STRUCTURE AND STRATEGY:

THE TWO FACES OF AGENDA POWER*

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In the last several years multidimensional voting models have become subtle and complex instruments for explicating social choice by majority rule. What has been learned from them is that little will be known about an institution governed by majority rule if the focus is exclusively upon the majority relationship between alternative outcomes built up from individual preferences over those outcomes. Given even a rather modest diversity in individual preferences, the dominance relation induced by majority preferences is cyclic. Represented as a noncooperative (cooperative, resp.) game, consequently, majority voting has no Condorcet winner (an empty core, resp.). In this paper we present the view that the equilibria of multidimensional voting models -- their existence and their properties -- depend upon institutional structure and the behavior such arrangements induce, advantage, or prescribe. In short, equilibrium depends upon structure and strategy.\(^1\)

One consequence of the cyclicity of the majority dominance relation, immediately recognized by theorists, is the importance of the agenda. If there is no Condorcet winner, the agenda is decisive. Under fairly plausible conditions one can associate a unique outcome with each order of the agenda and establish that this outcome varies with the order of voting. Perhaps the most important result obtained from this fact is the theorem of McKelvey (1976). In the context of a multidimensional spatial model he proved that, for any two points in the space, there exists a finite path of majority dominance between them. If a distinguished individual
-- the setter, convenor, proposer, chairman -- has complete and exclusive power to choose and order elements of the agenda, and is in no way constrained in his use of this power, then, as McKelvey noted, the consequence for majority rule in the context of dominance cyclicity is clear. The agenda-setter may extract all of the advantage from this situation by choosing the appropriate agenda to yield his own ideal point.

The McKelvey agenda-setter stands in stark contrast to the institution-free world of pure majority rule (PMR). Under PMR there is no institutional location for agenda-setting; indeed, the process by which alternatives are generated is never specified. In whatever manner alternatives arise, if \( x \) is a prevailing outcome, it is replaced by a proposal, \( y \), if \( y \) is preferred to \( x \) by a majority and is retained otherwise. With the generic cyclicity of the majority dominance relation, PMR is a process that may wander anywhere.\(^2\)

We have, then, two extreme views on majority rule. Without any agenda control, PMR processes may wander anywhere. With complete and exclusive agenda control, by contrast, majority rule systems are extremely predictable and well-behaved, terminating at the ideal point of the agenda-setter.

Neither of these views is compatible with the description or the operation of real majority-rule institutions. Such institutions with which we are familiar neither lack endogenous agenda-control mechanisms altogether nor centralize it completely in the hands of a single agent. Rather, complicated institutions are typically in place that have the effect of decentralizing agenda control and constraining its exercise in
various ways. In this paper we inquire about the effects of these institutional arrangements:

1. Do decentralized, constrained, agenda control mechanisms yield more definitive results than PMR?

2. Do they produce outcomes that differ from that of McKelvey's model of the central agenda-setter?

There are several underlying points about 'institutions which motivate these questions. First, a casual look at real institutions strongly supports the view that agenda-setting is neither the product of exogenous forces, as in PMR, nor the arbitrary and exclusive preserve of some distinguished agent, as in the McKelvey model. In fact, the range of agenda arrangements employed by different institutions is quite broad. Second, to treat agenda-setting as an organic, wholly integrated apparatus, as it has to date in the literature, is to conceive of this phenomenon in very narrow terms. Rather, agenda-setting may be seen as the conjunction of distinct mechanisms, and it is these mechanisms and the particular ways they are conjoined that need to be examined analytically. Yet no treatment of agenda-setting to date has allowed for it to be decomposed into more fundamental elements. Third, from the pervasiveness and genericity of majority-rule cycles established by previous theorems, it is apparent that the real game in a majority-rule institution does not involve voting, per se, but rather involves structuring choices (choosing an agenda). It is not surprising, then, that the structuring of choices is rarely left to exogenous forces or concentrated in the hands of a single agent. To understand how institutions arrange to structure choices, we must first understand how alternative arrangements operate. For this we require a more precise
and discriminating conceptualization of agenda-setting than is available in existing treatments.

The remainder of the paper is arranged as follows. In the first section we develop the theoretical context in which we model agenda-setting and link it to other theoretical studies. We then proceed to explore the various aspects of agenda power. We examine its decomposition into "adding alternatives," "deleting alternatives," and "ordering alternatives"; its distribution among agents (centralization vs. decentralization); and the strategic possibilities any such arrangement affords its agents. The third section presents a model of centralized agenda-setting in a strategically richer environment than that of McKelvey. Section 4 presents some results on decentralized agenda-setting. In Section 5, a decentralized model of agenda-setting is developed the precise features of which are drawn from some characteristics of the U.S. Congress. Finally, we review our results and discuss their implications for the general study of institutional equilibrium.

THEORETICAL CONTEXT

Preliminaries

Our treatment of agenda power is embedded in the context of multi-dimensional voting models of majority rule. We consider a committee or legislature consisting of \( n \) agents, \( N = \{1, 2, \ldots, n\} \), who must choose a single element from a compact policy space \( X \subseteq \mathbb{R}^m \). The preferences of each agent \( i \in N \) are represented on \( X \) by a type I ordinal utility function. That is, for each \( i \in N \) there is a point \( x^i \) called \( i \)'s ideal point with preferences between arbitrary points, \( y, z \in X \), derived from their respective Euclidean distances from \( x^i \), viz. \( U^i(y) > U^i(z) \) if and only if
\[ |y - x_i| < |z - x_i| \]. Here \( || \cdot || \) represents the standard Euclidean norm. Throughout we freely interchange \( U_i(y) > U_i(z) \) and \( y \succ_i z \), which should cause no confusion.

Decision making is orchestrated according to an agenda, \( V \subseteq X \). Specifically, we consider finite agendas -- \( V \) is a finite subset of \( X \) -- and shall use \( V \) to represent both the finite subset and some specific ordering of its elements. If there is the need we shall distinguish an agenda (subset) from an ordered agenda. The latter gives the voting order in which alternatives in \( V \) will be considered. We assume that once \( V \) is given, voting proceeds according to a majority rule, sequential-elimination procedure (also known as the amendment procedure). Thus, for the ordered agenda \( V = \{x_1, x_2, \ldots, x_k, x_{k+1}\} \) \( x_1 \) and \( x_2 \) are in the first division; the winner advances to the second division where it faces \( x_3 \); and so on, with the winner at the \( i^{th} \) division advancing to \( (i+1)^{st} \) division. The winner is the victor in the \( k^{th} \), or last, division. Finally, let us note that, unless otherwise specified, we assume that the agenda is set in advance of any voting and is known to all agents.

We assume that each agent regards a voting situation as an instance of a noncooperative game. The assumption of non-cooperation does not preclude bargaining, information exchanges, and other forms of pre-play communication. Rather, it reflects the inability of agents to make binding agreements. Owing to the complexity and multiplicity of 'contracting' contingencies, the absence of formal enforcement mechanisms (like a court of law which enforces contracts among economic agents), and the gross imperfection of self-enforcement, we regard our noncooperative treatment of legislative institutions to be appropriate.\(^3\)
One last qualification about this formulation needs to be noted. In the noncooperative game orchestrated by agenda \( V \) and any other rules that may be specified, we assume each agent is fully informed about this context. This does not necessarily require that each agent know the preferences of each other agent. We do require that each agent know the majority dominance relation among alternatives in \( X \), a somewhat weaker requirement. Moreover, we occasionally require that some specified agent(s) know the preferences of some other specified agent(s) (see sections 4 and 5 below).

A majority dominance relation, \( D \), is derived from agent preferences so that \( xDy \) if \( |\{i|x \succ_i y\}| > |\{i|y \succ_i x\}| \); \( yDx \) if the inequality is reversed. Here \( |A| \) is the cardinality of \( A \). If there is equality in the cardinality of the two sets of agents, then neither point dominates the other.\(^4\)

In general the points in \( X \) cycle; \( D \) is intransitive. Thus, there is no \( x \in X \) with \( xDy \neq y \in X \setminus \{x\} \). To characterize the dominance relation further, we define two correspondences (and their associated sets). The dominion of \( x \), \( x \in X \), is the set of points dominated by \( x \): \( D(x) = \{y|xDy\} \). The win set of \( x \), \( x \in X \), is the set of points which dominate \( x \): \( W(x) = \{y|yDx\} \).

For any finite agenda \( V \), the amendment procedure is a decision process that can be represented as a binary decision tree (Farquharson, 1969). We assume readers are familiar with these structures and shall not dwell on details here. In choosing elements of his strategy, an agent may think of his problem in terms of making binary choices at each node in the tree. We assume that each agent is tree-sophisticated (McKelvey and Niemi,
He associates with each node its "sophisticated equivalent" and chooses sincerely at each node between the two sophisticated equivalents implied by the binary choice.

For any finite agenda \( V \), tree-sophisticated voting by each agent supports a (set of) Nash equilibrium in the noncooperative voting game induced by agenda \( V \). If the agenda were \( V' \), a different noncooperative game would be induced with, possibly, a different Nash equilibrium outcome. Let \( \mathcal{U} \) be the family of finite agendas, with \( V \in \mathcal{U} \) a typical element, and let \( x_V^{**} \in V \) be the Nash equilibrium supported by tree-sophisticated voting when \( V \) is the agenda.\(^5\) If agenda-setting is centralized in agent \( i \) and unconstrained, then \((V, x_V^*)\) is an agenda-equilibrium if \( x_V^* \) is preferred by \( i \) to the Nash equilibrium associated with any other agenda in \( \mathcal{U} \). If agenda-setting is constrained by various rules, e.g. the status quo must be in the last division, and if \( \mathcal{U} \subset \mathcal{U} \) is the family of agendas consistent with the constraints, then the agenda equilibrium is \((V, x_V^*)\), \( V \in \mathcal{U} \), that is maximal for \( i \) compared to other agendas in \( \mathcal{U} \). Finally, and perhaps most interesting, if agenda power is decentralized -- for example, agent \( i \) picks the elements and agent \( j \) puts them in a voting order -- then a new game is defined among agenda-setters. In this case, an agenda equilibrium is one in which no agent with partial agenda power has any incentive to revise his use of that power.

