RADAR SCATTERING FROM TITAN AND SATURN’S ICY SATELLITES USING THE CASSINI SPACECRAFT

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FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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Abstract

Titan is the largest moon of Saturn and the second largest moon in the solar system. It has a thick atmosphere rich in nitrogen and hydrocarbons, analogous to the atmosphere of early, prebiotic Earth. This atmosphere inhibits observations of the surface using traditional optical methods. The Cassini-Huygens spacecraft (a joint endeavor of NASA/ESA/ASI) began orbiting Saturn in 2004, with a flyby of Titan nearly every month. Its RADAR instrument, with a 2.2 cm wavelength, penetrates the hazy atmosphere to detect the surface. RADAR operates near closest approach on roughly half of the Titan flybys. As of July 2011, the RADAR instrument has observed the surface on 41 of the 77 Titan flybys.

The RADAR instrument operates in several modes. It calculates surface height profiles, measures the emissivity, and also maps the surface at resolutions as fine as 300 m. The high-resolution maps reveal a surprisingly Earth-like physical surface, complete with icy mountains, dune fields, cryovolcanoes, flowing liquids, and hydrocarbon lakes. Another operation of the instrument, called scatterometer mode, measures the real aperture (beam-averaged) backscatter reflectivity as a function of incidence angle. The shape of this backscatter curve reveals much about the surface, such as material composition and roughness structure.

We develop a real aperture processor to reduce the scatterometer data, and also extend this reduction to the other active modes of the RADAR instrument. We calibrate the different modes in order to combine the data sets globally. We correct the measured backscatter for incidence angle effects to produce a global backscatter map (99.9% surface coverage) with real-aperture resolutions between 10 and 250 km.
This is the first time Titan has been mapped globally at cm wavelengths.

With all RADAR data processed and calibrated to the same scale, we obtain detailed scattering behavior at different locations on Titan over a range of incidence angles. This collective set of backscatter data allows us to measure the radar reflectivity as a function of viewing angle, i.e. the backscatter function, for specific Titan terrains. We model the backscatter functions with a superposition of classical facet scattering laws (Gaussian + Exponential generally gives the best fit) and an empirical cosine power law. The former describes the large-scale surface scattering term and the latter describes the significant diffuse volume scattering term. We invert the composite backscatter model to estimate surface composition and physical structure for a selection of surface features. We infer dielectric constant values consistent with solid hydrocarbons over much of Titan’s surface, but the brighter regions often appear more consistent with water ice bedrock. Almost all features are dominated by diffuse volume scatter, which comprises more than 80% of the radar echo. Comparison of the feature model results demonstrates the heterogeneity of the surface scattering parameters across Titan, and contributes to the understanding of the geological processes responsible for each feature’s formation and evolution.

We extend the real aperture processor and modeling technique to other moons of Saturn, specifically focusing on close targeted flybys of Enceladus, Rhea, and Iapetus. These surfaces appear almost entirely diffuse to the RADAR, but we detect a small quasispecular component on the dark side of Iapetus that we model with a Hagfors scattering law. This is the first quasispecular detection on an icy moon other than Titan. We compare the icy satellite backscatter models to those obtained for Titan features and detect similar diffuse scattering behavior. Enceladus and Rhea appear brighter than any surface on Titan, while Iapetus exists between the brightest and darkest Titan terrains.

We further analyze the backscatter from Titan’s largest southern lake, called Ontario Lacus. Altimetry observations over the lake reveal the first specular glints from
a liquid surface on Titan. We model this data with specular reflection theory to con- strain the height variation of the surface waves (if any). We find that Ontario Lacus must be very smooth to reproduce the observed scattering levels: the surface must have a root-mean-square height smaller than 3 mm across the 100 meter Fresnel zone. These results have direct implications for wind speed and wave generation on Titan’s liquid surfaces. We further investigate off-nadir imaging and real aperture data over Ontario Lacus, employing a two-layer scattering model to produce bathymetry maps and depth profiles across the lake. We find that the lake is shallow, with mean depths around 3-5 meters, and likely has a volume of 50-80 km$^3$. 
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It has been a great privilege to be part of the Cassini project, and an honor to be welcomed as a peer by the members of the Cassini RADAR Team. Titan is very much like a jigsaw puzzle, but one where the pieces are slow to take shape. To watch great scientists and engineers put these pieces together, and to occasionally contribute a piece of my own, has been an incredibly exciting experience.

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Chapter 1

Introduction

1.1 Background

The Cassini-Huygens spacecraft is the first spacecraft to visit Saturn’s system since the fly-by encounters of the Voyager probes in 1980 and 1981. The Cassini-Huygens spacecraft comprises two robotic components: the Cassini orbiter and the Huygens probe, both named after 17th century astronomers that contributed to the discovery and understanding of Saturn’s largest moons and rings. The orbiter was designed, developed, and assembled by Jet Propulsion Laboratory (JPL) for NASA, and JPL also manages the mission for NASA. The Huygens probe was developed by the European Space Agency (ESA). Seventeen countries contributed to the creation of the spacecraft and its instruments, for instance the Italian Space Agency (ASI) provided Cassini’s 4-meter high-gain radio antenna and the compact radar instrumentation.

The 6000 kg Cassini-Huygens spacecraft launched October 15, 1997 on a Titan IVB/Centaur rocket, the most powerful rocket available to NASA. To cover the 1,433,449,370 km distance to Saturn (roughly 80 light-minutes), the spacecraft required additional energy from planetary gravity assists. A gravity assist is an orbital technique used since the 1970’s that augments the speed, or kinetic energy, of a spacecraft at the expense of a small amount of potential energy from a massive body, such as a planet. Cassini mission designers planned four gravity assists to help the spacecraft
reach Saturn in seven years: the spacecraft first visited Venus in April 1998, followed by an Earth flyby in August 1999, a revisit to Venus in June 1999, and a final flyby of Jupiter in December 2000. The spacecraft reached the outer regions of the Saturnian system June 11, 2004, and it locked into orbit around Saturn on July 1, 2004. After passing through the gap between Saturn’s F and G rings, the onboard rocket engine helped to brake the spacecraft sufficiently for orbital capture. Subsequently, carefully designed flybys of Titan, whose large mass permits significant changes in Cassini’s direction and speed, have guided the spacecraft through the Saturnian system, permitting a variety of orbital inclinations and viewing geometries. Occasional burns from the main rocket engine help keep the spacecraft on a well-defined path. In this manner, Cassini loops from one Titan flyby to another, with occasional opportunities for close flybys of other interesting moons available along the way, as well as many opportunities for more distant observations.

As the Cassini orbiter approaches and passes by a targeted moon, it uses either its thrusters or its reaction wheels to keep the spacecraft’s instruments pointed steadily at the target, in spite of the large relative speeds involved. The observation time is carefully divided among Cassini’s instruments to maximize the science return of each flyby. The instruments are bolted to the side of the spacecraft, so operations typically have to be arranged sequentially rather than simultaneously. The collected data are stored onboard the spacecraft until the spacecraft is able to turn its high gain antenna to earth and relay the data to one of the antennas of the Deep Space Network on Earth. Several gigabytes of data are downlinked daily.

The Cassini orbiter contains 12 science instruments, including cameras, spectrometers, an imaging radar, and instruments to study the dust, gas, and plasma of the Saturnian environment. These 12 instruments are, in alphabetical order,

- Cassini Plasma Spectrometer (CAPS): measures the energy and electrical charge of particles encountered to study the composition, flow, and temperature of the particles in Saturn’s magnetosphere.
- Cosmic Dust Analyzer (CDA): measures the size, speed, direction, and chemical
composition of the tiny dust grains that impact the Saturn system.

- Composite Infrared Spectrometer (CIRS): measures the infrared emissions from atmospheres, rings and surfaces in the Saturnian system to study their thermal properties and compositions.

- Ion and Neutral Mass Spectrometer (INMS): analyzes charged particles and neutral particles to study the atmospheres of Titan and Saturn, Saturn’s magnetosphere and the rings.

- Imaging Science Subsystem (ISS): uses a wide-angle camera to take pictures of large areas and a narrow-angle camera to take pictures of smaller areas at higher resolution. Spectral filters attached to the cameras enable the instrument to take pictures at different wavelengths, from 0.2 to 1.1 \( \mu \text{m} \), although most imaging is done at wavelengths near 0.94 \( \mu \text{m} \).

- Dual Technique Magnetometer (MAG): measures the strength and direction of Saturn’s magnetic field to accurately model Saturn’s magnetosphere and study its interactions with the surrounding planetary environment. The magnetic fields yield information about the interior of Saturn and other bodies.

- Magnetospheric Imaging Instrument (MIMI): produces images of the plasma in Saturn’s magnetosphere and measures the charge and composition of the ions to study the configuration and dynamics of Saturn’s magnetic field.

- Radar (RADAR): uses the high gain antenna to transmit and receive Ku-band (13.78 GHz) microwave signals to produce high-resolution synthetic aperture images (300-500 m/pixel) and low-resolution scatterometer (100 km/pixel) images, measure the heights of surface objects, and quantify the strength of the backscatter reflectivity and the blackbody emissivity, yielding information about the compositional and structural characteristics of the target surface.

- Radio and Plasma Wave Science instrument (RPWS): measures the radio signals originating from Saturn and the electrical and magnetic fields around Saturn to
monitor the ionosphere and lightning from Saturn’s atmosphere as well as the plasma around Saturn.

- **Radio Science Subsystem (RSS):** uses the spacecraft X-band (8-12 GHz) communication link as well as S-band (2-4 GHz) downlink and Ka-band (26.5 to 40 GHz) uplink and downlink to transmit radio signals through objects, such as Titan’s atmosphere and Saturn’s rings, to receiving antennas on Earth. The bistatic radio signals reveal information about composition, atmosphere pressure and temperatures, ring particle size and structure, target masses, and gravitational waves.

- **Ultraviolet Imaging Spectrograph (UVIS):** captures images at wavelengths from 55.8 to 190 nm to learn more about the structure and composition of clouds and atmospheres.

- **Visible and Infrared Mapping Spectrometer (VIMS):** captures images of moon surfaces, Saturn’s rings, and the atmospheres of Titan and Saturn at 352 different wavelengths between 350 and 5100 nm with a typical resolution of 50 km/pixel. VIMS can move its primary mirror without moving the spacecraft to map spectral images from different areas on the targeted object. The spectral images reveal much about compositions and atmosphere layers.

In addition to the orbiter’s 12 science instruments, the spacecraft carried the 318 kg Huygens probe, built by the European Space Agency. The Huygens probe separated from the Cassini spacecraft on December 25, 2004 and coasted on its own trajectory for 22 days before successfully landing on the surface of Titan on January 14, 2005. The probe transmitted data to Cassini during its 2.5 hour descent through the atmosphere and continued to do so for about 90 minutes after landing, until Cassini disappeared behind the horizon. Cassini then relayed the probe’s data back to Earth. Huygens’ six instruments were designed to characterize the thermal properties and composition of the atmosphere, characterize the winds and aerosols, and determine the physical properties of Titan’s surface. Data transmitted from Huygens revealed that it had
landed in a dry area near the equator, yet Huygens’ images showed clear evidence for the presence of past liquids in the area: rounded pebbles cluttered the landing site and small-scale drainage channels cut through bright icy bedrock (Lebreton et al., 2005). Later data collected by the orbiter’s instruments confirmed the presence of liquid hydrocarbons on Titan (Brown et al., 2008; Stofan et al., 2007). These liquids are seen to persist in the polar regions, but they can also fleetingly appear in Titan’s equatorial regions after tropical storms (Turtle et al., 2011).

The Cassini orbiter continues to orbit and study Titan and the Saturn’s icy satellites. The Cassini-Huygens mission was originally designed to last only four years, from July 1, 2004 to June 30, 2008. But after completing 74 orbits around Saturn, including 44 close flybys of Titan, the spacecraft was still in perfect operating condition, and its success begged a continuation. The mission was extended for two more years, from July 1, 2008 to June 30, 2010, and then extended again for seven more years, from July 1, 2011 to September 18, 2017. The extended mission (XM), called the Cassini Equinox Mission due to the occurrence of the Saturnian equinox on August 11, 2009, contributes an additional 59 orbits around Saturn, including 26 Titan flybys, 7 Enceladus flybys, and one close flyby each of the icy moons Dione, Rhea, and Helene. The extended-extended mission (XXM), called the Cassini Solstice Mission due to the occurrence of Saturn’s northern summer solstice in May 2017, contributes 155 orbits around Saturn, including 56 Titan flybys, 12 Enceladus flybys, and several flybys of other icy satellites. Having arrived just after Saturn’s northern winter solstice in 2004, the additional extensions allow Cassini to observe the Saturnian system for a complete seasonal period, watching the progression from mid-winter to spring to mid-summer. The 27 degree tilt of Saturn’s pole, combined with its lengthy 29.4 Earth-year orbit around the sun, generates significant seasonal effects on Saturn and Titan (since Titan is nearly in synchronous orbit around Saturn, with hardly any inclination relative to its equatorial plane), and the Cassini spacecraft is in a prime position to monitor these long-term changes.

The last set of orbits in the mission have been carefully engineered to gently nudge
the Cassini orbiter into Saturn’s atmosphere, thereby safely disposing of the spacecraft and eliminating the risk for collision with and contamination of Saturn’s moons. The final orbits, called “proximal” orbits, pass just inside Saturn’s ring system and just above its cloud tops, enabling detailed studies of Saturn’s magnetic and gravitational fluctuations, and thus leading to a better understanding of Saturn’s internal structure.

1.1.1 Cassini-Huygens and the moon Titan

Much of the mission effort is focused on Saturn’s largest moon Titan. With a dense obscuring atmosphere composed largely of nitrogen, Titan has been the object of curiosity and wonder since its discovery by the Dutch astronomer Christiaan Huygens in 1655, especially since the revelation in 1944 that it has an atmosphere containing methane gas (Kuiper, 1944). A certain intrigue follows from the fact that all traces of methane should have been destroyed by ultraviolet light from the Sun early in Titan’s life, and thus the presence of methane today requires a source to replenish it. Before Cassini-Huygens arrived at Titan, one popular theory was that Titan was globally covered by an ocean of hydrocarbons, an ocean big enough to source the methane detected in the atmosphere. Other theories postulate that the methane is a by-product of biological organisms that might exist on or below the surface, or that the methane is replenished from cometary impacts. Most likely, the correct explanation involves the manufacture of methane from geological processes within Titan’s interior (Tobie et al., 2006). Whatever its source, methane is an essential component of Titan’s atmosphere; without it, the nitrogen would condense and the atmosphere would likely collapse (Lorenz et al., 1996). Additionally, methane is a major player on Titan’s surface, where it condenses into liquid and acts as water does on Earth, raining on and eroding the icy bedrock into the familiar Earth-like features that we observe (Jaumann et al., 2008). Furthermore, the methane in the atmosphere breaks down into heavier molecules that condense and rain out onto the surface, creating even more building blocks from which the surface takes shape.
Each Cassini flyby of Titan affords opportunities to study the surface and atmosphere of the enigmatic moon. The Titan flybys are tagged with the abbreviated target name “T” and the flyby number (e.g. “T3” refers to the 3rd flyby of Titan), with the exception of the first two Titan flybys, which are called “Ta” and “Tb”. Because of the extended atmosphere, the spacecraft can approach only to about 950 km altitude without experiencing significant drag. The closest approach point is the most coveted observation segment of the flyby, and the instruments typically take turns occupying this time.

Titan’s thick atmosphere precludes the detailed visual and infrared spectroscopic analysis often used to identify surface composition on other planetary surfaces, as only a few spectral windows through the atmosphere exist. The ISS and VIMS instruments exploit these atmospheric windows, with ISS recording images at 2-4 km resolution within the 0.94 $\mu$m window, and VIMS recording images at 50 km resolution within the 0.94, 1.08, 1.28, 1.6, 2.0, 2.8, and 5 $\mu$m windows. RADAR’s microwave radiation, with a wavelength of 2.18 cm, easily penetrates the atmosphere and interacts with the surface. The dependence of radar reflectivity on viewing geometry and polarization is directly related to surface dielectric and physical properties. Hence the RADAR data are one of the best means available for examining Titan’s constituent materials and their physical state.

At the time of this writing, the Cassini RADAR has completed 40 flybys of Titan (through T71 on 7-July-2010, the beginning of the XXM), and 15 RADAR Titan flybys remain in the XXM. Many features on Titan’s surface have been revealed. Major discoveries include indications of possible cryovolcanism (Lopes et al., 2007b), impact cratering (Wood et al., 2010), fluvial processes (Jaumann et al., 2008; Lorenz et al., 2008a), aeolian activity (Lorenz et al., 2006), mountain formation (Mitri et al., 2010; Radebaugh et al., 2007), and polar lakes (Stofan et al., 2007).
1.2 Motivation

Saturn’s system has long been a target of great interest for planetary exploration. In rapid succession, Saturn was first visited by Pioneer 11 in 1979 and then the two Voyager spacecraft in 1980 and 1981. Much was discovered about Saturn and its various moons during those early encounters, but very little was learned about the surface of Titan. The Cassini-Huygens spacecraft was specifically engineered to obtain information about this enigmatic surface. Four instruments in particular (CIRS, ISS, RADAR, and VIMS) were designed with special surface detection capabilities. The RADAR instrument is unique in that it can sense the surface with almost no atmospheric interference and no solar illumination constraints, and it is capable of producing images at very high resolutions (resolutions as fine as 300 m). The radar images offer one of the best tools for understanding the geological and geophysical processes on Titan. Yet, to correctly interpret the radar observations, we must understand the mechanisms controlling the radar scattering from the surface.

Surface scattering is a complicated process that depends on the composition and structure of the surface and subsurface. A surface is often characterized by its scattering function, which describes how the scattering strength varies with viewing angle. Analytically derived scattering models relate the form of the scattering function to physical surface characteristics, such as surface slopes and surface dielectric constants. In addition to their implications for the electromagnetic and statistical properties of the surface, the inferred surface parameters provide a quantitative means of classifying the remote surface relative to other surfaces and also contribute to our knowledge about the geophysical processes that shape the surface.

The motivation for this work is centered around accurately measuring, characterizing, and presenting the surface scatter properties of Titan and the other major moons of Saturn. We construct a real aperture processor that is capable of measuring the normalized radar cross section for all of the data collected by the RADAR instrument. The data combine to complement each other in viewing angle, enabling the
1.2. **MOTIVATION**

retrieval of complete backscatter functions for various locations on Titan’s surface. We discover that the form of the backscatter function varies widely with location. The diversity of features in the radar and optical images indeed indicate that the terrain on Titan is extremely heterogeneous, but the variation in the scattering function shapes further implies that the heterogeneity extends to the smaller scales responsible for the surface scattering behavior.

We combine the backscatter measurements to create comprehensive Titan reflectivity maps with 99.9% global coverage (0.1% is missing due to orbital geometry limitations). The 2 cm-λ reflectivity map correlates well with partial-global optical mosaics formed by the ISS and VIMS instruments, although stark areas of anticorrelation exist, perhaps suggesting a change in surface properties with depth. Because the RADAR instrument probes to decimeter depths and is sensitive to larger structural scales, while the optical instruments are more superficial and sensitive to smaller scales, the different reflectivity maps complement each other well and enhance geologic mapping and geophysical modeling efforts.

In general, we find that the measured radar reflectivities on Titan and Saturn’s icy satellites are often surprisingly large, especially when considered next to their thermal emissivity counterparts. The anomaly of the these measurements is difficult to understand with traditional rough surface scattering theory and perhaps indicates a more ordered structure to the surfaces of these moons (e.g. corner reflectors or rounded ice pebbles, see Janssen et al. (2011) and Le Gall et al. (2010) for examples).

The well-characterized radar dataset readily lends itself to more detailed applications, such as modeling the specular glints detected from Titan’s lakes in an effort to constrain the nature of wind-induced waves, or even converting the off-axis lake reflectivity into liquid depth maps.
CHAPTER 1. INTRODUCTION

1.3 Overview

This dissertation is structured as follows. We describe the operation of the RADAR instrument and its various modes (synthetic aperture imaging, altimetry, scatterometry, and radiometry) in Chapter 2. The nuances of the active modes are discussed, and in some cases the individual modes are decomposed into sub-modes. The chapter also includes a summary of radar observations of Titan and Saturn’s moons from ground-based instruments on Earth.

Chapter 3 describes the details of the real aperture radar processor that we develop and tune specifically for the RADAR system. We begin by introducing the reader to the area-extensive radar equation and the normalized radar cross section (NRCS) parameter that we ultimately measure. The real aperture radar equation is initially developed in its most common form, in terms of power quantities, but the variety of viewing geometries required for Cassini RADAR operation introduces complexities that quickly make this form unwieldy. We develop an alternative, much simpler form of the radar equation, in terms of energy quantities, and demonstrate the equivalence of the two perspectives. The simple energy form of the radar equation readily applies to the majority of the Titan RADAR data, but at large distances (>\sim 60,000 \text{ km}) and large incidence angles (>\sim 65^\circ), the effective-area approximation that we invoke no longer applies and integrals over the larger illuminated areas must be incorporated. The majority of the RADAR data collected for the icy satellites requires the modified version of the processor, as we discuss in Chapter 8 (note that, while Titan is a satellite composed of various ices, the term “icy satellite” refers to the other moons of Saturn).

A large component of the radar processing involves calibrating the measurements. We describe our calibration procedure, as well as certain mode-specific quantization corrections that we must apply, in the second half of Chapter 3. Chapter 3 concludes by discussing the errors and uncertainties in our measurements.
We provide a review of the existing theory of planetary surface scattering in Chapter 4. We introduce the reader to the concept of the backscatter function, or the relationship between NRCS and incidence angle. We describe the decomposition of the backscatter function into different scattering regimes, where quasispecular scattering dominates at low angles and diffuse scattering dominates at high angles. Several analytical quasispecular models exist that parameterize the physical characteristics of the surface. The models relate the shape of the backscatter function to the statistical roughness of the surface and the strength of the backscatter function to the dielectric constant of the surface. We consider a composite model comprising the linear combination of several of the most common quasispecular laws together with an empirical diffuse law. We describe our fitting technique to retrieve the surface parameters from the global collection of Titan backscatter function, as well as from the localized backscatter functions of specific surface features. The feature analysis results are presented and discussed in Chapter 5.

We use the best-fit composite model from the global collection of Titan data to correct the reflectivity measurements for incidence angles. In Chapter 6, we project the corrected result onto the surface, developing a weighting procedure to emphasize the higher resolution data. We present the individual maps created from each RADAR mode dataset, and we also present the global map created from the combined dataset. The global map yields more than 99.9% coverage of the surface of Titan and provides a good resource for geological modeling.

In Chapter 7, we present backscatter results related to Ontario Lacus, the largest lake in Titan’s south polar region. First, we describe the near-nadir altimetry data collected from the surface of the lake during the T49 flyby. These data represent the first clear detection of a specular glint from Titan’s liquid surfaces. The altimetry backscatter measurements are highly saturated within the radar receiver, but we develop a technique to partially correct the signal amplitude to retrieve a better estimate of the radar cross section (RCS). We then apply a specular roughness model to the RCS measurements to constrain the root-mean-square (rms) height of any surface
waves. Our results have direct implications for wind speed and wave generation on Titan’s liquid surfaces. We further investigate Ontario Lacus backscatter collected in the RADAR’s imaging and scatterometry modes during the T58 and T65 flybys. We apply a two-layer model to the backscatter measurements to constrain the depths of the liquid material across the entire lake. The resulting bathymetry map and height profiles depend strongly on the assumed dielectric properties of the liquid and the lake bed, so our results represent upper and lower bounds.

All of the work presented up to Chapter 8 focuses entirely on giant Titan. Yet, we have also collected RADAR data from 34 flybys of the other moons of Saturn, which provide valuable information about these moons. We list these flybys in Appendix C. Many of these data occur at such large distances that the echoes must be combined and filtered to produce a detectable signal that is larger than the noise. We have created a special frequency-domain processor for these distant data, but we do not discuss the results here in this work. Instead, we focus on icy satellite data collected on targeted flybys of Iapetus, Enceladus, and Rhea, at distances closer than 45,000 km. At this proximity, we can apply the real aperture processor developed for Titan, with some modifications to account for the increased surface curvature illuminated by the beam. The adapted processor yields nearly complete backscatter functions for these surfaces, to which we apply our backscatter modeling procedure. We compare the derived surface parameters to those derived for Titan’s surface features. We find that the reflectivities of Enceladus and Rhea are anomalously large, larger even than for Xanadu, the brightest feature on Titan. But the reflectivity of the dark Iapetus terrain closely resembles much of the dark terrain on Titan. The 2 cm-λ radar reflectivity measurements have implications for the structural maturity and compositional complexity of the targeted surfaces. They also complement the data collected at other wavelengths, and help to explain the geophysical development of the surfaces of these still mysterious moons and how they may be related to each other.
In addition to retrieving surface scatter parameters, we also use the best-fit backscatter models to correct the measured backscatter and produce real aperture reflectivity maps of Iapetus and Rhea. The real aperture images compliment the high-resolution synthetic aperture images that are formed from the same datasets. Furthermore, we use our derived backscatter models to correct the high-resolution images for incidence angle effects.

Some of the icy satellite observations use a frequency modulated chirp signal for transmission. We match filter these data to enhance the resolution and form range-Doppler images. In the last section of Chapter 8, we present range-Doppler images of five of the icy satellites.

Finally, in Chapter 9 we conclude with a synopsis of the presented work and opportunities for further development.

1.4 Contributions

The contributions of this work span a range of topics from specific details on data reduction to unique revelations about Titan’s largest southern lake, Ontario Lacus. Much of the work is centered around the measurement and characterization of the electrical and scattering properties of the surfaces of Saturn’s moons, with particular emphasis on the surface of Titan.

The principle contributions of this work are:

- Development of an absolutely calibrated real aperture radar processor for the Cassini RADAR instrument. The processor operates correctly on data collected in all active RADAR modes, over a variety of viewing geometries and receiver configurations. The processor produces a calibrated normalized radar cross section (NRCS, or $\sigma^0$) measurement for each radar burst transmitted. These data are stored in the Planetary Data System (PDS) for use by members of the scientific community.

- Formation of backscatter functions for a variety of surface features on Titan, as
well as features on Iapetus, Enceladus, and Rhea. The icy satellite functions require special consideration of the large distances and surface curvatures involved. We compare the forms of the backscatter functions to each other to demonstrate their diversity.

• Analysis of the surface backscatter functions using a variety of composite scattering laws. The retrieved model parameters are compared and carefully tabulated.

• Production of backscatter map products, including global reflectivity maps of Titan covering more than 99.9% of the surface and real aperture radar maps of Iapetus and Rhea. The 2 cm-\(\lambda\) reflectivity maps complement optical mosaics collected by Cassini’s ISS and VIMS instruments.

• Detection of a quasispecular scattering response on the dark leading hemisphere of Iapetus. Most icy satellite surfaces are dominated by volume scattering and any quasispecular scattering is negligible. The presence of quasispecular scattering on the dark terrain of Iapetus makes it similar to much of the Titan terrain.

• Production of first-stage range-Doppler images for several of Saturn’s icy satellites.

• Detection of the first specular glint from Titan’s liquids. Quantitative modeling of the measured backscatter tightly constrains the surface roughness of Ontario Lacus to less than 3 mm rms height.

• Production of bathymetry depth maps for Titan’s Ontario Lacus. Maps represent lower and upper bounds on lake depth for various assumed liquid properties.
Chapter 2

Cassini’s RADAR Instrument

2.1 The RADAR Instrument

Radar experiments first appeared around the turn of the twentieth century, but radar did not advance as a practical technology until the advent of World War II. The term radar is an acronym that stands for radio detection and ranging. It refers to the technique of transmitting pulses of radio waves or microwaves towards an object. A portion of the transmitted radiation is reflected by the object and received by the radar antenna. The received signal provides information about the range, altitude, direction, and speed of the object. In some cases, advanced post-processing of the relative motion data enables the formation of high-resolution radar images of the object. Further applications include interpreting the signal’s strength and angular scattering characteristics to gather information about the electrical properties of the object.

The radar instrument on the Cassini orbiter (called RADAR) comprises a set of microwave radar instrumentation primarily designed to investigate the surface of Titan. The spacecraft encounters Titan roughly once per month, but obtains close RADAR coverage only on selected passes. All RADAR equipment operate at a nominal wavelength of 2.18 cm (13.78 GHz), chosen for engineering reasons and also because microwave radiation readily penetrates Titan’s optically-thick atmosphere.
Standard operation of the Cassini RADAR consists of four unique data collection strategies. Starting at a distance of 100,000 km from the surface, it begins in radiometer mode, passively measuring the microwave emission radiating from the surface’s disk. Flying closer (9000-30,000 km), the scatterometer (SCAT) mode takes over, actively scanning the 4 meter high-gain antenna beam over the surface in a raster pattern to cover large areas as well as to sample the regional backscatter response. Closer yet, the altimeter (ALT) mode steers the beam towards nadir and records elevation profiles beneath the spacecraft with relative vertical accuracy between 90 and 150 meters and with horizontal resolutions of approximately 25 km. And around
closest approach (1000-5000 km), the SAR mode images swaths of the surface at resolutions between 300 m and 1 km. SAR imaging utilizes two different receiver filters depending on the observation geometry; for processing purposes, we qualify these as separate operational modes (L-SAR uses the lower bandwidth receiver filter and produces lower-resolution images than H-SAR). We illustrate this sequence in Figure 2.1. The observation sequence is often reversed on outbound to make a full RADAR Titan pass. The altitude distribution of each mode is plotted in Figure 2.2.

In addition to the four primary active modes listed above (SCAT, ALT, L-SAR, and H-SAR), there are occasionally two other modes that can push both SAR and scatterometry functionality out to larger distances than usual. Distant-SAR (D-SAR) mode produces images of 1 to 3 km resolution at altitudes between 10,000 and 25,000 km by using the lowest bandwidth receiver configuration (that of the scatterometer) to lengthen the size of the synthetic aperture and balance the azimuth resolution with
the range resolution (West et al., 2009). The second scatterometer mode, called the compressed scatterometer (C-SCAT) mode, can operate at altitudes between 25,000 and 100,000 km by combining the pulse echo data on board the spacecraft into a single echo profile to save data volume. This data collection strategy allows more pulses to be transmitted (>100 pulses as compared to the 8 pulses transmitted in standard scatterometry mode) without requiring more downlinked data volume. More pulses means higher SNR and better data quality.

During active RADAR operation, passive measurements are also acquired while the instrument awaits the return of echoes. These simultaneous passive and active measurements over the same antenna footprint region on the surface are essential to a thorough understanding of surface properties. Although we do not discuss the radiometric observations in this work, we have observed that the passive and active measurements are mostly anticorrelated (Wye et al., 2007), such that the radar bright regions are radiometrically cold and radar dark regions are radiometrically warm. This anticorrelation is consistent with the energy conservation relation that emission plus reflection equals unity. Janssen et al. (2011) quantitatively explore the energy-conservation relationship, called Kirchoff’s law of thermal radiation, for features on Titan. They find that radar-bright areas, such as Xanadu, can only satisfy the energy conservation relationship if the reflective characteristics of the putative volume scattering subsurface are highly constrained, or if there is organized structure on or in the surface that enhances the backscatter with respect to the passive emission.

All active modes use a same-sense linearly polarized five-beam radar antenna. The peak gain of the 4 meter parabolic antenna is 50.7 dB. A circular 0.37° width central beam is used by the scatterometer, radiometer, and altimeter, while the SAR modes use all five beams in a pushbroom geometry to increase coverage. Each radar mode observes Titan with a set of operational parameters unique to its experiment. All modes are designed for burst operation, transmitting a sequence of pulses that are linearly modulated in frequency, or “chirped”. The number of pulses, the length of
each pulse, and how the echo sequence falls within the receive window are different for each mode. Furthermore, the quantization method varies across the modes, with some using a Block Adaptive Quantization (BAQ; see Section 3.5) algorithm to conserve data rate. Signals entering the radar antenna are sampled at rates sufficient to preserve the bandwidths of the various modes.

The observational parameters of each active mode are also constrained according to the mode’s experiment. For instance, the SAR images present the finest surface detail available, revealing subtle changes in radar reflectivity for each resolution element on the surface. However, the SAR can only operate close to Titan (less than 4000 km in its low-resolution mode and less than 1500 km in its highest-resolution mode), and its Titan coverage is limited because it can only observe one narrow swath at a time (~1% of Titan’s surface is mapped in each flyby). Furthermore, its geometry is limited: it is restricted to a narrow range of incidence angles, between 25° and 45°, on either side of the nadir track. Similarly, altimetry mode operates only at incidence angles near nadir. Because of the restricted geometries, the inference of scattering mechanisms and surface electrical properties from SAR and altimetry data is limited.

On the other hand, the scatterometer mode’s operation supplements SAR and altimetry coverage. The large resolution elements (~100 km) obtained with the low scatterometer bandwidth provides a stronger reflected signal, increasing the maximum observational distance to about 30,000 km. From this larger distance, pattern-scanning the antenna across the surface, by slewing the spacecraft, permits imaging areas on the order of 5-10% of the surface in each pass, achieving near global coverage (more than 99% of Titan’s surface) at the time of this writing (through T71 on 7-Jul-2010, the beginning of the XXM). In addition to the larger surface coverage, scatterometry also allows backscatter measurements over a wider range of incidence angles, from near-nadir up to 80° or more. This angular coverage is crucial for parameterizing surface properties. The distributions of incidence angles used by the RADAR modes are illustrated in Figure 2.3

We summarize the operational and observational parameters for each of the six
Figure 2.3: Illustration of the distribution of incidence angles used by each of the six RADAR modes, using data from Ta through T71.

Table 2.1: RADAR Mode Parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>Bandwidth (kHz)</th>
<th>Sample Rate (kHz)</th>
<th>Time-Bandwidth Product</th>
<th>Altitude (1000 km)</th>
<th>Incidence Angles</th>
<th>BAQ Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCAT</td>
<td>117</td>
<td>250</td>
<td>500 ± 800</td>
<td>15.1 ± 35.2</td>
<td>0° ± 63°</td>
<td>8-8</td>
</tr>
<tr>
<td>C-SCAT</td>
<td>117</td>
<td>250</td>
<td>5300 ± 8800</td>
<td>26.9 ± 84.2</td>
<td>0° ± 67°</td>
<td>–</td>
</tr>
<tr>
<td>D-SAR</td>
<td>117</td>
<td>250</td>
<td>4200 ± 6900</td>
<td>11.0 ± 26.1</td>
<td>21° ± 58°</td>
<td>8-2</td>
</tr>
<tr>
<td>L-SAR</td>
<td>468</td>
<td>1000</td>
<td>5700 ± 10,200</td>
<td>2.5 ± 11.4</td>
<td>14° ± 29°</td>
<td>8-2</td>
</tr>
<tr>
<td>H-SAR</td>
<td>935</td>
<td>2000</td>
<td>4600 ± 9800</td>
<td>1.0 ± 3.9</td>
<td>14° ± 38°</td>
<td>8-2</td>
</tr>
<tr>
<td>ALT</td>
<td>4675</td>
<td>10,000</td>
<td>9500 ± 9500</td>
<td>3.1 ± 15.7</td>
<td>0° ± 1°</td>
<td>8-4</td>
</tr>
</tbody>
</table>

active RADAR modes in Table 2.1, where the range of parameters indicates the 10th to 90th percentiles.
In this work, we process each active mode of the Cassini RADAR instrument in its real aperture form by integrating the total received energy for each burst, ignoring the transmitted chirped waveform characteristics. This yields normalized radar cross sections (NRCS) values at beamwidth-sized resolutions. By processing all six active modes in a similar manner and calibrating them to the same scale, we can integrate the data into a single global backscatter map of Titan and also increase our multi-angle coverage of surface features for improved backscatter modeling. Analysis of the angular dependence of the radar cross section constrains the type and structure of surface materials by providing estimates of the dielectric constant, surface roughness and slopes over scales comparable to the wavelength.

The best parallel to our Titan Cassini RADAR experiment is the 12.6 cm-λ radar study performed by Campbell et al. (2003) and Black et al. (2011) using the Arecibo radio telescope in Puerto Rico. The Earth-based data measure the average scattering properties of the entire visible surface, but at a coarse resolution and at surface scales six times larger than our 2.2 cm-λ wavelength. We review the ground-based radar experiments in the next section and describe how they complement the more targeted studies performed by Cassini RADAR.

