ST Co-channel Interference Mitigation: Outline

- Sources
- Interference Channel Model
  - SIMO, MIMO, MISO
- Interference Mitigation
  - Reverse Link (SIMO)
  - MIMO
  - Forward Link (MISO)
- SS Modulation
  - ST RAKE, ST Pre-RAKE
- Interference Diversity
Sources of Interference

- Co-channel interference (CCI) arises from frequency reuse.
- Adjacent channel interference (ACI) arises from modulation waveform spectral spreading (and regrowth) and channel filter leakage - not considered
- Intersymbol interference (ISI) arises due to delay spread. CCI and ISI have dualities
Channel Reuse

Spatial Reuse
Robust to scattering
Fixed and mobile applications
High and low base stations
Sectoring can reduce co-channel

Angular Reuse
Fixed application
Directional antenna at CPE
Low scattering (high antenna, flat terrain)

Polarization Reuse
Fixed application
Directional antennas
Low scattering
Co-channel Interference

Co-channel from angular reuse comes from
- Beam pattern
- Angle scattering

Co-channel from polar reuse comes from
- Faraday rotation
- Tx
- Rx

Co-channel from spatial reuse comes from
- Path loss

Co-channel protected by spatial and polarization separation
- Angular separation
- Polarization separation

Co-channel protected by spatial separation
- Angular separation

Co-channel protected by angular separation
Handling Co-channel Interference

- CCI channel known (very hard in practice)
  - Multi user detection problem similar to ML in MIMO (same analysis)
- CCI channel unknown, but statistics known
  - Problem similar to MMSE (we only need channel from own signal and $R_{yy}$)
- CCI completely unknown
  - MRC processing for signal. Some suppression of CCI if channels for signal and CCI different.
**SIMO Interference Model (Reverse Link)**

- Reverse channel with 1 desired user and $N$ co-channel interferers
- Frequency Flat Channel

$$y = h_0 s_0 + \sum_{i=1}^{N} h_i s_i + n$$

- $y$ ($M_R \times 1$) is the received signal vector
- $s_0$ is the desired user’s signal with $\mathcal{E}\{|s_0|^2\}$
- $s_i$ is the $i^{th}$ interferer
Reverse Link - Frequency Selective Channel

\[ Y = H_0 S_0 + \sum_{i=1}^{N} H_i S_i + \mathcal{N}, \]

where

\[ H_i = \begin{bmatrix} h_{1,i}[L-1] & \ldots & h_{1,i}[0] \\ \vdots & \ddots & \vdots \\ h_{M_R,i}[L-1] & \ldots & h_{M_R,i}[0] \end{bmatrix}, \quad S_i = \begin{bmatrix} s_i[k-L+1] & \ldots & s_i[k+T-L] \\ \vdots & \ddots & \vdots \\ s_i[k] & \vdots & s_i[k+T-1] \end{bmatrix}, \]

with \( \mathcal{E}\{|s_i[k]|^2\} = E_{s,i} \).
- Desired and interfering signals are all from multiple transmit antennas.
- For a Frequency flat channel

\[ y = H_0 s_0 + \sum_{i=1}^{N} H_i s_i + n \]

- \( s_0, \mathbb{E}\{s_0 s_0^H\} = \frac{E_{s_0}}{M_T} I_{M_T} \) is the desired user’s signal.
- \( H_0, (M_R \times M_T) \) is the desired user’s channel
- \( s_i, \mathbb{E}\{s_i s_i^H\} = \frac{E_{s_i}}{M_T} I_{M_T} \) is the \( i^{th} \) interferer’s signal.
- \( H_i, (M_R \times M_T) \) is the \( i^{th} \) interferer’s channel
MISO Interference Channel (Forward Link)

- $N + 1$ interfering base-stations each serving one intended user per cell.

$$y_0 = h_0 w_0 s_0 + \sum_{i=1}^{N} h_i w_i s_i + n_0$$

- Linear pre-filtering performed at the transmitter
- Processing at the receiver not effective in the general case.
CCI Mitigation for Reverse Link (SIMO)