This completes our discussion of theoretical preliminaries. Our assumptions may be summarized as follows:

A.1 policy space \( X \subset \mathbb{R}^m \)

A.2 type I preferences

A.3 agenda \( V \in \mathcal{U} \) and the amendment procedure
A.4 noncooperative voting game over binary decision
tree defined by \( V \)

A.5 information: all agents know \( V \) and the majority
dominance relation at each division

A.6 majority dominance relation, \( D \), is cyclic;
hence \( W(x) \neq \phi \neq x \in X^6 \)

A.7 tree-sophisticated voting

A.8 agenda equilibrium.

Theoretical Forerunners

The story of disequilibrium in multidimensional voting models is, by
now, well known. Its high points have recently been summarized by Riker
(1980). Put briefly, if a distribution of ideal points in \( X \) (associated
with convex agent preferences) does not possess a total median,\(^7\) then there
is no majority rule equilibrium (MRE). Plott (1967) and, under slightly
different or more general conditions, Sloss (1973), McKelvey and Wendell
(1976), and Cohen and Matthews (1980) have provided the conditions
on preference profiles associated with the existence of MREs. In each of
these treatments, in one form or another, a point is a MRE if and only if
agent ideal points are distributed about it in a radially symmetric
fashion. Since this condition is highly implausible in most empirical
circumstances and, in any event, since the existence of equilibrium
according to this condition is not at all robust to slight perturbations,
the MRE concept is of little utility in explaining or predicting committee
behavior. For our purposes, the impact of these studies may be summarized
in the following disequilibrium theorem:
Disequilibrium Theorem: If there is no total median, then \( W(x) \neq \emptyset \ \forall x \in X \).

McKelvey (1976, 1979), Schofield (1978), and Cohen (1979) then went on to show precisely the pervasiveness of disequilibrium when a MRE fails to exist. Letting \( W(x) \) be defined as above, and defining its closure as \( R(x) \), now define

\[
R^i(x) = R(x), \\
R^i(x) = \{ y \mid y \in R(z), z \in R^{i-1}(x) \}
\]

and \( R^*(x) = \bigcup_{i=1}^{\infty} R^i(x) \).

\( R^*(x) \) consists of the set of points "reachable" from \( x \) via the majority dominance relation in a finite number of steps, where the elements of \( R^i(x) \) are those "reachable" in precisely \( i \) steps. The McKelvey-Schofield-Cohen result is

Top Cycle Set Theorem: If there is no total median, then \( R^*(x) = X \ \forall x \).

Thus, any point is reachable from any other in a finite number of steps according to the majority dominance relation. All points in the space are part of a single cycle. McKelvey (1976, Theorem 2) spelled out the implications of this result for agenda-setting:

McKelvey's Agenda Theorem: If there is no total median, then for any \( x, y \in X \), there exists an ordered agenda,

\( V = (\Theta_0, \Theta_1, \ldots, \Theta_k) \), with \( \Theta_0 = x \), \( \Theta_k = y \), and \( \Theta_i \in R(\Theta_{i-1}) \), \( i=1,\ldots,k \).
The McKelvey Agenda Theorem clarifies the strategic prospects of a centralized agenda-setter, even if he is constrained by the requirement that a specific alternative (viz., x) be included in the first division. But this is true only so long as actual behavior by all agents is accurately summarized by the majority dominance relation. Clearly, if agents vote sincerely at each division, then the result goes through, since this is precisely what the dominance relation describes. However, the dominance relation underestimates the strategic possibilities of agents, a point McKelvey noted in qualification of his theorem.

The McKelvey Agenda Theorem serves as a point of departure for our own work. In the next several sections we seek to discover, first, how a centralized agenda-setter is affected by constraints on his discretion and by strategic agent behavior. The latter has already been subjected to some scrutiny by Miller (1980), whose work we discuss below. Second, we begin to analyze decentralized agenda-setting as a game among agenda agents each possessing only partial agenda power. This more general framework, in which the McKelvey agenda-setting model is an important special case, is presented next.

ASPECTS OF AGENDA POWER

The agenda-setter of the McKelvey Theorem is unconstrained. And for this reason no student of real world institutions would recognize him. In most legislative institutions, agenda formation is constrained and decentralized. Agenda formation consists of structuring decision trees, i.e., adding, deleting, and ordering alternatives. Institutions constrain each of these activities by subjecting them to restrictions on order and
content. Institutions decentralize these activities by assigning partial control over them to different agents.

An agenda is a set $V \subseteq X$ accompanied by a prescribed order of voting (according to the amendment procedure). Any $V$ may be represented by a decision tree which depicts the possible paths the voting process can take. To illustrate, if $V = (A, B, x^0)$, where $A$ is an amended version of a bill, $B$, and $x^0$ is the status quo, then the decision tree is given in Figure 1.1. $A$ is voted against $B$ at the first division and the winner is pitted against the status quo at the second and final division. We first consider three classes of structural arrangements commonly employed by institutions -- order constraints, content constraints, and decentralization. We then consider the matter of strategy in centralized and decentralized settings.

**Structural Arrangements: Order Constraints**

Order constraints are institutional requirements on the possible ordered agendas that may be constructed from a collection of agenda elements. The most obvious, and in our judgment the one that most under-mines the direct application of the McKelvey Agenda Theorem to legislative institutions, is the requirement that the status quo be voted last. From our example above, the alternative agendas consisting of elements $(A, B, x^0)$ are $V = (A, B, x^0)$, $V' = (A, x^0, B)$, and $V'' = (B, x^0, A)$; all are displayed in Figure 1. With the above order constraint, only agenda $V$ is permitted. The consequence of this order constraint, in contrast to McKelvey, is that a final outcome must either be an element of $W(x^0)$ or $x^0$, itself.

The order constraint of voting the status quo last is a fairly common constraint; most legislative and committee procedures prescribe it. The form by which it is implemented may vary, but it usually involves the
(1) $V$

\[
A \text{ vs. } B
\]

\[
A \quad x^0 \quad B \quad x^0 \quad \text{WINNER vs. } x^0
\]

(2) $V'$

\[
A \text{ vs. } x^0
\]

\[
A \quad x^0 \quad B \quad x^0 \quad \text{WINNER vs. } B
\]

(3) $V''$

\[
B \text{ vs. } x^0
\]

\[
B \quad x^0 \quad A \quad x^0 \quad \text{WINNER vs. } A
\]

FIGURE 1
requirement that there be a "vote on final passage" at the ultimate division or, alternatively, that a "vote to table," a "vote to recommit," a "vote to strike the enacting clause," or a "vote to adjourn," be allowed at the final division.

Other constraints on order are possible. There are occasions in which the voting position of a privileged motion is stipulated by the rules. In the House of Representatives, for example, voting order is usually some variant of the following. The unordered agenda consists of the status quo \((x^0)\), a bill \((B)\), an amendment (so that, if passed, the amended bill is labeled \(A\)), an amendment to the amendment (so that, if passed, the amended amendment is labeled \(A'\)), a substitute bill \((S)\), and an amendment to the substitute (so that, if passed, the amended substitute is labeled \(S'\)). At the first division \(S\) is pitted against \(S'\). At the second division \(A\) is pitted against \(A'\) and the winner, at the third division, is pitted against \(B\). The winner at this last division \((A', A, \text{ or } B)\) is then paired against the winner at the first division \((S \text{ or } S')\), and the winner in this penultimate vote is then paired against \(x^0\).9

Structural Arrangements: Content Constraints

Content constraints, in contrast to order constraints, proscribe or permit motions because of what they say, not because of when they are offered (although example iv. below blurs the distinction). Put differently, order constraints restrict agenda-setter discretion with regard to the 'ordering' aspect of agenda construction, whereas content constraints are restrictions on the 'adding' aspect of agenda-setting. Consider the following examples drawn from the two houses of the U.S. Congress:
i. **dilatory tactics**: "It is not in order to offer an amendment that is substantially the same as an amendment that already has been offered and disposed of unfavorably. However, a senator may offer part of a previously rejected amendment as a separate amendment..." (Bach, 1980a, p. 29).

ii. **Germaneness**: Clause 7 of House Rule XVI states that "no motion or proposition on a subject different from that under consideration shall be admitted under color of amendment."

iii. **legislation in appropriations bills**: "The rules of the House impose a flat prohibition on unauthorized appropriations except for public works already in progress. By contrast, the requirements of Senate Rule XVI are far less demanding. The Senate may consider an amendment making an unauthorized appropriation if the authorization has passed the Senate alone or if the appropriation is pursuant to an estimate submitted in accordance with law. But most importantly, an amendment proposing an unauthorized appropriation is in order if recommended by the Committee on Appropriations." (Bach, 1980a, p. 33).

iv. **Budget Act restrictions**: As reported by Bach (1980b):

"Under the terms of the Budget Act, it is not in order for the House to consider:

1. a bill affecting budget or spending authority, revenues, or the public debt ceiling for a fiscal year before adoption of the first concurrent budget resolution for that fiscal year (section 303(s));
2. a bill that provides new spending authority that is not subject to appropriation acts (section 401(a));

3. a bill creating new entitlement authority that becomes effective before the beginning of the next fiscal year (section 401(b));

4. a bill providing new or renewed authorizations that is reported after May 15th preceding the beginning of the fiscal year in which the authorizations become effective (section 402(a)); or

5. a bill affecting revenues or budget or spending authority that would violate a second or subsequent concurrent budget resolution that has been adopted (section 311(a))."

Any motion, bill, resolution, or amendment that violates an order constraint or a content constraint is subject to a point of order. That is, any single agent may effectively veto (read: delete) an agenda element violating one of these constraints.

Structural Arrangements: Decentralization

In the Mckelvey Agenda Theorem, as well as in most other discussions of agenda power, all aspects of agenda-setting are centralized. A single agent -- the convenor, setter, chair -- has complete and absolute authority to add, delete, and order alternatives. As we have already seen, however, in our discussion of constraints, if an agenda element is in violation of a rule, then any agent may delete it by raising a point of order. Indeed, one can imagine various ways by which agenda power may be disaggregated and decentralized.

Instead of a single agent with all agenda power, suppose an agenda committee were charged with these powers. While hardly decentralized -- all powers are still exercised by a single agenda entity -- this arrangement replaces the optimization problem of an agenda-setter with a game of
strategy among members of an agenda committee. In some respects this game suffers from the ambiguities of the larger voting game induced by the cyclic majority dominance relation. However, the choice space of the agenda committee game is the set of feasible agendas (feasibility determined by the constraints, if any, in place), so that it differs from the larger game. Under the hypothesis of sophisticated voting by all agents, as noted earlier, each \( v \in V \) is mapped to a unique voting tree which, in turn, is mapped to a unique \( x_V^* \) (see note 5). In effect, then, the agenda committee game consists of picking an element from \( \{ x_V^*/v \in V \} \). Much will depend on how this process is structured, including how agenda-setting within the committee takes place. Although we do not analyze this form of decentralization below, we have briefly described it here as an alternative to the centralized agenda agent since many legislatures appear to utilize some form of agenda committee.