2.2 Ground-based Titan Radar Observations

Titan is a difficult radar target to observe from Earth. At about 8 AU (an AU, or astronomical unit, is the distance from the Earth to the Sun, or about 1.5e8 km, or 8.3 light minutes), its distance alone makes its echo extremely weak (the received echo power is proportional to the inverse of the distance to the fourth power). Only radio antennas with large collecting areas (such as the 305 meter diameter Arecibo telescope, or the 130 meter-equivalent Very Large Array) are sensitive enough to detect this signal. The large distance further causes Titan’s disk to appear very small in Earth’s sky; its diameter subtends an angle of only 0.8 arcseconds, which only very large antennas can resolve. The sensitivity issue can be mitigated by observing Titan
for long periods of time to improve the signal-to-noise ratio. However, the observation time is restricted to the amount of time that Titan spends above the Earth’s horizon, which depends on the positions of Saturn and Earth relative to the solar ecliptic. Saturn and Titan can spend almost two entire Earth decades at low declinations in the sky, during which it is only visible above the horizon for about 5 hours at a time (Muhleman et al., 1995). Since the round-trip travel time of the radar signal (to Titan and back) is about 2.5 hours, the integration time is limited to the difference, or just a little over 2 hours after accounting for calibration observations. The optimal ground-based observing conditions occur near Saturn opposition, when Titan is at its closest distance to Earth, and also when Titan is at northern-most declinations in Earth’s sky.

The first radar echo from Titan was detected in 1989 by Muhleman et al. (1990) using the VLA/Goldstone configuration. The 70-m Goldstone radio dish was used to transmit a 3.5 cm wavelength radio signal towards Titan and the VLA was used to receive the echo. Muhleman et al. (1995) repeated this experiment every year near the time of opposition from 1989 through 1993, although only the 1993 observation was free of possible pointing errors. In spite of the low declinations (Titan did not reach northern declinations near closest approach until 1997), Muhleman et al. (1995) detected the radar echo and measured the radar spectrum of Titan, albeit with a low signal-to-noise ratio of 3-5. Most notably, they determined that Titan’s spectral echo is broad, i.e. the scattering response with incidence angle is relatively flat (they measured an $n$ value of 1.4 for their $\cos^n$ power law fit to the Doppler spectral shape; see Section 4.3 for diffuse model details). In other words, Titan’s surface is radar rough and is not covered by a smooth, liquid reservoir of hydrocarbons, a popular post-Voyager theory that was hypothesized to explain the persisting presence of methane in the atmosphere (e.g. Lunine et al. (1983)). Furthermore, Muhleman et al. (1995) found that the strength of the radar echoes varies with Titan’s longitude, with the leading hemisphere appearing 50% more reflective than the trailing
hemisphere, again debunking the global-ocean theory. Muhleman et al. (1995) further estimated the mean 3.5 cm-λ radar albedo of Titan to be 0.125 ± 0.02 in the opposite-sense circular polarization (OC), a much smaller radar reflectivity than that of the Galilean satellites. The same-sense circular polarization (SC) albedo was generally much weaker than the OC albedo, or undetected, with the exception of two observations near 76° west longitude and 87° west longitude, when the SC albedo was larger than the OC albedo. We note that the initial albedo measurements of Muhleman et al. (1990) and Muhleman et al. (1993) were twice as large as those reported by Muhleman et al. (1995), apparently due to a doubling of the calibrator flux density by the VLA software.

Campbell et al. (2003) used the recently upgraded Arecibo 12.6 cm-wavelength system to observe Titan during opposition in 2001 and 2002. They discovered that small specular components are present in about 75% of the sub-radar locations sampled, but, like the Muhleman et al. (1995) results, they found that most of the echo power resides in the diffuse scattering component. Quasispecular scattering, or specular scattering from facets that are large relative to the wavelength, indicates the presence of regions that are smooth at scales greater than the wavelength; Campbell et al. (2003) use a Hagfors’ model to measure a root mean square (rms) surface slope for these regions that varies between 0.5° and 3.5°. Their model further requires low Fresnel reflection coefficients, with dielectric constants that vary between 1.4 and 2.2. A straightforward interpretation of these results implies the presence of liquid hydrocarbons over 75% of Titan’s equatorial region, an interpretation that is inconsistent with subsequent Cassini imagery. A fractal-based reanalysis of the Arecibo data by Sultan-Salem (2006) suggest that the quasispecular properties are more consistent with solid hydrocarbons.

Black et al. (2011) continued the annual Arecibo observation campaign through the opposition of 2008, after which Titan’s declination fell below that which is visible to the Arecibo Observatory. Because the Arecibo’s view of the sky is limited to zenith angles less than 20°, the experiment affords only 30 minutes of integration time, after
allowing for the 2h:15m round-trip travel time of the radar signal. In some cases, the
echo is also received by the 100-m Green Bank Telescope (GBT) for the full round-trip
travel time. In spite of the longer integration times, the smaller collection area of the
GBT limits the signal-to-noise ratio to only about half of that obtained by Arecibo,
or about <300 versus <650 (Black et al., 2011). But the longer integration times
 correspond to a longer sub-radar track on Titan’s surface, permitting the study of
larger areas (∼100 km is visible in a 2.5 hour observation versus the ∼20 km sampled
in a 30 minute observation).

Black et al. (2011) measure mean OC radar albedos of 0.16 and mean SC albedos
of 0.07, and thus a polarization ratio near 50%. The strength of the radar albedo
and the circular polarization ratio are frequently used as an indicator of the degree
and type of diffuse scattering. Diffuse scattering can originate from wavelength-
scale surface structure or multiple scattering from the surface or sub-surface volume.
Large radar albedos (greater than unity) combined with high polarization ratios (near
unity or higher), as observed for the Galilean satellites and some of the icy moons of
Saturn, are commonly thought to arise mainly from diffuse volume scattering (Black
et al., 2007; Ostro, 1993). The intermediate polarization ratio of Titan suggests
that volume scattering is contributing significantly to the echo, but the low radar
albedo values indicate that the volume scattering mechanism is less efficient than
on the other icy moons. Volume scattering requires that the medium be sufficiently
transparent, as with clean water ice, and also structurally mature to properly enable
higher order multiple scattering within the near sub-surface. Thus, the intermediate
volume scattering signatures on Titan suggest either that the sub-surface has greater
radar absorptivity than the other icy moons, or that there are structural differences
resulting in fewer sub-surface scattering centers.

The prevalence of quasispecular scattering signatures on Titan indicates that sur-
face scatter plays a greater relative role on Titan than on other icy solar system
bodies; since specular scattering is notably absent from radar observations of other
icy moons. Black et al. (2011) summarize their observations by noting that Titan
appears to be a hybrid to the 12.6 cm-\(\lambda\) radar: somewhere between the likeness of the rocky inner Solar System surfaces that are often dominated by surface scatter and the bright icy outer Solar System surfaces that are almost purely diffuse volume scatter.

### 2.2.1 Earth-based vs. Cassini radar observations

The 12.6 cm-\(\lambda\) albedo values are slightly larger than, but comparable to, the 3.5 cm-\(\lambda\) albedo values reported by Muhleman et al. (1995), suggesting that the diffuse scatter on Titan is largely frequency independent. On the contrary, our 2.2 cm-\(\lambda\) total power measurements for the Cassini RADAR, presented in Chapter 5, suggest radar albedo values that are larger than those obtained at 3.5 cm-\(\lambda\) and 12.6 cm-\(\lambda\). The discrepancy in the data from the two comparable wavelengths, 3.5 cm-\(\lambda\) and 2.2 cm-\(\lambda\), is difficult to reconcile. We note that the 3.5 cm-\(\lambda\) measurements are extremely weak compared to those made with Arecibo at 12.6 cm-\(\lambda\), with no visible specular component, and there also exists very large scatter among the data. The confusion over possible pointing errors and the presence of a calibrator flux density error in the initial VLA software makes the 3.5 cm-\(\lambda\) data further suspect. In the work that we present here, we constrain our comparison of results to those acquired by the 12.6 cm-\(\lambda\) Arecibo studies.

The larger surface scales and depths sensed by the 12.6 cm Arecibo wavelength complement those sensed by the Cassini RADAR 2.2 cm wavelength. In a mature clean icy regolith, radar will sound to depths of 10 to 20 wavelengths (Black et al., 2001), that is 2 to 5 decimeters for the Cassini radar and 1 to 2.5 meters for the Arecibo radar. Thus, any differences in measured reflectivity values at the two wavelengths reflect differences in radar transparency or structural heterogeneity with depth, or even a material whose absorption length is highly dependent on wavelength. Ostro et al. (2006) make the case that the structural requirements for efficient volume scattering are easily attained at both centimeter and decimeter scales, suggesting that “composition probably trumps structure as the source of radar albedo variation.” Ostro et al. (2006) list plausible candidates for water ice contaminants to be silicates,
metal oxides, ammonia, and polar organics, where ammonia is the only contaminant that would not reduce the visual albedo as it does the radar albedo.

Black et al. (2011) sense the radar echoes in both the same and opposite sense of circular polarization as that transmitted, yielding total power 12.6 cm-λ disk integrated reflectivity estimates (TP-13), or albedos, for many sub-radar Titan longitudes. Our 2.2 cm-λ disk integrated reflectivity estimates are only received in the same linear sense as that transmitted (SL-2). For comparison to the 12.6 cm-λ results, we must convert our SL-2 estimates to total-power (TP-2) estimates, as described in Section 4.6. We find that our TP-2 estimates are larger than the TP-13 estimates, sometimes by more than a factor of two. This difference does not stop at Titan, but rather it appears to be a general trend with most of the Saturnian satellites (Black et al., 2007). The absolute errors will be different between the datasets (the Arecibo radar results are calibrated with a systematic error of about 25% and the Cassini radar results are calibrated with a systematic error of about 12%), however there does not appear to be a single factor that can correct the results to the same values. Thus, the decrease in radar reflectivity with increasing wavelength likely reflects true surface variations with depth, such as increasing absorption, thereby helping to constrain the effective scattering layer of each moon.

The depth implications of the dual wavelength comparison originate largely from the diffuse scattering observations. Earth-based radar measures the mean diffuse scattering properties of the entire visible hemisphere, while Cassini RADAR is capable of assimilating the diffuse scattering measurements of specific terrains on Titan. The quasispecular scattering observations are more evenly matched in spatial resolution. Earth-based radar measures the quasispecular scattering properties of the sub-radar point, the point at the center of the visible disk, with a frequency resolution equivalent to 20-140 km on the surface (Black et al., 2011), similar to those surface resolutions obtained by the RADAR scatterometer mode at near-nadir angles. Quasispecular measurements are sensitive to surface scattering at scales comparable to the wavelength, thus the surface slopes retrieved by the Arecibo models refer to scales six
times larger than those relevant to the Cassini RADAR. In Chapter 5, we find that the rms slopes measured at 12.6 cm-λ are significantly lower than those measured at 2.2 cm-λ, supporting the long-standing observation that inferred rms slopes are strongly wavelength dependent, where surfaces are smoother at longer wavelengths (Muhleman, 1964; Simpson and Tyler, 1982). Sultan-Salem (2006) uses a fractal-based quasispecular law to demonstrate the horizontal scale dependence of the rms slope. Using the Cassini RADAR Titan dataset, he finds an rms slope near 10° at the horizontal scale corresponding to the Cassini wavelength, and an rms slope near 3° at the scale corresponding to the Arecibo wavelength, thus, in effect, explaining the differences in the reported results. It is also possible that the Arecibo modeling approach is most sensitive to the narrow quasispecular peak closest to nadir, which is of lower rms slope than the broader quasispecular component. Our approach for the Cassini data incorporates both quasispecular components, where the composite result is determined primarily by the broad quasispecular component (see Section 4.4). The dielectric constants retrieved for both wavelengths are comparable in value.

Aside from providing a wavelength baseline to help constrain the scattering properties of the Saturnian moons along three dimensional scales, the Cassini real aperture radar experiment contributes significantly in terms of spatial coverage; one goal is to recover and analyze the scattering response of localized features across most of the icy satellite globes. For example, Earth-based radar can only sense the quasispecular scattering component near the equator of Titan. Over time, with observations centered at many longitudes, the data can be aligned to produce estimates of the backscatter response along the equatorial belt of Titan, but with large north-south resolution widths. Cassini RADAR, on the other hand, is not as constrained in geometry and can sense all parts of the Titan’s globe at various resolutions and incidence angles. The variety of observational geometries allows us to create global 2.2 cm-λ RADAR reflectivity maps of Titan (Chapter 6), as well as backscatter responses recovered from specific surface features (Chapter 5).
Chapter 3

Cassini Real Aperture Radar Processor

In this chapter, we describe our procedure for estimating normalized radar cross section values using the Cassini RADAR instrument and real aperture processing. Real aperture radar processing is the integration of the echo power over the natural resolution cell of the radar, or essentially the beam footprint. While this processing is simple in theory, the actual system hardware and observation viewing geometry complicates its application to the Cassini RADAR data. This chapter discusses the difficulties encountered and the solutions derived.

We begin by developing the radar equation for an area-extensive target. This formulation requires that we properly define the contributing surface area, which will be either beam-limited or pulse-limited, depending on the viewing geometry. We then explain our data reduction procedure and how we account for overlapping pulses within the echo sequence as well as lost energy that falls outside of the receive window boundary. We characterize the noise response of the different RADAR receiver modes. We develop a calibration model that yields the equivalent system noise temperature for each of the receiver configurations. We conclude by characterizing the uncertainties inherent to the radar measurements.
3.1 The Radar Equation

The radar equation describes the fundamental relationships between the received echo signal and the radar instrument properties, the viewing geometry, and the target scattering characteristics. When the transmitting radar and the receiving radar are the same, such that they are effectively collocated, the radar equation takes its most common form,

\[
P_s \propto \frac{P_t G_t}{4\pi R^2} \sigma \frac{1}{4\pi R^2} A_r.
\]  

Eq. 3.1 is called the monostatic radar equation. Here, \( P_s \) represents the received echo signal power and is calculated from the total received power, \( P_r \), by removing the background noise contribution: \( P_s = P_r - P_n \), where \( P_n \) is the mean noise power. \( P_t \) is the total transmitted power (48.084 W), \( G_t \) is the on-axis antenna gain (50.7 dB), \( A_r \) is the effective aperture area of the receiving antenna (4.43 m\(^2\)), and \( R \) is the distance, or range, between the radar and the target. The terms in Eq. 3.1 can be simplified by considering the relationship between the effective area of an antenna and its gain:

\[
A_r = \frac{\lambda^2 G_t}{4\pi}.
\]  

The radar equation describes how the total power transmitted towards the target, \( P_t G_t \), is modeled as an isotropic spherical wave, incurring a spatial attenuation or spherical spreading loss \( 1/4\pi R^2 \) as the power spreads outwards from the antenna over a sphere of radius \( R \). This power density is intercepted by the target with an effective collecting area, \( \sigma \), and the intercepted power is then re-radiated by the target in various directions. The incident power that is not collected and re-radiated by the target is absorbed. The power re-radiated in the direction of the receiver is assumed to be the power level that would be radiated isotropically and thus incurs new spatial spreading losses. Finally, this power is intercepted by the effective collecting area of
the receiving antenna resulting in the measured received signal power.

3.1.1 Radar Cross Section: \( \sigma \)

The radar cross section (RCS) term models the scattering phenomena of collecting and re-radiating the incident power. The RCS will generally not be the same as the physical cross section, since a good portion of the incident energy may be absorbed by the target, and the target may also preferentially scatter power into certain directions. The RCS is more precisely defined as the hypothetical area required to intercept the incident power density at the target such that, if the collected power were re-radiated isotropically, it would reproduce the power density observed at the receiving radar. Quantitatively, the RCS is defined as

\[
\sigma = 4\pi R^2 \frac{S_s}{S_i} = 4\pi R^2 \frac{|E_s|^2}{|E_i|^2},
\]

(3.3)

where \( S_i \) is the incident power density measured at the target and \( S_s \) is the scattered power density measured at a distance \( R \) from the target. Similarly, \( E_i \) and \( E_s \) are the incident and scattered field amplitudes. The inclusion of the \( 4\pi R^2 \) scale factor in the definition of RCS indicates that the scattered power density received at the radar is presumed to have originated from an isotropic point source. In other words, the scatterer is modeled as an isotropic point source whether or not it really is one. The product \( 4\pi R^2 \times S_s \) then represents the total power that would have originated from the hypothetical point source. When this total reradiated power is divided by the incident power density in Eq. 3.3, the effective capture area of the target results. The RCS has units of \( \text{m}^2 \) and is intended to give a range-independent estimate of the perceived scattering strength.

The RCS is an inherent property of the target. It is determined primarily by the target’s reflectivity (controlled by its dielectric properties) and the target’s directivity (controlled by its physical structure, such as its size and shape, at scales relative to the illuminating wavelength). The RCS will also depend on the illuminating wavelength,
viewing geometry, and polarization configuration, but is otherwise independent of the radar system. The viewing geometry describes the illumination direction and the orientation of the target with respect to the radar. Proper characterization of the RCS’s response to incidence and azimuth angle variation helps to eliminate the viewing geometry dependence.

3.1.2 Normalized Radar Cross Section: $\sigma^0$

For area-extensive targets, the value of the RCS also depends on the size of the area illuminated by the radar antenna beam, which, in turn, depends on the range from the target to the radar. The larger the illuminated area, the more scatterers there are contributing to the scattered wave, and thus the larger the apparent RCS. Consequently, it is more revealing to define the RCS per unit area on the target’s surface. This quantity is called the normalized radar cross section (NRCS), or, sometimes, the differential or specific scattering coefficient. The NRCS is defined as the RCS normalized by the contributing surface area, or the area resolved by the system, $A_{\text{res}}$. Thus, the NRCS is dimensionless. We distinguish the NRCS symbol from the RCS symbol $\sigma$ by adding a superscript zero: $\sigma^0$. Thus, from Eq. 3.1, and incorporating the relationship from Eq. 3.2, we now have

$$P_s = \frac{P_t G_t^2 \lambda^2 \sigma^0 A_{\text{res}}}{(4\pi)^3 R^4}. \quad (3.4)$$

3.1.3 Real Aperture Radar Equation

For high-resolution synthetic aperture radar (SAR) imaging, the $A_{\text{res}}$ contributing to each pixel is generally small enough that the gain and range are roughly constant across the area, and Eq. 3.4 then holds at the pixel-level. For the real aperture radar (RAR) calculations that we focus on in this work, the received echo power originates from a much larger $A_{\text{res}}$ that is comparable to the size of the half-power beam footprint on the ground. This area is between ten and a few hundred kilometers in diameter for observations of Titan’s surface. The scatterers contributing to the received RAR echo
3.1. THE RADAR EQUATION

will be at different distances and will experience different antenna gains depending on their location within the beam. Similar to Ulaby et al. (1986), we divide $A_{\text{res}}$ into individual elements, $dA$, over each of which the gain and range are roughly constant. Then, as the element area goes to zero, the area-extensive radar equation takes the form

$$P_s = \frac{P_t G_t^2 \lambda^2}{(4\pi)^3} \int_{A_{\text{res}}} \frac{g^2 \sigma^0 dA}{R^4},$$

(3.5)

where $g$ is the relative gain of the antenna pattern at surface element $dA$, normalized by the peak gain $G_t$, and $R$ is the range to that element. We cannot resolve the received echo power within $A_{\text{res}}$, and thus we cannot know how $\sigma^0$ varies over this area. Instead, we factor $\sigma^0$ out of the integral and estimate a quantity that is the apparent average $\sigma^0$, which we define as

$$\sigma^0_{\text{avg}} = \frac{\int_{A_{\text{res}}} g^2 \sigma^0 dA}{\int_{A_{\text{res}}} g^2 dA / R^4}. \quad (3.6)$$

Substituting this definition into Eq. 3.5, and also noting that the relative change in range over the beam is insignificant compared to the change in gain, especially at the larger distances from which we observe, the real-aperture radar equation becomes:

$$P_s = \frac{P_t G_t^2 \lambda^2}{(4\pi)^3 R_0^4} \sigma^0_{\text{avg}} \int_{A_{\text{res}}} g^2 dA,$$

(3.7)

where $R_0$ is the range from the radar to the surface along the boresight, the main axis of the beam. The integral in Eq. 3.7 approximates an effective resolution area, $A'_{\text{res}}$, leading us to the near-final form of the real aperture radar equation:

$$P_s = \frac{P_t G_t^2 \lambda^2 \sigma^0_{\text{avg}} A'_{\text{res}}}{(4\pi)^3 R_0^4}. \quad (3.8)$$
Here, $A'_{\text{res}}$ is readily evaluated from knowledge of the antenna gain pattern together with knowledge of $A_{\text{res}}$. In Section 3.2.1, we define $A'_{\text{res}}$ in terms of two orthogonal components, one of which is subject to viewing angle effects and may be influenced by the pulsed nature of the transmitted signal. The form of the real aperture radar equation presented in Eq. 3.8 describes the relationship between the echo power measured at each individual receive sample and the NRCS of the resolved surface. A single power measurement, however, will have a large uncertainty due to inherent multiplicative speckle noise (see Section 3.6). We reduce this uncertainty and improve our estimate of the NRCS by averaging many measurements together, exploiting the redundancy in Cassini RADAR pulse transmissions. The Cassini RADAR transmits multiple pulses in a single radar burst, and, because the illuminated surface does not change significantly between pulses, the samples track across the pulse echoes; e.g. the kth sample of each pulse echo originates from essentially the same resolution cell on the surface. We could reduce the uncertainty of each sample measurement within the echo profile and simultaneously preserve the sample’s resolution by stacking the pulse echoes and averaging down the stack. Averaging $N$ pulses, which are presumed independent of each other, will reduce the statistical variance by a factor of $1/N$. $N$ is $\sim 50$ for SAR observations, but $N$ is only 8 and 15 for standard scatterometry and altimetry observations, respectively. Consequently, we choose to also average along the echo profile, i.e. average the echo samples together over the entire burst. This procedure gives us a single backscatter measurement for each radar burst, rather than multiple measurements along the echo profile, but the final measurement will have improved accuracy. We detail the measurement uncertainties obtained with this method in Section 3.6.

Thus, $P_s$ in Eq. 3.8 represents the average burst echo signal power rather than an individual echo sample power. We find that the average signal power for a pulsed system with a variety of viewing geometries is more complicated to compute than might be expected, as described in the next section. Given these complications, we develop an alternative form for Eq. 3.8 that is easier to apply to the Cassini RADAR.
real aperture experiment. The alternative form expresses the radar equation in terms of energy, rather than power, so that the intricacies of the pulsed power echoes can be largely ignored. The energy form of Eq. 3.8 becomes

\[ E_s = \frac{E_t F_c G_t^2 \lambda^2 \sigma_{0 \text{avg}}^0 A_{b\text{-eff}}}{(4\pi)^3 R_0^4}, \]  

(3.9)

where \( E_s \) is the total echo signal energy, \( E_t \) is the total transmitted energy, \( F_c \) is a correction factor that accounts for any lost echo energy falling outside of the receive window (see Section 3.2.2), and \( A_{b\text{-eff}} \) is the resolution area defined by the antenna’s effective beamwidth (see Section 3.2.1). Eq. 3.9 can be derived from first principles, but we demonstrate its equivalence to Eq. 3.8 in Section 3.2.5.

### 3.2 Data Reduction

Let us now consider how to apply the radar equations developed in Eq. 3.8 and Eq. 3.9 to the collected Cassini RADAR data. The Cassini RADAR receive window consists of \( N_{\text{rxw}} \) real voltage samples in Offset Video format (as opposed to complex pairs of samples in In-phase/Quadrature format). The analog-to-digital converter samples the data at a rate \( f_{\text{adc}} \) and the samples are spaced \( T_{\text{adc}} \) seconds apart. All data are quantized to 8 bits in their final form. However, depending on the radar mode, the data may first be compressed to 2 or 4 bits aboard the spacecraft and then decoded to 8 bits after downlink using a block adaptive quantizing algorithm. As a result, the data are not necessarily constrained to the expected dynamic range of \(-127.5\) to \(+127.5\) (see Section 3.5).

If \( V_{k\text{dv}} \) represents the kth sample voltage at the output of the receiver, then its square represents the received kth sample power, \( P_{k\text{dw}} \). As stated previously, we want to average the measurements together over the whole burst for improved accuracy.
The average received power is the mean of the $N_{rxw}$ sample powers, or

$$P_{rdW} = \frac{1}{N_{rxw}} \sum_{k=1}^{N_{rxw}} V_{k_dV}^2 = \frac{1}{N_{rxw}} \sum_{k=1}^{N_{rxw}} P_{k_dW} = \frac{1}{N_{rxw}} E_{r_dJ},$$  \hspace{1cm} (3.10)

where we recognize that the sum of the $N_{rxw}$ discrete sample powers within the receive window is equivalent to the total received energy $E_{r_dJ}$ (integrating power over time yields energy). The italicized second subscripts indicate that the physical quantity at the receiver input has been digitized, so that now voltage has units of $dV$, or digitized volts, power has units of $dW$, or digitized watts, and energy has units of $dJ$, or digitized Joules. We must relate the digitized units to the physical units before we can apply the radar equation. This is done using a calibration scale factor, e.g. $P_r = C \times P_{rdW}$. The calibration procedure is described in Section 3.4.

As pictured in Figure 3.1, the received signal is a sequence of pulse echoes that are typically interspersed with noise (called interpulse noise) and often have intervals of noise at the front and back of the sequence (called leading and trailing noise). Furthermore, the pulse echoes themselves reside on top of a floor of background noise. Thus, the average received power measured in Eq. 3.10 is the sum of the average noise power $P_{n_dW}$ and the average echo signal power $P_{s_dW}$, where the latter is weighted by the effective duration of the echo sequence ($N_{eseq}$) relative to the duration of the receive window ($N_{rxw}$):

$$P_{rdW} = P_{n_dW} + P_{s_dW} \times \frac{N_{eseq}}{N_{rxw}},$$  \hspace{1cm} (3.11)

Eq. 3.11 is best visualized by considering the combination of noise and signal in terms
Figure 3.1: We show the actual receive window for a T8-inbound scatterometry burst of 8.4° incidence, where we have applied a boxcar averaging filter 10 samples wide to smooth the signal for clarity. We detect the echo signal by correlating the transmitted pulse sequence with the received signal. The detected echo sequence is marked in red, where the width of each received echo is effectively equal to the pulse length τ, as demonstrated in Figure 3.10 for low incidence. The echo signal sits on top of a noise floor with measured power $P_n$, where $P_n$ is measured from the leading and trailing noise surrounding the echo sequence. The total received energy $E_r = P_r N_{rxw}$ is readily measured (the area under the pink line) and the total measured noise energy ($P_n N_{rxw}$, the area under the blue line) is subtracted from this to estimate the total echo signal energy $E_s$. From $E_s$ we determine $P_s$ by knowing the number of pulses received (here, $K_{prx} = 8$) and the effective width of each pulse, $N'_{echo}$. $N'_{echo}$ is defined in the block-echo model as the width of the pulse echo that gives height $P_s$ and area $E_s$.

of energy:

$$E_{t,dj} = E_{n,dj} + E_{s,dj}$$

$$P_{r,dw} N_{rxw} = P_{n,dw} N_{rxw} + P_{s,dw} N_{seq}.$$  

We measure $P_{n,dw}$ and $E_{n,dj}$ by considering the noise-only intervals within the
bursts. Because noise-only intervals are not always present in a burst, we average noise powers together across equivalent bursts (bursts with equivalent receiver configurations) prior to processing the data. If $V_{n_i\,dV}$ is the $i$th noise-only voltage sample, $P_{n\,dW}$ is measured as follows:

$$P_{n\,dW} = \frac{1}{N_n} \sum_{i=1}^{N_n} V_{n_i\,dV}^2.$$  \hspace{1cm} (3.13)

$E_{n\,dJ}$ follows from scaling $P_{n\,dW}$ by the length of the receive window: $E_{n\,dJ} = N_{rxw} P_{n\,dW}$.

We describe our noise estimation procedure more thoroughly in Section 3.3.

With measurements of $P_{r\,dW}$ and $P_{n\,dW}$ ($E_{r\,dJ}$ and $E_{n\,dJ}$) in hand, we are left to determine $P_{s\,dW}$ ($E_{s\,dJ}$). $E_{s\,dJ}$ readily results from Eq. 3.12. Thus, we can directly solve Eq. 3.9, the energy form of the radar equation, for the burst NRCS once we determine $A_{b\,eff}$ and $F_c$. We describe each of these determinations in turn in the succeeding sections.

Evaluating $P_{s\,dW}$ proves more complicated. From Eq. 3.11, we see that to find $P_{s\,dW}$ we need to first find $N_{eseq}$, the effective duration of the echo sequence. The echo sequence comprises $K_{prx}$ pulse echoes and each pulse echo is $N'_{echo}$ samples wide, thus

$$N_{eseq} = K_{prx} N'_{echo}.$$  \hspace{1cm} (3.14)

$K_{prx}$ may be less than or equal to the number of pulses transmitted $K_{ptx}$, depending on the duration of the receive window (RXW) relative to the total spread of the echo sequence and how the receive window is positioned relative to the start of the echo sequence. We calculate a correction factor $F_c$ that relates $K_{prx}$ to $K_{ptx}$, such that $K_{prx} = K_{ptx} F_c$. This is the same $F_c$ needed to relate $E_{s\,dJ}$ to $E_{t\,dJ}$ in Eq. 3.9, which we quantify in Section 3.2.2.

The effective duration of an individual pulse echo $N'_{echo}$ will depend on the viewing geometry and whether the pulse or the beam dominates the echo behavior. We explain the dependence and the calculation of $N'_{echo}$ in Section 3.2.3.

Once we quantify $K_{prx}$ and $N'_{echo}$, we can solve for $P_{s\,dW}$. Figure 3.1 illustrates the
calculation of $P_{sdW}$ for a real echo signal received during the T8-inbound scatterometry scan once $K_{prx}$ and $N_{echo}'$ are known. However, to solve for the burst NRCS in Eq. 3.8, the power form of the radar equation, we still need to determine the effective resolution area $A_{res}'$. In the next section, we quantify $A_{res}'$ for the Cassini real aperture system and then show the reduction to $A_{b-eff}$ in Section 3.2.5.

### 3.2.1 Real Aperture Resolution

$A_{res}'$ defines the target area over which individual scattered signals cannot be distinguished. These scattered signals combine to form the power measured at a particular sample in time. In a pulsed real aperture system, $A_{res}'$ is determined by the projection of the effective antenna beamwidth onto the target surface, except when the pulse projection onto the surface is smaller, such as occurs at larger viewing angles. The mean power quantity that we are interested in, $P_{sdW}$, represents an accurate measurement of the sample power and follows the same resolution considerations. However, because we are averaging echo samples together over the burst to determine $P_{sdW}$, we might intuit the corresponding resolution area to be constrained solely by the antenna beam ($A_{res}' = A_{b-eff}$); i.e. any pulse-defined resolution cells blur together.

In many cases, we can define the real aperture radar (RAR) resolution area as the product of the cross-range, or azimuth, resolution, $\delta a$, and the range resolution, $\delta r$, 

$$A_{res} = \delta a \cdot \delta r, \quad (3.15)$$

where the range direction coincides with the plane of incidence, and the cross-range direction is that which is perpendicular to the plane of incidence. The plane of incidence is the plane spanned by the target surface normal and the propagation vector of the transmitted wave. The range and cross-range dimensions are affected differently by changes in the viewing geometry, so choosing a rectangular resolution cell allows us to separate the viewing effects and evaluate the resolution area more simply. The actual shape of the resolution cell will be somewhere between an ellipse
and a trapezoid, depending on whether the beam or the pulse is limiting the range resolution, but the rectangular approximation holds as long as the area is kept the same.

To evaluate the effective beam resolution area (see Eq. 3.8) and eliminate the antenna pattern dependence, we solve for an effective antenna beamwidth, $\theta_{b\text{-eff}}$, for which the power is constant (set to the on-axis peak power gain, $G_t$) within the effective beam and zero without. The details of this calculation are presented in Appendix A. We calculate the rectangular effective beamwidth to be $0.29^\circ \times 0.29^\circ$ for the Cassini antenna pattern. As described in Section 2.1, the Cassini antenna is a 4 m diameter parabolic antenna with a 3-dB beamwidth of $0.37^\circ$ and a peak gain of 50.7 dB. The effective beam defines a beam-illuminated area on the ground, $A_{b\text{-eff}}$. $A_{b\text{-eff}}$ is the product of the ground resolutions of the effective beamwidth in the cross-range and range dimensions:

$$A_{b\text{-eff}} = \delta a_{b\text{-eff}} \times \delta r_{b\text{-eff}}.$$  \hspace{1cm} (3.16)

This is the form of the resolution area equation required to solve the energy form of the radar equation given in Eq. 3.9, and ultimately the average-power form of the radar equation given in Eq. 3.8 (see Section 3.2.5).

At low incidence angle radar configurations, the antenna beam will determine the real aperture resolution, so the effective resolution area will be the same as the effective beam-illuminated area ($A'_{\text{res}} = A_{b\text{-eff}}$). This resolution regime is termed “beam-limited”. At higher incidence angle radar configurations, the pulse duration will influence the real-aperture resolution, but in the range direction only. In this case, the effective resolution area, $A'_{\text{res}}$, will depend on the effective beamwidth in the cross-range direction and the pulse length in the range direction ($A'_{\text{res}} = \delta a_{b\text{-eff}} \times \delta r_{\text{pulse}}$). This resolution regime is termed “pulse-limited”. In the next sections, we illustrate the dependence of $A_{\text{res}}$ on viewing geometry and describe how to evaluate the effective resolution components in the cross-range and range dimensions. Because the Cassini RADAR viewing distance is typically much greater than the 2575 km radius of Titan,
3.2. DATA REDUCTION

Figure 3.2: (a) The cross-range, or azimuth, resolution is defined by the effective antenna beamwidth and the distance of the radar from the target. For clarity, (a) is rotated upwards, into the page, so that $\delta a$ appears to be directly below the spacecraft, but, in reality, it is offset from the nadir point by the viewing angle. (b) The range resolution incurs projection effects at off-nadir viewing angles and is thus a function of viewing angle in addition to distance. The real-aperture range resolution will either be defined by the effective antenna beamwidth or the pulse length, depending on the limiting case. (a) and (b) are viewed from planes perpendicular to each other.

we pay careful attention to how these resolution components map onto a curved surface.

3.2.1.1 Cross-Range Resolution

The resolution element in the cross-range direction is a simple projection of the antenna beam onto the ground (Figure 3.2a; for clarity, the target sphere in this cross-range view is rotated upwards, into the page, so that $\delta a$ appears to be directly below the spacecraft). It depends only on the viewing distance: an increasing distance spreads the beam proportionally over a larger area. The viewing distance, or slant
range, $R_0$, is the same as before, the distance from the radar to the target surface along the boresight. Accounting for the surface curvature, the cross-range resolution is readily computed as the arc length that subtends a target body centered angle $\alpha_a$, where $\alpha_a$ matches the spread of the effective antenna beam at the surface ($\theta_{b-eff} = 0.29^\circ$, the rectangular solution presented in Appendix A). The cross-range resolution then takes the form

$$\delta a = \delta a_{b-eff} = R_t \alpha_a = 2R_t \arctan \left( \frac{R_0}{R_t} \tan \frac{\theta_{b-eff}}{2} \right) \approx R_0 \theta_{b-eff}.$$  

The approximation $\delta a \approx R_0 \theta_{b-eff}$ holds when the viewing distance is comparable to the target radius, $R_t$; beyond this, the error increases quadratically. The approximation error would be only 0.1% at a slant range of 46,000 km (18 Titan radii) for observations of Titan. Most Cassini RADAR observations occur at distances much smaller than this, but we implement the full $\delta a$ evaluation nonetheless.

In Figure 3.3, we plot the distribution of cross-range resolution for each of the six Cassini RADAR operational modes, which were introduced and described in Section 2.1. The distributions are derived from data collected from the first Titan flyby of the prime mission (TA; 26-October-2004) through the first Titan flyby of the extended-extended mission (T71; 7-July-2010). The panels are sequenced in order of increasing resolution, or spacecraft altitude. The real-aperture cross-range resolution varies from as low as 5 km in the high-SAR mode to as high as 500 km in the compressed scatterometer mode.

