitive benchmark pricing to assess the extent of market inefficiencies in wholesale electricity markets. Borenstein *et al*. (2002), Joskow and Kahn (2002), Mansur (2003), and Bushnell *et al*. (2003) all use engineering-based estimates of these operational cost functions that assume no ramping constraints. These multi-output, genset-level cost functions can be used in place of the engineering estimates to capture the impact of ramping constraints in computing these counterfactual competitive benchmark prices. Mansur (2003), among others, has argued that taking these constraints into account may have important implications for the values of these competitive benchmark prices. He proposes and implements a regression-based procedure that suggests accounting for ramping constraints will yield an economically significant difference in the resulting competitive benchmark prices relative to the standard approach that ignores ramping constraints.

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