A more genuinely decentralized agenda-setting mechanism is one that parcells out the 'adding', 'deleting', and 'ordering' aspects of agenda-setting to different agents. Here, again, an agenda-setting game is defined, but one that is distinctly less symmetric than the agenda committee game. Rather, different agenda agents have distinctly different strategy sets. For example, parliamentary procedure commonly requires that a particular agent be designated as presiding officer. Any agent may move a proposal ('add' an element to the agenda) but, to do so he must obtain recognition from the presiding officer. Hence, while any agent may 'add', the presiding officer has the power to 'order' through the judicious use of his recognition power.\(^{10}\)
Below we analyze a variant of a decentralized agenda game which, in some respects, captures agenda-setting in the House of Representatives. Given a status quo, \( x^0 \), which is an element of the final division (order constraint), agent \( i \) (perhaps a legislative committee) picks a bill, \( B \), that alters the status quo and agent \( j \) (perhaps the Rules Committee), subject to the additional order constraint that \( B \) appear in the penultimate division, picks an amendment, \( A \) (or a sequence of amendments, \( A_1, \ldots, A_k \)). As constructed, the agenda is then voted on with all agents behaving in a sophisticated fashion.

**Strategy**

In the McKelvey Agenda Theorem there is only one strategic agent -- the agenda-setter. All other agents vote sincerely at each division in the voting tree, so that the majority dominance relation describes the behavior of majorities (not merely their preferences). Yet once an agenda (and hence a voting tree) is in place, an agent may be better off by deviating from his sincere strategy. Specifically, he may behave in a tree-sophisticated fashion, associating with each division its sophisticated equivalent and then voting sincerely between these equivalents at each division. In the context of centralized agenda-setting and structural constraints, our first task below is to explore the effect of fully strategic behavior on the consequences of the McKelvey Agenda Theorem.

In the context of decentralized agenda-setting, strategy appears not only in the sophisticated behavior of all agents in the voting game. As well, there is a game of strategy among agenda-setters, a game with a voting tree as its outcome. Our second task is to examine fully strategic behavior in a model of decentralized agenda-setting.
CENTRALIZED AGENDA-SETTING AND SOPHISTICATED BEHAVIOR

In order to study, in a sophisticated context, the effects of decentralization and institutional constraints, we first need to know what happens in their absence. Our first model of agenda-setting, then, is that of McKelvey except that all agents are sophisticated. Since we are interested in contrasting our results with those of McKelvey we address the same question he posed: Commencing at a point \( y \in X \), what outcomes can be produced with an appropriately chosen agenda? That is, where may majority rule "wander" when all agents are sophisticated?

No results have been published to date concerning the power of an agenda-setter in the context of sophisticated voting and a multidimensional alternative space, although there are some conjectures by Miller (1980). The thrust of Miller's analysis is that, as McKelvey (1976) noted in qualification to his theorem, sophisticated agents vastly limit the discretion of a centralized agenda-setter. He speculates (1980, footnote 8) that the agenda-setter will be restricted to a relatively small set of outcomes centrally located among the agent ideal points.\textsuperscript{11} We now demonstrate that the centralized agent-setter, somewhat surprisingly, is not so restricted as McKelvey and Miller anticipated.

**Principal Theorem**

We begin by defining the covering relation.

**Definition:** For \( x, y \in X \), \( y \) covers \( x \) if and only if
\[
D(x) \subseteq D(y). \quad x \in X \text{ is uncovered if and only if there is no } y \in X \text{ that covers it.}
\]

That is, \( y \) covers \( x \) if and only if every point dominated by \( x \) is also dominated by \( y \). Equivalently, \( y \) covers \( x \) if every point that dominates \( y \) also
dominates \( x, \text{viz. } W(y) \subseteq W(x) \). An immediate implication of this relation is that if \( y \) covers \( x \) there can be no \( z \) with \( x \) beating \( z \) and \( z \) beating \( y \), i.e., \( z \) with \( x \in W(z) \) and \( z \in W(y) \), since if \( x \) beats \( z \) then so, too, must \( y \).

We next generalize the notion of dominion to multi-step dominion.

\textit{Definition: } \( D^2(x) = \{ y/ y \in D(z) \cap z \in D(x) \} \). More generally,

\[
D^i(x) = \{ y/ y \in D(z) \cap z \in D^{i-1}(x) \}
\]

The \( D^2(x) \) set consists of all points dominated by any point dominated by \( x \).

Our first result is a slight variation on a theorem of Miller:

\textit{Proposition (Miller): } If \( y \) covers \( x \), then there is no agenda containing both \( x \) and \( y \) that yields \( x \) as a sophisticated outcome.

\textit{Proof: } Consider three cases: (i) \( x \) and \( y \) adjacent in voting order; (ii) \( x \) precedes \( y \) (not adjacent); and (iii) \( y \) precedes \( x \) (not adjacent).

\textit{case (i): } Suppose, to the contrary, that \( x \) is the sophisticated outcome. Then it defeats every sophisticated equivalent below it. But, since \( y \) covers \( x \), so does \( y \). Therefore at the division with \( x \) and \( y \), the two sophisticated equivalents are, in fact, \( x \) and \( y \). But \( y \in W(x) \). Therefore it beats \( x \) at this division, so \( x \) cannot be the sophisticated outcome.

\textit{case (ii): } If \( x \) is sophisticated outcome, then it was at the division at which it first appears and at all successive divisions. Specifically, it wins at the division containing \( y \). By the logic of case (i), with \( x \) and \( y \) at same division, this leads to a contradiction.
case (iii): If $x$ is sophisticated outcome, then it beats each sophisticated equivalent below it. Between the division at which it appears and the division at which $y$ appears, it must defeat each sophisticated equivalent it encounters. At the division where $y$ appears, $x$, by hypothesis, is the sophisticated equivalent of one branch, and, since $y$ beats anything that $x$ beats, $y$ must be the sophisticated equivalent of the other branch. Since $y \in W(x)$, $x$ cannot be the sophisticated outcome.

Q.E.D.

This proposition reveals the power of the covering relation: if $y$ covers $x$, then $\not\exists z$ with $x \in W(z)$ and $z \in W(y)$. Suppose, to the contrary, that such a $z$ existed (and hence $y$ did not cover $x$). Then even if $y \in W(x)$ there exists an agenda of $x, y \& z$ with $x$ as the sophisticated outcome, namely:

![Diagram]

If $y$ covers $x$, then no such $z$ exists and it may be verified that no three-element agenda containing $x$ and $y$ will yield $x$ as the sophisticated outcome. Miller's proposition demonstrates that this conclusion holds for an agenda of any length.

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With this proposition and two lemmas to be established next, we may obtain an answer to McKelvey's question in the sophisticated context. The following lemma generalizes an earlier result of Miller to the multi-dimensional setting.

Lemma 1: If x is uncovered, then D(x) U D^2(x) = X.

Proof: We must show that any y \in X is either dominated by x or is dominated "in two steps."

(i) x \in W(y). Then xDy and the consequence holds trivially.

(ii) x \notin W(y). Then either y \in W(x) and yDx or x and y tie.

In either case we must show there is a z \in X with z \in W(y) and x \in W(z), i.e., y \in D^2(x). Suppose the contrary.

That is, for every z with x \in W(z), it is also true that y \in W(z). This implies that if z \in D(x) then z \in D(y), i.e., D(x) \subseteq D(y). But this implies that y covers x, contrary to the premise.

Q.E.D.

This lemma establishes that from any initial point, an uncovered point is "reachable" via the dominance relation in at most two steps. Thus, if x is uncovered and y is any other point, then there is an agenda through y that yields x as the sophisticated outcome. If x \in W(y), then V = (x,y) trivially produces x as the outcome. If, on the other hand, x \notin W(y), then x uncovered implies, by Lemma 1, that there is a z \in W(y) with x \in W(z). Construct the agenda V = (x,y,z) with voting tree:
At node 3 z wins since $z \in W(y)$. At node 2 x wins since $x \in W(z)$. Therefore, the vote at node 1 is between the two sophisticated equivalents, x and z, and x wins since $x \in W(z)$. This lemma establishes the importance of the uncovered set of the entire space, UC(X). That is, for any $x \in UC(X)$, there is some agenda that allows x to be reached from any $y \in X$ by sophisticated voting. What we have yet to characterize are the set of points that may be reached from a given $y \in X$.

**Lemma 2**: Let x be any point not covered by y. Then there is an agenda beginning with y that yields x as the sophisticated outcome.

**Proof**: Pick a $z \in W(y)$ with $x \in W(z)$, which exists since y does not cover x. Construct the agenda $V = (x,y,z)$ and, as in the voting tree above, x is the sophisticated outcome.

Q.E.D.

Lemma 2 establishes the converse to Miller's proposition above.
The content of Lemmas 1 and 2, which provides the answer to McKelvey's question in the context with sophisticated agents, is summarized in our principal theorem.

**Theorem I:** In the class of agendas with \( y \in X \) in the first division, there exist agendas with any point not covered by \( y \) as the sophisticated outcome.

Thus, in contrast to majority rule under sincere voting which may wander anywhere, majority rule with sophisticated voting may only wander in the set of points not covered by \( y \). The implication of this theorem for the McKelvey agenda-setter confronting sophisticated agents is that he may construct an agenda which, in at most two steps, yields as the sophisticated outcome any point uncovered by \( y \). Specifically,

**Corollary I.1:** If the ideal point of the agenda setter is not covered by \( y \), then there is an agenda starting at \( y \) with it as the sophisticated outcome.

These are rather remarkable results in two respects. First, contrary to the anticipation of Miller (1980), sophisticated agent behavior may not constrain an optimizing agenda-setter to a small, centrally-located subset. This will depend, of course, on what constitutes the set of points not covered by \( y \) (which we examine below). Second, if the agenda-setter's ideal point, \( x^i \), is not covered by \( y \), then there is a three-element agenda, \( V = (x^i, y, z) \), with \( y \) in the first division, that produces \( x^i \) as the sophisticated outcome. Here \( z \in \mathcal{N}(y) \cap D(x^i) \neq \emptyset \), the existence of which has been established by Corollary I.1. Thus, in these circumstances, the task facing the agenda-setter simply entails finding such a point \( z \); this agenda is bound to be compatible with institutional rules that limit agenda size.\(^{12}\)

Stated formally, we have
Corollary 1.2 (The Two-Step Principle): Starting at \( y \), for any point that is the sophisticated outcome of a \( k \)-step agenda, there is a two-step agenda with the same sophisticated outcome.