- **Space-ML (S-ML) Receiver**
  - We solve the multiuser (joint decoding) detection problem
    \[
    \hat{s}_0 = \arg \min_{s_0, s_1, \ldots, s_N} \left\| y - h s_o - \sum_{i=1}^{N} h_i s_i \right\|_F^2
    \]
  - Extracts $M_R$ order diversity
  - Requires channel knowledge for all interferers
  - Has high decoding complexity
CCI Mitigation for Reverse Link (SIMO) - (ctd)

- Space-Minimum Mean Square Error (S-MMSE) Receiver
  
  - Uses an MMSE filter \( g_{MMSE} (1 \times M_R) \) given by

  \[
  g_{MMSE} = \arg \min_g \mathcal{E}\{ |gy - s_o|^2 \} = R_{s_0,y}R_{yy}^{-1} = E_{s,0}h_0^HR_{yy}^{-1}
  \]

  where \( R_{s,y} = \mathcal{E}\{ s_o y^H \} = E_{s,0}h_0^H \) and \( R_{yy} = \mathcal{E}\{ yy^H \} \)

  \[
  \text{SINR}_{MMSE} = \frac{\text{det}(R_{yy})}{\text{det}(R_{yy} - E_{s,0}h_0h_0^H)} - 1
  \]

  - Extracts \( M_R - N \) order diversity at high SNR and low to medium SIR.
  - Simpler Computationally, does not need CCI channel
  - Converges to S-MRC at high SIR with \( M_R \) order diversity
CCI Mitigation for Reverse Link (SIMO) - (ctd)

- Space Maximum Ratio Combining (S-MRC) Receiver
  - Ignores CCI but suppresses interference due to the different channel signatures, with the MRC filter $g_{MRC}$ ($1 \times M_R$) given by

  $$g_{MRC} = h^H$$

  $$\text{SINR}_{MRC} = \frac{E_{s,o}(h_o^H h_o)^2}{\sum_{i=1}^N E_{s,i} ||h_o^H h_i||^2 + h_o^H h_o N_o}$$

  - With dominant noise ($N_o \gg E_{s,i}$), $\text{SINR}_{MRC} \approx \rho h_o^H h_0 = \rho ||h_0||^2_F$
  - if $h_0 = h_w$ the S-MRC offers $M_R$ order diversity and an array gain of $M_R$
  - It can be shown that the SINR after processing is greater than the input SINR.
  - Exact gain depends on SINR and relative geometry of signal and interferers.
### Trade-offs - CCI Cancellation Receivers

<table>
<thead>
<tr>
<th></th>
<th>Diversity Order</th>
<th>Noise Enhancement Loss</th>
<th>Channel Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-ML</td>
<td>$M_R$</td>
<td>Nil</td>
<td>$h_i$ ($i = 0, 1, 2, \ldots, N$)</td>
</tr>
<tr>
<td>S-MMSE</td>
<td>$M_R - N$</td>
<td>Medium</td>
<td>$R_{yy}$, h</td>
</tr>
<tr>
<td>S-MRC</td>
<td>$M_R$</td>
<td>Nil</td>
<td>h</td>
</tr>
</tbody>
</table>
CCI Cancellation - Frequency Selective Channels

- **ST-ML Receiver**
  - ML receiver is a multiuser detection problem given by

  \[ \hat{S}_0, \hat{S}_1, \hat{S}_2, \ldots, \hat{S}_N = \arg \min_{S_0, S_1, S_2, \ldots, S_N} \left\| Y - H_0 S_0 - \sum_{i=1}^{N} H_i S_i \right\|_F^2. \]

  - Requires channel knowledge of desired signal and interferers
  - Offers space \((M_R)\) and path \((\approx L_{eff})\) diversity
  - Mitigates CCI and ISI but very complex
CCI Cancellation - Frequency Selective Channels