### 3.2.1.2 Beam-Limited Range Resolution

In the range dimension, the resolution is highly dependent on the viewing angle in addition to the viewing distance. The range cell is effectively stretched over the surface,
Figure 3.3: The distribution of cross-range resolution, or azimuth resolution, is plotted for each of the six modes of the RADAR instrument. The distribution is derived from TA-T71 data. The abscissa is in units of kilometers, and the ordinate is the frequency count normalized by the total number of bursts collected in that mode, and subsequently scaled into a percentage.

as the farthest defining edge vector, \( R_f \), must travel further than the nearest defining edge vector, \( R_n \), before it intersects the target (Figure 3.2b). For a flat surface, this projection effect would mean a lengthening of \( 1 / \cos \theta_i \) beyond the spreading introduced by \( R_0 \). For a curved surface, the resolution length is properly evaluated as the arc length that subtends a target body centered angle \( \alpha_r \), where \( \alpha_r \) is the difference in the angles subtended by the near and far range intersection points. Thus, we have

\[
\delta r = R_t \alpha_r \quad (3.18)
\]

\[
= R_t (\alpha_t - \alpha_n).
\]

To evaluate \( \alpha_n \) and \( \alpha_f \), we consider that we already know the radar distance from
the target body center, $R_{tbc}$, from the spacecraft ephemeris data. We also know the pointing vector of the antenna main axis, which is at a look angle $\theta_L$ relative to the $R_{tbc}$ vector. In the case where the antenna beam controls the range resolution (the range resolution is “beam-limited”, and $\delta r = \delta r_{b\text{-eff}}$), then the defining vectors $R_n$ and $R_f$ must be separated from each other by $\theta_{b\text{-eff}}$ $(0.29^\circ$ for the rectangular solution). In other words, the look angle to $R_n$ is: $\theta_{L-n} = \theta_L - \theta_{b\text{-eff}}/2$. Similarly, the look angle to $R_f$ is: $\theta_{L-f} = \theta_L + \theta_{b\text{-eff}}/2$. Once these are calculated, $\alpha_n$ and $\alpha_f$ are found from the law of sines and trigonometry, e.g.

$$\alpha_n = \pi - \theta_{L-n} - \beta_n$$

$$= \pi - \theta_{L-n} - (\pi - \arcsin (R_{tbc} \sin(\theta_{L-n})/R_t))$$

$$= \arcsin (R_{tbc} \sin(\theta_{L-n})/R_t) - \theta_{L-n},$$

where $\beta_n$ is the interior angle formed by $R_n$ and $R_t$. We know that $\beta_n$ must be an obtuse angle, so the law of sines solution is subtracted from $\pi$ radians to eliminate the ambiguity. Similarly,

$$\alpha_f = \arcsin (R_{tbc} \sin(\theta_{L-f})/R_t) - \theta_{L-f}.$$  \hspace{1cm} (3.20)

In Figure 3.4 we calculate and plot the effective beam-limited range resolution against incidence angle for the altitudes typical of Cassini RADAR observations. The six different RADAR modes each operate at specific altitudes over a certain set of viewing angles. Shaded in varying grays in Figure 3.4 are the achievable beam-limited range resolutions for each mode. The compressed scatterometry mode continues beyond the graph, up to an altitude of 84,000 km. The actual distributions of the effective beam-limited range resolutions are plotted for each mode in Figure 3.5.

In Table 3.1, we summarize the beam-limited resolutions obtained by each RADAR mode over Titan, using data from TA through T71. These resolutions are calculated using the rectangular effective beamwidth. We specifically call attention to the mean resolutions and the range of resolutions bounded by the 10th and the 90th percentiles.
Figure 3.4: The solid colored lines describe the variation of beam-limited range resolution with incidence angle for specific radar altitudes (in units of km). The individual RADAR modes operate at specific combinations of angles and altitudes, and their achievable resolutions are indicated with shaded blocks. These blocks are derived from the altitude and angle bounds presented in Table 2.1, representing the majority of data (10th to 90th percentiles) collected for each mode from Titan passes TA through T71.

Table 3.1: RADAR Mode Beam-Limited Real Aperture Resolutions
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Figure 3.5: The distribution of beam-limited range resolution, defined using the rectangular effective beamwidth, is plotted for each of the six modes of the RADAR instrument. The distribution is derived from TA-T71 data. The abscissa is in units of kilometers, and the ordinate is the frequency count normalized by the total number of bursts collected in that mode, and subsequently scaled into a percentage.

The stretching effects from projecting onto the ground in the range direction are readily apparent for those modes that observe at higher off-nadir angles. The altimetry mode, on the other hand, is used almost entirely in nadir-pointing geometry and so has equal range and cross-range resolutions.

Collectively, the RADAR modes average a cross-range resolution of 98 km and a beam-limited range resolution of 147 km. The distribution of the beam-limited resolutions for the total collection of active RADAR data is shown in Figure 3.6. These calculations are for the rectangular effective beamwidth of the central antenna beam data only (beam 3). The outer four SAR beams have different antenna patterns and thus different effective beamwidths and resolutions, as described in Appendix A. The exclusion of the outer 4 SAR beams also roughly compensates for the redundant
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Figure 3.6: The distribution of beam-limited range and cross-range resolutions is plotted for all central-beam data collected by the RADAR instrument, accumulated over all modes. The distributions are derived from TA-T71 data and use the rectangular effective beamwidth calculation. The abscissa is in units of kilometers, and the ordinate is the frequency count normalized by the total number of central-beam bursts collected by the radar, and subsequently scaled into a percentage.

spatial samples obtained in SAR mode, where consecutive bursts often sample the same area on the surface to improve the signal-to-noise ratio.

3.2.1.3 Pulse-Limited Range Resolution

If a pulsed signal is transmitted, then there will be a set of viewing angles where the pulse length will control the range resolution instead of the antenna beam (the range resolution will be “pulse-limited”, and \( \delta r = \delta r_{\text{pulse}} \)). In this case, \( \theta_{L-n} \) and \( \theta_{L-f} \) need to be redefined in Eq. 3.19 and Eq. 3.20 for the proper evaluation of the pulse resolution in Eq. 3.18.

To understand how the pulse affects the range resolution, consider a transmitted...
Figure 3.7: The leading edge of the pulse intersects point A on the ground at time \( T = t_0 \), launching a scattered wave, labeled with the red \( SW_A \). The trailing edge of the pulse reaches point A \( \tau \) seconds later, launching the blue \( SW_B \). However, at time \( T = t_0 + \tau \), the front of the pulse has already intersected point C and launched a scattered wave, the red \( SW_C \), which has traveled the \( c\tau/2 \) meters back towards the receiver and therefore coincides with the trailing edge’s blue \( SW_B \). The separation of points A and C, \( c\tau/2 \) meters apart in slant range, defines the resolution of the pulsed system; any points on the ground at smaller separations will be blurred together by the different segments of the pulsed waveform.

wave consisting of a single pulse of energy over a time duration \( \tau \). The pulse length maps to a slant range distance of \( c\tau \), where \( c \) is the speed of the propagating wave, or the speed of light in a vacuum. By the time the wave reaches the surface, it is effectively a plane wave, such that the wave front occupies a plane perpendicular to the propagation direction. If the transmitted wave arrives at an off-nadir angle, then the discrete pulse of energy illuminates only one patch of ground at a time. Thus, the measured scattered signal can be mapped to specific regions on the surface by considering the time of arrival. The uncertainty in the location, or the resolution of the pulse-limiting system, is determined by how the pulse waveform projects onto the surface, which in turn is controlled by the pulse length and the incidence angle, \( \theta_i \).

Consider Figure 3.7. The leading edge of the pulse waveform, in solid red, intersects point A on the surface at time \( T = t_0 \), emitting a scattered wave, shown
as the red dashed line labeled $SW_A$. The trailing edge, in solid blue, intersects the same point $\tau$ seconds later, emitting a scattered wave labeled with a blue $SW_A$. Previously, at time $T = t_0 + \tau/2$, the leading edge reached point C and excited a new scattered wave, marked with a red $SW_C$. At time $T = t_0 + \tau$, the leading edge’s red $SW_C$ has already traveled the $c\tau/2$ meters back towards the receiver and therefore coincides with the trailing edge’s blue $SW_A$. In other words, the leading and trailing pulse edges have the same round trip travel times from the radar to points A and C, respectively, and back to the radar. Thus, the scattered waves from the two points will be coincident at the receiver, and thus inseparable. The maximum separation of the ambiguous points on the ground defines the resolution of the pulsed system.

The pulse-limited resolution is then the projection of the slant range pulse resolution, $c\tau/2$, onto the surface. For a flat surface, like the one shown in Figure 3.7, the resolution on the surface is simply $1/\sin \theta_i$ times the slant range resolution. For a curved surface, we continue with the procedure described in Section 3.2.1.2 for the beam-limited range resolution case, except now the vectors bounding the resolution cell, $R_n$ and $R_f$, are defined by new criteria: 1) $R_n$ and $R_f$ are separated by an angle $\theta_p$ such that the known pointing vector $R$ bisects $\theta_p$, and 2) the difference in the length of the bounding vectors must equal the slant range pulse resolution, i.e. $R_f - R_n = c\tau/2$.

The first criterion insures that the resolution cell is properly centered. It yields the new look angles to $R_n$ and $R_f$:

$$\theta_{L-n} = \theta_L - \theta_p/2,$$

$$\theta_{L-f} = \theta_L + \theta_p/2.$$  \hspace{1cm} (3.21)

The second criterion identifies the relationship that will define $\theta_p$, and, by extension to Eq. 3.18, the projected range resolution. We first determine $R_n$ and $R_f$ from the law of cosines together with the quadratic formula:

$$R_n = R_{tbc}\cos(\theta_{L-n}) - \sqrt{\left(2R_{tbc}\cos(\theta_{L-n})\right)^2 - 4\left(R_{tbc}^2 - R_f^2\right)},$$ \hspace{1cm} (3.22)
Each of the bounding vectors is a function of incidence angle (through $\theta_L$), the range from the radar to the target body center, the target radius, and the separation angle $\theta_p$. When we insert Eq. 3.22 and Eq. 3.23 into the definition of the second criterion, there is also an additional dependency on the pulse length. All of these parameters are readily known except $\theta_p$. We find that it is difficult to evaluate an analytic function for $\theta_p$ that properly expresses its dependence on $\theta_i$, $R_{tbc}$, $R_t$, and $\tau$. For our purposes, it is sufficient to evaluate it numerically. Once $\theta_p$ is determined, we can calculate $\alpha_n$ and $\alpha_f$ from the substitution of Eq. 3.21 into Eq. 3.19 and Eq. 3.20. Using these results in Eq. 3.18 subsequently yields the pulse-limited range resolution, just as with the beam-limited range resolution. The pulse-limited range resolution does not depend on $R_{tbc}$ except for very low incidence angles, when the projection of the slant range resolution falls off the visible surface. In this case, the pulse-resolved surface area can be that of the entire visible sphere, which depends strongly on the radar distance (the visible surface area equals $R_t \arccos(R_t/R_{tbc})$). The visible area will be much larger than the beam-illuminated area, so this case is not of much interest. Thus, for a given target, the pulse resolution will vary only with incidence angle and pulse length. Its behavior is depicted with black dashed lines in Figure 3.8.

In Figure 3.8, we see that the beam-limited range resolution, identified by the solid colored lines, as in Figure 3.4, will dominate at lower incidence angles until the pulse ground projection is small enough to resolve sub-beam areas. The formal definition of the range resolution is thus the smaller of the two limiting resolutions:

$$\delta r = \min (\delta r_{\text{beam}}, \delta r_{\text{pulse}}).$$

(3.24)

The transition angle, where the pulse-limited resolution takes over, depends on the specific values of pulse length and altitude. The smaller the pulse length and the
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Figure 3.8: The solid colored lines describe the variation of the effective beam-limited range resolution with incidence angle for specific radar altitudes. The dashed black lines describe the variation of pulse-limited range resolution with incidence angle for specific pulse lengths. The individual RADAR modes operate at specific combinations of angles, altitudes and pulse lengths, and their achievable resolutions are indicated on the plot by shades of gray. In order of increasing lightness, or increasing distance, the shades correspond to high-SAR, low-SAR, altimetry, distant-SAR, scatterometry, and compressed scatterometry modes, as in Figure 3.4. Note that the resolution is limited by the beam at low angles, as depicted by the shaded blocks, but at higher angles, the resolution may be limited by the pulse length and is then no longer a function of distance. The resolution sets depicted here represent the resolutions for the majority of a mode’s data (10th to 90th percentiles), but there are cases where the combination of parameters may fall outside of this set.

larger the altitude, the lower the transition angle. We find that the transition angle is only reached in half of the RADAR modes.

Shaded by varying grays in Figure 3.8 are the final limiting range resolutions possible for each RADAR mode. The high-SAR, low-Sar, and altimetry modes, with mean pulse lengths near 150 $\mu$s, 250 $\mu$s, and 150 $\mu$s, respectively, are predominantly
beam-limited in their range resolution. The distant SAR, scatterometry, and compressed scatterometry modes, with mean pulse lengths near 500 µs, 550 µs, and 450 µs, respectively, have range resolutions that are almost equally split between pulse-limited and beam-limited, although the distant compressed scatterometry mode data, with their large altitudes, are weighted more towards the pulse-limited category.

To summarize, we presented an expression for the cross-range resolution \( \delta a_{b-eff} \) in Eq. 3.17 and an expression for the range resolution \( \delta r \) in Eq. 3.24, where the latter is evaluated from Eq. 3.18 together with the discussion in Sections 3.2.1.2 and 3.2.1.3. With these, we arrive at our final expression for the effective resolution area to use in Eq. 3.8:

\[
A_{res} = \delta a_{b-eff} \times \min(\delta r_{b-eff}, \delta r_{pulse}),
\]

(3.25)

### 3.2.2 Calculating \( F_c \)

The correction factor \( F_c \) accounts for any lost echo energy falling outside of the receive window. To calculate \( F_c \), we estimate the arrival time of the leading edge of the first echo using ephemeris and attitude data. We reconstruct the \( K_{ptx} \) echo positions relative to the start of the receive window, where each pulse echo is \( \tau \) seconds long, just like the actual transmitted sequence (this reconstructed signal is labeled “Tx Pulse Sequence” in Figure 3.9). As explained later in Figure 3.10, the true duration of each received echo results from convolving the transmitted pulse length with the effective temporal beam spread, \( T_{b-eff} \) (defined in Eq. 3.28). The resulting echo sequence, correctly positioned within the receive window is labeled “Rx Echo Sequence” in Figure 3.9. \( F_c \) is the ratio of the total echo sequence energy that falls within the confines of the receive window to the total echo energy that would be received if the receive window was unbounded. We demonstrate the calculation of \( F_c \) in Figure 3.9 for an echo signal that arrives late in the receive window, such that two pulses fall outside of the boundary. In addition to being ill-positioned, the high incidence angle and altitude creates a large echo spread: each individual pulse echo is widened by
Figure 3.9: Calculation of the effective number of pulses received through the correction factor $F_c$. The receive window is timed to capture the received echo sequence. However, the command instructions that reposition the receive window are only updated about every ten bursts, and the burst echo sequences will migrate across the receive window in that time. As a result, the beginning and ending bursts might have echo sequences that fall off the ends of the receive window boundary. Shown here is a model of the echo sequence captured for a scatterometry burst in the T8 inbound pass. This burst occurs shortly after an instruction update and thus is positioned towards the end of the receive window (subsequent bursts will be shifted to the left as the spacecraft moves closer). In addition to being ill-positioned, the pulse echoes incur considerable spread due to the high incidence angle (49°) and large distance (23,000 km) that causes the beam footprint to stretch over a larger area in the range direction. As a result, 30% of the reflected energy is lost ($F_c=0.696$), and effectively only 5.6 of the 8 pulse echoes are received.

50% of the transmitted pulse width, causing even more energy to be spread outside of the RXW boundary. As a result, only 70% of the reflected echo energy is received ($F_c=0.696$), i.e. only 5.6 of the 8 transmitted pulses are received ($K_{prx} = K_{ptx} F_c$).
Figure 3.10: Single pulse echo model used to demonstrate the effective time spread of the echo and its dependence on viewing geometry. A single pulse transmitted toward the surface with a given power and time length, and thus energy, projects onto the surface (in the range direction; it is infinite in cross-range). This projection convolves with the antenna beam projection (in the range direction) to produce the received pulse echo; e.g. the echo signal ramps up as the pulse surface-intersection moves into the beam footprint (time $t_1$), and stays constant at its average level across the footprint (time $t_2$ to $t_3$), and then ramps down again as it begins to move out of the footprint. The total duration of the echo is thus the sum of the pulse length and beam spread in time, but the effective echo duration, $T'_{\text{echo}}$, is the maximum of the two and will depend on the viewing geometry, as explained in the text. Note that, in both cases, the beam footprint is fixed to the surface, and the pulse energy moves across it in time, but in the upper panel we display the relative beam-pulse locations as if the pulse projection is fixed to the surface; this is for clarity of the figure only and could be interpreted as a switch in the reference frame. Also note that we display the pulse projection as finite in cross-range for clarity of the figure.

3.2.3 Calculating $N'_{\text{echo}}$

To calculate the effective echo spread $N'_{\text{echo}}$, or equivalently $T'_{\text{echo}}$ ($T'_{\text{echo}} = T_{\text{adc}} N'_{\text{echo}}$), consider the formation of a single pulse echo from the interaction of the transmitted pulse with the surface and the antenna beam, as depicted in Figure 3.10. Here, a single
pulse of width $\tau$, constant average power $P_t$, and energy $E_t$ is transmitted towards the surface. Note that we are now using the physical quantities that are derived from the digitized versions with the calibration constant presented in Section 3.4. The received echo signal forms from the convolution of the beam footprint and the pulse projection on the surface; e.g., the echo signal ramps up as the pulse surface-intersection moves into the beam footprint (time t1), and, in our model, stays constant at its average level ($P_{sW}$) across the footprint (time t2 to t3), and then ramps down again as it begins to move out of the footprint (time t3). Thus, the total duration of the echo in time is always the sum of the pulse length and the effective beam spread in time:

$$T_{\text{echo-total}} = \tau + T_{\text{b-eff}}. \quad (3.26)$$

Let us now model the pulse echo as a block by squaring it off (we call this our block-echo model). As shown in the figure, we can imagine moving the end-ramp piece (shaded gray in the full echo) to fill in the front-ramp piece of the echo. The area of the block-echo is unchanged and equals the signal energy $E_{sdJ}$. The height of the block-echo is also unchanged, held at its constant level that is equal to the unknown $P_{sW}$. The width of the block-echo is the desired effective width of the pulse echo, $T'_{\text{echo}}$ (or $N'_{\text{echo}}$).

The block-echo model described above reveals that $T'_{\text{echo}}$ can be calculated by reducing the total echo duration $T_{\text{echo-total}}$ by the duration of the end-ramp piece. The length of the end-ramp piece depends on the viewing geometry, as shown in Figure 3.10. At low incidence, the echo exists in the beam-limited regime, as shown in the upper panel of Figure 3.10. Here, the projection of the beam in the range direction is much less than the projection of the pulse, so the spread of the pulse in time controls the effective echo spread, as illustrated. At higher incidence, however, the echo exists in the pulse-limited regime, as shown in the lower figure panel. Here, the projection of the beam is much larger than that of the pulse, so the spread of the beam in time controls the effective echo spread. Thus, the effective echo spread is
defined as the maximum of these two quantities:

\[ T'_{\text{echo}} = \max (\tau, T_{\text{b-eff}}), \quad (3.27) \]

where the effective beam spread in time is computed as follows, with reference to Figure 3.2 and using the effective antenna beamwidth to define the near and far range vectors:

\[ T_{\text{b-eff}} = \frac{2 (R_f - R_n)}{c}. \quad (3.28) \]

We plot the distribution of the pulse length and beam time spread quantities in Figures 3.11 and 3.12 for each of the six modes of the Cassini RADAR instrument.
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![Graphs showing beam time spread and frequency (% of bursts) for different modes of the RADAR instrument.]

**Figure 3.12:** The distribution of beam spread in time ($T_{\text{b-eff}}$) used in each of the six modes of the RADAR instrument. The distribution is derived from TA-T71 data. The abscissa is in units of $\mu$s, and the ordinate is the frequency count normalized by the total number of bursts collected in that mode, and subsequently scaled into a percentage. Note that the beam spread grows larger for modes that operate at higher altitudes and at higher viewing angles.

As expected, the beam spread in time suffers the same dependence on the observation geometry as the beam-limited range resolution: namely that the time spread increases with increasing radar altitude and increasing viewing angles. This dependence is apparent in Figure 3.12, where the mode panels are positioned in order of increasing altitude. With the exception of the altimetry mode, which observes almost exclusively near-nadir and thus incurs minimal beam spreading effects, each mode has greater time spreads than the mode before it.

The single pulse model presented in Figure 3.10 extends readily to multiple pulses by visualizing the process of stacking the echo blocks side by side into one long block, as is effectively done in the definition of $N_{\text{seq}}$ ($N_{\text{seq}} = K_{prx}N'_{\text{echo}}$) in Eq. 3.12. By
When multiple pulses are transmitted, the echoes may blur together at higher incidence. This happens when the echo duration is greater than the pulse repetition interval ($T_{echo-total} > PRI$). We account for this overlap in our calculation of average echo power by “unwrapping” the echoes and stacking them side-by-side, where each pulse echo is modeled as a block with effective length $T_{echo}'$.

Stacking the block-echoes side by side, we are “unwrapping” any overlap that might occur at high-incidence geometry, where the beam gets larger and it takes longer for the pulse energy to travel across it (i.e. the beam spread $T_{b-eff}$ grows such that $T_{echo-total}$ becomes larger than the time interval between pulses). This is illustrated in Figure 3.13: the light-gray shaded triangles that rise above the constant level in the high-incidence angle picture are the cumulative powers where the echoes overlap.

The definitions of $N_{echo}'$ and $K_{prx}$ inherently account for pulse echo overlap and lost echo energy such that the signal power $P_{sdW}$ can be correctly separated from the measured received power. We are now prepared to adapt the traditional area-extensive radar equation given in Eq. 3.8 for operation with the Cassini RADAR system.
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3.2.4 Final Reduction Equation: Power Perspective

We calculate the signal power $P_{s_{dW}}$ by substituting our results for $K_{prx}$ (Section 3.2.2) and $T'_{echo}$ (Eq. 3.27) into Eq. 3.11,

$$P_{s_{dW}} = (P_{r_{dW}} - P_{n_{dW}}) \frac{N_{rxw}}{K_{prx} N_{echo}'} T_{rxw}$$  \hspace{1cm} (3.29)

$$= (P_{r_{dW}} - P_{n_{dW}}) \frac{K_{ptx} F_c \max (\tau, T_{b-eff})}{K_{prx}}$$

where we convert between digitized samples and seconds using the sample period $T_{adc}$ (e.g. $T_{rxw} = N_{rxw} T_{adc}$).

Substituting Eq. 3.29 into the radar equation given in Eq. 3.8, and including the calibration scale factor $C$ to convert the digitized powers into their physical representations, we arrive at the following formula for calculating the average NRCS over the radar burst:

$$\sigma^0_{\text{avg}} = C (P_{r_{dW}} - P_{n_{dW}}) T_{rxw} \frac{(4\pi)^3 R_0^4}{P_t G_t^2 A_{res} T'_{echo} K_{prx}}.$$  \hspace{1cm} (3.30)

We find that we can simplify Eq. 3.30 by further considering the quantity $A'_{res} T'_{echo}$ contained in the denominator. Substituting from Eq. 3.25 and Eq. 3.27, we have

$$A'_{res} T'_{echo} = \delta a_{b-eff} \cdot \min (\delta r_{b-eff}, \delta r_{pulse}) \cdot \max (\tau, T_{b-eff})$$.  \hspace{1cm} (3.31)

In the beam-limited case, Eq. 3.31 simplifies to

$$A'_{res} T'_{echo} = \delta a_{b-eff} \cdot \delta r_{b-eff} \cdot \tau.$$  \hspace{1cm} (3.32)

and in the pulse-limited case, the product takes the form

$$A'_{res} T'_{echo} = \delta a_{b-eff} \cdot \delta r_{pulse} \cdot T_{b-eff}.$$  \hspace{1cm} (3.33)

Let us now compare the quantities that differ in the two cases: $\delta r_{b-eff} \tau$ and
Figure 3.14: We demonstrate numerically, over a realistic set of incidence angles and altitudes, that the $A'_{\text{res}}T'_{\text{echo}}$ product in the beam-limited case (Eq. 3.32) is sufficiently equal to the product in the pulse-limited case (Eq. 3.33). The equality fails in the near-nadir regime, where our evaluation of the pulse-limited product deviates from the beam-limited product due to the early returns from the nadir point. Yet, because the near-nadir breakdown occurs in the beam-limited regime, our numerical demonstration justifies our assertion that using the beam-limited quantity within $A'_{\text{res}}T'_{\text{echo}}$ will work correctly for all geometries. Outside of the near-nadir regime, we find that the quantities differ by less than 0.006% on average, although the discritization of the calculation results in absolute differences as high as 0.4%; a finer numerical scale would remove much of the variation. The top two figure panels are displayed using identical color scales that stretch from 0 to 0.6.

$\delta r_{\text{pulse}}T_{\text{b-eff}}$. In the first case, $\delta r_{\text{b-eff}}$ is the projection of the effective beam, or $(R_f - R_n)$ in slant range, onto the surface. In the second case, $\delta r_{\text{pulse}}$ is the projection of the pulse, or $c\tau/2$ in slant range, onto the surface. Thus, we have $(R_f - R_n)/\text{proj} \cdot \tau$ in case one and $(c\tau/2)_{\text{proj}} \cdot 2(R_f - R_n)/c$ in case two, where we have substituted Eq. 3.28 for $T_{\text{b-eff}}$ in case two. The “proj” subscript signifies the operation of projecting the range onto the surface. Because the projection operation is nonlinear, it is not immediately
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obvious how these two products relate to each other. We compare the quantities numerically in Figure 3.14 and find that they differ by less than 0.006% on average, such that we can assume

\[ A'_{\text{res}} T'_{\text{echo}} = \delta a_{b-eff} (R_f - R_n)_{\text{proj}} \tau = \delta a_{b-eff} \frac{2(R_f - R_n)}{c} \left( \frac{c\tau}{2} \right)_{\text{proj}}. \]  

(3.34)

Thus we can remove the conditional relationships in Eq. 3.30 and replace \( A'_{\text{res}} T'_{\text{echo}} \) with \( A_{b-eff} \tau \), where \( A_{b-eff} \) was formally defined in Eq. 3.16 (\( A_{b-eff} = \delta a_{b-eff} \delta r_{b-eff} \)):

\[ \sigma_0^{avg} = \frac{C(P_{r_dW} - P_{n_dW}) T_{rxw} (4\pi)^3 R_0^4}{P_t G_t^2 \lambda^2 A_{b-eff} \tau K_{prx}}. \]  

(3.35)

In the above, we verify quantitatively what we had already intuited: averaging the echo sample returns together over the entire burst sequence, rather than considering the single samples individually, requires the measurement resolution to be that of the entire antenna beam footprint. This will diminish the resolution in the range direction only, and only at the combination of high incidence angles and high altitudes that might have otherwise been resolved by the transmitted pulse width. In Section 3.2.1.3, we noted that much of the collected data falls within the beam-resolved regime already.

In this section, we have directly applied the standard form of the radar equation given in Eq. 3.8 to the Cassini RADAR system. In the process, we gave special consideration to the pulsed transmissions, the possibility that the pulse echoes might overlap each other in time, and the occurrence of noise-only regions within the receive window. We next show that Eq. 3.35 readily converts to the energy form given in Eq. 3.9, where the previous considerations are not as complicated.

3.2.5 Final Reduction Equation: Energy Perspective

It is possible to express the burst NRCS in terms of echo energy rather than power. This is in many ways a simpler approach, and it is the one that we have used to reduce the Cassini Titan data set. In Eq. 3.9, we described the real aperture radar
equation in terms of energy units. Now that we have quantified the correction factor $F_c$ in Section 3.2.2, we can readily evaluate the burst NRCS:

$$\sigma_{avg}^0 = \frac{C' E_{sdj} (4\pi)^3 R_0^4}{E_{tadj} F_c G_c^2 \lambda^2 A_{b-eff}}.$$  (3.36)

where the calibration scale factor $C'$ is used to convert the digitized signal energy into its physical representation (see Section 3.4).

Eq. 3.36 is easy to comprehend; it compares the received echo energy to the total transmitted energy, accounting for the possibility of lost energy falling outside of the receive window. From the elaborate descriptions in the preceding sections, we can appreciate the simpler and more straightforward nature of the energy-based form of the radar equation. The energy form does not need to account for the variable weighting of signal and noise within the receive window, nor the possibility of overlapping signal echoes. As a result, consideration of the effective pulse echo spread formulated in Section 3.2.3 is unnecessary. Additionally, considering the total received energy instead of the individual received sample powers eliminates the need to consider beam-limited versus pulse-limited resolution computations; it is clear that the total received energy originates from the entire beam-illuminated area.

Let us now confirm that the energy-based expression is equivalent to the power-based expression given in Eq. 3.35. First, we note that the quantity $(P_{rdw} - P_{nrdw}) T_{rxw}$ in the numerator of Eq. 3.35 is equivalent to the total signal energy $E_{sdj}$ (see Eq. 3.12). Next, we note that the quantity $P_t \tau K_{prx}$ in the denominator of Eq. 3.35 is related to the total energy transmitted $E_{tadj}$. $P_t \tau$ is the transmitted energy in a single pulse, and $P_t \tau K_{ptx}$ is the total energy transmitted in all $K_{ptx}$ pulses. In Section 3.2.2, we calculate a correction factor $F_c$ that relates $K_{prx}$ to $K_{ptx}$: $K_{prx} = K_{ptx} F_c$. Thus, we have $P_t \tau K_{prx} = P_t \tau K_{ptx} F_c$, which makes the quantity in the denominator equal to $E_{tadj} F_c$. Substituting these expressions into Eq. 3.35 yields a result identical to Eq. 3.36.

In summary, we have independently derived separate expressions for the area-extensive radar equation from first principles and demonstrated here how the two
approaches relate to each other for the Cassini RADAR system. The variety of viewing geometries combined with pulsed transmissions complicates the development and application of the traditional power approach, whereas the energy approach proves more straightforward. Because of its simplicity, Eq. 3.36 is the final form of the area-extensive radar equation that we use in the Cassini RADAR real aperture processor.

### 3.3 Noise Response

As described previously, we solve for the burst NRCS by measuring the echo signal energy \( E_{s,dj} \) and evaluating Eq. 3.36. We calculate \( E_{n,dj} \) by removing the total noise energy \( E_{n,dj} \) from the total received energy \( E_{r,dj} \) (see Eq. 3.12). In this section we describe our process of measuring \( E_{n,dj} \).

Because the received signal is signal combined and interspersed with noise, we derive \( E_{n,dj} \) by estimating the average noise power \( P_{n,dw} \) from the noise sections and scaling the result by the length of the receive window. If the receive window consists of \( N_{rxw} \) digitized voltage samples, then \( E_{n,dj} = N_{rxw}P_{n,dw} \).

Estimating \( P_{n,dw} \) for a particular burst requires the presence of noise-only intervals within the temporal receive window (the transmitted signal is frequency modulated to fill the entire receiver bandwidth, so it is not usually possible to measure the noise power in the frequency domain). The noise-only intervals can take the form of “inter-pulse noise”, the regions between pulse echoes, or “leading/trailing noise”, the regions before or after the echo sequence. To illustrate these regions, we show example receive window measurements for three of the radar modes in Figure 3.15. The scatterometry receive window, displayed in two ways in the upper row, comprises 14 pulse repetition intervals (PRI). Only eight pulses are typically transmitted in scatterometry mode, allowing for approximately six empty PRI that are pure noise. In the case pictured, the echo sequence is positioned such that the first \( \sim 2.5 \) PRI and last \( \sim 3.5 \) PRI are only noise. The relatively high incidence angle (20.6°) causes the scatterometry echo to spread in time and precludes an accurate measurement of any noise between the
**Figure 3.15:** Sample receive windows for three of the four receiver filter modes. The L-SAR receive window is similar to that of H-SAR. For each receiver filter, the full receive window is shown as sample power versus time in the left column, and is stacked by pulse intervals into an image in the right column (i.e. each row in the image represents one pulse, and each column in the image is a sample power in that pulse). The receive windows illustrate the unique challenge of locating noise for each receiver mode, as explained in the text.
3.3. **NOISE RESPONSE**

Echos. In contrast, the altimetry receive window, in the middle row of the figure, occurs near nadir and has very well defined interpulse noise regions. However, the altimetry receive window typically lacks leading or trailing noise-only intervals because the length of the transmit window is longer than the length of the receive window, i.e. each of the 15 PRI in the receive window will contain echo signal. The H-SAR receive window, in the lower row of the figure, exemplifies a case in between scatterometry’s leading/trailing noise scenario and altimetry’s interpulse noise scenario. The lower altitude of H-SAR limits the echo spread so that interpulse noise is abundant and detectable even at 18.7° incidence. Additionally, there are several noise-only intervals preceding and succeeding the ~50 PRI-long H-SAR echo sequence. Unfortunately, the leading/trailing noise-only samples are unusable because they are biased by the 8 bit to 2 bit block adaptive quantization (8-2 BAQ) algorithm (see Section 3.5). This means that, for any mode that utilizes the 8-2 BAQ compression algorithm (H-SAR, L-SAR, and D-SAR modes), the average noise power of the burst can only be determined if interpulse noise-only regions are present.

We identify noise-only intervals by first estimating the total echo spread in time $T_{echo-total}$ (see Eq. 3.26) using a conservative approximation of the 6-dB effective beamwidth (0.7° instead of 0.29°) to eliminate the possibility of scattered signals entering the antenna sidelobes and contaminating the noise. We then identify the bursts with $T_{echo-total}$ less than 95% of the PRI. This guarantees that there will be some interpulse noise and also that the echo sequence can be readily located within the receive window. We locate the echo sequence by correlating the received signal with a pulse train comprising $K_{prx}$ pulses (see Section 3.2.2), where each pulse has a width equal to the transmitted pulse length $\tau$ (note: $\tau$ will be smaller than the true echo width $T_{echo-total}$, but the smaller width will locate the strongest centers of the echoes). Any data samples present in the receive window prior to the identified start of the echo sequence are marked as leading noise, and any data samples following the end of the echo sequence are marked as trailing noise. The interpulse noise-only width in samples is approximately $N_{ip} = f_{adc} \times (PRI - T_{echo-total})$, where $f_{adc}$ is the sample
rate. We conservatively mark the $0.8N_{ip}$ data samples centered between the identified echo pulses as interpulse noise. If $V_{ni_{dV}}$ is the $i$th noise-only voltage sample, then we measure the average noise power $P_{n_{dW}}$ as

$$P_{n_{dW}} = \frac{1}{N_n} \sum_{i=1}^{N_n} V_{ni_{dV}}^2. \quad (3.37)$$

Noise-only intervals are not present in every burst, and when they are present, they are not always long enough for an accurate measurement of the mean level (the variance of the noise power estimate is inversely proportional to the square root of the number of noise samples). Thus, instead of attempting to estimate $P_{n_{dW}}$ for each individual burst, we integrate the noise-only measurements over all bursts with identical receiver configurations over all Titan passes (from Ta through T71). The receiver configuration is identified by (1) the bandpass filter used (there are four filters in the Cassini RADAR receiver, see Section 2.1 and (2) the total attenuation setting.

As explained more thoroughly in the next section, the mean noise power level originates largely from thermal noise generated by the receiver electronics ($P_{rec}$) and, to a lesser extent, thermal noise from target radiation, as collected by the radar antenna ($P_a$). The combination of the two noise sources yields the system noise power that we observe and measure: $P_{sys} = P_{rec} + P_a$. The ideal receiver system has a large front-end gain so that the receiver thermal noise power is unaffected by any back-end gain changes. In this scenario, $P_{rec}$ would be constant for a particular receiver bandwidth, and we would only need to calculate the noise power once for each receiver filter. However, because the gain of RADAR’s leading low noise amplifier is not large enough for it to dominate the receiver noise power, we observe a strong dependence of the receiver noise power on the total gain settings, where the total gain is controlled primarily by the back-end attenuator settings.