This set of results underscores the importance of the covering relation. First, from the Miller Proposition, any point in an agenda covered by some other point in that agenda cannot be the sophisticated outcome. Second, starting at a predetermined point, there is a two-step agenda yielding any point not covered by the predetermined point as the sophisticated outcome.

Characterizing Uncovered Sets

Because the covering relation plays such an important role in sophisticated voting, we need to characterize the following correspondences:

Definition: \( C(y) = \{ x/y \text{ covers } x \} \)

\( UC(y) = \{ x/y \text{ does not cover } x \} \)

While we offer the following characterization, little is known about these correspondences. For example, for a given \( y \in X \), what is the relationship of \( UC(y) \) to the distribution of ideal points and hence the Pareto surface? More specifically, for a given \( y \in X \) and a distinguished agent \( i \in N \), what is the relationship of \( UC(y) \) to \( x^i \)? To learn more about agenda-setting and to provide a more detailed answer to the McKelvey question, further research would be fruitful here.

An analytical characterization for the points not covered by \( y \) is given by the following theorem:
Theorem II (Characterization Theorem for UC(y)): A point $x \in X$ is uncovered by $y \in X$ if and only if

(i) $x \notin \mathcal{W}(y)$

or (ii) $x \in \bigcup_{z \in \mathcal{W}(y)} \mathcal{W}(z) = W^2(y)$.

Proof. Sufficiency: If (i) then $x \not\in D(y)$. Hence $y \not\in D(x)$ and, since $y \not\in D(y)$, $D(x)$ is not contained in $D(y)$. Thus, $x \notin UC(y)$.

If (ii), then $x \in \mathcal{W}(z)$ for some $z \in \mathcal{W}(y)$. Then $z \not\in D(x) \cap W(y)$ which implies $D(x)$ not contained in $D(y)$. Thus $x \notin UC(y)$ and sufficiency is established.

Necessity: Suppose $x \in UC(y)$. Then either $x \not\in D(y)$, $y \not\in D(x)$, or the two tie. If the first, then (i) holds. If either the second or the third, then, from $x \in UC(y)$, $D(x) \not\subseteq D(y)$. Then

$\exists z \in D(x) \cap W(y)$. But $z \not\in D(x)$ implies $x \in \mathcal{W}(z)$. Hence $x \in \mathcal{W}(z)$ for a $z \in \mathcal{W}(y)$ and (ii) holds.

Q.E.D.

The following corollary is apparent:

Corollary II.1: A point $x \in X$ is covered by no $y \in X$ if and only if either

(i) it is a MRE,

or (ii) $x \in UC(X) = \bigcap \left[ \bigcup_{y \in X} \mathcal{W}(z) \right]_{z \in \mathcal{W}(y)}$

$= \bigcap_{y \in X} W^2(y)$.

Finally, we note one last obvious consequence of the characterization theorem.

Corollary II.2: $UC(X) \subset UC(y) \iff y \in X$. 

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The characterization theorem describes the fact that the uncovered set of a point \( y \in X \) consists of those points that dominate \( y \) directly or that dominate points that in turn dominate \( y \). The uncovered set for the entire alternative space \( X \) consists of those points that are not covered by any \( y \in X \) (Corollary II.1). Importantly, however, the set of points not covered by a specific \( y \in X \) contains \( UC(X) \), but typically contains other points as well (Corollary II.2).

Miller (1980) speculates about the set we have labeled \( UC(X) \), conjecturing that it is a relatively small subset of the convex hull of agent ideal points. Inasmuch as it must satisfy the rather stringent intersection property given in Corollary II.1, our intuition about \( UC(X) \) is compatible with Miller's conjecture. Indeed, this intuition is supported by the following generalization of a result of Miller (1980):

*Theorem III*: \( UC(X) \subseteq PO(X) \), where \( PO(X) \) is the convex hull of agent ideal points.

*Proof*: Suppose the contrary. Then \( y \in UC(X) \) but \( y \notin PO(X) \). By definition, \( \exists y' \in PO(X) \) with \( y' \succ_i y \forall i \in N \). Let \( z \in D(y) \).

Since \( y' \succ_i y \) for all \( i \) and \( y \succ_i z \) for a majority, then there is a majority for which \( y' \succ_i z \) (this follows from the transitivity of type I preferences). Since this is true for any \( z \in D(y) \), \( D(y) \subseteq D(y') \) and \( y' \) covers \( y \).

Hence \( y \notin UC(X) \), contrary to hypothesis. Q.E.D.

But it is clear that \( UC(X) \) is not the theoretically relevant set (if, indeed, it exists at all). It gives a very conservative estimate, as
Corollary II.2 above establishes, of the strategic possibilities of an agenda-setter, since for any given \( y \in X \), UC(\( X \)) will normally be a proper subset of the set of sophisticated outcomes for which a feasible agenda exists. Consequently, UC(\( X \)) is not the appropriate answer to McKelvey's question. Rather, the power of an agenda-setter is fully specified by the family \{UC(\( y \)): \( y \in X \); once \( y \) is given UC(\( y \)) describes the potential sophisticated outcomes.

Aside from Theorem II and its corollaries, we have not established precise properties for UC(\( y \)) for a given \( y \in X \). A few examples, however, will give the reader some sense of the possibilities. In Figure 2 we have \( N = \{1,2,3\} \) and \( y \in PO(X) \), where PO(\( X \)) is the triangle connecting the three ideal points. The three petal-shaped surfaces, labeled A, B, and C, comprise W(\( y \)). The larger, more outlandishly shaped, surface containing A, B, and C is UC(\( y \)).\(^{13}\) Thus, an agenda-setter who must include \( y \) in the first division of an agenda may nevertheless construct an agenda with any point in UC(\( y \)) as the sophisticated outcome.

Note in this example that UC(\( y \)) is 'large' in comparison to PO(\( X \)); it contains most of PO(\( X \)). Note, secondly, that it contains non-Pareto points. Third, if either of Mssrs. 1 and 2 is the agenda-setter, he can obtain his ideal point while Mssr. 3 cannot. Thus, neither 1 nor 2 is constrained by sophisticated agent behavior, but 3 is. Nevertheless, Mssr. 3's most-preferred point in UC(\( y \)), call it \( \hat{x}^3 \), lies on the contract locus between his and 2's ideal point. Consequently, an optimizing agenda-setter whether 1, 2, or 3, will secure a Pareto optimal point through the judicious selection of his optimal agenda. Finally, UC(\( y \)) is a proper subset of \( X \) so that, unlike PMR, the process cannot "wander anywhere."
Figure 3 is an analysis of the same situation except that $y \not\in \text{PO}(X)$. The shaded petals constitute $W(y)$ and the oddly shaped outline containing $W(y)$ is $\text{UC}(y)$. Again note that elements of $\text{PO}(X)$ are covered (though this time by a non-Pareto point) and that one prospective agenda-setter will be unable to obtain his ideal point.

These examples (and others we have constructed) strongly suggest that sophisticated agent behavior is not so binding a constraint as Miller believed and, for some prospective agenda-setters, it is not binding at all. However, it would appear that the more extreme an agenda-setter's ideal point is in comparison to the distribution of ideal points, the less likely it is that he can obtain it as the sophisticated outcome of some agenda beginning at $y$. On the other hand, in qualification, the more distant $y$ is from $\text{PO}(X)$, the more likely it is that any $i \in \mathbb{N}$ can obtain his ideal point if he optimally chooses the agenda. In sum, $\text{UC}(y) \subseteq X$ in contrast to PMR; and $x^i \in \text{UC}(y)$ for some $i$, but perhaps not for all $i$, in contrast to McKelvey's Agenda Theorem.

**Constraint: Voting y Last**

In the world of sophisticated agents, we have seen that if $y \in X$ must be an element of the first division, then any $x \in \text{UC}(y)$ is available to an agenda-setter. In many legislatures, however, we have not that $y$ is voted first, but rather that it is voted last, where $y = x^0$, the status quo ante. This proves to be a much more binding constraint than that of voting $y$ first.

**Theorem IV:** If $x^0 \in X$ is in the last division, then there is an agenda with $x \in X$ as a sophisticated outcome only if $x \in W(x^0)$.
Proof: Suppose \( x \notin W(x^0) \). Then either \( x \) is eliminated in an earlier division, or \( x \) survives to the last division. At the last division, sophisticated behavior and sincere behavior are the same so, in the vote between \( x \) and \( x^0 \), \( x \) loses. This establishes necessity. 

Q.E.D.

The difference in requiring \( y \) to be in the first division and in the last division is easily seen in Figures 2 and 3. Under the former constraint, the larger, oddly shaped area comprises the set of points that are the sophisticated outcomes of appropriately chosen agendas. Under the latter constraint, on the other hand, only the subset of points in the petal-shaped area are possible sophisticated outcomes.\(^{16}\)

Constraint: General Order Requirements

We seek here to generalize the notion of an order constraint presented in the previous subsection. For example, suppose there is a fixed alternative, or bill, \( b \), that is required to appear in the penultimate division with the winner there pitted against \( x^0 \) in the final division. Under these two order constraints, what is the set of potential sophisticated outcomes?