- **ST-MMSE Receiver**
  - The filter weights $\mathbf{g}_{MMSE}$ ($1 \times M_R T$) are given by
    \[
    \mathbf{g}_{MMSE} = \arg \min_{\mathbf{g}} \mathcal{E}\{[\mathbf{g} \mathbf{y}[k] - s_0[k - L + \Delta_D]]^2\}
    \]
    \[
    = E_{s,0} \mathbf{1}_{\Delta_D,T+L-1} \overline{\mathbf{H}_0^H} \mathbf{R}_YY^{-1},
    \]
  - The temporal taps primarily mitigate ISI while the spatial taps (antennas) mitigate CCI.
Performance of ST-MMSE

ST-MMSE receiver for one user and single interferer with one Tx antenna each

Base Station has two receivers
CCI Cancellation - Frequency Selective Channels

- **ST-MMSE-ML Receiver**
  - Two stage receiver - ST-MMSE in first stage and MLSE in second
  - ST-MMSE cancels CCI alone, with \( M_R - N \) spatial diversity
  - MSLE handles ISI and captures \( L_{eff} \) order path diversity
  - Compromise design.

ST-MMSE-ML receiver, \( Y \) — Rx signal, \( w_{opt} \) — optimal filter
### Trade-offs - Frequency Selective CCI Mitigation Rx

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<th>Channel Knowledge</th>
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<tr>
<td>ST-MLSE</td>
<td>$M_R L_{eff}$</td>
<td>Nil</td>
<td>$H_0, H_i$</td>
</tr>
<tr>
<td>ST-MMSE</td>
<td>$(M_R - N)L_{eff}$</td>
<td>Medium</td>
<td>$H_0, R_{yy}$</td>
</tr>
<tr>
<td>ST-MMSE-ML</td>
<td>$\approx M_R L_{eff} - N$</td>
<td>Low</td>
<td>$H_0, R_{yy}$</td>
</tr>
</tbody>
</table>
CCI Mitigation for MIMO with Diversity Coding

- The $M_T$ antennas on each co-channel source are treated as $M_T$ independent interferers
- At the receiver we need $M_R = M_T N + M$ antennas
  - $M_T N$ antennas for CCI removal
  - $M$ antennas for the desired diversity of the user signal
- The requirement can be reduced if the co-channel users employ space-time diversity.
Alamouti Coded Signal and Interference

- System is from $M_T = 2$ (desired signal, CCI) and $M_R = 2$
- The signal model is given by

$$y = \sqrt{\frac{E_{s,0}}{2}} H_{0,\text{eff}} s_0 + \sqrt{\frac{E_{s,0}}{2}} H_{1,\text{eff}} s_1 + n$$

- $y$ and $n$ are $4 \times 1$ received signal and noise vectors, (time and space)
- $H_{i,\text{eff}} (i = 1, 2)$ - $4 \times 2$ orthogonal matrices; $H_{i,\text{eff}}^H H_{i,\text{eff}} = \|H_{i,\text{eff}}\|_F^2 I_2$
- $s_0$ and $s_1$ are the $2 \times 1$ signal and interference vectors
Alamouti Coded Signal and Interference - (ctd)

- The MMSE CCI cancelling receiver is

\[ G = \arg \min_g \| G y - s_0 \|_F^2 \]

- The MMSE solution is

\[ G = \sqrt{\frac{E_{s,0}}{2}} H_{0,\text{eff}}^H R_{yy}^{-1} \]

- The receiver cancels the interference with one antenna leaving the other antenna to support Alamouti coded reception.

- At high SIR, it yields fourth order diversity while at low to medium SIR it provides second order diversity.

- The number of Rx antennas required per interferer is the effective spatial rate \( r_s \).
CCI Mitigation Forward Link (MISO)

- The base station tries to reduce/eliminate CCI at the interferer while maximizing signal power at the desired user.