The RADAR receiver uses a series of back-end variable attenuators to keep the received amplitudes on-scale and prevent clipping. The total attenuation amount can vary by more than 15 dB over all Titan observations collected in a particular receiver
mode, causing $P_{rec}$ to vary by 30%-60% of its mean value. The attenuation settings used in each receiver mode and their frequency of use are documented in Figure 3.16. Figure 3.16 shows that some modes regularly use only a handful of the attenuation settings (i.e. scatterometry mode predominantly uses an 8.4 dB attenuation loss, but if a stronger signal is anticipated, such as would occur when the spacecraft is at closer altitudes, then higher attenuation settings are needed, the most common ones being 14.2 dB, 17.1 dB, and 21.1 dB). Even though Figure 3.16 shows that the extreme attenuation settings are less commonly used, we need to be able to estimate the noise power over all attenuation settings to correctly calibrate all of the data.

To characterize how the receiver noise power changes with attenuation, we collect together the Titan noise-only measurements as a function of attenuation setting and receiver bandwidth filter, where the noise is identified following the echo-location procedure described above and using data from Ta through T71. We then compute the mean noise power at each available attenuation setting. Some attenuation settings have more noise samples, and thus more accurate mean noise power measurements, than others. To eliminate some of the uncertainty in the measurements, we fit a model curve to the mean noise power measurements to better describe the noise power dependence on attenuation. Then, rather than use the individual mean noise power measurements to derive the correct $E_{n,d}$ value, we evaluate the noise model result at the desired attenuation setting. With this model approach, we are able to estimate $P_{sys}$ for any attenuation value in a particular receive mode, even if we were never able to actually measure the noise directly from the data. For example, the H-SAR data collected at 13.3 dB (1% of the H-SAR data) and 16.1 dB (3% of the H-SAR data) do not have distinct interpulse sections that can be easily measured. The attenuation settings that are completely missing noise measurements are outlined with red circles in Figure 3.16. Missing noise occurs only for the SAR modes, where the incidence angles are often too high for well-defined interpulse noise regions.

We find that the noise model is well described as the sum of two exponentials when the attenuation loss is expressed in dB, which is equivalent to the sum of two
Figure 3.16: The distribution of attenuation settings used for each of the four receiver modes. The D-SAR and compressed-scatterometry modes utilize the 8.4 dB attenuation setting with the scatterometry receiver filter. The attenuation settings that are missing proper noise measurements are outlined with red circles, illustrating the need for a noise model to estimate the missing noise powers. We explicitly call attention to the most commonly used settings in each mode. 87% of scatterometry bursts occur at 8 dB, 11% at 14 dB, 1% at 17 dB, 1% at 21 dB, and the other bursts occur at less than 0.03% of the time. 88% of altimetry bursts occur between 23 dB and 33 dB, and the remaining 12% of bursts occur between 34 dB and 45 dB. 90% of L-SAR bursts occur at 13.3 dB, 15.2 dB, 17.1 dB, 19.1 dB, and 21.1 dB. 89% of H-SAR bursts occur at 17.1 dB, 19.1 dB, 21.1 dB, and 23.1 dB.

power laws when the attenuation loss is left in its linear form. This model reflects the known mean power measurements within 4% (see Section 3.4.1 for an explanation
of the thermal noise power variability). The two model expressions take the form of

\[ y = ae^{bx} + ce^{dx} \]

\[ y = aL^{b'} + cL^{d'} \]

where \( L \) is the total linear attenuation loss (the inverse of the attenuation gain) and \( x \) is the logarithm of \( L \) (\( x = 10\log_{10}(L) \); \( x \) should be positive since \( L > 1 \)). \( y \) is the measured noise power in dW. \( b' \) is related to \( b \) through the relationship \( b' = 10b/\ln(10) \), and \( d' \) is related to \( d \) in the same manner.

We summarize the best-fit parameters of the noise response model for each of the four receiver bandwidth filters in Table 3.2.

We also show the noise measurements and the model fits in Figure 3.17, where each panel represents one of the receiver bandwidth filters and the results are organized counter-clockwise in order of increasing bandwidth. The Titan noise measurements are plotted as black squares, where the error bar represents the standard deviation of the measurements. The best-fit model results are plotted as solid black lines. The gray lines on either side of the black model lines represent model fits to calibration noise measurements collected around Saturn during a portion of the Ti29 distant Titan observation, as explained in the next section. The upper gray line is the model fit to Saturn noise measurements and the lower gray line is the model fit to the background sky noise measurements. Titan’s mean microwave brightness radiation is in between that of Saturn and the background sky, and thus we expect the Titan noise measurements to be bounded by the two calibration noise curves where the attenuation settings overlap. This expectation holds true for scatterometry data at

<table>
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<th>Bandwidth (kHz)</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( b' )</th>
<th>( d' )</th>
</tr>
</thead>
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<tr>
<td>117</td>
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<td>-0.2179</td>
<td>81.291</td>
<td>-0.0617</td>
<td>-0.9462</td>
<td>-0.2677</td>
</tr>
<tr>
<td>468</td>
<td>8351.6</td>
<td>-0.2280</td>
<td>51.075</td>
<td>-0.0418</td>
<td>-0.9330</td>
<td>-0.2108</td>
</tr>
<tr>
<td>935</td>
<td>9382.4</td>
<td>-0.2100</td>
<td>7.3930</td>
<td>0.0029</td>
<td>-0.9120</td>
<td>0.0127</td>
</tr>
<tr>
<td>4675</td>
<td>43407.0</td>
<td>-0.2148</td>
<td>46.876</td>
<td>-0.0485</td>
<td>-0.9903</td>
<td>-0.1814</td>
</tr>
</tbody>
</table>
Figure 3.17: The Titan noise power measurements for each attenuation setting are shown as black squares, where the error bars represent the measurement standard deviation values. The best-fit model curve for each receiver mode is shown as a solid black line. The bounding gray lines represent fits to calibration noise measurements, as discussed in the next section. The Titan noise model parameters are given in each figure panel.

Attenuations less than 21 dB, altimetry data at attenuations less than 32 dB, and L-SAR and H-SAR data at attenuations less than 25 dB. Beyond these upper attenuation values, calibration measurements do not exist and the calibration model curves are less accurate. For instance, the Titan altimetry noise, which utilizes attenuation values up to 45 dB, is increasingly higher than the calibration model bounds above
3.4 CALIBRATION SCALE FACTOR

32 dB, and the Titan SAR noise, which utilize attenuation values up to 29 dB, are increasingly lower than the calibration model bounds above 25 dB.

We note that the scatterometry and SAR noise curves fall at the same relative level between their respective calibration noise curves, about half-way between, whereas the altimetry noise curve is very close to its upper calibration curve. The relative level between the different noise curves is related to the antenna noise power $P_a$, or equivalently the antenna noise temperature, as explained in the next section. The altimetry noise curve suggests a much higher antenna noise temperature than the other modes. However, we find that if we use the 8-bit straight altimetry data available from various engineering tests over Titan, then the altimeter noise curve level is more consistent with the other modes. This suggests that there might be a possible bias in the noise floor of the 8 to 4 bit block adaptive quantized data (8-4 BAQ; see Section 3.5), i.e. the altimetry data. If this is the case, we do not have an explanation for the apparent increase in 8-4 BAQ interpulse noise - it is not an effect that appears in the 8-2 BAQ interpulse noise or the 8-bit straight interpulse noise, and it does not appear to be caused by the observation geometry (i.e. there is no correlation with altitude, incidence angle, etc.). We accept the observed noise power as inherent to the 8-4 BAQ implementation and use this measured noise power for the altimetry mode processing in spite of its relative inconsistency with the other modes. An inter-mode comparison of the derived NRCS values shows that the final results are consistent with each other at the 10% level (see Figure 3.22).

In the next section, we explore the implications of the measured noise power response for the calibration of RADAR Titan data.

3.4 Calibration Scale Factor

The signal quantities that we measure directly from the received signal have unphysical units (digitized units) and are referenced to the output of the RADAR receiver. To apply the measurements to the real-aperture radar equation (Eq. 3.35 and Eq. 3.36),
we must interpret them in their physical sense and reference them to the input port of the radar antenna. This is possible by relating the measured noise power, $P_{\text{ndW}}$, to its theoretical thermal noise equivalent.

We know from thermodynamics that a resistor with resistance $R$ ohms, at a physical temperature $T$ Kelvin, has a voltage variance of $4kTR$ volts per Hz, where $k$ is the Boltzmann constant ($k = 1.3807 \times 10^{-23}$ J/K). This resistor delivers a noise power of $kTB$ to an impedance-matched load, where $B$ is the bandwidth in Hz over which the noise is measured. The resulting noise is white, with a flat power spectral density, and the noise amplitude follows a zero-mean Gaussian distribution. We can model any thermal noise source with its resistor equivalent: the noise temperature of the noise source is defined as the physical temperature of the hypothetical resistor that generates the same thermal noise power per unit bandwidth. Thus, knowledge of the equivalent noise temperature yields knowledge of the physical noise power.

The total thermal noise power $P_n$ originates from two sources: the receiver electronics ($P_{\text{rec}}$) and the background radiation collected by the RADAR antenna ($P_a$). Thus, we have

$$P_n = P_{\text{rec}} + P_a, \quad (3.38)$$

or equivalently,

$$T_{\text{sys}} = T_{\text{rec}} + T_a, \quad (3.39)$$

where the system noise temperature $T_{\text{sys}}$ is related to the total noise power $P_n$ through the Boltzmann constant and the receiver bandwidth:

$$P_n = kBT_{\text{sys}}. \quad (3.40)$$

The total noise power described in Eq. 3.40 is referenced to the input terminals of the receiver system. The noise power that we measure, $P_{\text{ndW}}$, represents the noise power at the output of the receiver. The input and output noise quantities are related
through a scale factor $G_{\text{rec}}$ that represents the total gain of the system as well as the analog-to-digital conversion between digitized watts and true watts:

$$P_{n_{dW}} = G_{\text{rec}} \cdot P_n.$$  \hspace{1cm} (3.41)

The total gain is controlled by a series of variable attenuators at the back-end of the RADAR receiver, where the total attenuation loss $L$ is chosen to keep the anticipated echo amplitudes on-scale and prevent clipping. The value of $L$ is known to within about 0.2 dB (the one-sigma level; see West et al. (2009)). We separate $G_{\text{rec}}$ into a constant component $G_c$ and the known variable component $L$, where $L$ represents the loss, or the inverse of the gain:

$$G_{\text{rec}} = \frac{G_c}{L}.$$  \hspace{1cm} (3.42)

$G_c$ is the quantity that we need to determine to complete the signal calibration. We diagram the RADAR receiver system in Figure 3.18, a modified version of Figure 7 from West et al. (2009).

We can evaluate $G_c$ by first solving for the theoretical input noise power $P_n$ through knowledge of $T_{\text{sys}}$ (Eq. 3.40) and comparing the result to the measured noise power $P_{n_{dW}}$, where the latter is scaled by the total loss $L$ to remove the gain variation:

$$G_c = \frac{LP_{n_{dW}}}{kBT_{\text{sys}}}.$$  \hspace{1cm} (3.43)

We estimate $T_{\text{sys}}$ by observing two reference targets of known microwave brightness and comparing the measured noise powers. We use the engineering test data on Ti29 (21-Sep-2006), one of the few engineering tests with attenuation diversity that was fully warmed up at the time of the observation. West et al. (2009) find that receiver temperature increases by 15 K and the receiver gain drops by 1 dB over a 3 hour warm up period, so it is essential to have completed the warm up before performing the experiment.
Figure 3.18: Simplified diagram of the active path of the Cassini RADAR receiver system. The physical input noise power is related to the measured output noise power through the receiver gain factor, which is comprised of a constant component \((G_c)\) and a variable component \((L = L_1L_2L_3)\). We determine the equivalent noise temperature of the receiver \((T_{\text{rec}})\) through the Friis equation, as described in Section 3.4.1. Once \(T_{\text{rec}}\) is known, we can solve for the constant gain factor \(G_c\). The characterizing parameters \((G_F, T_F)\) and \((G_B, T_B)\) lump together the frontend gains and noise temperatures and the backend gains and noise temperatures, respectively.

The Ti29 engineering experiment comprises a series of on-Saturn/off-Saturn noise measurements, where the off-Saturn measurements observe the cosmic microwave background (CMB, or “sky”), which has a microwave radiometric temperature of 2.7 K at 2.2 cm-\(\lambda\) \((T_{\text{sky}} = 2.7 \text{ K})\). Saturn’s microwave disk-averaged brightness temperature is estimated to be between 144 K and 148K at 2.2 cm-\(\lambda\) (de Pater and Massie, 1985; West et al., 2009). During the Ti29 observation, Saturn fully fills the main lobes of the Cassini RADAR antenna. However, about 35% of the collected power originates from far sidelobes positioned over the cold sky (Janssen et al., 2009; West et al., 2009), bringing the sensed antenna temperature down to about 106 K \((T_{\text{sat}} = 106.25 \text{ K})\).
The on-Saturn noise power measurement, $P_{\text{nSat}}$, takes the following form, following the relationships presented above in Equations 3.38-3.43:

$$P_{\text{nSat}} = \frac{G_c k B}{L} (T_{\text{rec}} + T_{\text{sat}}). \tag{3.44}$$

Similarly, the off-Saturn CMB noise power measurement is expressed as

$$P_{\text{nSky}} = \frac{G_c k B}{L} (T_{\text{rec}} + T_{\text{sky}}). \tag{3.45}$$

We plot the noise power measurements of Saturn and the CMB from the Ti29 engineering test for each receiver filter as a function of the variable gain setting ($L$) in Figure 3.19. Sixteen attenuation settings are sampled for three of the receiver filters, and eleven attenuation settings are sampled for the fourth (the altimetry filter with a bandwidth of 4675 kHz). The measurements are given in Table 3.5 at the end of this section, where the errors given are derived from the standard deviations of the noise measurements. We find that the noise power dependence on attenuation loss is best described as the sum of two power laws (or the sum of two exponentials if the attenuation loss is expressed in dB), where the best-fit parameters are given in the figure panels. A large part of this variation is due directly to the changing attenuation. If we normalize out the attenuation variation, computing $P_{\text{n}} L$, we find that $P_{\text{n}} L$ increases steadily as the attenuation loss increases. According to Eq. 3.43, $P_{\text{n}} L$ should be proportional to the system noise temperature, where the proportionality constant is $G_c k B$. If we can measure the system noise temperature, we can retrieve the calibration constant $G_c$.

We derive the receiver temperature from the relative difference of the power measurements, where the gain factors cancel out:

$$T_{\text{rec}} = \frac{P_{\text{nSky}}} {P_{\text{nSat}}} \frac{T_{\text{sat}} - P_{\text{nSat}} T_{\text{sky}}} {P_{\text{nSat}} - P_{\text{nSky}}}. \tag{3.46}$$

The receiver temperature results are plotted as gray circles in Figure 3.20 for each
Figure 3.19: The calibration noise power measurements collected during the engineering experiment on Ti29 for each of the four receiver filters. The scatterometry and altimetry measurements are sampled at 2 dB attenuation intervals, and the SAR measurements are sampled at 1 dB attenuation intervals. The on-Saturn noise powers are plotted as red dots, with the errorbars representing the measurement standard deviation, and the off-Saturn CMB noise powers are plotted in blue. The solid lines represent the model solution. The best-fit model parameters are given, where the first value in the parenthesis is the Saturn model parameter and the second value is the corresponding CMB model parameter.

receiver filter. We also evaluate Eq. 3.46 for the Saturn and Sky noise power curves and derive the receiver temperature curve shown as the solid gray line in Figure 3.20. The ratio between the scaled noise powers $P_{n,dW} \times L$ and the derived system temperature
3.4. **CALIBRATION SCALE FACTOR**

![Graphs of receiver temperatures for different bands](image)

**Figure 3.20:** The receiver noise temperatures derived from the Ti29 calibration experiment are plotted as gray circles. The receiver temperature model derived from the Saturn and CMB noise models is plotted as a solid gray line. The Ti29 Saturn and CMB scaled noise models are shown in red and blue, respectively, where the noise power is scaled $L$ to remove the gain variation and then scaled by the proportionality constant $1/(G_c k B)$ to normalize it to system temperature units. The target antenna temperature is subtracted from the result to yield the receiver temperature estimate. The mean values of $G_c$, as reported in the figure panels, are used in the proportionality constant.

$(T_{rec} + T_a)$ yields the proportionality constant $G_c \times k B$ (see Eq. 3.43). In this manner, we point-wise estimate the $G_c$ value for each attenuation setting in the experiment. If we use the mean value of $G_c$ as the proportionality constant and scale $P_{ndw} \times L$
accordingly, we retrieve the red and blue lines in Figure 3.20, for the Saturn and sky noise curves respectively. The mean value of $G_c$ for each receiver filter is given in each figure panel and also in Table 3.3. The scaled noise curves match the receiver temperature trend with attenuation, as expected, but there are differences due to the assumption of a constant $G_c$.

The individual $G_c$ and $T_{rec}$ values fluctuate between the individual attenuation settings by about 6-7% of the mean value (see Table 3.5 at the end of this section). We develop a receiver model to explain this fluctuation in the next subsection. The receiver model suggests that if we characterize the system noise response according to the total attenuation loss $L$, as opposed to the individual attenuator component values, then we should expect some fluctuation about our mean calibration models. We incorporate this fluctuation into our error analysis in Section 3.6.

To calibrate the RADAR data using Eq. 3.35 or Eq. 3.36, we need to know $C$ or $C'$. $C$ ratios the predicted noise power referenced to the antenna input ($P_n$, in Watts) to that which we measure at the receiver output ($P_{n_dW}$, in digitized watts), thereby forming a calibration scale factor that converts $dW$ to $W$:

$$C = \frac{P_n}{P_{n_dW}}. \quad (3.47)$$

Comparing Eq. 3.47 to Eq. 3.43 shows that $C$ equals $L/G_c$.

$C'$ converts digitized output energy, in units of $dJ$, into input joules. $C'$ is related to $C$ through the analog-to-digital sample period $T_{adc}$:

$$C' = \frac{P_n \times N_{rxw} T_{adc}}{E_{n_d}} = \frac{P_n \times T_{adc}}{P_{n_dW}} = C \times T_{adc} = \frac{L}{G_c} \times T_{adc}, \quad (3.50)$$

where the power and energy quantities are readily interchanged by knowing the receive
window duration \((N_{rxw} \text{ samples or } N_{rxw}T_{adc} \text{ seconds})\), i.e. \(E_{ndJ} = N_{rxw} \times P_{ndW}\) and \(E_n = N_{rxw}T_{adc} \times P_n\).

We can apply the \(G_c\) results derived from the calibration data above to the Titan noise curves measured in Section 3.3. The resulting receiver temperatures should match those of the Ti29 calibration experiment, thus verifying that the Ti29-derived \(G_c\) values are appropriate for calibrating the Titan data. To solve for the receiver temperatures from the Titan noise data, we first need to know the mean Titan antenna temperature. The mean brightness temperature measured for Titan is close to 86 K, with a standard deviation of 3.5 K (Janssen et al., 2009). The 95 percentile bounds for the Titan brightness temperature measurements are 78 K and 89 K, and the 99 percentile bounds are 74 K and 90 K. The Titan observations are close enough for the target to fill the main beam, but the average location of the far sidelobes, accounting for 35% of the collected energy, is unknown. If the far sidelobes are positioned over Titan, then the antenna temperature will be higher than the brightness temperature, but if they are on cold sky, then the antenna temperature will be lower. By comparing the relative Titan noise power levels with those of the known calibration sources, we can derive the mean Titan antenna temperatures that are compatible with the observations. We perform this calculation by recognizing that

\[
\frac{P_{nSky,dW} - P_{nSat,dW}}{T_{sky} - T_{sat}} = \frac{P_{nSky,dW} - P_{nTi,dW}}{T_{sky} - T_{ti}},
\]

where the noise powers of Saturn, the CMB sky, and Titan are considered with respect to their apparent antenna temperatures. Manipulation of the relationship in Eq. 3.51 leads to

\[
T_{ti} = T_{sky} - (T_{sky} - T_{sat}) \frac{P_{nSky,dW} - P_{nTi,dW}}{P_{nSky,dW} - P_{nSat,dW}}.
\]

We apply Eq. 3.52 to the noise power measurements collected from the Ti29 engineering experiment and to the noise power measurements collected from the Titan flyby observations and measure a mean Titan antenna temperature near 55 K for the
scatterometry data, 64 K for L-SAR data, 42 K for the H-SAR data, and 105 K for the altimetry data. The low antenna temperatures suggest that some cold sky radiation is entering the far lobes of the RADAR antenna, or that there is an equivalent lower-noise bias. The high antenna temperature of the altimetry mode suggests that some of the Titan’s radiation is entering the far lobes of the RADAR antenna, or that there is an equivalent extra noise bias, perhaps from the 8-4 BAQ algorithm, as we suggested in the previous section.

We then use the derived mean Titan antenna temperatures together with the $G_c$ calibration constants retrieved from the Ti29 calibration experiment to measure the corresponding receiver temperature from the Titan noise power measurements as follows:

$$T_{rec} = \frac{LP_{naw}}{G_c k B} - T_a. \quad (3.53)$$

The $T_{rec}$ results thus derived from the Titan noise measurements are shown as black squares in Figure 3.21, where the black solid line represents the result derived from the Titan noise model curve. The gray circles are, as before, the Ti29 calibration results, and the $G_c$ value used is given in each figure panel in units of $kB$. We see that the Titan-derived receiver temperatures match the Ti29-derived receiver temperatures very well. Furthermore, the Titan-derived results complete the receiver temperature response at high attenuation values, values that are missing from the Ti29 calibration experiment. Thus, we feel justified in using the $G_c$ values derived from the Ti29 engineering experiment to calibrate the Titan data. Knowledge of the calibration constant $G_c$ (Table 3.3) for each receiver mode and the attenuation loss values are all that are needed to evaluate $C$ or $C'$ and complete the calibration of the real-aperture RADAR data.

The red dots in Figure 3.21 represent solutions derived from compressed data embedded within the Ti29 engineering experiment, whereas the gray dots were derived from 8-bit straight quantized data. The compressed data are similar to the 8-bit straight measurements, but they were compressed on-board the spacecraft before
Figure 3.21: The black lines are derived from the Titan noise curves, using $G_c kB$. The black squares are similarly derived from the individual Titan noise power measurements. The gray circles are the same results as in Figure 3.20. The red circles are derived from the compressed data in Ti29; they match the results reported in West et al. (2009). The red dashed line is the $T_{\text{rec}}$ model that we formerly used to process the RADAR data. Large error bars in the first panel reflect the few number of noise-only samples collected at those attenuation settings.

downlink. The two different data types alternate with each other and have similar integration times. However, the power levels of the compressed data measurements underestimate those of the 8-bit straight data by about 5-10% (see Appendix B).
Table 3.3: RADAR Calibration Constants

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>$G_c$ (1e18)</th>
<th>Previous $G_c$ (1e18)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>117</td>
<td>1.9724 ± 0.114</td>
<td>1.8788 ± 0.092</td>
<td>-4.75</td>
</tr>
<tr>
<td>468</td>
<td>1.9600 ± 0.142</td>
<td>2.0778 ± 0.264</td>
<td>+6.01</td>
</tr>
<tr>
<td>935</td>
<td>1.4641 ± 0.080</td>
<td>1.5130 ± 0.103</td>
<td>+3.34</td>
</tr>
<tr>
<td>4675</td>
<td>1.5172 ± 0.105</td>
<td>1.6805 ± 0.040</td>
<td>+10.76</td>
</tr>
</tbody>
</table>

Fortunately, the bias in power essentially cancels out in the $T_{rec}$ calculations, yielding valid results.

West et al. (2009) also measure $T_{rec}$ using data from Ti29, together with other engineering experiments. However, their analysis focuses solely on the compressed data for consistency between the experiments (the other engineering tests contain only compressed data). However, the compressed data in Ti29 sample very few attenuation settings (2 to 4 settings), and the other engineering tests typically sample even fewer attenuator settings. As a result, the behavior of $T_{rec}$ with attenuation loss is not thoroughly characterized by the West et al. (2009) results. The 8-bit straight Ti29 data offer the best opportunity to describe the behavior of $T_{rec}$, and this document provides the first record of these results.

We note that, prior to this analysis, we used the West et al. (2009) $T_{rec}$ results to compute the calibration scale factor. We also used a different noise locator procedure, yielding slightly different noise curves. We then solved for the calibration scale factor through the proportionality constant required to match the measured attenuation-scaled noise curves to the West et al. (2009) $T_{rec}$ values. The result, the $T_{rec}$ curve derived from the scaled Titan noise curve, is shown as the red dashed line in Figure 3.21. As expected, the resulting $T_{rec}$ curve lies directly between the red compressed $T_{rec}$ measurements, which is slightly lower than the level of the gray 8-bit straight $T_{rec}$ measurements for the altimetry and SAR mode data. Furthermore, we see that the curvature of each of the SAR mode $T_{rec}$ curves is improved by the more
3.4. **CALIBRATION SCALE FACTOR**

Table 3.4: Revised NRCS Compared to Previous NRCS (in PDS)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Bandwidth (kHz)</th>
<th>Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scat</td>
<td>117</td>
<td>94%</td>
</tr>
<tr>
<td>C-Scat</td>
<td>117</td>
<td>96%</td>
</tr>
<tr>
<td>D-SAR</td>
<td>117</td>
<td>95%</td>
</tr>
<tr>
<td>L-SAR</td>
<td>468</td>
<td>104%</td>
</tr>
<tr>
<td>H-SAR</td>
<td>935</td>
<td>104%</td>
</tr>
<tr>
<td>Alt</td>
<td>4675</td>
<td>115%</td>
</tr>
</tbody>
</table>

accurately determined noise. We list the previous calibration factors in Table 3.3; these were the calibration results that we used for calculating the NRCS values that are stored in the Planetary Data System (PDS). Our new analysis yields a more accurate \( G_c \) estimate, and the subsequent NRCS values will be different on average from those stored in the PDS by about -0.27 dB to 0.6 dB, depending on the mode. In Table 3.4, we tabulate the mean percent ratio of the new NRCS values compared to the previous NRCS values (those that are stored in the PDS) for each mode. The revised NRCS values for modes that use the scatterometry receiver filter (scatterometry, compressed scatterometry, and distant SAR) are about 5% smaller than the previous NRCS values, whereas the L-SAR and H-SAR NRCS values are about 4% larger than the previous NRCS values, and the altimetry NRCS values are as much as 15% larger than the previous altimetry NRCS values. These errors are on par with the one-sigma fluctuation in \( G_c \) that we measure from the calibration data listed in Table 3.5.

In Figure 3.22, we plot the mean Titan backscatter curves for each of the receiver modes to check that the mode NRCS results are consistent with each other. If we use the scatterometry curve as a reference, we calculate that the other curves agree on average within 10-20%. Much of this variation originates from nonuniform sampling of the different terrains on Titan. If instead we compare the backscatter collected
Figure 3.22: The mean backscatter curves measured from Titan, colored by receiver mode. Even though the modes sample the different terrains on Titan nonuniformly, the results agree on average within 10-20%.

over a homogeneous area, like over Titan’s dunes, we measure inter-mode consistency at the 5% level. The overall consistency of the mode data is well-illustrated in the feature analysis chapter (Chapter 5), where we plot the backscatter response of each surface feature and color the backscatter measurements according to their radar mode (e.g. Figure 5.10).

3.4.1 RADAR Receiver Model

The Friis formula for noise temperature says that the total receiver noise temperature is the sum of the individual element noise temperatures, where the individual temperatures are inversely weighted by the preceding element gain values (Kraus, 1966). For the simple Cassini receiver model depicted in Figure 3.18, the receiver temperature
3.4. CALIBRATION SCALE FACTOR

Figure 3.23: The receiver temperature models derived from the Ti29 noise power models, displayed as a function of total attenuation loss for each of the receiver filter bandwidths.

would have the following form:

$$T_{rec} = T_F + \frac{T_1}{G_F} + \frac{T_2 L_1}{G_F} + \frac{T_3 L_1 L_2}{G_F} + \frac{T_B L}{G_F}, \quad (3.54)$$

where $L$ represents the total attenuation loss, the inverse of the attenuation gain, created from the three individual attenuator components ($L = L_1 L_2 L_3$). Since we assume that only the attenuation gain $L$ is changing, we can lump together the front-end gains ($G_F, T_F$), as well as the back-end gains ($G_B, T_B$).

Eq. 3.54 shows that $T_{rec}$ is approximately equal to $T_F$, i.e. a constant, if the front-end gain $G_F$ is large. This is the ideal receiver response. In Figure 3.23, we plot the $T_{rec}$ curves together against $L$ for each of the receiver bandwidths. We evaluate the $T_{rec}$ curves by applying the noise power curves (described in Figure 3.19) to Eq. 3.46. Figure 3.23 clearly shows that $T_{rec}$ is not only a strong function of the value of $L$, but it also decreases with increasing receiver bandwidth. West et al. (2009) attribute these variations to an unknown narrowband noise source in the back-end
of the active receiver. As the bandwidth increases, the narrowband contamination has less of an effect, and as a result the $T_{\text{rec}}$ value converges to the high-bandwidth passive receiver value, a value between 550 K and 600 K according to West et al. (2009). The conjecture that the unknown noise source is located in the back-end of the receiver originates from the observed positive correlation between the measured receiver temperature and the attenuation loss settings: as the total attenuation loss $L$ increases, so does $T_{\text{rec}}$. According to Eq. 3.54, this dependence would largely arise only if the back-end equivalent noise temperature $T_B$ is nonzero, as would occur in the presence of an unknown back-end noise source. Figure 3.23 demonstrates that as the attenuation loss goes to zero, the $T_{\text{rec}}$ values appear to originate from a common value between 550 K and 600 K, i.e. the high-bandwidth radiometry measurement.

Eq. 3.54 also indicates that the value of $T_{\text{rec}}$ depends on the exact breakdown of the attenuation loss between the individual attenuator elements. For example, if $L_1 = 12 \text{ dB}, L_2 = 6 \text{ dB},$ and $L_3 = 2 \text{ dB}$, then $L = 20 \text{ dB}$ and $T_{\text{rec}}$ equals some value $Z$. But if $L_1 = 10 \text{ dB}, L_2 = 6 \text{ dB},$ and $L_3 = 4 \text{ dB}$, $L$ still equals 20 dB, but $T_{\text{rec}}$ will now be smaller than $Z$ due to the lesser weight of $L_1$ on $T_2$. The dependence of $T_{\text{rec}}$ on the configuration of the individual attenuator elements explains much of the observed fluctuation in $T_{\text{rec}}$ (and $P_{n_dW}$) with respect to $L$, and likely also explains the fluctuation in $G_c$.

We demonstrate the fluctuating behavior of $T_{\text{rec}}$ with respect to the configuration of the individual attenuator elements by simulating the receiver model pictured in Figure 3.18. Assuming that the individual attenuator noise temperatures are similar, i.e. $T_1 \simeq T_2 \simeq T_3$, a value that we call $T_e$, and assuming that $T_F \approx 550K$, as suggested by the observations, we apply Eq. 3.54 and numerically solve for the parameters $L_1$, $L_2$, $L_3$, $T_e$, and $T_B$ that minimize the squared error between the model solution and the measured $T_{\text{rec}}$ values derived from the Saturn/CMB data. We find that we can replicate the behavior of $T_{\text{rec}}$ very well, as shown by the red diamonds in Figure 3.24, using the model parameters given in Figure 3.25. The solution for $T_B$ behaves as expected, decreasing in value with increasing bandwidth, but the solution for $T_e$ is
not as steady between the different receiver modes as one might expect, jumping from 17.5 K at 117 kHz bandwidth to 5 K at 4675 kHz bandwidth. An additional constraint in the simulation might include holding $T_e$ constant across the modes.

The model results we present above are not a unique solution, but serve only
Figure 3.25: The best-fit model parameters used to evaluate the receiver model and recreate the observed receiver temperature behavior (see the results in Figure 3.24). The $T_e$ and $T_B$ parameters are given in the figure panel titles and the individual attenuator values ($L_1, L_2, L_3$) are plotted as a function of the total attenuation loss $L$ (note: only the circle points are valid - the connecting lines are for clarity only). This combination of parameters is capable of explaining the observed $T_{rec}$ at each value of $L$, but they are not a unique solution.

to illustrate that the exact value of $T_{rec}$ will depend on how the attenuation loss is divided among the particular components, and this is information that we do not readily have. Although the intended individual attenuator values are stored with the data record, only the total attenuation value is calibrated before launch to its true
value. If we knew the exact decomposition of attenuation, we could solve the receiver model for $T_B$ and $T_e$ and thereby solve for $T_{rec}$ more precisely, eliminating much of the error caused by the fluctuation. Instead, we settle for describing the mean response of $T_{rec}$ with the total $L$ value, finding that the resulting relative errors are less than 10%.