If \( b \notin W(x^0) \) then, since it will never be the sophisticated outcome (Theorem IV), it is irrelevant. The set of possible sophisticated outcomes is \( W(x^0) \) or \( x^0 \), itself. If, on the other hand, \( b \in W(x^0) \), then in order for our \( x \in X \) to be the sophisticated outcome it must be capable of beating \( b \) at the first division and must beat \( x^0 \) at the final division. That is, the set of sophisticated outcomes is \( W(b) \cap W(x^0) \).
We now consider a general order constraint in which there is a fixed, partial, ordered agenda, \((b_1, b_2, \ldots, b_k, x^0)\) and determine the set of motions, \(A = \{a/\alpha\in X\}\), that could be the sophisticated outcomes of the agenda \((a, b_1, b_2, \ldots, b_k, x^0)\). Since any alternative that survives to the final division must defeat \(x^0\), we know that \(\alpha \in W(x^0)\) is a necessary condition. Suppose that \(b_k\), the penultimate element, cannot beat \(x^0\), i.e., \(b_k \notin W(x^0)\). Then, if \(a\) reaches the penultimate division and \(a \in W(x^0)\), then sophisticated agents will vote a over \(b_k\) (whether \(a \in W(b_k)\) or not), since a vote for the latter is in effect a vote for \(x^0\) and a majority prefers \(a\) to \(x^0\). Thus, if \(b_k \notin W(x^0)\) it is an innocuous motion in no way constraining the set of possible sophisticated outcomes. If \(b_{k-1}\), on the other hand, beats \(x^0\), then at the first pre-penultimate division, a vote between \(a\) and \(b_{k-1}\) is just that in the eyes of sophisticated agents so that \(a \in W(b_{k-1})\) is a necessary condition for \(a\) to be the sophisticated outcome. We now have that \(a\) must be an element of \(W(b_{k-1}) \cap W(x^0)\). If \(b_{k-1} \notin W(x^0)\) then, at the division where \(a\) faces \(b_{k-1}\) sophisticated agents will vote for \(a\), even if \(a \notin W(b_{k-1})\), so \(b_{k-1}\), like \(b_k\), is an innocuous motion.

Our theorem says that the set of sophisticated outcomes, subject to the partial agenda \((b_1, \ldots, b_k, x^0)\), consists of those elements preferred by a majority to each noninnocuous motion in the partial agenda. A noninnocuous motion is one that can defeat all those motions following it in the voting order that, in turn, are capable of defeating every motion following them. We label this ordering of noninnocuous motions \((z_1, \ldots, z_T)\), called the sophisticated agenda, and construct it as follows. Since \(x^0\) trivially satisfies the definition of a noninnocuous motion (it defeats every motion following it), \(z_T = x^0\). If \(b_k \in W(z_T)\), then \(b_k = z_{T-1}\);
if not, then $b_k$ is deleted. If $b_k$ is not $z_{i-1}$, then $b_{k-1}$ is if $b_{k-1} \in W(z_i)$; if $b_k = z_{i-1}$, then $b_{k-1} = z_{i-2}$ if $b_{k-1} \in W(z_{i-1}) \cap W(z_i)$.

And so on.

**Theorem V (Intersecting Win Sets Theorem):** Given the partial, ordered agenda $(b_1, b_2, \ldots, b_k, x^0)$, and the associated ordered agenda of noninnocuous motions $(z_1, \ldots, z_T)$, $a$ is the sophisticated outcome of the agenda $(a, b_1, \ldots, b_k, x^0)$ if and only if

$$a \in \bigcap_{i=1}^{T} W(z_i).$$

This theorem provides the set of elements that can be sophisticated outcomes of an agenda subject to a general order constraint, viz.

$$A = \bigcap_{i=1}^{T} W(z_i).$$

As an example, consider $(b_1, b_2, b_3, x^0)$ with $b_3 \not\in W(x^0)$, $b_2 \in W(x^0)$ but not in $W(b_3)$, and $b_1 \in W(x^0) \cap W(b_2)$ but not in $W(b_3)$. That is, we have the dominance cycle $b_3 \triangleright b_1 \triangleright b_2 \triangleright x^0 \triangleright b_3$. According to our algorithm for constructing the sophisticated agenda, $z_T = x^0$; since $b_3$ is dominated by $x^0$ it is an innocuous motion and is deleted; but since $b_2$ is not, $z_{T-1} = b_2$; since $b_1$ dominates both $b_2$ and $x^0$, $z_{T-2} = b_1$. Our sophisticated agenda is $(z_1, z_2, z_3) = (b_1, b_2, x^0)$ and $a$ is the sophisticated outcome of $(a, b_1, b_2, b_3, x^0)$ if it is an element of the intersecting win sets given in the theorem:

$$a \in W(b_1) \cap W(b_2) \cap W(x^0).$$

With $a$ as given, the agenda $V = (a, b_1, b_2, b_3, x_0)$ is represented by the following voting tree:
Working up the voting tree, attaching sophisticated equivalents to each node, we see that a wins. Since a never faces b₁ as a sophisticated equivalent, a need not be an element of W(b₃).

If agent i is a centralized agenda-setter with ideal point xᵢ, then we have from Theorem V:

**Corollary V.1:** For generalized order constraint (b₁, ..., bₖ, x⁰), and sophisticated agenda (z₁, ..., zₚ), there is an agenda with xᵢ the sophisticated outcome if and only if

\[ xᵢ \in \bigcap_{j=1}^{T} W(z_j). \]

It is, of course, entirely possible that xᵢ is not contained in the intersection of relevant win sets; this intersection may define a very small set (in fact, it will be empty if, and only if, any point on the partial agenda is a MRE -- a circumstance we have excluded by assumption). To see what Theorem V implies when xᵢ is not an element of the relevant intersection, define ₗᵢ as the maximal element of A = \( \bigcap_{i=1}^{T} W(z_i) \). (Since A is open, ₗᵢ is taken to be the supremum on A). We have
Corollary V.2: For generalized order constraint
\((b_1, b_2, \ldots, b_k, x^0)\), and sophisticated agenda \((z_1, \ldots, z_T)\),
agent \(i\)'s induced optimum, \(\mathring{x}_i^i \in \bigcap_{i=1}^T W(z_i)\) is the
sophisticated outcome of the agenda \((\mathring{x}_i^i, b_1, \ldots, b_k, x^0)\).

Finally, we observe that Theorem V is in fact more general, allowing
us to characterize the set of sophisticated outcomes for a broader class
of order constraints. Fix the partial agenda \((b_1, \ldots, b_k, x^0)\) as before,
but now let \(a\) be placed at any point preceding \(x^0\).

Theorem VI: Given partial agenda \((b_1, \ldots, b_k, x^0)\)
and sophisticated agenda \((z_1, \ldots, z_T)\), \(a\) is the
sophisticated outcome of the agenda
\((b_1, b_2, \ldots, b_{j-1}, a, b_j, \ldots, b_k, x^0)\) if and only if
\(a \in \bigcap_{i=1}^T W(z_i)\).

DECENTRALIZED AGENDA-SETTING

We have explored in some detail the effects of strategy and structure
on centralized agenda-setting in the last section. Our broad conclusion
there was that strategy, in the form of sophisticated agent behavior,
need not much constrain the opportunities open to an optimizing agenda-
setter. But structural constraints, coupled with sophisticated agent
behavior, may often be much more restrictive on the agenda-setter's opportu-

The message for institution-building is clear: strategic
adaptation alone is often insufficient to protect agents from possible
exploitation by an agenda-setter. Structural restrictions are often required.

In many institutions the problem of agenda-setter exploitation is dealt with more directly. Instead of (or in addition to) constraining the practice of agenda-setting in various ways, the components comprising agenda power are decentralized. It is something of an irony that, while most of the analytical literature on agenda-setting has dealt with the phenomenon as though it must, of necessity, be centralized in a single agent, most real institutions have dealt with it by carving it up and passing it around!

The most important effect of decentralizing agenda power is the transformation of an optimization problem for a centralized agenda-setter into a game among agenda agents. Partial control over the various aspects of agenda-setting (adding, deleting, ordering -- see below), once institutionalized among agents, defines both well-specified strategy sets for agenda agents and the various moves of an extensive-form game. The product of a play of the agent game is an ordered agenda \( V \) and, on the hypothesis of sophisticated agent voting behavior, each agenda strictly determines a sophisticated outcome \( x_1^*eV \) (see note 5). Thus, the decentralized agenda game may be seen, in its normal form, as a mapping from agenda agent strategy sets to a sophisticated equivalent \( x_1^*eX \). It is important to see, then, that once agenda-setting is decentralized and institutionalized in a particular fashion, and all agents are endowed with the capacity for sophisticated responses, the voting aspects of legislative institutions may be suppressed; everything follows once agenda agents select their respective strategies. In short, institutional outcomes are completely determined by agent agenda strategies.

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Agenda-setting consists of three distinct activities: adding alternatives to the agenda, deleting (vetoing) alternatives from the agenda, and ordering the elements of an agenda. These activities are institutionalized by (i) bundling them in various ways; (ii) distributing the bundles among agents; and (iii) sequencing the various moves (creating an extensive form game). For example, agent i may be empowered to select as a "proto-agenda" a subset of X at the first move (add elements to the agenda); agent j, then, may select a subset of this subset, thereby deleting some elements initially added by i; finally, agent k is empowered to order the remaining elements, possibly after adding additional elements. In this example, agents i and k share the adding power; j possesses the deleting power; and k has exclusive ordering power. In principle, there may be several rounds of agenda-setting in which an ordered agenda at the end of one round is subjected to a new flurry of adding, deleting, and ordering in a subsequent round.

However institutionalized, each activity defines a set of actions or strategies for agenda agents of each type:

Adding. Select \( F(X) \subseteq X \)

Deleting. Select \( G(F(X)) \subseteq F(X) \)

Ordering. Select \( H(G(F(X))) \subseteq \text{Perm}(G(F(X))) \), where \( \text{Perm}(A) \) is the various permutations of elements of A.

We now turn to a brief examination of the components of decentralized agenda-setting. We emphasize that the precise choice of bundling, distributing, and sequencing constitutes the choice of a particular institutional game. Here, however, we only describe the components. In the
next section we show how different combinations of bundling, distributing and sequencing induce different institutional games.