- **Transmit-MRC or Matched Beamforming**
  - Reference base-station has knowledge of the channel $\mathbf{h}_0$ to its own user.
  - The beamforming weight vector and SINR to the reference user become
    \[
    \mathbf{w}_{MRC,0} = \mathbf{h}_0^H / \sqrt{\| \mathbf{h}_0 \|_F^2}; \quad SINR_0 = \frac{E_{s,0} \| \mathbf{h}_0 \|_F^2}{\sum_{i=1}^{N} E_{s,i} \| \mathbf{w}_{MRC,i} \mathbf{h}_i \|_F^2 + N_0}
    \]
  - With an increasing number of antennas ($M_T$), the beampattern becomes more spatially selective with an improvement in SINR.
CCI Mitigation Forward Link (MISO) - (ctd)

- Transmit ZF or Nulling Beamformer
  - Places nulls in the direction of interfered users so no CCI delivered.
  - With $M_T \geq N + 1$ and $\mathbf{H}$ full rank, the pre-filtering vector is

$$\mathbf{w}_{ZF,0} = \frac{\mathbf{h}_0^{(\dagger)}}{\sqrt{\|\mathbf{h}_0^{(\dagger)}\|_F^2}}$$

$\mathbf{h}_0^{(\dagger)}$ is the first column of $\mathbf{H}^{\dagger}$
  - The diversity offered is $M_T - N$
CCI Mitigation Forward Link (MISO) - (ctd)

- **Max. SINR Beamforming with Co-ordination**
  - Trades signal power delivered to desired user for interference generated to others.
  - If power co-ordination possible, also adjusts power to minimize mutual interference.
  - The SINR at the reference user becomes
    \[
    \text{SINR}_0 = \frac{E_{s,0} \| \mathbf{w}_0 \mathbf{h}_0 \|_F^2}{\sum_{i=1}^{N} E_{s,i} \| \mathbf{w}_i \mathbf{h}_i \|_F^2 + N_0}
    \]
  - A centralized processor searches for optimal \( \mathbf{w}_i, E_{s,i} \) so that the the SINRs are equal or exceed a target SINR subject to a power constraint
  - The diversity offered is \( M_T - N \)
SS Modulation

- Perfect auto-correlation of the spreading codes $\Rightarrow$ Ignore inter-chip interference.
- Model Multiple Access Interference (MAI) as white noise.
- Final model is temporally white but spatially colored noise.
- As there are a large number of co-channel users, we assume that the interference field is quasi-isotropic.
ST-RAKE

- Assume zero ICI, and MAI and CCI are temporally white additive noise

\[ g_{MMSE} = E_{z,0} h_0^H R_{yy}^{-1} \]

- \( R_{yy} \) is the spatial covariance matrix of signal, interference and noise at the Rx, \( h_0 \) is the channel for the desired signal

- Assuming two paths with one chip delay given as \( h_0[i], (i = 1, 2) \) and perfect autocorrelation, the output of the ST-RAKE is

\[ z = g[0]y[0] + g[1]y[1] \]

with \( y[j], (M_R \times 1) \) the vector outputs at lag \( j \) and \( g[j], (1 \times M_R) \) the weight vectors at lag \( j \)

\[ g[j] = E_{s,0} h_0[i]^H R_{yy}^{-1}, j = 1, 2 \]

- Offers array gain of \( M_R \) and diversity gain of \( 2M_R \) with 2 paths.
ST Pre-RAKE

- Transmit analog of ST-RAKE
- Pre-RAKE output at the transmit antennas ($\tilde{s}[i]$) after spreading is
  \[ \tilde{s}[i] = s(c[i]h_0[0]^H + c[i + 1]h_0[1]^H) \]
- Functions as a matched beamformer.
- Transmit with beamforming and delay pre-correction/phasing to ensure coherent combining at the receiver.
- Offers array gain of $M_T$ and diversity gain of $2M_T$ with delay pre-correction and co-phasing.
Interference Diversity and Multiple Antennas

- Uses selection to choose signal with least interference.
- Assuming a frequency flat Rayleigh channel for signal and interference

\[ y = \sqrt{E_{s,0}h_0s_s} + \sqrt{E_{s,1}h_1s_1} + n \]

- We have \( M_R \) Rx antennas and assume the signal is constant and the interference varies.
- The antenna with the maximum SIR, minimum interference power is selected.
- From the results, there is no change in slope