### Table 3.5: Ti29 Calibration Experiment Results

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>Attenuation (dB)</th>
<th>Saturn $P_n$ (dW)</th>
<th>CMB $P_n$ (dW)</th>
<th>$G_c$ (1e18 dW/W)</th>
<th>$T_{rec}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>117</td>
<td>6.4</td>
<td>585.71 ± 5.21</td>
<td>512.80 ± 6.83</td>
<td>1.90 ± 0.04</td>
<td>726 ± 36</td>
</tr>
<tr>
<td>117</td>
<td>7.4</td>
<td>485.13 ± 1.35</td>
<td>423.27 ± 3.88</td>
<td>2.03 ± 0.08</td>
<td>706 ± 39</td>
</tr>
<tr>
<td>117</td>
<td>8.4</td>
<td>394.52 ± 2.38</td>
<td>346.69 ± 2.65</td>
<td>1.97 ± 0.01</td>
<td>748 ± 39</td>
</tr>
<tr>
<td>117</td>
<td>9.4</td>
<td>340.06 ± 1.63</td>
<td>303.27 ± 2.85</td>
<td>1.90 ± 0.06</td>
<td>851 ± 45</td>
</tr>
<tr>
<td>117</td>
<td>10.3</td>
<td>268.30 ± 2.86</td>
<td>239.22 ± 2.15</td>
<td>1.88 ± 0.05</td>
<td>849 ± 40</td>
</tr>
<tr>
<td>117</td>
<td>11.3</td>
<td>231.06 ± 2.26</td>
<td>205.28 ± 1.95</td>
<td>2.09 ± 0.03</td>
<td>822 ± 39</td>
</tr>
<tr>
<td>117</td>
<td>12.3</td>
<td>186.78 ± 2.33</td>
<td>167.65 ± 1.55</td>
<td>1.94 ± 0.08</td>
<td>905 ± 41</td>
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<tr>
<td>117</td>
<td>13.3</td>
<td>164.29 ± 0.74</td>
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<td>883 ± 47</td>
</tr>
<tr>
<td>117</td>
<td>14.2</td>
<td>129.46 ± 1.35</td>
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<td>1.88 ± 0.10</td>
<td>1020 ± 48</td>
</tr>
<tr>
<td>117</td>
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<td>103.41 ± 0.60</td>
<td>2.02 ± 0.07</td>
<td>1040 ± 51</td>
</tr>
<tr>
<td>117</td>
<td>16.1</td>
<td>92.30 ± 0.44</td>
<td>85.12 ± 0.81</td>
<td>1.76 ± 0.09</td>
<td>1224 ± 65</td>
</tr>
<tr>
<td>117</td>
<td>17.1</td>
<td>82.64 ± 0.79</td>
<td>76.17 ± 0.61</td>
<td>1.97 ± 0.06</td>
<td>1217 ± 59</td>
</tr>
<tr>
<td>117</td>
<td>18.1</td>
<td>65.87 ± 0.80</td>
<td>60.89 ± 0.65</td>
<td>1.92 ± 0.06</td>
<td>1263 ± 58</td>
</tr>
<tr>
<td>117</td>
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<td>55.64 ± 0.61</td>
<td>1.91 ± 0.01</td>
<td>1456 ± 69</td>
</tr>
<tr>
<td>117</td>
<td>20.1</td>
<td>48.09 ± 0.59</td>
<td>44.84 ± 0.59</td>
<td>1.98 ± 0.01</td>
<td>1427 ± 65</td>
</tr>
<tr>
<td>117</td>
<td>21.1</td>
<td>44.56 ± 0.74</td>
<td>41.70 ± 0.33</td>
<td>2.20 ± 0.31</td>
<td>1506 ± 62</td>
</tr>
<tr>
<td>468</td>
<td>10.4</td>
<td>860.08 ± 11.53</td>
<td>742.59 ± 7.52</td>
<td>1.90 ± 0.06</td>
<td>652 ± 38</td>
</tr>
<tr>
<td>468</td>
<td>11.3</td>
<td>707.17 ± 12.02</td>
<td>612.79 ± 10.14</td>
<td>1.91 ± 0.04</td>
<td>670 ± 37</td>
</tr>
<tr>
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<td>569.40 ± 3.90</td>
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<td>1.95 ± 0.01</td>
<td>660 ± 43</td>
</tr>
<tr>
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<tr>
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<td>702 ± 45</td>
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<td>763 ± 48</td>
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</table>

Continued on next page
<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>Saturn $P_n$ (dW)</th>
<th>CMB $P_n$ (dW)</th>
<th>$G_c$ (1e18 dW/W)</th>
<th>$T_{rec}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>468</td>
<td>211.03 ± 2.63</td>
<td>184.29 ± 1.51</td>
<td>2.04 ± 0.09</td>
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<tr>
<td>468</td>
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<tr>
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<tr>
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<td>468</td>
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<td>59.64 ± 0.22</td>
<td>1.77 ± 0.10</td>
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<tr>
<td>468</td>
<td>52.91 ± 0.37</td>
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<td>1.87 ± 0.09</td>
<td>1022 ± 67</td>
</tr>
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<td>1225.9 ± 11.82</td>
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<td>1.46 ± 0.04</td>
<td>597 ± 27</td>
</tr>
<tr>
<td>935</td>
<td>1006.3 ± 11.08</td>
<td>860.5 ± 8.41</td>
<td>1.48 ± 0.03</td>
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<td>1.49 ± 0.01</td>
<td>570 ± 35</td>
</tr>
<tr>
<td>4675</td>
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<td>682.6 ± 9.15</td>
<td>1.48 ± 0.07</td>
<td>577 ± 37</td>
</tr>
<tr>
<td>4675</td>
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<td>451.3 ± 6.23</td>
<td>1.62 ± 0.01</td>
<td>553 ± 32</td>
</tr>
<tr>
<td>4675</td>
<td>323.7 ± 5.05</td>
<td>280.5 ± 1.53</td>
<td>1.32 ± 0.11</td>
<td>671 ± 36</td>
</tr>
</tbody>
</table>

Continued on next page
3.5 Block Adaptive Quantization Effects

The Cassini RADAR observations often involve large data volumes, especially during the imaging and altimetry experiments, and data compression is required to reduce the downlink data rate. To this end, a block adaptive quantization (BAQ) algorithm is implemented in software in the on-board flight computer unit (FCU) and applied to data collected in H-SAR, L-SAR, D-SAR, and altimetry modes. The Cassini BAQ design is based on the compression algorithm used for Magellan radar (see Kwok and Johnson (1989)). The BAQ algorithm used by Cassini RADAR has two instantiations, an 8 bit to 2 bit (8-2 BAQ) version and an 8 bit to 4 bit (8-4 BAQ) version, where 8-2 BAQ is used for imaging and 8-4 BAQ is used for altimetry. The 8-2 BAQ algorithm is specified in Appendix C of the Cassini RADAR Digital Subsystem (DSS) High Level Design (HLD) handbook. We summarize the 8-2 BAQ and 8-4 BAQ algorithms here.

Ideally the BAQ algorithm is encoded and decoded behind the scenes, in the pre-processing steps before forming the burst ordered data products (BODPs, or the official data sets produced by the Cassini RADAR team; the BODPs are organized as time-ordered records for each radar burst of operation). However, we uncover some BAQ-effects that permeate into the final data. For example, consider the fact that the receiver window often incorporates some receive-only noise before and after the

### Table 3.5 – continued from previous page

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>Attenuation (dB)</th>
<th>Saturn $P_n$ (dW)</th>
<th>CMB $P_n$ (dW)</th>
<th>$G_c$ (1e18 dW/W)</th>
<th>$T_{rec}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4675</td>
<td>25.1</td>
<td>219.3 ± 1.62</td>
<td>187.8 ± 1.60</td>
<td>1.53 ± 0.01</td>
<td>613 ± 38</td>
</tr>
<tr>
<td>4675</td>
<td>27.1</td>
<td>142.0 ± 0.51</td>
<td>121.0 ± 0.84</td>
<td>1.61 ± 0.03</td>
<td>595 ± 39</td>
</tr>
<tr>
<td>4675</td>
<td>29.1</td>
<td>96.2 ± 1.16</td>
<td>82.0 ± 1.06</td>
<td>1.71 ± 0.01</td>
<td>597 ± 34</td>
</tr>
<tr>
<td>4675</td>
<td>31.0</td>
<td>65.6 ± 0.67</td>
<td>57.5 ± 0.75</td>
<td>1.54 ± 0.02</td>
<td>726 ± 43</td>
</tr>
<tr>
<td>4675</td>
<td>32.0</td>
<td>54.0 ± 5.24</td>
<td>47.9 ± 5.21</td>
<td>1.45 ± 0.01</td>
<td>798 ± 22</td>
</tr>
</tbody>
</table>
received echo sequence. Yet the BAQ algorithm samples the pulsed echo signal only at the beginning and end of the receive window to estimate the signal levels. Mixing noise-only data with echo-data in these intervals introduces a variable bias, and the decoded signal is an underestimate of the true signal. We model the effects of the noise-only intervals on the BAQ algorithm output and develop a correction factor to remove the BAQ bias. As another example of processing complications introduced by the BAQ compression, which we discuss further in Chapter 7, the BAQ algorithm behaves unexpectedly when the dynamic range of the RADAR receiver is not large enough to contain the received signal and the signal saturates. In Section 7.2.1, we model the effects of saturation on the BAQ algorithm in an attempt to correct some of the clipping incurred.

3.5.1 8-2 BAQ

The Cassini BAQ algorithm assumes three things about the received data: 1) the individual samples have a zero-mean Gaussian distribution, 2) the Gaussian statistics vary slowly with range (or time delay), and 3) the spacecraft does not move substantially over the course of a burst sequence. The first assumption proves true for most rough surfaces, where there are a large number of scatterers reflecting signal back towards the radar, but fails for smooth surfaces that are dominated by a single specular echo (e.g. Section 7.2). The second assumption is valid because the scatterers that contribute to each echo sample are spread over a large area defined by a combination of the antenna beam and the pulse projection, thus the set of scatterers contributing to each sample does not change significantly from one sample to the next. The third assumption holds true for the spacecraft velocities (<6 km/s) and burst lengths (<0.02 s) typical of Cassini RADAR observations. The third assumption implies that the pulses transmitted within a burst will each sample the same area on the surface, that a specific ground target will not move by more than a sample or two within the echo profile over the course of the burst. Consequently, the echo profiles are effectively periodic - each measured echo profile is an instantiation of the true echo profile.
The Cassini BAQ method characterizes the true echo profile by segmenting the received burst into pulse repetition intervals (PRI). The statistics at each position within the PRI profile are estimated from the set of samples collected across all PRIs at corresponding positions (using assumption 3). Since the distribution at each profile position is assumed zero-mean Gaussian, only the variance $\sigma^2$ needs to be calculated (using assumption 1). Rather than considering every position within the PRI, the BAQ algorithm implemented for Cassini divides the PRI into 24 uniform blocks and calculates the variance for each block (using assumption 2). The 24 variance values effectively describe the mean echo power profile. Each individual 8-bit sample $X$ is compared to a threshold $Th$ related to the variance of its containing block and the sample is encoded into a 2-bit word $Y$:

$$Y = \begin{cases} 
11 & \text{if } X < -Th, \\
10 & \text{if } -Th \leq X \leq 0, \\
00 & \text{if } 0 < X \leq Th, \\
01 & \text{if } Th < X.
\end{cases} \quad (3.55)$$

The optimal 8-2 BAQ algorithm recovers the original sample from the encoded word $Y$ with minimal distortion as follows:

$$\hat{X} = \begin{cases} 
-\frac{1.5104}{\sqrt{0.884}} \cdot \frac{1}{0.98} Th & \text{if } Y = 11, \\
-\frac{0.4528}{\sqrt{0.884}} \cdot \frac{1}{0.98} Th & \text{if } Y = 10, \\
\frac{0.4528}{\sqrt{0.884}} \cdot \frac{1}{0.98} Th & \text{if } Y = 00, \\
\frac{1.5104}{\sqrt{0.884}} \cdot \frac{1}{0.98} Th & \text{if } Y = 01.
\end{cases} \quad (3.56)$$

where $\hat{X}$ is the decoded estimate of the original sample $X$, and the threshold $Th$ is defined as 0.98 times the standard deviation $\sigma$ of the containing block, i.e. if the sample exists in block $k$ of its PRI ($1 < k < 24$), then $Th_k = 0.98\sigma_k$. The 24 $Th_k$ values are unique to each burst and are downlinked as 8-bit words together with the
2-bit data words representing each sample.

Calculating a block’s standard deviation involves summing squares of the samples and taking the square root, operations that can be computationally intensive. For simplicity, the BAQ algorithm is implemented by approximating the variance \( \sigma^2 \) of a zero-mean Gaussian input analog signal from the average magnitude \( \mu = E(|X|) \) of its 8-bit digitized version \( X \) by using the following equation:

\[
\mu = 127.5 - \sum_{n=1}^{127} \text{erf}\left(\frac{n}{\sigma\sqrt{2}}\right).
\]  

(3.57)

The Cassini DSS HLD refers to the right hand side of Eq. 3.57 as \( F(\sigma) \) and shows that \( F(\sigma) \) can be directly inverted to solve for \( \sigma \). Thus, the BAQ threshold of block \( k \) is calculated as \( Th_k = 0.98 F^{-1}(\mu) \).

We illustrate the segmentation of the burst into PRI and the segmentation of a PRI into blocks in Figure 3.26. Using the same nomenclature as the Cassini RADAR DSS HLD, each received echo sequence comprises \( N_e \) PRIs, and each PRI consists of \( N_p \) digital samples. The 8-2 BAQ algorithm segments each PRI into \( N_b \) uniformly separated blocks, where \( N_b \) is always set to 24. Each block has \( N_s \) samples, except for the last block which will include any additional samples if \( N_s N_b \) does not divide evenly into \( N_p \).

In calculating the threshold \( Th \), or variance, of each block, the 8-2 BAQ algorithm only considers the first and last eight samples of each block within the first and last eight PRI. This choice of subsamples keeps the algorithm computations simple while retaining enough samples for a useful threshold calculation (256 samples for each of the 24 blocks, or 256 samples for each variance estimation). While this algorithm saves computation time, we find that it introduces a bias if the PRI periodicity assumption fails, i.e. if there are noise-only intervals surrounding the echo sequence. In Figure 3.27, we see that non-zero noise-only intervals are a common occurrence for the 8-2 BAQ modes: H-SAR bursts typically have 3 to 5 noise-only PRI surrounding the echo sequence, L-SAR bursts often have 1 to 4 noise-only PRI, while D-SAR bursts
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**Figure 3.26:** Illustration of the segmentation of the burst receive window into $N_e$ pulse repetition intervals (PRI) and the segmentation of each PRI into $N_b$ blocks. The first eight and last eight samples in each block, across the first eight and last eight PRIs, are used to estimate the variance of the block and thus the threshold value. In this way, 24 threshold values are derived for each burst, and every sample within the burst is compared to its block’s threshold value to properly decode it into a quantized word.

most commonly have 4 to 6 noise-only PRI. The noise-only intervals result largely from transmitting fewer pulses than the length of the receive window, but migration of the echo sequence within the receive window can increase the number of noise-only intervals as well.

To demonstrate the effects of leading/trailing noise-only intervals on the 8-2 BAQ operation, we implement the 8-2 BAQ algorithm and apply it to simulated signals with varying amounts of noise-only intervals. Figure 3.28 illustrates the results for zero-mean Gaussian signals with different signal-to-noise ratios. As the number of noise-only PRI increases, the 8-2 BAQ algorithm increasingly underestimates the simulated signal’s average power levels. The bias is more significant at high SNR. For
Figure 3.27: Frequency of occurrence of noise-only pulse repetition intervals (PRI) for each 8-2 BAQ mode (using data from Ta through T71). Noise-only PRI at the front and/or back of a burst’s receive window will introduce a bias in the 8-2 BAQ algorithm.

example, with eight noise-only PRI (half of the number of PRI used to determine the threshold values), the signal power is underestimated by about 40% at 10 dB SNR and 50% at 20 dB SNR. More commonly, for four or five noise-only PRI the error is about 25% or 30%, respectively, at 20 dB SNR.

To correct the bias introduced by the presence of noise-only PRI, we evaluate a parameter $X$ that is a measure of how different the leading/trailing PRI are from the rest of the burst. Following the BAQ algorithm’s use of the average magnitude to determine the block threshold values (see Eq. 3.57), we define $X$ as the ratio of the average signal magnitude computed over the first and last eight PRI to the average magnitude over all the PRI in between. Together with the Cassini RADAR engineers at JPL, we then determine a correction factor $Q_{baq}$ that characterizes the 8-2 BAQ
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Figure 3.28: Effect of leading/trailing noise-only pulse repetition intervals (PRI) on 8-2 BAQ estimation. As the number of noise-only PRI increase, the 8-2 BAQ algorithm increasingly underestimates the simulated signal’s average power levels. The noise-only intervals pull down the measured threshold values and result in a lower decoded signal level. The bias is more significant at higher SNR. At low SNR, the signal is indistinguishable from the noise, so the variance/thresholds computed for each of the 24 blocks across the 16 PRI will be correct. But at high SNR, the large contrast between the noise-only intervals and the echo intervals within the 16 PRI results in distorted variance/threshold calculations and thus larger errors.

Error in 8-2 BAQ Operation as a function of Noise-Only PRI and SNR

\[ Q_{\text{baq}} = -2.430X^2 + 5.7853X - 2.3936 \]  \hspace{1cm} (3.58)

\( Q_{\text{baq}} \) is an estimate of the reconstructed signal energy after block adaptive quantization compared to the true received signal energy (or, equivalently, the ratio of the average powers), as plotted along the ordinate in Figure 3.28. We note that \( Q_{\text{baq}} \) is defined only for 0.77 \( \leq X \leq 1.026 \), the range of values used to create Eq. 3.58. We find that most measured \( X \) values fall within this range (Figure 3.29A), and any
Figure 3.29: We measure the parameter $X$ over all bursts collected in H-SAR, L-SAR, and D-SAR modes from Ta through T71 in (A). The histograms are calculated for uniformly spaced bins with widths of 0.0051. The histogram results are normalized for each mode to determine the percent likelihood of occurrence. $X$ approximates how different the leading/trailing PRI are from the rest of the burst. Since the 8 leading and 8 trailing PRI are used for the 8-2 BAQ threshold calculations, if they do not represent the rest of the burst the BAQ output will be in error. We map $X$ into an error estimate $Q_{baq}$ in (B); $Q_{baq}$ estimates the variance of the BAQ-signal compared to the variance of the true signal. We find that the power levels of the Cassini 8-2 BAQ received signals are about 70% to 98% underestimated. We correct the received signal levels by dividing their calculated power or energy by $Q_{baq}$.

Values measured outside of this range are simply clipped to the nearest valid $X$ value, with minimal effects. We compute the $Q_{baq}$ correction results for each 8-2 BAQ burst from Ta through T71 and display their frequency of occurrence for each of the three 8-2 BAQ modes in Figure 3.29B.

If we define $E'_{rd}$ as the received energy measured by Cassini RADAR while in one of the 8-2 BAQ modes (H-SAR, L-SAR, or D-SAR), then $E_{rd} = E'_{rd}/Q_{baq}$ is the corrected received energy, with the BAQ bias removed. Similarly, if $P'_{rdw}$ is the
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Figure 3.30: Demonstration of the 8-2 BAQ bias and correction for Titan T12 and T15 transition data. Here, the RADAR switches from 8-2 BAQ D-SAR mode (on the left) to regular 8 bit scatterometry mode (on the right). The received power measurements are plotted in black. Although the observation geometry is nearly identical before and after the mode transition, and even though the receiver setup is the same, a large jump (∼7% for T12 and ∼16% for T15) is readily observed in the received power measurements where the mode transition occurs. We apply our 8-2 BAQ correction procedure to the D-SAR power measurements. The results, shown in red, appear to line up perfectly with the 8-bit straight scatterometry data suggesting that the 8-2 BAQ bias has been removed.

We now demonstrate the BAQ bias and our correction technique for data collected during two Titan D-SAR observations: T12 and T15. Here, the RADAR switches from 8-2 BAQ D-SAR mode to regular 8 bit scatterometry mode. The observation received power measured for 8-2 BAQ estimated data, then $P_{r_{dW}} = P'_{r_{dW}} / Q_{baq}$ is the corrected received power. We make this correction before estimating the signal-only energy $E_{s_{dJ}}$ and signal-only power $P_{s_{dW}}$ for use in the radar equations developed in Eq. 3.36 and Eq. 3.35. In this manner, we remove the unforeseen effects introduced by the presence of noise-only PRI in the receive window.

We now demonstrate the BAQ bias and our correction technique for data collected during two Titan D-SAR observations: T12 and T15. Here, the RADAR switches from 8-2 BAQ D-SAR mode to regular 8 bit scatterometry mode. The observation
geometry is nearly identical before and after the mode transition, and even the receiver bandwidth is the same, but a large jump (≈7% for T12 and ≈16% for T15) is readily observed in the received power measurements where the mode transition occurs, as shown in black in Figure 3.30. Our measured values of $X$ for the T12 D-SAR data vary between 0.93 and 0.98, implying a $Q_{baq}$ error value between 0.89 and 0.94. The T15 D-SAR data has $X$ measurements between 0.88 and 0.92, which map to a $Q_{baq}$ between 0.81 and 0.87. We divide the received power measurements by the computed $Q_{baq}$ to correct the D-SAR data. The results are colored in red in the figure and appear to line up perfectly with the 8-bit straight scatterometry data, suggesting that the 8-2 BAQ bias has been correctly removed. There are many other such transition points in the Titan data set that confirm the correction procedure; T12 and T15 serve as just two examples.

### 3.5.2 8-4 BAQ

The 8-4 BAQ algorithm is similar to the 8-2 BAQ except that it encodes the data into 16 4-bit words and estimates the block variance $\sigma^2$ from the first and last eight samples of the corresponding block within the first and last four PRI, rather than the first and last eight PRI. Each 8-bit sample $X$ within the receive window is compared to the threshold $Th$ calculated for its containing block, where $Th = 0.98 F^{-1}(\mu)$ as before, and encoded into a 4-bit word $Y$ according to the following relationships:
\( Y = \begin{cases} 
1111 & \text{if } X < -1.1000 Th, \\
1110 & \text{if } -1.1000 Th \leq X < -0.8400 Th, \\
1101 & \text{if } -0.8400 Th \leq X < -0.6550 Th, \\
1100 & \text{if } -0.6550 Th \leq X < -0.5000 Th, \\
1011 & \text{if } -0.5000 Th \leq X < -0.3650 Th, \\
1010 & \text{if } -0.3650 Th \leq X < -0.2375 Th, \\
1001 & \text{if } -0.2375 Th \leq X < -0.1175 Th, \\
1000 & \text{if } -0.1175 Th \leq X < 0, \\
0000 & \text{if } 0 \leq X < 0.1175 Th, \\
0001 & \text{if } 0.1175 Th \leq X < 0.2375 Th, \\
0010 & \text{if } 0.2375 Th \leq X < 0.3650 Th, \\
0011 & \text{if } 0.3650 Th \leq X < 0.5000 Th, \\
0100 & \text{if } 0.5000 Th \leq X < 0.6550 Th, \\
0101 & \text{if } 0.6550 Th \leq X < 0.8400 Th, \\
0110 & \text{if } 0.8400 Th < X < 1.1000 Th, \\
0111 & \text{if } 1.1000 Th < X. 
\end{cases} \)
The optimal 8-4 BAQ algorithm recovers the original sample from $Y$ with minimal distortion as follows:

$$\hat{X} = \begin{cases} 
-1.2490 Th & \text{if } Y = 1111, \\
-0.9455 Th & \text{if } Y = 1110, \\
-0.7395 Th & \text{if } Y = 1101, \\
-0.5740 Th & \text{if } Y = 1100, \\
-0.4305 Th & \text{if } Y = 1011, \\
-0.3000 Th & \text{if } Y = 1010, \\
-0.1775 Th & \text{if } Y = 1001, \\
-0.0585 Th & \text{if } Y = 1000, \\
1.2490 Th & \text{if } Y = 0111, \\
0.9455 Th & \text{if } Y = 0110, \\
0.7395 Th & \text{if } Y = 0101, \\
0.5740 Th & \text{if } Y = 0100, \\
0.4305 Th & \text{if } Y = 0011, \\
0.3000 Th & \text{if } Y = 0010, \\
0.1775 Th & \text{if } Y = 0001, \\
0.0585 Th & \text{if } Y = 0000. 
\end{cases}$$

(3.60)

The 8-4 BAQ algorithm is used exclusively by RADAR altimetry mode data. Just like the 8-2 BAQ algorithm, the 8-4 BAQ algorithm is susceptible to errors introduced by leading/trailing noise-only intervals, perhaps more so because it uses fewer PRI to estimate the threshold values. Fortunately, most RADAR altimetry experiments transmit more pulses (21 pulses) than would fit within the receive window (15 pulses). On these occasions, even with the migration of the echo sequence through the receive
window from one burst to the next, the receive window is completely full of pulse echoes, and there are zero noise-only PRI. Very rarely (0.7% of the time), the RADAR may transmit fewer pulses (9 pulses) than it is capable of receiving (15 pulses), and these bursts will be prone to BAQ error. Of the 18,356 altimetry bursts measured from Ta through T71, 18,178 have zero noise-only PRI (99%), 53 have one noise-only PRI (0.3%), and 120 bursts have six noise-only PRI (0.7%). Only 5 bursts have two, three, or five noise-only PRI. The severity of the BAQ bias introduced by the six noise-only PRI will depend on the signal-to-noise ratio of the pulse echo sequence. We compute the $X$ parameter, where $X$ measures how different the 4 leading and 4 trailing PRI are from the rest of the burst, for the bursts with six noise-only PRI and find $0.5 < X < 0.9$. Given the small number of altimetry bursts that are affected by the BAQ bias, we choose to ignore the bursts with non-zero noise-only PRI rather than correct them.

Understanding how the 8-4 BAQ algorithm operates is important for reasons other than the possibility of a noise-only introduced bias. Because of the near-nadir geometry, altimetry echoes are very sensitive to changes in the surface roughness and dielectric properties, and the signal levels may change rapidly. Typical RADAR observations use the receiver’s auto-gain feature, where the attenuation setting of a new group of bursts is increased or decreased in 2 dB increments depending on the signal level of the last burst in the previous group. The auto-gain update typically occurs about every 50 bursts, which is sometimes not frequent enough to catch up with a suddenly increasing signal level. Thus, occasionally, an altimetry echo may be larger than expected and may saturate the RADAR receiver. This is particularly true for specular liquid detections, as described in Chapter 7.

When an 8 bit digitized sample saturates, it is clipped to a maximum absolute amplitude of 127.5 $dV$. This clipping alone will introduce errors in the measured signal, yet the clipped signal will incur additional saturation effects from the 8-4 BAQ algorithm. For example, the maximum threshold value that can occur in the BAQ algorithm is $Th = 254$. When enough of the 8-bit input samples (>38%) in block $k$
are clipped to $\pm 127.5\, dV$, the 8-4 BAQ algorithm will always measure $Th_k$ to be at its highest value, $Th_k = 254$. $Th_k$ may even assume its maximum value when the ratio of clipped input samples is as low as 18%, depending on the values of the non-clipped samples in the containing block. The encoding formula in Eq. 3.59 says that if $X$ is constrained to have a magnitude less than 127.5, and $Th_k = 254$, then $X$ can only map to one of ten 4-bit words; it will always satisfy $|X| < 0.655Th$. As a result, the signal saturation results in an underutilization of the 8-4 BAQ capabilities (six words are not used when $Th_k = 254$). Furthermore, since the algorithm described in Eq. 3.60 decodes the words by scaling the threshold by a constant value that depends on the word, and the threshold value is fixed to $Th = 254$, the ten words will always decode to the same ten fixed values: $\pm 14.859, \pm 45.085, \pm 76.2, \pm 109.347, \text{ or } \pm 145.796$.

We demonstrate these effects in Figure 3.31. Here, we simulate Gaussian signals with a standard deviation that varies between $1 \, dV$ and $200 \, dV$. We clip any samples with absolute amplitudes greater than $127.5 \, dV$, which starts to occur at an input rms amplitude of $40 \, dV$. In the upper left panel, we observe the 8-4 BAQ threshold results. At input rms amplitudes beginning around $90 \, dV$ the threshold values start to saturate at their maximum value. The upper right panel shows the corresponding 8-4 BAQ decoded output data in their stacked-histogram form. The bottom panels show the individual histogram results for input rms amplitudes $25 \, dV$ (row 25 of the stacked-histogram image) and $200 \, dV$ (row 200 of the stacked-histogram image). The stacked-histogram image demonstrates the tendency towards ten fixed decoded signal levels, starting at an input rms amplitude around $100 \, dV$, when about 20% of the input samples are clipped and the thresholds begin saturating. For input rms amplitudes greater than $\sim 145 \, dV$, more than 38% of the input samples are clipped so that $Th = 254$ for all blocks, and the output signal is entirely isolated to the ten fixed decoded levels. Furthermore, we observe that the number of encoded words decreases from 16 to 14 at an input rms amplitude around $60 \, dV$, from 14 to 12 at around $80 \, dV$, and from 12 to 10 words at around $110 \, dV$.

All of the saturation effects described above will distort the measured 8-4 BAQ
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Figure 3.31: To illustrate how the 8-4 BAQ algorithm behaves for a saturated signal, we simulate Gaussian signals with rms voltage amplitudes between 1 dV and 200 dV. The signal sample values are clipped to exist between $-127.5$ dV and $127.5$ dV, as an 8-bit digitized sample would be. The upper left panel shows the 8-4 BAQ threshold results and the upper right panel shows the corresponding 8-4 BAQ decoded output histograms, stacked vertically such that the first row is the histogram of the output signal corresponding to an input rms amplitude of 1 dV, and the last row is the histogram of the output signal corresponding to an input rms amplitude of 200 dV. The bottom panels plot the individual output histogram results for input rms amplitudes 25 dV and 200 dV.

signal when the echo is larger than the dynamic range of the RADAR receiver. Fortunately, of the 18,356 altimetry bursts measured from Ta through T71, only 185 bursts saturated their 8-4 BAQ threshold values, implying that at least $\sim20\%$ of the 8-bit input samples had been clipped by the analog-to-digital converter. Furthermore, 151 of the 185 bursts occurred during the T49 nadir-looking observation over Ontario.
Figure 3.32: The observed effect of 8-4 BAQ saturation on output amplitude discretization. On the left are the stacked histograms of the 34 bursts (outside of T49) that have output samples with amplitudes equal to their extreme representations ($\pm145.796\ dV$), where the color indicates the percent occurrence. On the right, the measured percentage of saturated samples (amplitudes equal to $\pm145.796\ dV$) is plotted against the measured percentage of discretization (the percentage of samples confined to the ten fixed amplitudes). The red circles indicate the 34 saturated bursts outside of T49 while the non-circled black dots indicate the T49 measurements. The gray line is the predicted behavior from simulations.

Lacus. In Chapter 7, we deduce that the 151 Ontario Lacus bursts represent specular reflections from the liquid surface, hence the larger-than-expected received power. The other 34 saturated bursts indicate the presence of an area in the beam that is smoother than its surroundings, since the attenuation setting is tuned according to the signal strength of regions sensed in previous adjacent bursts. Yet, the saturation of the non-T49 bursts is not generally extreme and the detected areas do not appear smooth enough to generate coherent specular reflections.

We plot the histograms of the received voltage amplitudes for the 34 non-T49 saturated altimetry bursts in Figure 3.32. On the right of this figure, we also plot the amount of saturation (the percentage of received echo samples with amplitudes equal to $\pm145.796\ dV$) versus the amount of discretization (the percentage of received echo samples that are confined to the ten fixed amplitudes described above). In agreement with our simulations (shown as the gray line), as the percentage of saturated samples approaches 40%, the samples are increasingly isolated within the ten discrete bins.
The figures show that, except for a group of 14 bursts, the measured amplitudes are not extremely saturated (<10% of the samples have amplitudes equal to their extreme representations) or extremely discretized (<30% of the samples are confined to the ten fixed bins). The 14 exceptions do appear to have extreme saturation, but closer scrutinization reveals that these bursts simply have a very low attenuation setting (33 dB) given their close altitude (1100 km), a situation likely resulting from non-optimal commanding when RADAR SAR switched sides during the T41 closest approach. Typically, the attenuation has increased to 45 dB by the time the spacecraft nears 1000 km altitudes. However, the T49 measurements, indicated with non-circled black dots in the right panel of Figure 3.32, show a great deal of saturation and discretization. These data show some deviations from the predicted simulated behavior as the amount of saturation increases. The deviations likely result from nonlinearities introduced by the saturated receiver elements which make it difficult to model and correct the amount of saturation.

In Chapter 7, we make an effort to correct some of the saturation effects incurred by the specular echoes from Ontario Lacus, an undertaking that requires detailed understanding of how the 8-4 BAQ algorithm behaves for clipped signals, as we’ve documented in this section. We go to great lengths to improve our estimation of the true specular reflection signal strengths so that we can better model the degree of smoothness across the lake, and by extension, the prevalence of wind-generated waves. We do not go to such effort to correct the 34 other saturated altimetry bursts because of the uncertainties remaining in the correction procedure and the infrequency of their occurrence. Instead, we choose to simply ignore these 34 bursts and remove them from our dataset, flagging them as potentially smooth sites of interest.
3.6 Measurement Uncertainties

Error in the measured radar cross section is controlled by several factors: systematic instrument errors determine the absolute uncertainty, while noise statistics and receiver variation determine the relative uncertainty. (West et al., 2009) estimate the absolute error to be 12% at the one-sigma level due primarily to receiver gain variability. In Section 3.4, we determine that the relative uncertainty in the data due to the variation of the thermal noise with attenuator setting is less than the 10% level. On average, the relative uncertainty between the modes appears to be closer to the 5% level. In addition to the relative calibration uncertainties, the statistical measurement uncertainty affects the relative uncertainty of the measured mean signal level, which we explain next.

The echo from a resolution cell comprises the reflected signals from numerous individual scatterers. Each scattered signal has its own amplitude and phase such that the total signal may be represented as a random phasor sum. Because the number of scatterers contributing to the total signal is generally very large for a distributed surface, the central limit theorem applies, and the real and imaginary components of the total signal will be normally distributed. It is readily shown that these Gaussian distributed components will be independent and of equal variance (Ulaby et al., 1986). As a result, the net field magnitude follows a Rayleigh distribution, whereas the square of the magnitude, or the power, follows an exponential distribution. For exponentially distributed signals, the standard deviation of the signal is equal to its mean value. Thus, the variation, or noise, in a measurement is multiplicative: as the signal gets larger, so does the uncertainty. This is called fading, or speckle noise. We can reduce large uncertainties by averaging multiple echo measurements together: the standard deviation of the mean received power estimate decreases in proportion to the inverse square root of the number of independent samples averaged.

The mean echo power estimate ($P_{s_{4W}}$) that we are interested in is derived from the mean received power estimate ($P_{r_{4W}}$) and the mean noise power estimate ($P_{n_{4W}}$), as in
Eq. 3.12. As described above, we can reduce the uncertainty in the $P_{dW}$ estimate by averaging measurements together, but how does this relate to the uncertainty in the $P_{sdW}$ estimate? Ulaby et al. (1986, Section 7-2.4.) show that the standard deviation of the $P_{sdW}$ estimate is given by

$$\sigma_s = P_{sdW} \sqrt{\frac{1}{N_r} \left(1 + \frac{1}{S_n}\right)^2 + \frac{1}{N_n} \left(\frac{1}{S_n}\right)^2}$$

(3.61)

where $N_r$ is the number of samples averaged to estimate the mean received signal power, $N_n$ is the number of samples averaged to estimate the mean thermal noise power, and $S_n$ is the ratio of the measured signal power to the measured thermal noise power. The number of noise samples $N_n$ is usually much larger than $N_r$ since we have many receive-only bursts that go into the noise power estimate. Thus, the second term in Eq. 3.61 is negligible relative to the first, and the relative standard deviation of the signal estimate, which is called $K_{pc}$ by West et al. (2009), becomes

$$K_{pc} = \frac{\sigma_s}{P_{sdW}} = \sqrt{\frac{1}{N_r} \left(1 + \frac{1}{S_n}\right)^2}$$

(3.62)

Evaluating this relationship yields a statistical measurement uncertainty of about 5% at the one-sigma level for the real aperture observations.

### 3.7 Summary and Conclusion

In this chapter, we described the details of the real aperture radar processor that we develop and tune specifically for the RADAR system. We introduced the reader to the area-extensive radar equation and the normalized radar cross section (NRCS; $\sigma^0$) parameter that we are interested in measuring. We initially developed the real aperture radar equation in its most common form, in terms of power quantities, but the variety of viewing geometries required for Cassini RADAR operation introduces complexities...
that quickly made this form unwieldy. We thus developed an alternative, much simpler form of the radar equation, in terms of energy quantities, and demonstrated the equivalence of the two perspectives. A large component of the radar processing involves calibrating the measurements. We described our calibration procedure, as well as certain mode-specific quantization corrections that we must apply. Finally, we concluded by discussing the errors and uncertainties in our measurements.

The simple energy form of the radar equation readily applies to the majority of the Titan RADAR data, but at large distances (>∼60,000 km) and large incidence angles (>∼65°), the effective-area approximation that we invoke no longer applies and integrals over the larger illuminated areas must be incorporated. The majority of the RADAR data collected for the icy satellites requires the modified version of the processor, as we discuss in Chapter 8.
Chapter 4

Surface Scattering Theory and Models

Radar scattering occurs i) at surface interfaces and ii) within a volume of material if the loss (represented by an imaginary dielectric constant) is small and scatterers are contained therein. Regions will appear dark to the radar if they are smooth, sloped away from the direction of illumination, or if they are made of microwave-absorbing materials. Regions will appear bright if they are of rougher terrain, have slopes facing the radar, are made of more reflective materials, or if significant volume scattering is present. We distinguish between different scattering mechanisms by analyzing the strength of the backscatter as a function of angle, a property called the backscatter function or the backscatter response. We describe two types of scattering mechanisms in this chapter: surface specular scatter, which dominates the signal for small incidence angles, and diffuse scatter, which dominates at larger angles. We use quasispecular laws, or models, to describe the specular return and an empirical law to describe the diffuse return.

We begin this chapter by reviewing existing theories that describe radar scattering from rough surfaces, including the Kirchoff approximation from which the quasispecular laws are constructed. We define roughness and explain how the surface can be spatially decomposed into a large-scale facet-based component and a small-small roughness component. The large-scale component is commonly described by its statistical slope distribution (or equivalently, its height distribution and correlation
function). The classical quasispecular laws differ from each other mainly in the surface distributions assumed. We define the three classical quasispecular laws that we use in our backscatter analysis. We form a composite model using a linear combination of two of these quasispecular laws together with a diffuse law; this combination is required to properly describe the scattering behavior observed from Titan’s surface. We describe the modeling technique that we apply to the backscatter responses measured from Titan’s various terrains. Finally, we finish by explicitly defining radar albedo, one of the key surface parameters determined by backscatter modeling.