Adding

Suppose agent i is charged with selecting an \( F(X) \subseteq X \). He cannot know precisely how the \( F(X) \) he chooses ultimately is transformed into a determinate \( x_y^* \) until the strategies and preferences of other agenda agents are known. Since his optimal strategy depends upon how his partial agenda power is embedded in a game, we de-combine only some obvious implications of this component. First, if \( x \notin F(X) \), it cannot be the sophisticated outcome. Hence, centralized 'addition' implies extreme veto power and, if \( x^0 \notin F(X) \) is required, then it also guarantees no harmful change for i in the status quo ante, i.e., i will not include an \( x \in F(X) \) if \( x^0 \succ x \). Second, certain strategic purposes may be served by including "inferior" alternatives (see below). Subject to deletion strategies, the Miller Proposition implies that i may assure that y is not the sophisticated outcome, not only by not including \( y \in F(X) \) -- perhaps he is constrained against this -- but also by adding an \( x \) that covers \( y \). Thus, in considering whether to choose \( F(X) \) or \( F(X) \cup \{ x \} \) for some \( x \notin F(X) \), i must consider both the feasibility of \( x \) becoming the sophisticated outcome (is \( x \) covered by some \( y \in F(X) \)?) and the set of \( y \in F(X) \) that cannot be the sophisticated outcome in the presence of \( x \) (is \( y \) covered by \( x \)?) This suggests a more general point about strategy. An agent may make a motion either because he prefers that motion to be the final outcome or, somewhat more subtly, because he seeks to cover some motion he wishes to veto. In games without deleting, this tactic is available to any agent with adding power.
Deleting

Given an \( F(X) \) by agents with adding power, deleters must consider which elements, if any, to delete in light of potential ordering moves. Like adding, deleting alternatives serve two distinct purposes. In choosing \( F(X) - \{ y \} \) for some \( y \in F(X) \), a deleter vetoes \( y \) and potentially uncovers some \( x \in F(X) - \{ y \} \) that might previously have been covered by \( y \). In the latter case, \( x \) becomes a possible sophisticated outcome that, in light of Miller's Proposition, it could not have been had \( y \) not been deleted. Since the deleting power can render an agent's problem trivial -- e.g., delete everything but a deleter's maximal element in \( F(X) \) -- it is normally embedded in a system of constraints, much as the adding power is. For example, a common constrained form of deletion power limits it to a veto power as the final move in the game. Thus, no \( x^*_V \in V \) can be the sophisticated outcome if \( x^0 \succeq_i x^*_V \), where \( i \) is the veto agent and \( x^0 \) is the reversion point. The committee chairman whose discretion permits him to decide whether or not to pass on a voting decision of his committee to the full legislature is an illustration of this form. The conference committee plays an analogous role with regard to a bill passed by the full legislature.

Ordering

Given a set of agenda elements \( G(F(X)) \), this component of agenda-setting provides a strict voting order. We know that no \( x \in G(F(X)) \) can be the sophisticated outcome if there is a \( y \in G(F(X)) \) that covers it in \( G(F(X)) \). Miller (1980, p.82) conjectures, but has not proved, that for any \( x \in G(F(X)) \) not covered by another element of \( G(F(X)) \), there is an agenda for which \( x \) is the sophisticated outcome. If true, then the orderer arranges
an agenda that yields his maximal element in $UC[G(F(X))]$; if not, he does the best he can from among those uncovered elements that can be sophisticated outcomes. Whichever element, $x$, the orderer places last in the voting order, the sophisticated outcome, say $x^*$, is either $x$, itself, or must satisfy $x^* \in W(x)$. Moreover, if $x'$ is next to $x$, and $x' \in W(x)$, then either $x'$ is the sophisticated outcome or $x^* \in W(x') \cap W(x)$. That is, the Intersecting Win Set Theorem must guide the ordering component.

We turn now to a rather specific application of some of these principles by looking at a progression of models, beginning with a highly centralized and unconstrained agenda-setting circumstance and progressively strengthening constraints and decentralizing agenda components.

**Properties and Equilibria in Specific Majority Rule Legislative Institutions**

We have emphasized the various ways in which agenda formation occurs under different institutional structures. Since the outcome is known once the agenda is formed, the real locus of outcome choice takes place in agenda formation, not voting. Most real world institutions have a myriad of particular features constituting agenda formation: committees, motions, rules governing amendments and motion-making, power of recognition, etc. The preceding four sections have satisfactorily demonstrated that these features have significant bearing on outcomes. Therefore, in order to understand policy choice in different legislatures, we need a theory of legislative institutions. This requires the development of a theory of agenda formation under constraints and decentralization, i.e., how different mixes of these features lead to different outcomes. Indeed, we argue that it is precisely these features which differentiate institutions based upon majority rule voting from one another.
In this section, we turn the analysis around to see what we can demonstrate about the way in which different combinations of structures restrict the outcome. We provide a series of models of institutions to illustrate how the various constraints, studied above in isolation, work when they are imposed simultaneously. The motivation for the analysis is twofold. First, we show that under plausible mixes of constraints, equilibria may exist. Second, we show that even if they do not, the combination of constraints and other relevant features may significantly restrict the location of the final outcome to some well-defined subset of the entire space.

The analysis begins with the study of a simplified legislature with a committee structure that afford committees (and committees only) proposal power. In combination with the requirement that the status quo be voted last, this institution resembles the House of Representatives operating under the closed rule, i.e., no amendments to a committee motion are in order. Other institutional details, such as amendment possibilities, are then added to the model one by one ending with a committee structure and a "rules committee" that determines which amendments will be considered and in what order. In addition, we consider the case in which the legislature may vote on the choice of a "rule" governing consideration of the issue, e.g., the choice between the closed rule and other rules allowing varying restrictions over amendments.

In each of the institutions studied below, we consider a legislative committee system. We assume that the legislature is divided into committees with each committee having exclusive power to initiate motions
over alternatives within its jurisdiction, a subspace (possibly multi-dimensional) of the full issue space. We continue to assume A.1 - A.8, given earlier, except here we interpret $X$ as the relevant committee's policy jurisdiction rather than as the entire issue space. Throughout we assume that the rules of the legislature privilege the status quo, $x^0$, which must be in the final division. We also assume that the legislative decision is final and that the game may not be repeated with the outcome of the previous play becoming the status quo of the new play. Finally, we assume that the committee has a continuous, transitive utility function, $U^L(x)$, over policy alternatives. This assumption will be relaxed in the course of our models. In each of the models which follow, we add specific institutions governing agenda formation.

The Closed Rule

Consider the legislative committee with jurisdiction $X$, where it has complete agenda power with the sole restriction that the status quo must be voted on last. As Theorem IV demonstrated, this requirement insures that any replacement for the status quo must lie in $W(x^0)$. Since any point in this set beats the status quo, the committee simply chooses that point in this set that it most prefers, i.e., $x^* = \text{Max } U^L(x), x \in W(x^0)$. This is depicted in Figure 4 for a five-person legislature and legislative committee, $L$. $x^*$ is the best the committee can do under the rules. If it gets more utility from $x^0$ than $x^*$, then it simply will not initiate consideration of this issue. Similarly, since all agents not on the committee have no power to offer amendments, $x^*$ is the best they can do; therefore, $x^*$ is seen to be a Nash equilibrium.
Behavior under the closed rule is straightforward and is useful as a starting point in our analysis. With the requirement that the status quo be voted last, it tells how much can be achieved by a centralized agenda setter. Moreover, it shows the existence of an equilibrium even though $W(x) \neq \phi$ everywhere. The basic principle underlying this equilibrium is that those who prefer elements which beat $x^*$ have no opportunity to place them on the agenda. While this example institutionalizes this principle in a very stringent form -- closed rule -- we shall see that equilibrium in more flexible institutions results from precisely the same set of circumstances.

The Open Rule: The LCRC Game

The only action allowed under the closed rule was through the committee's centralized agenda power. Most legislatures and committees allow for consideration of amendments and substitutes to motions. For example, in contrast to the closed rule noted above, various forms of the open rule are typically employed in Congress. These allow, to varying degrees, other members to offer motions, amendments, and substitutes to a particular committee bill.

We shall study as models of decentralized agenda power two important forms of the open rule. The first is called the LCRC game for the Legislative Committee-Rules Committee game, defined as follows. As before, the status quo must be voted last. In addition, the Legislative Committee (LC) may make a proposal to which amendments may then be offered. A second committee, called the Rules Committee (RC) is given the power to order, add to, and delete from the set of amendments offered by other members. The motivation for this institution draws on the House of Representatives wherein any bill offered by a committee to replace a given
status quo must first go through the Rules Committee of the House. In general, the Rules Committee grants a rule, i.e., it specifies the constraints governing the amendment process, which amendments are in order, how they will be sequenced, and so on.

Since we have assumed that the RC has full power to order, delete from and add to the set of amendments, it will simply allow the single amendment that it most prefers which beats both \( x^0 \) and the proposal made by the LC, \( B \). Thus, we may ignore all other amendments. This condition subsequently will be relaxed.

This defines a game in extensive form: Given a status quo, \( x^0 \), that must be in the final division, LC chooses \( B \), an element of the first division, and RC chooses \( A \), the other element of the first division. By the Intersecting Win Sets Theorem, we know that the LC proposal \( B \) must be in \( W(x^0) \), and that RC will offer an amendment that is in both \( W(x^0) \) and \( W(B) \) in order to beat both the status quo and \( B \).

The strategy set available to LC is \( W(x^0) \) while that available to the RC, once \( B \) is chosen, is \( W(x^0) \cap W(B) \). Their respective choice problems in this game are as follows. For any bill \( B \in W(x^0) \) offered by LC, RC will formulate its best reply, \( A = R(B) = \text{Max } U^R(x) \) such that \( x \in W(x^0) \cap W(B) \). Thus, for any \( B \), \( R(B) \) is RC's best response to LC's choice.\(^{19}\) It is clear that because of the fixed sequencing of the agenda, when the LC offers a proposal \( B \), it is really choosing an alternative \( R(B) \), the RC's best response to \( B \). Thus it will not choose \( B \in W(x^0) \) for the utility derived from it, but rather for its instrumental value in gaining the appropriate \( R(B) \). Thus, LC chooses over all \( B \in W(x^0) \), according to \( U^L(R(B)) \). Let \( B^* = \text{Max } U^L(R(B)) \), where \( B \in W(x^0) \), and let \( A^* = R(B^*) \) be RC's best response to \( B^* \).
FIGURE 5
\[ R(B) = \begin{cases} 
  x^R & \text{for } B \in [x^0, x^R] \\
  B & \text{for } B \in [x^R, x^m] \\
  x'' & \text{for } B \in [x^m, x^i] \\
  \quad \text{(where } x'' = x^m - B \text{ and } |x^i - x^m| = |x^m - x^R|) \\
  x^R & \text{for } B \in [x^i, \bar{x}] 
\end{cases} \]
Notice that \((B^*, A^*)\) is a Nash equilibrium for the two committees. LC chooses B knowing that it will be beaten by \(A = R(B)\), and, given this behavior on the part of RC, therefore chooses that B which does the best for it according to its utility for \(R(B)\). Similarly, since \(R(B)\) was defined as the best RC can do given B, we have that \(A^* = R(B^*)\) is the best either committee can do given the other's strategy choice.