4.1 Fundamentals of Rough Surface Scattering

Wave scattering from rough surfaces is a complex problem that is not well understood, despite more than half a century of serious study (e.g. Hagfors, 1964; Rayleigh, 1945; Rice, 1951). A scattered signal forms when an incident electromagnetic (EM) wave interacts with the surface and induces currents on or within the surface, causing the re-radiation of EM fields. The interaction of the incident EM wave with the rough surface is not easily computed, thus scattering theories often invoke approximations to make the calculations more manageable. One such approximation is called the Kirchoff approximation (KA). KA uses a physical optics approach to solve the EM boundary condition problem; it treats each point on the local surface as though it were an infinite tangent plane made of the same material (e.g. Beckmann and Spizzichino, 1963; Ogilvy, 1991). The EM waves induced on a planar surface are easily computed and serve as a good approximation to the actual fields that would occur at each point as long as certain criteria are satisfied. These criteria include: 1) a local radius of curvature that is greater than the wavelength (see Barrick, 1965; Tyler, 1976), and 2) the surface correlation length must be larger than the wavelength (see Ogilvy, 1991). For small slopes, Papa and Lennon (1988) show that satisfying the second criterion also satisfies the first criterion. The accuracy of KA depends on how well these criteria are satisfied. For surfaces that are increasingly rough on wavelength scales, KA grows
increasingly less valid. Overall, KA is highly tolerant of height variations that are large compared with the wavelength, and this is its primary advantage in planetary studies.

Quasispecular scattering models (or “laws”) follow from the application of KA to conventional stochastic surfaces, where the surface statistics are assumed known, but the exact surface profile is not. The term quasispecular comes from visualizing the surface as a series of facets ($\gg \lambda$ in size) that are tilted perpendicular to the incident radiation. The facets function as perfect mirrors such that the observed echo strength is proportional to the Fresnel reflection coefficients for a specular geometry. Thus, the strength of the echo at a given incidence angle depends on 1) the dielectric constant of the surface, and 2) the number of facets that are oriented perpendicular to the incident direction, which in turn depends on the surface slope probability distribution. Quasispecular scatter will typically dominate the low-incidence angle end of the backscatter function, forming a quasispecular peak. Quasispecular models are designed to measure the strength and width of the quasispecular peak and relate those parameters back to the surface composition (dielectric constant) and surface roughness (rms surface slope). We utilize three common quasispecular models in our Cassini RADAR analysis, and we describe those models in Section 4.2.

Because they are based on KA, quasispecular models are only capable of describing surface scattering behavior from surfaces that are relatively smooth with respect to the wavelength (i.e. where the effective local radius of curvature is greater than the wavelength). To account for scattering from rougher surfaces, we utilize a diffuse scattering model. This model encompasses all scattering behavior not accounted for by KA, which includes multiple scattering, volume scattering, and scattering from small-scale ($\ll \lambda$) roughness on the surface, e.g. rocks. We employ a cosine power law to empirically describe the diffuse scatter term, as is common practice (e.g. Ostro, 1993). The parameters of the diffuse model are not explicitly related to the surface properties, but are empirically consistent with measurements from planetary objects. We describe this model further in Section 4.3.
From the above discussion, we see that the scattering behavior of surfaces will depend on how smooth or how rough the surface appears to the radar. Let us now quantify these terms.

### 4.1.1 Surface Roughness

The roughness of a surface depends on how the surface fluctuates in elevation, or height $h$, relative to a mean surface. The height standard deviation, or root-mean-square height ($h_0$), is a measure of this fluctuation. The Rayleigh criterion says that a surface is smooth if the rms height deviation of a random surface is

$$h_0 < \frac{\lambda}{8 \cos \theta_i},$$

(4.1)

where $\lambda$ is the wavelength of the incident EM radiation and $\theta_i$ is the incidence angle of the EM wave. This relationship originates from considering the relative phase difference ($\Delta \phi$), or path length difference, of incident rays reflected from different points on a corrugated surface, as illustrated in Figure 4.1. A delimiting phase difference value of $\pi/2$ is used to define the Rayleigh criterion, but it is not uncommon to consider other values, such as $\pi/4$, when identifying a smooth surface from a rough surface. Eq. 4.1 shows that roughness is not an intrinsic property of the surface, but rather depends on the observing conditions. A surface will appear smoother at longer wavelengths, and it will also appear smoother at larger incidence angles.

As depicted in Figure 4.1 (right-hand side) and Figure 4.2, the scattering behavior of a surface, or how the scattered signal strength varies with viewing angle, depends on how smooth or rough it appears to be. To understand the scattering response, consider the implications of Huygens’ principle, which says that each point on the surface is a source of secondary wavelets such that an observer in a given direction will see the cumulative combination of these wavelets. For a smooth surface, where the height difference between surface points is small, the strength of the scattered signal depends strongly on viewing angle. An observer in the specular direction will
see a small relative phase difference between the scattered wavelets, such that the signals interfere constructively to produce a strong reflection in the specular direction. However, an observer in any other direction will see large phase differences between the scattered wavelets, such that their contributions will average to zero and produce a weak scattered field at non-specular viewing angles. The smooth surface scenario
Figure 4.2: Comparison of the the scattering responses of the surfaces types introduced in Figure 4.1. At one extreme, we have the Flat Surface that is a specular spike visible only at zero incidence (the specular direction). At the other extreme, we have the diffuse Very Rough Surface that is almost equally visible at all incidence angles (it essentially has a flat surface response). The Slightly Rough Surface backscatter function consists mostly of a quasispecular response, where the height differences are small enough to contain most of the energy in the specular direction. The Rough Surface backscatter function is a combination of diffuse and quasispecular responses. Here, incidence angle has units of degrees.

produces a scattering response that behaves like a delta function, as depicted in Figure 4.1 and Figure 4.2 (where it is labeled “Flat Surface”). The smooth surface scattering behavior is called *specular* scattering. It is also called *coherent* scattering because the phase of the scattered signal is coherent with the incident field.

For a very rough surface, where the height difference between the surface points is large, the strength of the scattered signal varies less drastically with viewing angle. An observer in the specular direction will see larger phase differences between the scattered wavelets than would be seen for a smooth surface, such that the signals interfere destructively and produce a weaker signal in the specular direction. However, as less energy is scattered in the specular direction, more energy gets scattered into the other directions, and an observer in a non-specular direction will see a comparable amount of scattered energy as the observer in the specular direction. This causes the scattering response to appear more omnidirectional, or less dependent on viewing angle, as depicted in Figure 4.1 and Figure 4.2 (where it is labeled “Very Rough
Figure 4.3: Two-scale surface representation. A planetary surface can be spatially decomposed into a small-scale component and a large scale component, i.e. small-amplitude fluctuations superimposed on a slowly varying surface. Different scattering mechanisms occur from each component. Quasispecular scatter arises from the facet model and diffuse scatter arises from the small-scale structure (although we note that diffuse scatter may also arise from volume scatter, which is not depicted in this two-scale representation).

Surface”). The very rough surface scattering behavior is called diffuse scattering. It is also called incoherent scattering because the relative phase between the scattered field and the incident field is random (the phase of the scattered signal is the random vector sum of the individual wavelets, such that it varies abruptly and randomly with direction).

A surface may have roughness in between the two extremes described above. The “Slightly Rough Surface” illustrated in Figure 4.1 and Figure 4.2 might be modeled as a collection of flat surface facets, such that the scattered energy is contained in a lobe about the specular direction. This scattering behavior is called quasispecular scattering. The “Rough Surface” illustrated in the same set of figures might be modeled as a collection of roughened surface facets, such that the scattering response is a combination of the diffuse and quasispecular scattering behaviors.

A typical planetary surface consists of roughness at multiple scales, but the incident EM wave interacts predominantly with the surface at scales that are similar to the incident wavelength and scales that are much larger than the wavelength. Thus, the surface is typically visualized as a small-scale component (with height fluctuations $\approx \lambda$) superimposed on a large scale component (that slowly varies over distances $\gg \lambda$). The decomposition of a rough surface into these two spatial components is illustrated in Figure 4.3. The large scale surface profile contributes quasispecularly to the scattered radiation pattern, while the small scale surface profile contributes
diffusely. Thus, the scattering response might appear most similar to the “Rough
Surface” type described above. We next discuss the large-scale roughness information
that is contained within the quasispecular scattering component.

4.1.2 Large-scale roughness

The large-scale ($\gg \lambda$) surface profile is defined by its vertical and horizontal rough-
ness. The exact profile is not known, so the surface is modeled statistically. The
height probability distribution function $p(h)$ represents the vertical roughness and is
characterized by the height standard deviation, or root-mean-square (rms) height, $h_0$. The surface correlation function $C_f(R)$ represents the horizontal roughness and
is characterized by the autocorrelation length, $R_0$, i.e. the distance over which the
correlation function falls by $e^{-1}$.

It is common to assume Gaussian and exponential forms for the height probability
functions and correlation functions, as follows:

$$p_G(h) = \frac{1}{h_0 \sqrt{2\pi}} \exp \left( \frac{-(h - \bar{h})^2}{2h_0^2} \right) \quad (4.2)$$

$$p_E(h) = \frac{1}{h_0} \exp \left( -\frac{h}{h_0} \right) \quad (4.3)$$

$$C_{fG}(R) = \exp \left( -\frac{R^2}{R_0^2} \right) \quad (4.4)$$

$$C_{fE}(R) = \exp \left( -\frac{R}{R_0} \right) \quad (4.5)$$

where $h$ is the height deviation from the mean surface, $\bar{h}$ is the average height, and
$R$ is the horizontal separation of two points.

The surface slope probability distribution encompasses both the vertical and hori-
zontal roughness and is the more common measure of large-scale surface roughness. In
4.1. **FUNDAMENTALS OF ROUGH SURFACE SCATTERING**

In fact, the surface slope distribution links directly to the backscatter normalized radar cross section value (NRCS, or $\sigma^0$, see Section 3.1) using geometric optics (Barrick, 1968):

$$
\sigma^0(\theta_i) = \pi \rho P(\theta_i) \sec^4 \theta_i,
$$

where $P(\theta_i)$ is the slope distribution function and $\rho$ is the Fresnel reflection coefficient at normal incidence. Eq. 4.6 reveals that the backscatter is proportional to the slope distribution for small angles of incidence.

The rms surface slope, $s$, characterizes the slope distribution and is related to the surface rms height and correlation length, depending on the distributions assumed. The adirectional rms slope is equal to

$$
s = h_0 \sqrt{-2C_f(0)'},
$$

where $C_f(0)'\prime$ is the second derivative of the correlation function, evaluated at $R = 0$ (Ogilvy, 1991, p. 21). For a Gaussian correlation function, the adirectional rms surface slope is then the scaled ratio of the rms height to the correlation length (Ulaby et al., 1986),

$$
s = h_0 \frac{\sqrt{2}}{R_0}
$$

For an exponential correlation function, the adirectional rms slope is

$$
s = h_0 \sqrt{\frac{\kappa_{\text{max}}}{R_0}}
$$

where $\kappa_{\text{max}}$ is the maximum surface wave number equal to the reciprocal of the minimum horizontal scale, $R_{\text{min}}$, that significantly contributes to the scattering process (see Appendix B of Sultan-Salem (2006) for details). In other words, $\kappa_{\text{max}}$, or $R_{\text{min}}$, arises because the scattering process is not sensitive to roughness on scales that are smaller than the incident EM wavelength, so that the surface appears naturally
smoothed or low pass filtered (as foretold by the Rayleigh criterion). $R_{min}$ is typically chosen such that the rms surface slope $s$ equals the width parameter $C$ used in the quasispecular models (see Eq. 4.14).

In the next section, we describe the form of the classical quasispecular laws that we use to model the measured backscatter from the surface of Titan and Saturn’s other icy satellites. We describe the parameters of these models and how they relate to the properties of the surface.

### 4.2 Quasispecular Scattering Models

We rely on three classical scattering laws to describe the quasispecular behavior observed in the measured backscatter responses; these include Hagfors’ law, the Gaussian law, and the exponential law. Each scattering law assumes different statistical descriptions of the surface. The Hagfors law ($\sigma_0^H$) assumes a Gaussian height distribution and an exponential correlation function, the Gaussian law ($\sigma_0^G$) assumes a Gaussian form for both the height distribution and the correlation function, and the exponential law ($\sigma_0^E$) assumes an exponential surface slope distribution. Applying these surface descriptions to the KA-simplified EM scattering integral yields the following forms for each law (see Beckmann and Spizzichino, 1963; Hagfors, 1964, 1966; Ogilvy, 1991):

$$\sigma_0^H(\theta_i) = \frac{\rho C}{2} \left( \cos^4 \theta_i + C \sin^2 \theta_i \right)^{-3/2}$$ (4.10)

$$\sigma_0^G(\theta_i) = \frac{\rho C}{\cos^4 \theta_i} \exp \left( -C \tan^2 \theta_i \right)$$ (4.11)

$$\sigma_0^E(\theta_i) = 3 \frac{\rho C}{\cos^4 \theta_i} \exp \left( -\sqrt{6C} \tan \theta_i \right)$$ (4.12)

where $\rho$ is the Fresnel reflection coefficient at normal incidence and $C$ is the width parameter (defined below). The derivation of the exponential law more simply comes
4.2. QUASISPECULAR SCATTERING MODELS

from substituting the exponential slope distribution into Eq. 4.6.

The Fresnel reflection coefficient $\rho$ is independent of incidence angle and polarization angle because only slopes that are perpendicular to the incident radiation contribute to the scattered signal (Hagfors, 1966). The bulk dielectric constant of the surface, $\epsilon$, thus derives from $\rho$ as follows

$$\epsilon = \left(\frac{1 + \sqrt{\rho}}{1 - \sqrt{\rho}}\right)^2. \quad (4.13)$$

The width parameter $C$ provides a measure of the angular extent of the backscattering lobe and roughness of surface. $C$ is defined as

$$C = \frac{\lambda^2 R_0^2}{16 \pi^2 h_{rms}^4}. \quad (4.14)$$

where $R_0$ is the correlation length, and $h_{rms}$ is the rms height, and $\lambda$ is the incident EM wavelength. The inverse square root of $C$ is commonly interpreted as the rms surface slope (Hagfors, 1967, 1970), as follows:

$$s = \tan \phi_{rms} = \frac{1}{\sqrt{C}}. \quad (4.15)$$

where $\phi_{rms}$ is the rms tilt angle of the facet whose slope corresponds to the rms slope $s$. In our model analysis, when we report the inferred values of $s$, we are actually reporting the rms tilt angle values.

According to each of these quasispecular models, the backscatter strength at normal incidence to the mean surface is a function of $\rho C$. Thus, as the surface gets rougher and the slope increases, $C$ decreases, causing the amplitude of the backscatter response to decrease and the width of the quasispecular scattering lobe to broaden. In other words, surfaces with larger $s$ will have more facets aligned in a specular geometry farther away from nadir than surfaces with smaller $s$. This results in higher backscatter values at larger incidence angles, and by conservation of energy, this
Figure 4.4: Comparison of the behavior of the three quasispecular models. The left panel shows how drastically the behaviors differ from each other, even when assuming the same surface parameters. The right panel shows how different the inferred surface parameters are from each other, even when describing the same dataset.

requires the total amplitude of the backscatter to decrease. Similarly, smoother surfaces result in larger values of \( C \), and subsequently a larger backscatter amplitude and narrower width.

We compare the behavior of the three models in Figure 4.4 for two cases. In the first case, the quasispecular models are evaluated with the same parameter values \(( \rho = 0.01, C = 200 \) \( \Rightarrow \) \( \epsilon = 1.5, s = 4^\circ \))). In the second case, the quasispecular models are fit to the same data set, and their best-fit parameter values are compared. In the first case, illustrated in the left panel of the figure, we observe that the amplitude and width of the models vary greatly from each other, even though their surface parameters are identical. The exponential model peaks at a higher amplitude, and the Hagfors at a lower amplitude. Furthermore, the Gaussian has a larger width than the other two. Now, when we restrict the models to behave similarly, with each matched to the same surface data, as illustrated in the right panel...
of the figure for the second case, we find that the inferred surface parameters vary greatly from each other. The exponential model requires a larger dielectric constant and larger slope, the Gaussian model requires a smaller dielectric constant and smaller slope, and the Hagfors model parameters are in-between the other two. From this comparison, we show that the absolute values of the surface parameters are highly dependent on the model used to describe the surface.

An analysis by Gunnarsdottir (2009) quantifies the relationships between the inferred surface slopes. For identical half-power widths (the incidence angle at which the scattering model has decreased to half of its maximum value), the rms tilt angles inferred from the models are related as follows:

\[
\phi_{\text{rmsH}} = 1.09\phi_{\text{rmsG}}
\]
\[
\phi_{\text{rmsE}} = 2.75\phi_{\text{rmsH}}
\]
\[
\phi_{\text{rmsE}} = 3.00\phi_{\text{rmsG}}
\]

While it is thus possible to relate the inferred parameters to each other, we instead minimize inter-model ambiguity by using only a single model form in our analysis, as discussed in Section 4.5. Some ambiguity in the absolute surface values will still exist, but the relative comparison between the model results of different surface features will hold true.

A further complication in interpreting the slope parameter is that the KA-based quasispecular models do not explicitly define the slope’s dependence on scale, which is inconsistent with the observation that most natural surfaces have scale-dependent properties. Fractal surface models offer an alternative approach to determining the scale-dependent height and slope properties of a surface (Franceschetti et al., 1999; Shepard et al., 2001)). Sultan-Salem (2006) use a generalized fractal surface model together with the Kirchoff approximation to construct a versatile quasispecular scattering model that is capable of describing the scattering behavior while also measuring the scale dependence of the derived surface slopes. We acknowledge that this
approach would yield more physical slope descriptions, but we choose to utilize the classical quasispecular approach for its simplicity and consistency with other planetary analyses. Thus, due to their lack of scale-explicit information, the inferred slope values that we present in this work are best interpreted relative to each other, rather than absolutely.

We focus our attention on these three quasispecular models, because they each have a long history for characterizing planetary radar scattering (Tyler et al., 1992). The Hagfors model yields the best-fit to lunar data (Simpson and Tyler, 1982), and the Gaussian and exponential models characterize much of the scattering from Venus data (60% of the surface is best matched by the Gaussian law and 38% of the surface is best matched by the exponential law, as reported by Sultan-Salem (2006)). However, it is possible to derive other quasispecular laws by assuming alternative surface distributions (e.g., the Rayleigh quasispecular law).

In this section, we have described how the quasispecular component helps to constrain the dielectric constant and surface slopes. The strength of the quasispecular peak relates to the dielectric constant and the rms surface slope, and the angular width of the quasispecular peak relates just to the rms surface slope. Because a quasispecular signal originates only from facets oriented perpendicular to the incident wave, and because it is unusual for a natural surface to have slopes tilted beyond 30° incidence, we do not expect quasispecular scatter to exist at incidence angles larger than this angle. Thus, we require a separate scattering mechanism to explain the large scattered powers often observed from planetary surfaces at higher incidence angles. We discuss this scattering mechanism next.

4.3 Diffuse Scattering

Diffuse scatter encompasses the scattered signal that cannot be described by quasispecular models. The diffuse scattering phenomena might include small-scale structure on the surface or in the near sub-surface (Rice, 1951; Ulaby et al., 1986), or coherent
volume scatter from inhomogeneities embedded in a low-loss medium (Hapke, 1990; Peters, 1992). We isolate the diffuse scattering contributions by modeling the relative strength of the backscatter at higher angles. As is common practice, we describe the measured diffuse scattering cross section with a cosine power law:

\[ \sigma_D^0(\theta_i) = A \cos^n \theta_i, \]  

(4.17)

where the amplitude \( A \) and shape parameter \( n \) are similar to typical descriptors used in planetary radar literature and are empirically consistent with measurements from planetary objects (Ostro, 1993). The parameter \( n \) describes the rate of falloff with angle and quantifies how focused the diffuse scatter is. Isotropic surfaces will have \( n = 1 \), whereas Lambertian surfaces have \( n = 2 \). It is unusual for a surface to scatter with \( n > 2 \). Because it is empirically-based, the diffuse model does not explicitly relate to the properties of the surface. Rather it is largely meant to give a sense of the radar brightness, or albedo (see Section 4.6).

We find that diffuse scattering from wavelength-scale surface roughness is likely prevalent on Titan, as suggested by the 10-15 cm stones imaged by the Huygens probe (Tomasko et al., 2005), but the large magnitude of the diffuse scatter term appears to be more consistent with volume-scatter mechanisms than rough-surface models. The “unusual” scattering (high radar albedo and polarization inversion) reported for the icy satellites of Jupiter (see, for example, Ostro (1993)) and Saturn (Ostro et al., 2006) or from polar region craters on Mercury (Harcke, 2005; Harmon et al., 2001) is generally interpreted as evidence for volume scattering in water ice. However, any material can provide anomalously high volume scatter if it is a very low-loss material and structural heterogeneities exist (Black et al., 2001; Peters, 1992). The loss tangents of solid hydrocarbons, liquid hydrocarbons, and water ice have an upper bound of \( 10^{-4} \) and \( 10^{-5} \) at the conditions on Titan, allowing for significant surface penetration (on the order of hundreds of wavelengths), so that volume scattering is the most probable explanation for the high backscatter that we observe on Titan. Even water ice and carbon-dioxide ice, with slightly higher loss tangents of 0.01 and 0.005,
CHAPTER 4. SURFACE SCATTERING THEORY AND MODELS

Figure 4.5: Illustration of the surface scattering mechanisms and their dependence on viewing geometry. Quasispecular scatter originates from facets that are oriented perpendicular to the incident wave, thus they occur at low incidence angles. Diffuse scatter (from either small-scale surface structure or inhomogeneities in a low-loss volume) occurs at all angles, but it dominates only at the higher incidence angles where quasispecular scatter cannot occur.

could allow penetration of tens of wavelengths and provides the likely explanation for the high backscatter observed on Saturn’s other moons.

From the data that we present here, we cannot definitively say that multiple scattering from the subsurface dominates over that from the surface, but we believe that it is the most likely possibility. Analysis of the polarization properties of the RADAR radiometry data also suggests that volume scattering controls the diffuse scattering mechanism (Janssen et al., 2009; Zebker et al., 2008).

4.4 Composite Model

The scattering mechanisms heretofore described will behave differently depending on the viewing angle. As illustrated in Figure 4.5, quasispecular scatter will dominate at low incidence angles ($\theta_i < 30^\circ$), while diffuse scatter will dominate at higher incidence
angles. Thus, the scattering response of a surface consists of the superposition of both terms. We describe this behavior with a composite model, as follows:

\[
\sigma^0(\theta_i) = \sigma^0_{QS}(\theta_i) + \sigma^0_D(\theta_i),
\]

(4.18)

where \(\sigma^0_{QS}\) represents one of the three quasispecular models described in Eqs. 4.10-4.12, and \(\sigma^0_D\) represents the diffuse scattering law described in Eq. 4.17. This composite model (which we label CM1) is illustrated in the left panel of Figure 4.6.

However, as first noted by Sultan-Salem (2006), a single quasispecular scattering law fails to properly capture the low-angle scattering behavior from the surface of Titan (the same happens to be true for Venus). Instead, Sultan-Salem (2006) introduces a composite model that is the superposition of two quasispecular laws together with the diffuse law:

\[
\sigma^0(\theta_i) = \sigma^0_{QS1}(\theta_i) + \sigma^0_{QS2}(\theta_i) + \sigma^0_D(\theta_i).
\]

(4.19)

According to geometric optics, the model parameters of each individual quasispecular law combine to give the total model parameters as follows (see Sultan-Salem, 2006, Appendix D):

\[
\rho = \rho_1 + \rho_2
\]

(4.20)

\[
s = \sqrt{\frac{\rho_1}{\rho} s_1^2 + \frac{\rho_2}{\rho} s_2^2} = \sqrt{\frac{\rho_1}{\rho} \frac{1}{C_1} + \frac{\rho_2}{\rho} \frac{1}{C_2}}
\]

(4.21)

where \(\rho_1\) and \(\rho_2\) are the Fresnel reflection coefficient parameters associated with the first and second quasispecular term, \(\sigma^0_{QS1}\) and \(\sigma^0_{QS2}\), respectively. We apply the total \(\rho\) value to Eq. 4.13 to infer the surface dielectric constant \(\epsilon\). Similarly, \(s_1\) and \(s_2\) are the slope surface parameters inferred from the model width parameters \(C_1\) and \(C_2\) using Eq. 4.15, for the first and second quasispecular terms respectively. Eq. 4.21 shows that the square of the total inferred slope parameter \(s\) is the weighted sum of
The two-quasispecular composite model describes the data very well, but suffers from multiple significant fits (i.e. several statistically significant models of the squares of the individual slope parameters.

We illustrate the two-quasispecular form of the composite model (which we label CM2) in the right panel of Figure 4.6. In our implementation of Eq. 4.19, $\sigma_{QS1}^0$ effectively describes the behavior near nadir ($\theta_i < 2^\circ$), and $\sigma_{QS2}^0$ effectively describes the behavior at low angles away from nadir ($2^\circ < \theta_i < 30^\circ$). Each component can assume any one of the three quasispecular laws, meaning that there are nine possible forms of Eq. 4.18. For example, the HHD composite model is composed of a Hagfors law near-nadir ($\sigma_{QS1}^0 = \sigma_H^0$), a Hagfors law at low angles away from nadir ($\sigma_{QS2}^0 = \sigma_H^0$), and the diffuse law ($\sigma_D^0$), while a GED composite model is formed from a Gaussian law near nadir ($\sigma_{QS1}^0 = \sigma_G^0$), an exponential law at low angles away from nadir ($\sigma_{QS2}^0 = \sigma_E^0$), and the diffuse law.

The two-quasispecular form of the composite model describes the data very well, but suffers from multiple significant fits (i.e. several statistically significant models
will match the data equally well). The reason for this, we find, is that the goodness of fit is controlled primarily by the second quasispecular term, $\sigma_{QS2}^0$, and the exact form of the $\sigma_{QS1}^0$ model is less important, as long as it provides some type of near-nadir peak. Typically we will have three model forms (one for each possible $\sigma_{QS1}^0$) that will match the data. Fortunately, the total model parameter values depend mostly on the model parameter values associated with the second quasispecular term, $\rho_2$ and $C_2$ (i.e., $\rho_1$ is typically be small in comparison to $\rho_2$, so that $\rho$ is dominated by $\rho_2$ in Eq. 4.20, and $s$ is dominated by $C_2$ in Eq. 4.21). Thus, the model-dependent variation of the parameters associated with the first quasispecular term, $\rho_1$ and $C_1$, is not so important. The insignificance of the first quasispecular term relative to the second quasispecular term is well-illustrated by their relative areas in the right panel of Figure 4.6. Nevertheless, to minimize any parameter ambiguity associated with the different models, we use a single consistent model form when we compare the surface parameters across feature classes. We find that the GED model fits most of the Titan features best, so this is the model that we use for feature comparison.

The CM2 composite model defined in Eq. 4.19 allows us to properly describe the behavior of the surface backscatter response. In addition to enabling the characterization of the surface parameters, a proper description allows us to correctly account for the angle dependence of the radar reflectivity. With the angle dependence properly recorded, we can form backscatter maps that reflect the intrinsic brightness variations of the surface, regardless of viewing geometry (see Chapter 6).

Like Sultan-Salem (2006), we use geometric optics to interpret the need for two quasispecular components in the composite model. Eq. 4.6 says that the surface backscatter cross section is proportional to the slope distribution function. Thus, the linear combination of quasispecular laws suggests that the radar is illuminating a patch of surface that is the linear superposition of two or more slope distributions. The accompanying $\rho$ parameters represent the relative weights of those individual slope distributions. The results then imply that the main slope distribution suggested by the second quasispecular model does not proffer enough flat facets with near-zero
slope, and the first quasispecular model fills this void.

In the next section, we describe our implementation of the CM2 composite model defined in Eq. 4.19 and our technique for fitting the model to the data.

4.5 Model Fitting Technique

We wish to model the backscatter of various surface features using the CM2 composite model described by Eq. 4.19. To eliminate any bias from data points concentrated at certain incidence angles (e.g., SAR data usually occur at 20°-35°, so there is typically a cluster of backscatter data within that incidence angle range), we first reduce the measured backscatter response to its mean backscatter curve. We evaluate the mean backscatter curve by dividing the incidence angle range into 0.5° bins (i.e., the first bin is 0°-0.5°, the second bin is 0.5°-1°, etc.), and calculating the average of the backscatter data that fall within each bin.

We fit the individual composite model components to the mean backscatter data in segments in order to ensure that each scattering mechanism is properly modeled. We first fit the diffuse law (Eq. 4.17) to the mean backscatter data with incidence angles in the range of 30° to 75°. This range of angles is chosen to accurately determine the diffuse scattering signal; below 30° we risk including quasispecular contributions, above 75° we risk including errors from the effective beam area approximation (see Appendix A). Once we have the diffuse scattering component properly modeled, we extrapolate it over all incidence angles and subtract the result from the mean backscatter data. The remaining signal can be attributed purely to quasispecular scattering mechanisms. We fit the main quasispecular model ($\sigma^0_{QS2}$, which can equal anyone of the three quasispecular laws defined in Eqs. 4.10-4.12) to the remaining data that have incidence angles greater than 2° so that we can properly describe the off-nadir low angle data without distortion from the near-nadir peak. We then extend the best-fit model over all incidence angles and remove it from the signal so that only the near-nadir quasispecular data remains. We apply the low-angle quasispecular
model \( \sigma_{QS1}^0 \), which can also equal anyone of the three quasispecular laws defined in Eqs. 4.10-4.12) to the remaining data; the resulting best-fit model describes the signal contributed by the locally smooth patches in the nadir region (i.e. the facets with near-zero slopes).

We perform each fitting stage using nonlinear least squares, where we minimize \( \delta \), the sum of the weighted squared errors:

\[
\delta = \sum_n w_n (y_n - \hat{y}_n)^2
\]  

(4.22)

where \( y_n \) is the nth mean backscatter measurement, \( \hat{y}_n \) is the model evaluated at the center angle of the nth angle bin, and \( w_n \) is the weight applied to the squared residual. The weighting is necessary to maintain equal focus across the range of incidence angles. The larger signal values that occur at lower incidence angles will naturally have larger variance, i.e. larger residuals, than the smaller signal values that occur at higher incidence angles. Thus, the minimization procedure will favor the lower angle data unless the residuals are properly equalized. We empirically find that \( w_n = y_n^{-1} \), i.e. the inverse of the mean signal value, yields a properly balanced fit. The weights are required for fitting the main quasispecular model \( \sigma_{QS2}^0 \) and the diffuse model, but we find that they are not necessary for the near-nadir model \( \sigma_{QS1}^0 \).

The sum of the best-fit diffuse model, the main quasispecular model, and the near-nadir backscatter model yield the best-fit composite model that describes the mean backscatter response of the surface. We repeat the fitting procedure for all combinations of the quasispecular laws, thus measuring the best-fit composite model for nine different forms. We calculate the sum of the squared errors (sse) for each best-fit model form, compare the sse values across all of the features, and find the model form that minimizes the sse for the most features. The model that best describes most of the Titan surface features is the GED composite model, where \( \sigma_{QS1}^0 = \sigma_G^0 \) and \( \sigma_{QS2}^0 = \sigma_E^0 \).
For each surface feature on Titan, we calculate the GED best-fit parameters and their confidence intervals (see Chapter 5). These parameters include the diffuse amplitude and shape exponent ($A$ and $n$), the dielectric constant and rms surface slope ($\epsilon$ and $s$), and the 2.2 cm-$\lambda$ same-sense linear polarization radar albedo ($\hat{\sigma}$). We described the diffuse model parameters in Section 4.3 and the combined quasispecular parameters in Section 4.2 and Section 4.4. The radar albedo calculation requires a more detailed description, which we report in the next section.

4.6 Albedo Calculations

The term *albedo*, Latin for “whiteness”, is a dimensionless reflection coefficient - a measure of the reflecting power of a nonluminous object. In its general form, it is defined as the fraction of incident radiation that is reflected or scattered from a surface. In astronomy, *albedo* can acquire more specific definitions depending on the wavelength(s) of the measurement and the directional distribution of the incident radiation (c.f. de Pater and Lissauer (2010)). For instance, if the reflected radiation intensity is measured over all directions, relative to the direction of illumination, and over all wavelengths, and is then compared to the total electromagnetic intensity incident upon the target, the result is called the *bond albedo*, or sometimes *spherical albedo*. The bond albedo is important for characterizing the energy balance of the target, and its value cannot exceed unity. When the reflected radiation intensity is instead measured only along the direction of illumination, the resulting reflection coefficient is termed *normal albedo*. If the reflected radiation intensity measured along the direction of illumination is compared to the reflected intensity expected from a perfectly reflecting, and perfectly diffuse (isotropic) surface that is the same size and distance from the source as the target, then the result is the *geometric albedo*. The geometric albedo may be defined for specific wavelengths, such as the *visual geometric albedo* that only accounts for radiation in the visible part of the electromagnetic spectrum. The measured geometric albedo may be very different from the measured
bond albedo and may even exceed unity, such as when there is a strong opposition effect and light is preferentially scattered back toward its source.

The above terminology is usually reserved for targets that are illuminated by a natural source, such as the Sun. When the target is illuminated by an artificial source, such as microwave radiation from a radar, alternative albedo definitions arise. We use radar albedo ($\hat{\sigma}$) to describe the reflectivity of a radar target. The radar albedo is the most basic property of the object and is defined as the ratio of a target’s disk-integrated radar cross section (RCS or $\sigma$) to its geometric projected area:

$$\hat{\sigma} = \frac{\sigma_{\text{disk}}}{\pi R_t^2}, \quad (4.23)$$

where $R_t$ is the radius of the spherical target. $\sigma_{\text{disk}}$ represents the RCS of the target’s entire visible hemisphere that would be measured by a monostatic radar far enough away for its incident rays to appear parallel. Section 3.1.1 describes the meaning of the RCS: it is the projected area of a perfectly reflective, isotropic target that would return an identical echo power to what is measured, if it was observed at the same distance from the radar and with the same transmitted and receive polarizations. Thus, when we normalize the disk RCS by its geometric projected area, or equivalently by the RCS of a perfectly reflective target such as a smooth metallic sphere, we have a measure of how reflective the target is in the backscatter direction.

We recognize a similarity to the definition of geometric albedo in the definition of radar albedo. In fact, for a given target, the radar albedo and optical geometric albedo are often compared to each other to characterize any differences between the optical surface and the depths probed at microwave wavelengths (e.g., Black et al., 2007; Ostro et al., 2010). A strong radar-optical correlation can imply that the surface processes and/or contaminants at the top surface layer are similar to those at depths of 10 to 20 wavelengths, or 0.2 to 0.5 meters for 2.2 cm-$\lambda$. The values of radar penetration depth are derived for coherent volume scattering from a clean and mature icy regolith, as modeled by Black et al. (2001) for the icy Galilean satellite echoes. We require similar scattering mechanisms to produce the range of radar albedos we
observe for Saturn’s moons (see Chapter 5 and Chapter 8).

Cassini RADAR receives radar echoes in the same linear polarization (SL) as it transmits. We follow the convention of Ostro et al. (2010) and identify our 2 cm-λ SL measurements using the notation SL-2. We calculate the disk-integrated SL radar cross section ($\sigma_{SL-2}$) by integrating the specific radar cross section ($\sigma^0_{SL-2}$) over the surface area that is visible to an observer far enough away that the incident radar rays are parallel, i.e. the surface area of the entire visible hemisphere $A_{hem}$, or $2\pi R^2_t$.

$$\sigma_{SL-2} = \int_{A_{hem}} \sigma^0_{SL-2} (\theta_i) dA_{gr} (\theta_i), \quad (4.24)$$

where $dA_{gr}$ is the surface element projected along the target ground, which will vary according to the local incidence angle $\theta_i$, the angle between the incident radar ray and the element’s surface normal. The integration is simplified by converting the ground surface area to the equivalent perpendicular area, $dA_\perp$, using the relationship

$$dA_\perp = dA_{gr} \times \cos \theta_i. \quad (4.25)$$

We express the perpendicular disk area using polar coordinates $(r, \phi)$, where $r$ and $\phi$ are illustrated in Figure Figure 4.7, and substitute Eq. 4.25 into Eq. 4.24 to obtain

$$\sigma_{SL-2} = \int_{A_{disk}} \sigma^0_{SL-2} (\theta_i) \frac{dA_\perp}{\cos \theta_i} = \int_0^{R_t} \int_0^{2\pi} \sigma^0_{SL-2} (\theta_i) r dr d\phi \frac{r dr d\phi}{\cos \theta_i}. \quad (4.26)$$

Assuming azimuthal homogeneity in the mean backscatter, the polar coordinate $\phi$ integrates out of Eq. 4.26. We express the polar coordinate $r$ remaining in the RCS integral in terms of the local incidence angle: $r = R_t \sin \theta_i$ and $dr = R_t \cos \theta_i d\theta_i$.