The a priori sequencing of the LCRC game precludes endless cycling. It is not that there do not exist motions that beat RC's response, \(A^*\), but rather that the legislative rules constrain LC from placing these alternatives on the agenda in a position that would beat \(A^*\) and leave it better off. For example, suppose \(B'\) beats \(A^*\) (and \(U^L(B') > U^L(A^*)\)). LC may not place it on the agenda after RC has offered \(A^*\). On the other hand, if it places it on the agenda prior to RC's choice, RC will offer \(A' = R(B')\), not \(A^*\). By definition of \(B^*\), this strategy must leave LC worse off. This illustrates how decentralized and constrained agenda formation determines the outcome of the game. We offer the following existence result the proof of which is found in an appendix.

Theorem VII: An equilibrium strategy pair, \((B^*, A^*)\), with \(A^* = R(B^*)\), exists for the legislative committee and the rules committee, respectively, in the LCRC game.

We illustrate this game for the one dimensional case in Figure 5. Here, \(x^0, x^m, x^R, x^L\), are the status quo, and the median voters', RC's, and LC's ideal points, respectively. \(W(x^0)\), the set of points that beat the status quo, is the interval \((x^0, \bar{x})\). Under the closed rule, the committee may choose any element of this set and hence would offer \(\bar{x}\) which
remains unchallengeable and therefore replaces the status quo. However, this option is not available in the LCRC game.

In Figure 6 we have depicted RC's reaction function, \( R(B) \forall B \in W(x^0) \). The best LC can do is to propose \( x^m \) and get \( x^m = R(x^m) \). As may be seen, this is a Nash equilibrium since for all \( B \in W(x^0) \), \( U^L(R(B)) < U^L(x^m) = U^L(R(x^m)) \). Similarly, for RC, since \( W(x^m) = \emptyset \), \( x^m \) is its only possible reply and therefore is trivially its best reply. While this example depicts only the simplest case (\( X \) is one-dimensional), the proof of the above theorem (see the Appendix) shows this is essentially what occurs in higher dimensions.

Finally, let us compare the outcome of the LCRC game with that under the closed rule. First, LC can never do better under LCRC than under the closed rule. Since the final choice under both institutions must be in \( W(x^0) \), \( x^* \), the outcome of the closed rule, is the best the committee can hope for from the LCRC game; it is not clear in LCRC that there exists some \( B \) such that \( R(B) = x^* \). Second, as long as the power to propose subsumes the power to initiate the process, the committee would never start the process if for all \( B \in W(x^0) \), \( U^L(x^0) \geq U^L(R(B)) \). Therefore, under LCRC, LC is at least as well off as it is with the status quo and, if it chooses to start the game, it is strictly better off.20

Another Open Rule

The LCRC game adds an important level of realism and complexity to the closed rule model. However, the number of members who can make motions is still extremely proscribed. Consider, therefore, the following augmented LCRC game. In addition to RC's amendment \( A = R(B) \), suppose one other substitute amendment \( S \), must be considered in the first division.
(going against A, the winner against B, and the penultimate winner against x^0). We continue to assume that RC may add additional amendments. In order to assess the restrictiveness of the assumption in the LCRC game that there be only one amendment, we now ask when the existence of other amendments can change the outcome of this game. Put differently, when can RC assure victory of A = R(B) over a substitute?

By the Intersecting Win Sets Theorem, any successful amendment must be ... the win sets of every noninnocuous agenda element that follows it. Therefore, we have a necessary condition that S beat A,

\[ C_1: \ S \in W(x^0) \cap W(B) \cap W(A). \]

Second, assuming that C1 holds, RC, by virtue of its power to add amendments, may, under certain circumstances devise an agenda to defeat S and yield A. This requires both that S does not cover A, and that RC can find some alternative Y \in W(x^0) \cap W(B) such that Y \in W(S) and A \in W(Y).

\[ C_2: \ W(x^0) \cap W(B) \cap W(S) \cap D(A) \neq \emptyset. \]

Using the Intersecting Win Sets Theorem and Theorem I, we can see that if condition C1 holds, then condition C2 is both necessary and sufficient for A to beat S. Or, in terms of RC's problem, if this intersection is non-empty, then RC may devise an agenda, including the appropriately chosen Y, with A as the outcome under this more general open rule. In this sense, these conditions tell us how restrictive the assumptions are in LCRC that prohibited all other amendments: only amendments S satisfying condition C1 for which condition C2 fails will beat all other agenda items before them. Thus, even at the level of only four alternatives, decentralized agenda formation may significantly reduce the
set of potential outcomes, as implied by the number of intersections embodied in C1 and C2.

Finally, we note that an interesting variant of this game often occurs in the House: some member of a legislative committee, an advocate of B, is designated the "floor manager" with the power to recognize agents offering amendments. Since, in the House, specific individuals must submit amendments in writing in advance, this recognition power gives the floor manager the power to order the amendments. For an interesting reconstruction of two important cases where the floor manager manipulated the final outcome, see Blydenburgh (1971). He shows that, in the only two times in the mid-twentieth century that the House Ways and Means Committee used the open rule instead of the closed rule on tax legislation, the outcome was manipulated by the floor manager (in the presence of a cycle) with an appropriately chosen order of amendments (read: order of recognition).

Choice Among Rules

Riker (1980) points out in his discussion of institutionally-induced equilibrium that, even if these equilibria exist, cycles over various institutional rules may reduce the importance of equilibrium within institutions. That is, the "Riker Objection," as we called it elsewhere (Shepsle and Weingast, 1981), asserts the prospect of a cycle among institutions. To address this point, we offer one small counterexample. In the House, the Rules Committee offers a particular rule which will govern debate and voting over a bill. But this rule is subject to majority approval and, itself, may be amended. Thus, consider choice among three rules, each of which leads to a different unique outcome: (1) the closed
rule yields $x^*$; (2) a special open rule in which the only amendment allowed is $A = R(B)$, i.e., the LCRC game, yields $A$; and (3) an open rule yields $S$ offered by some agent as a substitute to $A$ and $B$. Moreover, suppose the outcomes associated with each rule cycle: $A$ beats $x^*$, $x^*$ beats $S$, and $S$ beats $A$. What occurs if the body must choose among these rules?

Here we note that rules also govern the choice among rules. In fact, for the choice among rules, the rule offered by the Rules Committee -- (2) above -- takes the place of the committee bill in our earlier analysis; that rule is then subject to amendments -- (1) and (3) above. But the rules governing deliberation over a rule operate in a precisely analogous fashion to those governing substantive deliberations. In particular, the sequence of voting over rules is: $x^0$ (no rule) last, the Rules Committee proposal ((2) above) in the penultimate division, and the two amendments (1) and (3) in the first division. The sophisticated outcome of this agenda is the open rule. The logic of sophisticated equivalents has merely pushed the analysis back to a level twice removed from actual voting.21

Choice Within Committees

We have omitted any discussion of the fact that a committee in Congress does not act as a single agent with transitive preferences but is, itself, composed of individuals. The reader, by now, will appreciate that the same logic that applies to institutional restrictions at the level of the entire legislature applies to committees as well. In this final generalization of the LCRC game, it is apparent that this complication simply adds one more level of complexity but does not change the
basic form of the above conclusions. Here, too, rules govern committee choice procedures. For example, the status quo is voted last in committee; the chairman has agenda powers through his power of recognition; and the chairman possesses veto power inasmuch as he may elect not to forward a committee decision to the full legislature. For a proposal B that is assigned to the committee, committee bill, i.e., the outcome of committee deliberation, must, by the Intersecting Win Sets Theorem, be some element in $W'(x^0) \cap W'(B) \cap P_{\text{chmn}}(x^0) \cap W(x^0)$, where primes indicate committee majority win sets and $P_{\text{chmn}}(x^0)$ is the set of elements preferred to $x^0$ by the committee chairman. The outcome of the game within the committee then goes before the entire legislature as the committee bill, the logic of which follows that studied above. Thus, adding a committee essentially nests a committee version of the LCRC game within the legislative version of LCRC.

DISCUSSION

The implications of our approach to agenda-setting are central to the proper study of political institutions. We shall forego a detailed elaboration here, instead simply listing several points that we regard as most important.

1. Existing treatments of agenda-setting are very special cases of the manner in which it may be institutionalized. The centralized, unconstrained agenda-setter of the McKeelvey Agenda Theorem, operating in a nonstrategic environment, is but one (and a very special one at that) of the ways in which an institution's agenda may be constructed.
2. Precisely because McKelvey's agenda-setter is so powerful, he rarely appears in any real institution. Hence, to make positive theories of agenda power empirically applicable, there is an obvious need to examine agenda power in a more general framework.

3. An agenda-setting arrangement, in the multidimensional context with sophisticated agents, defines an extensive game form. On the hypothesis of sophisticated voting, each agenda is determinate (up to a lottery in the event of ties). Hence, legislative voting may be suppressed and it follows, then, that the agenda game is the model of legislative institutions. Given fully anticipated strategic adaptation, structure governing the agenda fully characterizes legislative games.

4. In the more general framework presented above, majority rule games are not so indeterminate as PMR; nor need they be so fully exploited (and, hence, so determinate) by any single agenda agent. Nevertheless, agenda games have equilibrium or the outcomes are restricted (though not necessarily to a "nice" subset of X).

5. Thoughtful analyses are required of the components of agenda-setting -- adding, deleting, ordering -- and the manner in which they are institutionalized -- bundling, distributing, sequencing. The McKelvey Agenda Theorem, it now appears, is the starting point in the study of institutions, not the final analysis.

6. The politics of institution-building may be represented as choices among institutional games. To analyze these choices, it is necessary to model the equilibrium outcomes of these games under different assumptions. The choice among games is then seen as the choice among equilibria.22
7. Institutional games may be modeled under alternative behavioral assumptions. The McKelvey model permitted a single strategic actor. Miller's model, and our own, allow for fully strategic behavior and complete anticipation by all agents of fully strategic behavior. Yet there are several empirical studies (Riker, 1965; Enelow and Koehler, 1980) which suggest that, in any specific play of an institutional game, some agents may afford themselves of strategic opportunities while others are constrained (by extra-institutional factors) to vote sincerely. In representative legislatures, for example, it appears that one condition necessary for sophisticated behavior is an agent's ability to "explain his vote to the folks back home" (Fenno, 1978). Further analysis of alternative mixed-behavior circumstances would certainly interest and provoke congressional scholars. A more fundamental exploration of the factors necessary and sufficient for particular manifestations of mixed behavior circumstances is of great theoretical interest.

8. Ours has been a noncooperative view of institutions. Our justification for this view, as noted earlier, was made convincing by Kramer (1972). Yet there may be circumstances in which partial or more fully cooperative models provide new insights. The phenomena constituting agenda-setting may prove to be fertile soil for such inquiries.

In sum, institutional analyses are in their infancy. Much of pre-behavioral political science consisted almost wholly of institutional description. The 'behavioral revolution' was something of an interlude (if not a distraction). Analytical approaches now seem to have rediscovered institutions and hold the promise of bringing some theoretical
rigor to earlier description. We conclude, on the basis of the results offered above and these final comments, that institutions are characterized by their agenda-setting mechanisms and that this constitutes the focus for future research.
APPENDIX

We seek to prove the following result:

Theorem: \( \exists \) an equilibrium strategy pair \((B^*, A^*)\),

with \( A^* = R(B^*) \), in the LCRC game.

Our strategy of proof is to first show that \( W(x) \) is both upper and lower
hemicontinuous \((uhc and lhc, resp.)\). From this, we can then use the Be:je
Maximum Principle to show that RC's response function, \( R(B) \), is uhc. This
implies that it has a closed graph contained in \( W(x^0) \) and is therefore
compact. This, in turn, implies that any continuous function over it
attains a maximum so there exists a \((B^*, R(B^*))\) that maximizes LC's utility
over \( W(x^0) \).

Since \( W(x) \) as defined in the text is an open set, we consider its
closure in the proofs. This follows the strategy of McKelvey (1977, 1978)
in similar circumstances.

Lemma A: \( W(x) \) is uhc.

Proof: A correspondence from \( X \) to \( Y \) \((Y \) nonempty and compact) is uhc if and only if it is closed. We first show that for \( i \in \mathbb{N} \),
agent i's 'preferred to' set, \( R_i(x) = \{y \in X : U_i^i(y) \geq U_i(x)\} \) is uhc.

We must show that if (1) \( x_n + x \),

(2) \( y_n \in R_i(x_n) \),

and (3) \( y_n + y \),
then \( y \in R_i(x) \). (2) implies \( U_i^i(y_n) \geq U_i^i(x_n) \), for all \( n \), and,
by the continuity of \( U_i^i \) (follows from A.2), (1) and (3) imply
\( U_i^i(y) \geq U_i^i(x) \). Hence \( y \in R_i(x) \) and \( R_i(x) \) is uhc. Now, let
\( V_S(x) = \bigcap_{i \in S} R_i(x) \), where \( S \subseteq \mathbb{N} \) and \(|S| > n\). \( V_S(x) \) is uhc since it is the finite intersection of uhc correspondences (Berge, Theorem 2', p.114).
But, $W(x) = \bigcup_{S \subseteq N, |ST| > n} V_S(x)$ Thus, $W(x)$ is now constructed as the closure of what we called $W(x)$ in the text. Since $W(x)$ is the finite union of uhc correspondences, it is uhc (Berge, Theorem 3', p. 114).

Q.E.D.

**Corollary:** For any $B \in W(x^0)$, $F(B) = W(B) \cap W(x^0)$ is uhc.

**Lemma B:** $F(B) = W(B) \cap W(x^0)$ is lhc.

**Proof:** We follow McKelvey (1979, Lemma 4). A correspondence $\gamma(x)$ is lhc if $\forall x \in X$, and every open $G \subseteq X$ with $G \cap \gamma(z) \neq \emptyset$, there exists a neighborhood $N(z)$ of $z$ such that $x \in N(z)$ implies $\gamma(x) \cap G \neq \emptyset$.

So, let $B_0 \in W(x^0)$ and let $G \subseteq W(x^0)$ be open with $G \cap F(b_0) \neq \emptyset$.

We must show that there is an open neighborhood $N(B_0)$ such that $B \in N(B_0)$ implies $G \cap F(B) \neq \emptyset$. So, let $y \in G \cap F(B_0)$ and define

$N(B_0) = \{x \in W(x^0) | y \in D(x) \} = D(y) \cap W(x^0)$. So, $N(B_0)$ is open since $D(y)$ is open. Moreover, $B_0 \in D(y) \cap W(x^0)$ since $y \in G \cap F(B_0)$ and this intersection is equal to $G \cap W(B_0) \cap W(x^0)$. Now, $\forall B \in N(B_0)$, $y \in W(B)$; hence $W(B) \cap W(x^0) \cap G \neq \emptyset$.

Q.E.D.

From the Corollary and Lemma B, $F(B)$ is a continuous correspondence.

We may use the Berge Maximum Principle (Berge, 1963, p. 116) to show that RC's response function $R(B)$ is uhc.

**Berge Maximum Principle:** Let $X \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{R}^m$ and let $\gamma: X \rightarrow 2^Y$ be a compact-valued correspondence. Let $f: Y \rightarrow \mathbb{R}$ be continuous. Define $\mu: X \rightarrow 2^Y$ as $\mu(x) = \{y \in \gamma(x) | y$ maximizes $f$ on $\gamma(x)\}$. If $\gamma$ is continuous at $x$, then $\mu$ is uhc at $x$. 

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Thus, in order to show that \( R(B) \) is uhc, we make the following identifications to apply the theorem.

\[
X = W(x^0)  \\
Y = W(x^0)  \\
\gamma(x) = F(B)  \\
f(y) = U^R(x)  \\
u(x) = R(B).
\]

Therefore, we have:

**Lemma C:** \( R(B) \) is uhc.

Since \( R(B) \) is uhc it has a closed graph which is bounded by \( W(x^0) \) and is therefore compact. This insures that any continuous function on this set attains a maximum. Therefore, LC has a well-defined maximum,

\[
B^* = \max_{B \in W(x^0)} U^L(R(B)).
\]

This proves the theorem.
FOOTNOTES

1. It would be hasty to conclude that, since pure majority rule equilibrium is nonexistent, the search for equilibrium of some sort should be abandoned. In fact, the nonexistence of a single alternative with very strong equilibrium properties (e.g., it defeats each other available alternative in a pairwise comparison) has stimulated some impressive creative efforts to redefine the notion of equilibrium. These efforts include such set-wise (as opposed to point-wise) equilibrium concepts as the top-cycle set (Schwartz, 1977), the minmax set (Simpson, 1969; Kramer, 1977), the competitive solution (McKelvey, Ordeshook, and Winer, 1978), the uncovered set (Miller, 1980), the least vulnerable set (Ferejohn, Fiorina, and Weisberg, 1978), and the least alternative set (Ferejohn, Fiorina, and Packel, 1980). For a general discussion of alternative equilibrium concepts, see Fiorina and Shepsle (1982).

2. Recently, Ferejohn, McKelvey and Packel (1981) began exploring the limiting behavior of a probability distribution over outcomes when agenda-setting is formalized as a Markov process. It remains true that PMR can wander anywhere but, in the limit, some outcomes (more "centrally located" ones) are more probable than others.

3. The most compelling justification for the noncooperative treatment of institutional games is found in Kramer (1972). For a dissenting view, see Koford (1978).

4. This convention, while typical in the literature, occasionally poses technical difficulties for us since we will wish to find a maximum on a set which, by the above convention, is open. We will indicate this problem and defend our resolution of it whenever it arises.

5. If, owing to ties, there are several possible outcomes associated with $V$, then $x^*$ is interpreted as the lottery over those outcomes prescribed by tie-breaking procedures.

6. While the theory we develop below is compatible with the existence of a Condorcet winner, we assume $W(x) \neq \emptyset$ to emphasize that our equilibrium results do not depend on the existence of an undominated point.

7. A total median is a point with the property that any hyperplane containing it divides $X$ into two half-spaces the closures of each of which contain at least half of the voter ideal points.

8. A $y \in X$ is an element of $R(x)$ if $|\{i/y \succ_i x\}| \geq |\{i/x \succ_i y\}|$. 

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9. The vote order is given by the following tree:

It appears that this procedure is different from the simple sequential elimination process entailed by the amendment procedure.

10. We construe these components of agenda-setting broadly so that, for example, "adding" includes "making it possible to add." Thus, the chairman's power of recognition, in which he recognizes an agent for the purpose of moving a specific amendment, constitutes an agenda-setting power. Similarly, "deleting" includes the chairman's power to decline recognition to an agent or to uphold a point-of-order against an agent's motion (which has the effect of removing it from the agenda).

11. Specifically, Miller focus attention on the "uncovered set" and proves a number of results about the uncovered set of a finite agenda $V$. This set proves less consequential in our context than Miller believed, but the covering relation on which it is based turns out to be indispensable.

12. A troubling feature of McKelvey's Agenda Theorem, further developed in McKelvey (1977, 1978), is that the result guarantees a finite agenda but says nothing about its length. This may be of only limited value in institutions that impose a length constraint on agendas. However, Corollary I.2 has some implications for sincere voting in this respect. If $y$ does not cover $x$, then, under sincere voting, there exists a three-element agenda with $y$ in the first division and $x$ the sincere outcome.

13. UC($y$) is constructed from W($y$) by taking the union of win-sets for the extreme points, $a$, $b$, and $c$ of the petals, $A$, $B$, and $C$, respectively. We conjecture that the union of these win-sets contains the win-sets of any other $z$ ε W($y$), though we have not established this result analytically.

14. This can be pictured in Figure 2 or 3 by moving Mssr. 3 away from Mssrs. 1 and 2 in a southeasterly direction.
15. A related result is established by Kramer (1972) and Miller (1980).


17. Miller (1980, Theorems 2 and 3) establishes that if no MRE exists, then there are at least three uncovered elements in any agenda and that they cycle. We have proved Miller's conjecture for exactly three uncovered elements but, while it appears to be generally true, a proof of this theorem remains an open question.

18. Since preferences are separable we shall not label the different committees but merely consider choice by a given committee within its jurisdiction. Under A.2 (which implies separable preferences), actions in one jurisdiction do not affect decisions in other jurisdictions.

19. In order for this response to be well-defined, we must look at the closure of $W(x_0) \cap W(B)$. Since these sets are always bounded, this qualification is not heroic. McKelvey (1977, 1978) deals with a similar problem and we have chosen to adopt his solution.

20. In a different context, G. Miller (1980) devises a very similar model.

21. In fact, by not making its own preferred rule the main vehicle for debate, the Rules Committee instead may allow its preferences as an amendment and can assure its adoption.

22. Even in less determinate situations -- situations with no equilibrium but in which the outcome must be contained in some bounded intersection of several sets -- there is a promising methodology for informing expectations about outcomes. Elegantly presented in Ferejohn, McKelvey, and Packel (1981), this methodology yields a limiting distribution for the probability that a given element is the final outcome.


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