The incidence angle varies from $0^\circ$ at the disk center to $90^\circ$ at the limb. Thus, the disk-integrated RCS takes the following form:

$$\sigma_{SL-2} = 2\pi R^2_t \int_0^{\pi/2} \sigma^0_{SL-2} (\theta_i) \sin \theta_i d\theta_i. \quad (4.27)$$
4.6. ALBEDO CALCULATIONS

Figure 4.7: The albedo calculation integrates the backscatter over the entire target surface visible to a far away observer, i.e. the visible hemisphere. The integral is expressed in terms of the perpendicularly projected area in polar coordinates, \( dA_\perp = rd\phi \), where \( r \) and \( \phi \) are defined in the illustration on the right. The leftmost panel illustrates the dependence of \( r \) on the local incidence angle \( \theta_i \).

The SL-2 albedo (\( \hat{\sigma}_{SL-2} \)) is calculated by normalizing Eq. 4.27 by the visible disk area:

\[
\hat{\sigma}_{SL-2} = \frac{\sigma_{SL-2}^{0}}{\pi R_t^{2}} = 2 \int_{0}^{\pi/2} \sigma_{SL-2}^{0}(\theta_i) \sin \theta_i d\theta_i. \tag{4.28}
\]

We evaluate Eq. 4.28 by assuming a particular model to describe the dependence of \( \sigma^0 \) on incidence angle. This model may comprise a combination of quasispecular laws and a diffuse cosine power law (Section 4.4), or just the diffuse cosine law (Section 4.3). Icy satellites other than Titan are usually well represented by the diffuse model alone, while Titan backscatter typically requires the composite model that can also describe the quasispecular signatures observed at low incidence angles. If just the diffuse model is assumed, then Eq. 4.28 simplifies to

\[
\hat{\sigma}_{SL-2} = \frac{2a}{n + 1}. \tag{4.29}
\]
This result is used regularly in the icy satellite data analysis (see Chapter 8).

Typical ground-based observations transmit a circularly polarized signal and receive simultaneously both the same circular (SC) and opposite circular (OC) polarizations. To be able to compare our SL-2 albedo measurements with those of other instruments, such as the ground-based 13 cm-\(\lambda\) Arecibo radar measurements (e.g. Black et al. (2007) and Campbell et al. (2003)), we need to estimate the total-power (TP) albedo, \(\hat{\sigma}_{TP}\). The total-power albedo is the sum of the albedos observed in two orthogonal polarizations:

\[
\hat{\sigma}_{TP} = \hat{\sigma}_{SL} + \hat{\sigma}_{OL} = \hat{\sigma}_{SC} + \hat{\sigma}_{OC}. \tag{4.30}
\]

Cassini RADAR does not measure the opposite linear (OL) polarization signal. Thus, we are forced to constrain the TP albedo from probable estimates of the OL/SL albedo ratio. We follow the strategies outlined by Ostro et al. (2006) and Ostro et al. (2010). Ostro et al. (2006) observe that most Solar System bodies have OL/SL \(\leq 0.7\), requiring \(\hat{\sigma}_{SL} \leq \hat{\sigma}_{TP} \leq 1.7\hat{\sigma}_{SL}\). Ostro et al. (2010) further note that the OL/SL ratio of the Galilean icy satellites have similar OL/SL ratios to each other (Europa and Ganymede have OL/SL \(\simeq 0.47 \pm 0.08\) and Callisto has OL/SL \(\simeq 0.55 \pm 0.1\)) that are also apparently wavelength independent (Ostro et al., 1980). Since the Galilean icy satellites and the Saturnian icy satellites have similar scattering behavior to each other at 13 cm-\(\lambda\), e.g. large albedo values and unusually large circular polarization ratios (Black et al., 2007), Ostro et al. (2010) go on to assume that the Saturnian icy satellites at 2.2 cm-\(\lambda\) and the Galilean icy satellites at 13 cm-\(\lambda\) have a similar mean linear polarization ratio, implying

\[
\hat{\sigma}_{TP-2} = (1.52 \pm 0.13)\hat{\sigma}_{SL-2}. \tag{4.31}
\]

We apply the latter formula to our Cassini RADAR measurements, recognizing that the TP albedo may vary slightly about this value depending on its actual linear polarization ratio at 2.2 cm-\(\lambda\).
4.7 Conclusion

In this chapter we reviewed the existing theories that describe radar scattering from rough surfaces. We focused on the Kirchoff approximation, and the resulting quasi-specular laws, as well as on diffuse scattering theory. These scattering mechanisms explain the majority of the backscatter behavior observed from the surface of Titan and Saturn’s icy moons. Yet, scattering anomalies remain, such as the extreme radar brightness of some features on Titan (Janssen et al., 2011; Le Gall et al., 2010) and the extreme radar albedos of some of the icy moons (Ostro et al., 2006, 2010, ; see also Chapter 8). Other scattering theories exist that might help to explain the extreme scattering behavior (e.g., Mie scattering, small perturbation models, and integral equation models; see the likes of Ulaby et al. (1986) and Ogilvy (1991) for details), but further work is needed to fully understand the surface and sub-surface scattering behavior from these moons.
Chapter 5

Titan Surface Feature Analysis

In this chapter, we report the backscatter functions of various Titan feature classes. We apply the composite backscatter model developed in the previous chapter (Eq. 4.19) to the feature data to retrieve the scattering parameters. Because we only consider data that have more than 90% of their beam footprint filled by feature terrain, we are able to measure the parameters that are specific to that surface type. The analyzed features represent the majority of Titan’s surface and allow us to explore the heterogeneity contained therein.

5.1 Introduction

We identify five feature classes to study in this chapter: the bright Xanadu province, bright hummocky and mountainous terrain, dark dune fields, homogeneous gray plains, and cryovolcanic terrain. We define the boundary of Xanadu from the real aperture radar map (we describe the creation of this map in the next chapter), but the other features are identified by their geomorphological boundaries in the high-resolution SAR images. The SAR-based mapping is performed by colleagues and is reported primarily in Lopes et al. (2010) and Le Gall et al. (2011).

We begin the chapter by modeling the global Titan backscatter response to study the average surface properties of the moon. The global backscatter investigation also serves as a detailed backscatter modeling example, and we apply the lessons learned
to the subsequent feature analysis sections. The backscatter model yields parameters related to the surface dielectric constant and the surface rms slope. Before we begin our analysis, we describe the context in which these parameters may be interpreted.

5.1.1 Interpreting the Dielectric Constant Parameter

We compare the dielectric constants ($\epsilon$) inferred from our backscatter models to laboratory measurements performed on candidate surface materials at conditions similar to those on Titan’s surface (94 K, 147 kPa). The dielectric properties of candidate Titan surface materials have been investigated and reported by Lorenz (1998); Lorenz et al. (2003); Paillou et al. (2008); Thompson and Squyres (1990), among others. Dielectric constants between 2.0 and 2.4 are in the range expected for simple solid hydrocarbon materials (Paillou et al., 2008), although complex hydrocarbons can also have dielectric constants near 4.5 and above (Thompson and Squyres, 1990). Liquid hydrocarbons will have dielectric constant values between 1.6 and 2.0 (Paillou et al., 2008; Thompson and Squyres, 1990). Dielectric constants near 3.1 and 4.5 are consistent with those of solid water ice and water-ammonia ice respectively (Lorenz, 1998; Paillou et al., 2008). We summarize the dielectric properties in Table 5.1.

While it is tempting to simply map our modeled dielectric constant values to those recorded in Table 5.1, it is important to remember that we are sensing the bulk dielectric constant, which would include the effect of porosity of the surface materials. In other words, a dielectric constant measurement of 2.0 could result from either solid material of $\epsilon = 2.0$ or from a highly porous material of considerably higher dielectric constant. For example, bulk lunar radar dielectric constants are in the range of 2.8 to 3.1 (Tyler and Howard, 1973), even though the rocky material forming the surface has a dielectric constant in the range of 6 to 9. Breakdown of the lunar material to dust and soil lowers the effective dielectric constant to a value near 3. It is possible that similar processes on Titan lower the observed values of the dielectric constant. In this case, our measurements must be considered lower bounds. For example, if we measure a dielectric constant of 3.0 for a region on Titan’s surface, we know that it
Table 5.1: Dielectric properties of candidate materials at Titan’s surface. The range of values reflect different compositions (e.g. the solid hydrocarbon Benzene has a dielectric constant of 2.4, but the solid hydrocarbon Heptane has a dielectric constant closer to 2.0, as measured by Paillou et al. (2008)), and the range also reflects the measurement uncertainties (e.g. Paillou et al. (2008) measure the loss tangent of ammonia-water ice to be $\sim 10^{-3}$, while Lorenz (1998) measure the loss tangent of ammonia-water ice to be $\sim 10^{-2}$, for roughly the same amount of ammonia ($\sim 30\%$); the two dielectric constant measurements (4.4 and 4.5, respectively) are in close agreement). We further note the discrepancy for carbon-dioxide ice as measured by Simpson et al. (1980) and Paillou et al. (2008) – we put the measurements from the latter in parenthesis.

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant ($\epsilon$)</th>
<th>Loss Tangent ($\tan\delta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Hydrocarbons</td>
<td>1.6–1.9</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Solid Hydrocarbons</td>
<td>2.0–2.4</td>
<td>$10^{-2} - 10^{-3}$</td>
</tr>
<tr>
<td>CO2 Ice</td>
<td>2.2 (1.6)</td>
<td>$10^{-3}$ ($10^{-4}$)</td>
</tr>
<tr>
<td>Water Ice</td>
<td>3.1</td>
<td>$10^{-4} - 10^{-5}$</td>
</tr>
<tr>
<td>Water-Ammonia Ice</td>
<td>4.5</td>
<td>$10^{-2} - 10^{-3}$</td>
</tr>
<tr>
<td>Organic heteropolymers</td>
<td>4.5–5.5</td>
<td>$&lt; 10^{-5}$</td>
</tr>
<tr>
<td>Meteoric Material</td>
<td>8.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

cannot consist of pure simple hydrocarbons, but we cannot easily say if it is mainly solid water ice or a porous ammonia-ice mixture.

The inferred dielectric constant is a lower limit for another reason as well. By removing the estimated diffuse contribution, we are assuming that the remaining low-angle quasispecular return is from a surface that is smooth on wavelength scales. The inferred dielectric constant is derived solely from this surface. It is impossible to recover the surface characteristics from the empirical diffuse model, thus the diffuse contribution to the dielectric constant goes unmeasured (Sultan-Salem, 2006). This presents a challenge when the diffuse component is very large, such that the remaining quasispecular component is small in comparison. In this scenario, without a rigorous correction based on the diffuse parameters, the inferred dielectric constant will appear to approach unity and will thus be underestimated. Consequently, we present our results as lower bound estimates of the true surface dielectric constant.
Furthermore, there is some uncertainty in the inferred surface parameters due to the presence of the significant diffuse component. Remember that we measure the diffuse contribution at incidence angles greater than 30° and then extend the diffuse model down to nadir, assuming the same cosine power law dependence, to determine the strength of the quasispecular component that is left (see Section 4.5). We believe this approach to be sufficiently accurate, but further work is needed to test the precise sensitivity of the measured surface parameters on the strength of the diffuse component.

A final word of caution in interpreting the retrieved dielectric constant values: we observe that the inferred dielectric constant can vary by as much as 10% depending on the assumed model form. In our analysis, we find the single best model that is capable of accurately describing all of the feature backscatter functions so that we can accurately compare the results in their relative sense.

5.1.2 Interpreting the RMS Surface Slope Parameter

The rms surface slope parameter derived from the quasispecular models is difficult to physically interpret because there is no horizontal scale associated with them. Fractal-based quasispecular models offer a method of resolving this ambiguity (Sultan-Salem, 2006). Further complicating the interpretation is the large variability (∼30%) observed in the inferred slope parameter depending on the assumed model form.

In our analysis, we choose to utilize the slope estimates in their relative sense, rather than attempt to relate them absolutely to the physical surface. The relative interpretation gives us a sense of how the different surface terrains compare in terms of their large-scale roughness, as long as the slopes are derived from the same composite model form.
5.2 Global Titan: Model Fitting Example

We apply the composite backscatter models to the global collection of Titan data following the fitting procedure of Section 4.5 to determine the average surface scattering properties. We perform this analysis in a detailed manner to illustrate the modeling process. Xanadu data is excluded from the global Titan dataset because Xanadu is much brighter than the average global data, and it is predominantly sampled only at diffuse incidence angles between 15° and 30°; thus Xanadu would skew the global results if it was left in. The fitting results for each of the nine two-quasispecular composite model forms (CM2; Eq. 4.19) and the fitting results for each of the three single-quasispecular composite model forms (CM1; Eq. 4.18) are listed in Table 5.2.

Table 5.2: Composite model fitting results to the global set of Titan data, ordered by their percent relative fitting errors. The diffuse model result is the same for all: \( a = 0.30 \pm 0.01 \), \( n = 1.79 \pm 0.03 \).

<table>
<thead>
<tr>
<th>Model</th>
<th>( \epsilon )</th>
<th>( s (°) )</th>
<th>( \hat{\sigma} )</th>
<th>( \delta_r ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GED</td>
<td>2.37 ± 0.04</td>
<td>16.45 ± 0.51</td>
<td>0.26 ± 0.01</td>
<td>0.55</td>
</tr>
<tr>
<td>HED</td>
<td>2.40 ± 0.04</td>
<td>16.20 ± 0.51</td>
<td>0.26 ± 0.01</td>
<td>0.59</td>
</tr>
<tr>
<td>EED</td>
<td>2.38 ± 0.04</td>
<td>16.33 ± 0.51</td>
<td>0.26 ± 0.01</td>
<td>0.59</td>
</tr>
<tr>
<td>GHD</td>
<td>2.59 ± 0.06</td>
<td>8.54 ± 0.34</td>
<td>0.27 ± 0.01</td>
<td>0.97</td>
</tr>
<tr>
<td>HHD</td>
<td>2.62 ± 0.06</td>
<td>8.43 ± 0.34</td>
<td>0.27 ± 0.01</td>
<td>1.03</td>
</tr>
<tr>
<td>EHD</td>
<td>2.60 ± 0.06</td>
<td>8.50 ± 0.34</td>
<td>0.27 ± 0.01</td>
<td>1.03</td>
</tr>
<tr>
<td>HGD</td>
<td>2.25 ± 0.06</td>
<td>11.25 ± 0.52</td>
<td>0.25 ± 0.01</td>
<td>1.44</td>
</tr>
<tr>
<td>EGD</td>
<td>2.23 ± 0.06</td>
<td>11.41 ± 0.52</td>
<td>0.25 ± 0.01</td>
<td>1.46</td>
</tr>
<tr>
<td>GGD</td>
<td>2.20 ± 0.06</td>
<td>11.46 ± 0.52</td>
<td>0.25 ± 0.01</td>
<td>1.49</td>
</tr>
<tr>
<td>ED</td>
<td>2.33 ± 0.04</td>
<td>16.73 ± 0.51</td>
<td>0.26 ± 0.01</td>
<td>13.92</td>
</tr>
<tr>
<td>HD</td>
<td>2.55 ± 0.06</td>
<td>8.68 ± 0.34</td>
<td>0.27 ± 0.01</td>
<td>16.55</td>
</tr>
<tr>
<td>GD</td>
<td>2.15 ± 0.06</td>
<td>11.93 ± 0.52</td>
<td>0.25 ± 0.01</td>
<td>22.04</td>
</tr>
</tbody>
</table>

* The parameter uncertainties are based on the 95% confidence intervals that are calculated from the least squares solution, the fit residuals, and the Jacobian matrix using MATLAB’s `confint()` function. They are relative to each model form and do not reflect the absolute uncertainties that exist between the models.
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Figure 5.1: The CM1 models fit to the global backscatter response of Titan (see caption for Figure 5.2). The ED composite model fits better than the HD and GD models. The differences between the CM1 models reflect the differences between the CM2 model groups.

The CM1 model solutions are tabulated in the last three rows of Table 5.2 and are illustrated in Figure 5.1. Figure 5.1 demonstrates that the CM1 models perform well over all incidence angles except those closest to nadir (<2°). The CM2 models differ from the CM1 models only in the inclusion of the near-nadir quasispecular model, which can take the form of any of the three quasispecular laws defined in Eqs. 4.10-4.12. We group the CM2 model forms according to their second quasispecular component, such that we have the _ED group of models where $\sigma_{qs2}^0 = \sigma_E^0$ (these are GED, HED, EED), the _HD group of models where $\sigma_{qs2}^0 = \sigma_H^0$ (these are GHD, HHD, and EHD), and the _GD group of models where $\sigma_{qs2}^0 = \sigma_G^0$ (these are GGD, EGD, HGD). The behavior of each group of CM2 models is primarily described by the behavior of its corresponding CM1 model (e.g. the behavior of the _ED group follows the behavior of the ED model). This is similar to saying that the total composite model parameters are controlled by the parameters of the second quasispecular model component, as we first explained in Section 4.4. We find that the model results are consistent with each other by ~1% within a particular grouping (e.g. the _ED models
all have dielectric constants of 2.38 ± 0.02), but that they can vary from each other by 10% to 30% between the different groupings.

Table 5.2 illustrates that the different model groups suggest very different surface characteristics from each other. The HD model (or _HD group) yields rms slopes that are about half of the ED (_ED) model slopes and dielectric constant values that are slightly greater than the ED (_ED) dielectric constant values. The GD (_GD) model yields rms slopes that are in between the other two and dielectric constants that are lower than the other two. The relationship between the model parameters is different than that portrayed in Figure 4.4, probably due to the presence of the diffuse scattering component. The variability in the inferred parameters between the different model groups presents its own source of uncertainty. To minimize this uncertainty, we find the common model that best describes all of the analyzed surface features, rather than considering the best-fit models for each individual feature. It turns out that the common best-fit model across all of the features is the same as the best-fit model of the global Titan data.

To determine the best-fit model, we compute the relative fitting error ($\delta_r$) of each composite model, which we calculate from the mean normalized squared residual as follows:

$$\delta_r = \frac{1}{n} \sum_n \frac{(y_n - \hat{y}_n)^2}{\hat{y}_n^2},$$

(5.1)

where $y_n$ is the nth mean backscatter measurement, and $\hat{y}_n$ is the composite model evaluated at the center angle of the nth angle bin (see Section 4.5 for details on the mean backscatter curve). Eq. 5.1 is similar to Eq. 4.22, the fitting minimization criteria imposed on each model component, but Eq. 5.1 evaluates the fitting performance of the total model over all incidence angle bins. The results in Table 5.2 are ordered according to their relative fitting errors.

We find that the GED composite model fits the global set of Titan data slightly better than the other _ED composite models. We show the best-fit GED model in Figure 5.2 together with the best-fit parameters. The HED and GED best-fit models
Figure 5.2: Global backscatter response of Titan on a logarithmic scale. The shaded red area represents the 20-80th percentile range of NRCS, computed for angle bins of 0.5° width. The best-fit global model is a Gaussian-Exponential-Diffuse (GED) composite model (black solid line), with the best-fit parameters as shown. The Xanadu core data, which are sampled predominantly at diffuse incidence angles between 15° and 30°, are filtered out so as to not skew the backscatter curve.

look very similar to the GED best-fit model. The _ED best-fit composite models are followed by the _HD best-fit composite models and then the _GD best-fit composite models. We contrast the appearance of the different best-fit model groups through a comparison of the CM1 best-fit models in Figure 5.1. This simplified comparison works because the main differences between the CM2 models occur in the second quasispecular component; the near-nadir quasispecular contributions look very similar to each other.

5.2.1 Global Titan Model Result Discussion

The best-fit model results for Global Titan suggest a surface with a mean bulk dielectric constant larger than 2.0 and probably less than 3.0 (the upper bound is uncertain because we do not understand how the significant diffuse component might influence
the perceived dielectric constant). Table 5.1 suggests that this range of dielectric constant is consistent with several materials: solid hydrocarbon, carbon dioxide ice or porous water ice, or a mixture of the above. Reflectance spectra produced by the VIMS instrument show signatures of hydrocarbons, nitriles, and carbon dioxide frost across Titan’s surface, but areas of pure water ice are less infrequent (see, for example, McCord et al., 2008; Soderblom et al., 2007). The mean dielectric constant measured by the RADAR radiometer (Janssen et al., 2009) suggests a value closer to 1.6 over the globe of Titan. The lower dielectric constants implied by the passive data hint at a well-known discrepancy that dates back to the original lunar observations (e.g. Heiles and Drake, 1963). The discrepancy is likely due to differences in the assumed models. Zebker et al. (2008) construct a model that considers both the active and passive datasets to solve for a single surface solution, but additional investigation is still needed.

The global Titan model results suggest rms surface slopes between 8° and 16°, depending on the model. These values imply a rather high large-scale roughness, but it is difficult to physically interpret the slope values without knowing the horizontal scale associated with them. In our analysis, we choose to utilize the slope estimates in their relative sense, rather than attempt to relate them absolutely to the physical surface. The relative interpretation gives us a sense of how the different surface terrains compare in terms of their large-scale roughness (as long as the slopes are derived from the same composite model form).

The retrieved global mean dielectric constants and rms slopes are not consistent with those deduced from ground-based measurements. Campbell et al. (2003) report a lower mean dielectric constant (1.8) and rms slope (0.5° to 3.5°) for Titan at 13 cm-\(\lambda\). The rms slopes measured at 13 cm are significantly lower than those measured at 2 cm, supporting the long-standing observation that inferred rms slopes are strongly wavelength dependent, where surfaces are smoother at longer wavelengths (Muhleman, 1964; Simpson and Tyler, 1982). Sultan-Salem (2006) uses a fractal-based quasispecular law, derived from the linear superposition of standard quasispecular laws,
to demonstrate the horizontal scale dependence of the rms slope. Using Cassini data from the first few Titan passes, he finds an rms slope near 10° at the horizontal scale corresponding to the 2.2 cm Cassini wavelength, and an rms slope near 3° at the scale corresponding to the 13 cm Arecibo wavelength, thus, in effect, explaining the differences in the reported slope results. Sultan-Salem (2006) also reanalyzes the Arecibo data with his composite model and finds dielectric constant values closer to 2.1 on average. The residual discrepancy between the 13 cm-\( \lambda \) results and the 2 cm-\( \lambda \) results might be attributed to the different depths sensed by the different wavelengths or the different regions sampled (the 13 cm-\( \lambda \) nadir data occur at low latitudes, while the 2 cm-\( \lambda \) low-angle data occur over all latitudes).

We apply the radar albedo equation (Eq. 4.28) to the global Titan composite model results and obtain SL-2 (same-sense linear polarization at 2 cm-\( \lambda \)) albedo values between 0.25 (using the _GD model group) and 0.27 (using the _HD model group). The best-fit model group (_ED) yields an SL-2 albedo of 0.26. We calculate that roughly 82% of the 2.2 cm albedo derives from diffuse backscatter mechanisms, with the remaining 18% from quasispecular backscatter. That is, the diffuse SL-2 albedo estimate is 0.21 and the quasispecular SL-2 albedo estimate is 0.05. This average diffuse estimate is actually a lower bound since we exclude the Xanadu data from our global Titan dataset (due to uneven angular sampling). Such a large fractional diffuse component is consistent with the theory that the diffuse scattering is predominantly volume in origin.

To compare our albedo values with ground-based radar measurements of Titan, which are typically obtained in both senses of circular polarization, we need to estimate the total power (TP) albedo, which is the sum of the albedos in two orthogonal polarizations (see Section 4.6). No dual-linear radar measurements exist for Titan, but we estimate the relationship between the 2 cm-\( \lambda \) total power albedo (TP-2) and the SL-2 albedo for the Saturnian satellites by observing the linear polarization behavior of the Galilean satellites (Eq. 4.31). With this approach, we estimate the Titan TP-2 albedo to be 0.40 ± 0.04 from the _ED best-fit composite model group.
Campbell et al. (2003) report a TP-13 albedo (12.6 cm-\(\lambda\)) near 0.21, which is about half of our TP-2 albedo value. We would expect the 3.5 cm ground-based albedo measurements to also be larger than the 12.6 cm values, but they are not. Muhleman et al. (1995) report an average 3.5 cm OC albedo for Titan near 0.125. The discrepancy in the data from the two comparable wavelengths, 3.5 cm-\(\lambda\) and 2.2 cm-\(\lambda\), is difficult to reconcile. We note that the 3.5 cm-\(\lambda\) measurements are extremely weak compared to those made with Arecibo at 12.6 cm-\(\lambda\), with no visible specular component, and there also exists very large scatter among the data. The confusion over possible pointing errors and the presence of a calibrator flux density error in the initial VLA software makes the 3.5 cm-\(\lambda\) data further suspect. As a result, we constrain our comparison of results to those acquired by the 12.6 cm-\(\lambda\) Arecibo studies.

The difference between the TP-13 and TP-2 albedo results does not stop at Titan, but rather it appears to be a general trend with most of the Saturnian satellites (Black et al., 2007). The absolute errors will be different between the datasets (the Arecibo radar results are calibrated with a systematic error of about 25% and the Cassini radar results are calibrated with a systematic error of about 12%), however there does not appear to be a single factor that can correct the results to the same values. Thus, the decrease in radar reflectivity with increasing wavelength likely reflects true surface variations with depth, such as increasing absorption, thereby helping to constrain the effective scattering layer of each moon. In a mature clean icy regolith, radar will sound to depths of 10 to 20 wavelengths (Black et al., 2001), that is 2 to 5 decimeters for the Cassini radar and 1 to 2.5 meters for the Arecibo radar. Thus, any differences in measured reflectivity values at the two wavelengths reflect differences in radar transparency or structural heterogeneity with depth, or even a material whose absorption length is highly dependent on wavelength.

We find that the average surface of Titan has a similar total albedo to the leading side of Iapetus (~0.44; see Section 8.2.2), but a smaller total albedo than most of the other major Saturnian icy satellites (Ostro et al., 2006). Titan is also darker
than the Galilean satellites at similar wavelengths; the 3.5 cm-\(\lambda\) TP albedo values of Callisto, Ganymede, and Europa are measured to be 0.72, 1.55 and 2.31, respectively (Ostro et al., 2006). Yet, the global Titan measurement is an average measurement. If we model subsets of the surface, we discover a variety of different surface types are present, as we discuss next.

5.3 Xanadu

We begin our feature analysis with Xanadu, the brightest and most sizable surface feature on Titan. Xanadu is about 4000 km \(\times\) 2500 km, or about the size of Australia, and is centered on the equator at about 100° W longitude, i.e. near the apex of orbital motion. Its brightness is in such contrast to the surrounding terrain that it readily stands out in the first near-infrared images of Titan. Hubble Space Telescope images of Titan acquired in 1994 (at 940 and 1080 nm wavelengths) identify it as a “large bright, roughly rectangular” continent-like region (Smith et al., 1996). Subsequent imagery from Cassini reveals Xanadu to be a rugged, mountainous terrain dessicated by river channels, hills, and valleys. Xanadu seems to be one of the oldest terrains on Titan and its geomorphology is consistent with it being exposed and eroded fractured bedrock (Radebaugh et al., 2011).

Xanadu appears geologically diverse in the available high-resolution radar images. Radebaugh et al. (2011) find that Xanadu can be divided into four major areas based on location and morphology: the western region consists of extensive river and drainage systems, the middle region consists of rugged mountain peaks interspersed with dark valleys, the eastern region consists of arcuate mountain belts interrupted by drainages and impact craters, and the southern region consists of basins and ridges, but is dominated by interleaved, lobate flows. The northern region of Xanadu is notably absent from the high-resolution radar dataset, although an image of the northern tip acquired on the T77 flyby (21-June-2011) reveals bright wispy terrain that ends abruptly in dark dunes.
Figure 5.3: Maps of Xanadu observed by the RADAR instrument in real aperture mode (top left), synthetic aperture mode (top right), radiometer emissivity at normal incidence (middle left), radiometer dielectric constant (middle right), the ISS instrument (bottom left), and the VIMS instrument (bottom right). The black boundary depicts the outer extent of Xanadu, while the inner, purple boundary defines what we call the Xanadu core. The two boundaries are chosen by following the joint contours in the radiometry and real aperture radar maps.

The diversity of surface features within Xanadu means that it is not uniformly bright. As a result, the boundaries of Xanadu are not easily defined. To complicate matters, the extent of Xanadu appears to vary with wavelength. We illustrate the different appearances in Figure 5.3, where we display RADAR, ISS, and VIMS maps of the Xanadu region. The RADAR maps consist of low resolution (real aperture radar, or RAR) and high resolution (synthetic aperture radar, or SAR) backscatter
maps, as well as emissivity and dielectric constant maps from the radiometry data (see Janssen et al. (2009) for details on the latter). The active RADAR maps are corrected for incidence angle effects using the global Titan backscatter model and scaled to the model value at 32° incidence (see Chapter 6 for details). We identify two Xanadu boundaries from a joint comparison of the RAR backscatter and radiometer emissivity maps: one boundary defines the outer extent of the main region of Xanadu-type brightness (outlined in black in the figure), the other boundary defines the brightest, or least emissive, “core” of Xanadu (outlined in purple in the figure). It is difficult to correlate the chosen boundaries with the SAR data due to the incomplete high-resolution coverage. Comparison of the boundaries with the optical maps reveal large discrepancies; the radar-defined boundary generally appears smaller than the boundary suggested by the optical data. Furthermore, certain areas to the south of our boundary are dramatically anticorrelated. For example, the orange (or 5 µm-λ bright) elongated feature near 20° S and 125° W in the VIMS map appears bright in the ISS map but dark in our radar maps (this feature is officially named Tui Regio; we investigate it in more detail in Section 5.7). Thus, the Xanadu boundary that we define does not apply uniformly across the datasets.

We apply the collection of composite models to the Xanadu and Xanadu Core backscatter data and find that the ED model group fits well (1.9% relative error for Xanadu and 0.8% relative error for the Xanadu Core; see Eq. 5.1). While we characterize the GED model for comparison to other surface features (see Figure 5.10), we also analyze just the CM1 (single-quasispecular composite model) ED model results. We focus on the latter analysis here for two reasons: 1) low angle data are missing from the Xanadu Core region, and 2) the near-nadir altimetry data for Xanadu may come mostly from a dark basin near the south that may not represent Xanadu as a whole. Recent altimetry data over the width of Xanadu acquired on the T77 flyby may more properly describe the average nadir response of Xanadu, but, at the time of this writing, the data are still in their preliminary form and are not ready for analysis.

The ED model results are shown in Figure 5.4. We find that the Core of Xanadu
5.3. XANADU

Figure 5.4: ED backscatter model results for Xanadu (left) and the Xanadu Core (right). The backscatter data points are colored by their radar mode. The decomposition into diffuse and quasispecular components (lower row) shows the significant portion of diffuse scatter that dominates the backscatter response of Xanadu. The Xanadu Core is about 6% more diffusive than the rest of Xanadu.

is about 20% more radar-bright than the rest of Xanadu, and is about 5.5% more diffusive. The large diffuse component (>88% of the radar reflectivity) suggests that Xanadu is predominantly volume scatter in origin, an interpretation also favored by the radiometry data (Janssen et al., 2009). Yet, we do detect a clear, yet small, surface scattering signature in the form of the quasispecular component.

The dielectric constant associated with Xanadu’s quasispecular component appears to be most consistent with water ice that may be slightly porous and may also contain a minor contaminant. We find that the Xanadu Core ($\epsilon > 2.7$) has a slightly
lower inferred dielectric constant than the rest of Xanadu, perhaps indicating that
the Xanadu Core is composed of more pure ice, and the rest of Xanadu \((\epsilon > 3.4)\) may
contain more contaminant (such as ammonia). The large-scale surface roughness of
the Xanadu Core also appears to be lower than that of the rest of Xanadu, but this
parameter has a large uncertainty associated with it due to the absence of low-angle
Xanadu Core data.

To the VIMS instrument, Xanadu appears to be composed of organic surface
deposits, with little or no exposed water ice (Soderblom et al., 2007). Yet, the VIMS
and RADAR data are also sensitive to very different depths, and it is possible that
Xanadu is composed of water ice bedrock covered by a radar-transparent organic
coating that is not much more than a few \(\mu m\) thick. This interpretation is consistent
with VIMS observations of mountain chains at the eastern end of Xanadu that appear
to have a higher proportion of exposed water ice (Barnes et al., 2007). Radebaugh
et al. (2011) suggest that perhaps fluvial erosion in these areas wases away some of the
organic coating to expose the underlying water ice. Radiometry measurements find
an unrealistically low effective dielectric constant for Xanadu \((\epsilon \approx 1)\) that Janssen
et al. (2009) attribute to a graded-density porous layer at least a centimeter thick
that depolarizes the emitted microwaves.

We further note the presence of Xanadu-bright extensions directly south and also
south-east of the main province that we’ve been discussing. The Xanadu “extensions”
do not readily appear in the ISS and VIMS images, but their high radar brightness
and low radiometric emissivity bear a strong resemblance to Xanadu in the RADAR
data. These data collectively occur over all incidence angles and appear to have a
backscatter response consistent with the rest of the Xanadu data. Indeed, the ED
model results yield surface parameters \((\epsilon > 3.3, s \approx 21^\circ)\) that are very similar to
those derived for Xanadu.
5.4 Hummocky Mountains

The radar-bright hummocky and mountainous terrains are very similar to that of Xanadu, but they occur in isolated patches that are small in area or in long mountain chains. The hummocky terrain units appear textured and are interpreted as likely tectonic in origin. Lopes et al. (2010) have mapped the hummocky terrain through the T30 Titan flyby, and recently the authors have also incorporated data from more recent passes. We determine the backscatter function for the hummocky terrain based on these mapping results. While Lopes et al. (2010) include Xanadu as a sub-unit of the hummocky terrain, we consider the two terrains separately due to their different backscatter strengths. The mapped hummocky terrains that we analyze are pictured in Figure 5.5.

We plot the best-fit GED Hummocky backscatter model in the upper right panel of Figure 5.10. We find that the hummocky terrain appears to have a dielectric constant
similar to that of the Xanadu Core, but a large rms slope \( s \approx 22^\circ \) that is more similar to average Xanadu. However, the integrated radar brightness, or albedo, of the Hummocky terrains \( \hat{\sigma} = 0.42 \) is only about half of the total Xanadu albedo value. The hummocky terrains also appear slightly less diffusive (86% diffuse scattering, or a diffuse albedo of \( \hat{\sigma}_D = 0.36 \)), and their diffuse exponent is larger \( (n \approx 2) \) than the Xanadu terrain.

The hummocky modeling results suggest that the volume scattering mechanism is not as potent on the mountainous terrain as it is on Xanadu; perhaps the bright icy bedrock is not completely exposed to the incident radar wave, or perhaps the exposed ice is not as clean and low-loss. Another possibility is that the ice has not properly matured, so that it is lacking a sufficient number of embedded scattering centers within the volume. Whatever the explanation, the similarities between the hummocky terrain and Xanadu are clear; the scattering properties of the former are just a less extreme version of the latter.

### 5.5 Dunes

With the exception of the bright Xanadu continent, the tropics of Titan were known to be dark at microwave and optical wavelengths long before Cassini began orbiting Saturn (Campbell et al., 2003; Smith et al., 1996). Detailed RADAR imagery acquired during the first few Titan passes revealed the source of the dark material: long parallel streaks comprise dune fields that cover much of the equatorial latitudes (Elachi et al., 2006; Lunine et al., 2008). Typically, the dunes are spaced about 3 km apart, and are probably about 100 m high (Kirk et al., 2009; Lorenz et al., 2006; Neish et al., 2010). The dunes appear to be of longitudinal type, where the dunes are aligned parallel to the vector sum of wind directions, and they appear to move around brighter elevated terrain, suggesting that a westerly surface wind is responsible for their formation (Lorenz et al., 2006; Lorenz and Radebaugh, 2009; Radebaugh et al., 2008). While the dunes appear younger than other terrain – they are observed to
The dune terrain mapped by Le Gall et al. (2011) through the T55 Titan flyby are outlined in black over the real aperture radar map.

cross over terrain of similar elevation – it is not clear if they are still actively forming. The RADAR radiometry data show the dune fields to be the radiometrically warmest feature on Titan (only the liquid lakes and seas appear warmer). The mean brightness temperature of the dunes is 88.7 K ± 1.3 K, or about 3-5 K brighter than the dune surroundings, and the average dune emissivity is then ∼0.95. Such a high emissivity value suggests a low permittivity surface that is smooth at wavelength scales (Le Gall et al., 2011).

The dune fields appear to be confined between -30° and +30° latitude and may cover as much as 12.5% of Titan’s surface, or about 10 million km², a similar area to that of the United States (Le Gall et al., 2011). The dune fields have been mapped within the SAR imagery (up to the T55 flyby) by (Le Gall et al., 2011); these boundaries are depicted over the real aperture radar map (see Chapter 6) in Figure 5.6. The SAR images show regional variations in the dune characteristics: some areas are darker to the radar, with wider dunes that are closely spaced, and some areas are brighter, with dunes that are spaced farther apart. In the first case, it is likely that there is a more abundant sand supply, such that a thicker layer of sand covers
the interdune and the bright icy substrate is less exposed to the penetrating radar wave. In the second case, the sand supply is likely more restricted such that the volume scattering signature from the interdune is more apparent. Le Gall et al. (2011) observe a latitudinal trend in the dune radar reflectivity and emissivity, with the dunes becoming more reflective and less emissive farther from the equator, and Savage and Radebaugh (2011) find that the fraction of interdune increases towards northern latitudes. Both observations point to there being a general reduction of sand supply and/or an increase in ground humidity that inhibits the mobilization of the sand-sized particles north of the equator (Le Gall et al., 2011).

We measure the backscatter function of the mapped dune terrains using our collective set of real aperture radar data. We only utilize data with a beam footprint that covers more than 90% of the dune field to avoid contamination from bright inselbergs or surrounding terrain. We consider the average dune response from the total collection of dune data, and the dune response from just the Fensal region (near 5° N, 40° W). The Fensal dune fields tend to have more widely spaced dunes and brighter interdunes than the average dune field (Le Gall et al., 2011). The best-fit GED models for these two features are pictured in the middle row of Figure 5.10, and the results are tabulated in Table 5.8. We find that the Fensal dunes are about 5% more radar-reflective than the average dune and also appear to have a larger dielectric constant ($\epsilon > 2.14$ for the Fensal Dunes vs. $\epsilon > 1.97$ for the average dune), consistent with there being more bright interdune exposed. Overall, the dunes are highly diffusive (>82%), suggesting that volume scattering (probably primarily from the interdune) is prominent. The Fensal dunes are slightly more diffusive than average, consistent with the likelihood that volume scattering from the interdune is less inhibited by sand cover.

The retrieved dielectric constant values are consistent with VIMS observations that find the dune material to be largely composed of solid complex organics (Barnes et al., 2008; Clark et al., 2010; Soderblom et al., 2007). Furthermore, the RADAR radiometry data also suggest that the dune dielectric constant lies between 1.5 and
2.5, depending on the model used, where the variability is attributed to subsurface scattering effects (Le Gall et al., 2011).

The backscatter study of the Titan dune fields is complicated by the fact that the dunes are highly geometrical and directional features. The slope of the dune face is estimated by radarclinometry to be $6^\circ \pm 3.5^\circ$ (Lorenz et al., 2006; Neish et al., 2010), but visible glints in SAR imagery suggest steep faces oriented at $26^\circ$ (Le Gall et al., 2011). Thus, the local incidence angle, and azimuth angle, may strongly affect the perceived backscatter behavior, particularly when it comes to interpreting the inferred rms slope. However, without knowing the exact dune profile, it is difficult to correct for these effects. To further complicate matters, the real aperture beam footprint cannot separate the dune and interdune material, so what we measure here is a combination of the two surface types. If the interdune material is largely exposed icy bedrock, then the inferred dielectric constant will be pulled higher than if we were sensing the organic material by itself (as illustrated by the comparison of Fensal dunes to the average dunes).

The dune radar backscatter response contains a bright signature near nadir. While the near-nadir peak is common across the Titan feature set, its relative strength is most striking within the dune fields. A possible explanation is that the interdune, oriented perpendicular to the radar at zero incidence, is being polished smooth by the saltating organic sand particles. Or perhaps the interdune sand cover is flatly compacted, so that wavelength-scale roughness is reduced. It is also possible that flat nadir-oriented areas exist within the dunes themselves.

To summarize, the dune radar backscatter signature is consistent with the dunes being composed largely of organic material. This material may have been deposited as rain from the atmosphere and then eroded into sand-sized particles. While they are restricted to Titan’s tropics, the dune fields represent the largest organic reservoir on Titan (Lorenz et al., 2008b).
5.6 Gray Plains

Between the dune-filled tropics and the liquid-populated poles, the surface of Titan is covered by a distinctly bland and homogeneous radar-gray terrain that is identified as “gray plains” (they are also called “undifferentiated plains” by Lopes et al. (2010)). These plains are mapped by Lopes et al. (2010) from SAR imagery, and their boundaries are pictured in Figure 5.7 on top of the real aperture backscatter map (see Chapter 6). We measure the backscatter response from these plains and apply our composite backscatter model. The best-fit GED model is shown in the lower left panel of Figure 5.10, and the results are tabulated in Table 5.8.

We find that the plain scattering parameters are practically indistinguishable from the Fensal dune parameters, i.e. a low radar reflectivity ($\hat{\sigma} = 0.20$), a dielectric-constant consistent with solid hydrocarbons ($\epsilon > 2.1$), moderate large-scale roughness ($s \approx 14.1^\circ$), and a large diffuse component (82.6%). In other words, to radar, the mid-latitude plains appear similar to the brighter dunes that have more prominent interdune segments. Perhaps the plains are simply an extended version of the interdune
terrain (i.e. an icy substrate covered by an even and thin coating of organic dune-type material)? Le Gall et al. (2011) suggest a genetic relationship between the plains and the dunes: “they could correspond to regions where the dunes are too narrow to be resolved by SAR, where sand-sized grains are cemented by a wetter climate and thus cannot be sculpted into fields of dunes, or where winds do not blow sufficiently hard to organize sands into dunes”. If the plains are simply an unformed dune-field, or an organic-covered substrate, then the similarities in the scattering properties of the plains and the dunes may suggest that geometric nature of the dunes need not be considered when interpreting the dune scattering properties.

The microwave similarities between the mid-latitude plains and the equatorial dunes does not extend to optical wavelengths. VIMS observes the mid-latitudes to be more highly reflective at 5 \( \mu \text{m} \) wavelengths than the equatorial surface (Barnes et al., 2007). This suggests that the top tens of micrometers probed by VIMS in the mid-latitude zone are distinct from the those probed in the equatorial zone, and that the similarity between the two zones is a slightly deeper (< 1 meter) phenomena.

5.7 Cryovolcanic Terrain

The term cryovolcanism refers to volcanism with low-density, low-melting point materials such as ammonia water hydrates and methane-clathrate hydrates (see, for example, Mitri et al., 2008; Tobie et al., 2006). The RADAR SAR images provide evidence for cryovolcanic activity on Titan in a number of locations; Ganesa Macula, Winia Fluctus, Sotra Facula, Tui Regio, and Hotei Regio are some of the major features associated with cryovolcanic activity. Ganesa Macula is a circular radar-dark feature with associated radar-bright flows located near 50° N, 87° W (Lopes et al., 2007b). Winia Fluctus, located near 45° N, 30° W, is the largest identified cryovolcanic flow on Titan and is notable for its diffuse edges (Lopes et al., 2010). Sotra Facula is a 60 km subcircular cryovolcanic edifice near 15° S, 40° W from which bright-edged lobate flows radiate northward (Lopes et al., 2010). Stereo analysis of overlapping
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Figure 5.8: Images of Tui Regio (left) and Hotei Regio (right) as seen by the different RADAR modes and the ISS and VIMS instruments. The two features are outlined in green. The real aperture radar (RAR) color scale goes from 0 (blue) to 1 (red) and represents the NRCS normalized to 32° incidence (see next chapter for details). The radiometry color scale goes from 72 K (blue) to 92 K (red) and represents the brightness temperature measurements. The VIMS image is colored according to wavelength: the red channel is 5 μm-λ, the green channel is 2 μm-λ, and the blue channel is 1.6 μm-λ. For each feature, we zoom in on the high-resolution SAR image to produce a detailed callout box.

SAR images reveal that Sotra Facula consists of a 1000-m high peak adjacent to a 1500-m deep pit, and that the flows themselves can grow to about 800 m thick (Kirk et al., 2010).

Tui Regio (24° S, 125° W) and Hotei Regio (26° S, 78° W) are two regions identified as bright at visible wavelengths and anomalously bright (30% brighter than anything else on Titan) at 5 μm wavelengths (Barnes et al., 2005, 2007). We illustrate the appearances of Tui Regio and Hotei Regio at different wavelengths in Figure 5.8. Both are located at similar latitudes along the southern margin of Xanadu (Tui is at Xanadu’s southwestern end, and Hotei is at its southeastern end), perhaps suggesting the presence of a tectonic border along the south of Xanadu (Wall et al., 2009). They
appear radar-dark on average, in sharp contrast to the bright Xanadu terrain north of them. Interleaved, lobate flows appear to exist in both VIMS and RADAR images of the features, and, at least for Hotei, they are strikingly well correlated (Barnes et al., 2006; Soderblom et al., 2009; Wall et al., 2009). Stereo analysis of the SAR images over Hotei reveal that the flows are 100-200 m thick (Kirk et al., 2009). The spectroscopic signatures of Tui and Hotei are consistent with a depletion of water ice and an enrichment of carbon-dioxide ice (Hayne et al., 2008; McCord et al., 2008).

Photometric spectral variability was reported over Hotei Regio, as well as the western tip of Xanadu, by Nelson et al. (2009), indicating that cryovolcanism on Titan may be currently ongoing.

The cryovolcanic features described above have been mapped by Lopes et al. (2010), as depicted over the real aperture map in Figure 5.9. Our backscatter analysis of these cryocolcanic terrains indicate a surface that is unique from the other Titan terrains that we have studied. Most notably, the cryovolcanic surfaces have a very large diffuse exponent ($n \approx 3$), indicating the presence of a very focused diffuse scattering mechanism. As a result, the diffuse component falls off much more steeply.
with incidence angle than is otherwise common on Titan (see Figure 5.11). Furthermore, the measured diffuse echo is only 70% of the total echo power, in contrast to the >82% diffuse fractions observed elsewhere. The diffuse scattering behavior that we’ve described suggests that volume scattering is not as prevalent in the cryovolcanic terrain, either because the medium is more radar-absorptive or an insufficient number of scattering centers exist in the volume. It is also possible that the diffuse scattering mechanism is different altogether.

In all iterations of our analysis, the dielectric constant of the cryovolcanic terrain is consistently higher than any other that we’ve observed. The GED composite model results that we present here indicate $\epsilon > 3.5$, a dielectric constant that is consistent with the possible presence of ammonia-water ice. If ammonia is a component of the cryovolcanic material, that may also explain the poorer diffuse scattering levels. Further joint analysis of the different datasets is needed to explore and understand this unique terrain.

5.8 Modeling Results and Conclusions

We present a summary of the backscatter results from the analyzed Titan surface features in Table 5.8 and in Figure 5.10. A direct comparison of the backscatter model curves from the features is illustrated in Figure 5.11. With the exception of the cryovolcanic terrain, the backscatter shapes of the different features are surprisingly similar: all have diffuse exponents between 1.5 and 2.0 and all have greater than 80% diffuse scattering fractions. The cryovolcanic terrain stands out for having a large diffuse exponent ($n = 3$) and a small diffuse scattering fraction (70%). It’s steep backscatter slope is apparent in Figure 5.11 and suggests a different diffuse scattering mechanism than is common elsewhere on Titan.

The quasispecular parameters show a consistent trend: bright features tend to have larger dielectric constants and larger rms surface slopes than dark features. To the RADAR, the bright features are consistent with an icy substrate, while the dark
Figure 5.10: The GED backscatter model results are displayed for each of the six analyzed Titan surface features. The cross section data points are colored by their radar mode and show that the collective set of data is well-calibrated to the same scale.
Table 5.3: GED Composite model fitting results for various features on Titan’s surface.

<table>
<thead>
<tr>
<th>Feature</th>
<th>$a$</th>
<th>$n$</th>
<th>$\epsilon$</th>
<th>$s$ (°)</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{\sigma}_{D}/\hat{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xanadu</td>
<td>0.90 ± 0.03</td>
<td>1.48 ± 0.07</td>
<td>3.44 ± 0.34</td>
<td>20.72 ± 1.79</td>
<td>0.82 ± 0.04</td>
<td>88.3%</td>
</tr>
<tr>
<td>Xanadu Core*</td>
<td>1.25 ± 0.05</td>
<td>1.56 ± 0.10</td>
<td>2.69 ± 0.53</td>
<td>17.50 ± 5.19</td>
<td>1.04 ± 0.08</td>
<td>94.1%</td>
</tr>
<tr>
<td>Hummocky</td>
<td>0.54 ± 0.03</td>
<td>1.97 ± 0.12</td>
<td>2.59 ± 0.22</td>
<td>22.73 ± 2.36</td>
<td>0.42 ± 0.03</td>
<td>86.0%</td>
</tr>
<tr>
<td>Dune (All)</td>
<td>0.18 ± 0.02</td>
<td>1.76 ± 0.18</td>
<td>1.97 ± 0.06</td>
<td>12.34 ± 0.88</td>
<td>0.16 ± 0.02</td>
<td>82.1%</td>
</tr>
<tr>
<td>Dune (Fensal)</td>
<td>0.29 ± 0.04</td>
<td>2.22 ± 0.39</td>
<td>2.14 ± 0.10</td>
<td>14.81 ± 1.33</td>
<td>0.21 ± 0.04</td>
<td>83.0%</td>
</tr>
<tr>
<td>Gray Plains</td>
<td>0.25 ± 0.01</td>
<td>1.94 ± 0.09</td>
<td>2.12 ± 0.07</td>
<td>14.10 ± 0.94</td>
<td>0.20 ± 0.01</td>
<td>82.6%</td>
</tr>
<tr>
<td>Cryovolcanic</td>
<td>0.44 ± 0.05</td>
<td>2.99 ± 0.26</td>
<td>3.49 ± 0.27</td>
<td>15.19 ± 1.35</td>
<td>0.32 ± 0.04</td>
<td>70.0%</td>
</tr>
</tbody>
</table>

* The Xanadu Core model results are given for the ED model form since low angle data is missing.
** The parameter uncertainties are based on the 95% confidence intervals that are calculated from the least squares solution, the fit residuals, and the Jacobian matrix using MATLAB’s `confint()` function. They are relative to the GED model form and do not reflect the absolute uncertainties that exist between the models. The last column reflects the fraction of the echo power that is diffuse.

Figure 5.11: Comparison of the backscatter GED model curves for the six different Titan surface features analyzed in this chapter. The backscatter shapes appear similar to each other, with the exception of the cryovolcanic feature.
features have a more organic appearance. The one exception to this trend is, again, the cryovolcanic terrain, which is radar-dark but is also the location of the highest dielectric constant that we observe on Titan.

We note that many more surface features exist on Titan than the six that we present here. Lopes et al. (2010) present other geologic units that include empty lake basins, labyrinthic terrains, craters and crater-like structures, and fluvial features. In most cases, we do not have sufficient angular coverage to properly determine these features’ backscatter responses. In other cases, we do not have the spatial sensitivity to resolve the features using the real aperture radar beam footprint. The acquisition of future data may help to resolve the first issue, while the development of a higher resolution scatterometry processor (Wye and Zebker, 2006; Zebker et al., 2011) may help to resolve the second issue. We also note that these backscatter features are all identified by their geomorphological appearances within the radar SAR images. It is possible to also analyze the backscatter from terrains characterized by their optical albedos (e.g. Shangri-La, Adiri, Senkyo, Quivira, Aztlan, and Tsegihi), as we have done in Wye et al. (2007).

The backscatter model curves that we measure aid in the development of backscatter map products. In the next chapter, we demonstrate how we use the global Titan backscatter model to correct the total collection of radar data for incidence angle effects. The resulting near-global backscatter mosaic reveals the changes in radar reflectivity across the surface due to surface effects alone.
Chapter 6

Global Titan Backscatter Maps

In this chapter, we project the normalized radar cross section (NRCS or $\sigma^0$) measurements onto the surface of Titan, producing global map products that record the variation of 2.2 cm-$\lambda$ reflectivity. We correct the NRCS measurements for incidence angle effects by normalizing them by the best-fit global backscatter model and scaling the residual by the model value evaluated at 32° incidence. We mosaic together the measurements collected from each Titan flyby in each of the six RADAR modes. In this manner, we produce a global reflectivity map covering 99.93% of Titan’s surface, with resolutions ranging from 6 km to $\sim$250 km. We display the global map in different projections: a simple equirectangular cylindrical projection, pseudocylindrical equal-area projections, and the polar and equatorial stereographic projections. Collectively, the different projections yield a nearly complete portrayal of the surface reflectivity. We further decompose the global mosaic into the individual mode mosaics to show the coverage and resolution achieved by each.

6.1 Correcting for Incidence Angle Effects

Surface backscatter measurements depend strongly on viewing geometry, particularly the incidence angle of the observation (see Chapter 4). Thus, to produce meaningful backscatter reflectivity maps, the measurements must first be “flattened”, or corrected for incidence angle effects. We utilize the best-fit global backscatter model
Figure 6.1: The global backscatter response of Titan is shown on a linear scale in the left panel. The shaded red area represents the 20-80th percentile range of NRCS, computed for angle bins of 0.5° width. The best-fit global model is a Gaussian-Exponential-Diffuse (GED) composite model (black solid line), with the best-fit parameters as shown. We correct the global backscatter dataset using Eq. 6.1 together with the GED model. The “corrected” backscatter data are plotted in the right panel of the figure. The red solid line is the corrected mean backscatter curve, and the solid black line is the ideal flattened response for uniform terrain (equivalent to the model value at 32°, or a value of 0.23). The Xanadu core data, which are sampled predominantly at diffuse incidence angles between 15° and 30°, are filtered out so as to not skew the backscatter curve. A hump remains in the mean corrected backscatter curve near 20° that likely represents non-uniform sampling of bright terrain (perhaps the bright terrain are sampled more by the SAR modes, which operate around 20°, than by the other modes).

(a Gaussian-Exponential-Diffuse composite model or GED; see Section 4.4 for model details) to perform this correction. Each NRCS measurement is normalized by the model solution evaluated at the same incidence angle, as follows:

$$\sigma^0_{corr} = \sigma^0_{model}(32^\circ) \frac{\sigma^0_{avg}(\theta_i)}{\sigma^0_{model}(\theta_i)},$$

where $\sigma^0_{avg}$ is the beam-averaged NRCS measurement evaluated from the radar equation given in Eq. 3.9, $\theta_i$ is the incidence angle along the boresight axis, and $\sigma^0_{model}$ is the backscatter model function. The normalized residual is scaled by the model solution at 32° incidence (equal to 0.23 for the best-fit GED global backscatter model) to put the corrected result back onto the NRCS scale.
6.1. CORRECTING FOR INCIDENCE ANGLE EFFECTS

We plot the GED global backscatter model as a solid black line in the left panel of Figure 6.1 (this is the same figure as Figure 5.2 in Section 5.2, except that it is now displayed on a linear scale). The 20 to 80 percentile range of the global backscatter data is shaded in red. The corrected backscatter values, after applying Eq. 6.1 to the global backscatter measurements, are displayed in the right panel of the figure, where the solid red line represents the mean backscatter response and the solid black line is the ideal flattened response for uniform terrain (equal to the model value at 32°, or 0.23 for the GED global model). The 20 to 80 percentile range of the corrected backscatter data is again shaded in red. We find that, for incidence angles between 0.5° and 65° incidence, the mean corrected backscatter fluctuates about the ideal flat level with an rms error of 0.018, i.e. it is flat to within about 7.6%. At incidence angles lower than 0.5°, the data is highly variable, as it exists in the specular scattering regime and is more sensitive to the surface reflection properties within the resolved area. We remove these very low angle data from our mapping dataset. At incidence angles higher than 65°, when the areas illuminated by the beam can grow very large, the data is prone to errors from our effective-area simplification (see Appendix A). We also remove these high angle data from our mapping dataset.

Variations in the corrected backscatter about the ideal flat response represent specific features on the surface. These features will have backscatter responses that differ both in magnitude and in shape from the mean global backscatter response. The magnitude differences will result in the feature appearing either brighter or darker than the mean level. But the shape differences will result in residual incidence angle effects, even after correcting with the mean backscatter model. In many cases, the residual incidence angle effects appear subtle and not significant. The effects will be most apparent when there is overlapping coverage from very different incidence angle sets. This will cause seams to appear in the map mosaics, although the seams can in some cases be attributed to resolution effects. Large features could be corrected separately, if their backscatter response is known and adequately sampled, but we find that using the mean global model for the angle correction works well enough to
distinguish and accurately portray the different terrains.

To illustrate the backscatter correction procedure, consider the scatterometry mode measurements collected from the inbound segment of the T8 Titan flyby (T8-Inb). The observation is centered about the nadir direction, over the dark dune-covered area named Shangri-La. The beam is steered in an east-west raster scanning pattern, capturing the western part of Xanadu and the eastern part of Adiri. Bright faculae appear throughout the Shangri-La region (facula is the official term for bright spots on planets and moons). The backscatter response from the T8-Inb observation is plotted in the upper left of Figure 6.2, with data from the Xanadu region colored in red to demonstrate its above-average brightness. The backscatter data are mapped directly to the surface in the image labeled A (see Section 6.2 for the mapping procedure). This backscatter image is dominated by the bright quasispecular response that occurs at low incidence angles (near the center of the image). However, Xanadu is so bright, even at incidence angles greater than 25°, that it still peeks through on the right side of the image. We use the global best-fit Titan model (plotted as a solid black line over the backscatter response) to correct the data for incidence angle effects. The results from Eq. 6.1 are plotted in the upper right panel of Figure 6.2 and show that the non-Xanadu data (dark gray dots) are flattened well against incidence angle. The corrected backscatter are then mapped to the surface in the same manner as before (Figure 6.2B). In the corrected backscatter image, the geometric effects are largely removed and the surface features are no longer obscured. We can readily see Adiri, the details of Xanadu, and the faculae embedded within Shangri-La. A comparison to the corresponding ISS optical mosaic (Figure 6.2C) shows that the identified features are in very good agreement. Furthermore, there is a high correlation between the strength of the 2 cm-λ radar reflectivity and the 0.94 μm-λ optical reflectivity, with the exception of a region just south of western Xanadu, near the feature identified as Tui Regio (see Section 5.7 for discussion).

We note that a local best-fit model to the non-Xanadu T8-Inb data would yield an even flatter (improved) corrected response. Furthermore, the Xanadu backscatter
6.1. CORRECTING FOR INCIDENCE ANGLE EFFECTS

Figure 6.2: Illustration of the radar backscatter correction procedure for scatterometry data collected during the T8 inbound flyby (T8-Inb). The upper left panel represents the measured backscatter response, with the global GED backscatter fit shown as the black solid line. The data are corrected according to the global backscatter curve, with the results of Eq. 6.1 displayed in the upper right panel. The uncorrected data are mapped in image A and the corrected data in image B. The co-registered ISS optical map is displayed in image C.
response has a different form than the average Titan response, and thus may not
be optimally corrected with the global model curve. However, any residual incidence
angle effects are difficult to detect in the T8-Inb corrected backscatter plot shown here
because the western side of Xanadu (at lower incidence angles, \( \sim 30^\circ \)) is inherently
darker than the rest of Xanadu, especially the core of Xanadu that is observed at
angles closer to 60° (see Section 5.3 for backscatter analysis of the Xanadu Feature).
By lumping all of the Xanadu data together as we have here, we cannot separate a
trend in inherent brightness from a trend in residual incidence angle effects. However,
our purpose here is solely to demonstrate the angle-correction procedure, so this issue
is not important.

6.2 Forming the Radar Reflectivity Map

We project the corrected backscatter onto the surface to create a map of the surface
reflectivity. To do this, we begin by creating a Cartesian grid of the surface. We
calculate the intersection of the antenna beam with the surface grid for each burst
and then map the burst’s corrected NRCS value to the intersected area, performing
a weighted average with any overlapping data. For example, consider the corrected
backscatter from burst \( k \), \( \sigma_{\text{corr}}^0(k) \), as derived from Eq. 6.1. We accumulate this
result onto the map by creating a weight function \( w \) that depends on the location
within the map \((i,j)\) and the burst number \( k \). In other words, if there are \( N_B \) bursts
contributing to the map, and \( N_i \) rows and \( N_j \) columns within the map, then we form
the backscatter map according to the following equation:

\[
\text{Backscatter Map} = \frac{\sum_{k=1}^{N_B} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} w(i,j,k) \sigma_{\text{corr}}^0(k)}{\sum_{k=1}^{N_B} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} w(i,j,k)}.
\]  (6.2)
We typically assign $N_i = 2020$ and $N_j = 4040$ to create bins with widths of 0.09°, or about 4 km (along the equator). This bin size is smaller than the best-case resolution achievable with real aperture processing on Titan (the best-case RAR resolution is 5 km; the atmosphere extends to more than 950 km above the surface, thus restricting the orbiter to altitudes greater than that). Thus, with these map dimensions, we will preserve the fidelity of the data.

The weight function $w$ depends on the value of the burst’s resolution area ($A_{b\text{-eff}}$; see Eq. 3.16 and the discussion in Section 3.2.5) and also the angular separation of a particular grid point relative to the burst’s boresight location. For example, for burst $k$, the angular distance of the intersection point $(i,j)$ from the boresight axis is equal to the look angle $\theta_{\text{look}}(i,j)$. We wish to concentrate the weight towards the center of the beam, where the gain is strongest, and gradually taper the signal away from the beam center, similar to the behavior of the actual beam pattern. Thus, we, somewhat arbitrarily, apply a cosine function that falls to zero when the look angle is off the boresight axis by one full beamwidth ($\theta_{\text{beam}}$). This is a more gradual taper than would occur with the actual beam pattern. In addition to the taper, we restrict the data to only exist within an angular radius equal to one half of the beamwidth ($\theta_{\text{beam}}/2$), centered about the beam main axis (i.e. one full beamwidth in diameter). In this manner, we derive a mask where the weight tapers off from unity along the boresight axis to 0.707 at $\theta_{\text{look}}(i,j) = \theta_{\text{beam}}/2$, and beyond $\theta_{\text{beam}}/2$ the weight equals zero. We further weight the data according to the inverse square of the burst’s resolution area, which puts greater emphasis on the higher resolution data. We scale this result by $10^{10}$ so that most of the area weights lie between zero and unity (the areas are measured in m²). Thus, the formula for computing the weight of the $k$th burst at grid location $(i,j)$ is

$$w(i, j, k) = \begin{cases} \cos \left( \frac{\pi \theta_{\text{look}}(i,j)}{2 \theta_{\text{beam}}(k)} \right) \times 10^{10} A_{b\text{-eff}}(k)^{-2} & \text{if } \theta_{\text{look}}(i,j) \leq \theta_{\text{beam}}(k)/2, \\ 0 & \text{if } \theta_{\text{look}}(i,j) > \theta_{\text{beam}}(k)/2. \end{cases}$$  

(6.3)

The value of $\theta_{\text{beam}}$ depends on the antenna beam used. For data collected with
the central antenna beam (beam 3), we use the one-way half power beamwidth ($\theta_{\text{beam}} = 0.373^\circ$). For SAR data that are collected with the outer antenna beams, we approximate the oblong beam patterns (see Appendix A) with a symmetrical beam that has a diameter equal to the longest dimension of the pattern ($\theta_{\text{beam}} = 2^\circ$). The longest dimensions of the outer beam patterns are oriented in the Y-direction of the radar antenna coordinate system and are typically positioned perpendicular to the along-track direction in order to image a larger swath of the surface, i.e. the longest dimension is oriented across the waist of the SAR swath. As such, the $2^\circ$ symmetrical beam approximation works well to map the data across the width of the swath, but will blur the data in the along-track dimension by about a factor of three, an effect that is hardly noticeable at the scale of real aperture resolutions. The small sacrifice in along-track resolution simplifies the mapping computations by eliminating the need to project the beam coordinate system onto the surface.

We note that we derive the set of $N_B$ bursts used to form the global backscatter maps only from those bursts with boresight incidence angles between $1^\circ$ and $65^\circ$ (see the discussion in the previous section). We further limit the set of bursts to those with resolution areas less than $\sim$1% of Titan’s hemispheric surface area ($A_{\text{b-eff}} < 2 \times 10^{11}$ m$^2$). The latter restriction reinforces the $65^\circ$ limitation: measurements from bursts with areas larger than $2 \times 10^{11}$ m$^2$ (or angles higher than $65^\circ$), incur errors from applying the effective-area simplification to areas of large curvature. As a result, the altimetry mode dataset and about half of the compressed scatterometry mode dataset are not considered in the global map mosaic products.

### 6.3 Backscatter Map Products

We form separate backscatter reflectivity maps for each of the RADAR modes in Figure 6.3, following the map formation formula presented in Eq. 6.2. The scatterometry mode (SCAT) mosaic achieves the greatest surface coverage, with almost 92% of Titan’s surface mapped. The compressed scatterometry mode (C-SCAT) mosaic
Figure 6.3: Real aperture radar (RAR) backscatter mosaic maps, grouped by RADAR mode and displayed with an equatorial equirectangular cylindrical projection. The RAR data are normalized by the global backscatter model curve following Eq. 6.1. The color scale of the backscatter maps runs from zero (dark blue) to unity (dark red). Except for some very dark lakes in the polar latitudes, the dark blue largely indicates missing data. The coverage of each mode is indicated in parenthesis in the figure titles. The mean resolution varies between the modes from 10 km (H-SAR) to 300 km (C-SCAT).

achieves almost 90% coverage, although at a coarser resolution (the mean C-SCAT resolution is around 300 km, compared to the mean SCAT resolution that is around 150 km, see Table 3.1). The high-resolution SAR mode (H-SAR) mosaic also covers a substantial portion of the surface (42.5%), with mean resolutions around 10 km. Low-resolution SAR (L-SAR) and distant-SAR (D-SAR) produce maps of similar
coverages (11%-13%), with mean L-SAR resolutions near 35 km and mean D-SAR resolutions near 100 km. Finally, the altimetry mode (ALT) covers only 2% of the Titan’s surface, but with resolutions near 45 km, all at near-nadir incidence angles.

We form a global map mosaic by combining the data from the various RADAR modes. We again follow the map formation formula presented in Eq. 6.2, looping over all data with boresight incidence angles between $1^\circ$ and $65^\circ$ and resolution areas less than $2 \times 10^{11} \text{ m}^2$. The weighting function (Eq. 6.3) effectively keeps the high resolution H-SAR data on top. Collectively, the RADAR data combine to cover 99.93% of Titan’s surface. The global real aperture map mosaic is presented as an equirectangular cylindrical projection in Figure 6.4, together with a mosaic of the optical images from the Cassini ISS instrument. Two areas in the north polar region comprise the missing RADAR coverage (missing due to orbital geometry limitations). These “holes” are colored dark blue in the mosaic, not to be confused with the dark blue near-zero backscatter from the lakes and seas. To clarify this point, we mark the holes with white arrows.

Some artifacts are visible in the global radar mosaic of Figure 6.4. The artifacts look like “seams” where the data overlap and are in part due to combining data of a variety of resolutions, but mostly are due to the inability of the global model function (Figure 6.1) to simultaneously correct the incidence angle effects of the different terrain types on Titan, especially when overlapping coverage occurs at very different incidence angles. The global model function does a great job correcting the “average” terrain for incidence angle effects, but, as discussed in Chapter 5, the features on Titan can have very different backscatter forms.

We also display the global RADAR backscatter mosaic using two equal-area pseudocylindrical projections. The pseudocylindrical projection is similar to the cylindrical projection in that it has straight and horizontal parallels, as if a cylinder was wrapped around the globe and the globe was projected onto it, but the pseudocylindrical projection has arbitrary curves for meridians instead of straight lines. The pseudocylindrical projection attempts to compensate for some of the east-west stretching
Figure 6.4: Real aperture radar (RAR) backscatter mosaic of Titan, displayed with an equatorial equirectangular cylindrical projection. The total collection of RAR data, from all modes, are normalized by the global backscatter model curve following Eq. 6.1. The color scale of the backscatter maps runs from zero (dark blue) to unity (dark red). 99.93% coverage is achieved. The dark blue “holes” marked with white arrows indicates the missing data (the other dark blue data at polar latitudes coincide with lakes and seas, and the dark blue data at equatorial latitudes coincide with sand dunes).

inherent to cylindrical projections. For example, the Mollweide projection (upper panel of Figure 6.5) depicts the globe of Titan as a proportional 2:1 ellipse and is capable of accurately representing area at the expense of moderate shape distortion. In the Mollweide projection, the relative area between any given parallel and the equator on the ellipse is the same as the relative area between that parallel and the equator on
Figure 6.5: The global real aperture radar (RAR) backscatter map displayed using two equal-area pseudocylindrical projections: the Mollweide projection (upper panel) and the sinusoidal projection (lower panel). The two projections accurately depict area and relative sizes, but sacrifice fidelity to angles and shapes. The distortion is most severe at the edges of the maps, with the sinusoidal projection incurring more distortion than the Mollweide. The total collection of RAR data, from all modes, are normalized by the global backscatter model curve following Eq. 6.1. The color scale of the backscatter maps runs from zero (dark blue) to unity (dark red).

The true globe. The sinusoidal projection (lower panel of Figure 6.5) depicts the globe of Titan using straight parallels and sinusoidal meridians. The sinusoidal direction preserves distances along the parallels and also preserves areas, but at the expense of severe distortion of shapes and directions, especially away from the central meridian and the equator.

In Figure 6.6, we display stereographic projections at 90° longitudinal rotations centered along the equator. Here, the exposed hemisphere of Titan is projected onto
Figure 6.6: The global real aperture radar (RAR) backscatter map displayed as hemispheric stereographic projections along the equator of Titan, spaced at 90° intervals in longitude. The features at the projection point are accurately depicted, but the distortion is severe away from the center. The total collection of RAR data, from all modes, are normalized by the global backscatter model curve following Eq. 6.1. The color scale of the backscatter maps runs from zero (dark blue) to unity (dark red).

a plane such that shapes and sizes are accurately depicted at the centered projection point, but are distorted elsewhere. The upper left panel is centered at 0° latitude, 0° longitude over the features Senkyo and Quivira. The upper right panel is centered
Figure 6.7: The global real aperture radar (RAR) backscatter map displayed as hemispheric stereographic projections at the poles of Titan. The features at the projection point are accurately depicted, but the distortion is severe away from the center. The total collection of RAR data, from all modes, are normalized by the global backscatter model curve following Eq. 6.1. The color scale of the backscatter maps runs from zero (dark blue) to unity (dark red). The parallels are concentric circles spaced 30° apart and the meridians are radiating straight lines at 45° intervals. 0° longitude is downward and 90° west longitude is leftward.

at 0° latitude, 90° west longitude over Xanadu. The lower left panel is centered at 0° latitude, 180° longitude over Shangri-La and Dilmun. The lower right panel is centered at 0° latitude, 270° longitude over Belet.

The different projections of Figure 6.6 accurately represent the equatorial terrain of Titan. To show the polar terrain, we present stereographic projections centered on the polar axes of Titan. The north polar hemisphere is shown on the left of Figure 6.7, and the south polar hemisphere is shown on the right. Here, the parallels are concentric circles spaced 30° apart and the meridians are radiating straight lines at 45° intervals. 0° longitude is downward and 90° west longitude is leftward. The large hydrocarbon seas Kraken Mare and Ligeia Mare darken the northern terrain, while Ontario Lacus is barely visible along the 180° meridian in the southern hemisphere (about halfway between the pole and the 60° parallel).
6.4 Conclusion

In this chapter, we describe our method of normalizing and combining the real aperture backscatter measurements collected by the Cassini RADAR instrument to form reflectivity maps of the surface of Titan. The normalization technique corrects the majority of the incidence angle effects incurred at the global scale, however local variations in backscatter responses inhibit the complete elimination of angle effects. Future work might modify the correction procedure to consider localized backscatter models rather than a single mean global backscatter model. Currently, some artifacts are introduced in the final mosaic maps, especially where overlapping data occur at very different incidence angles, but some of these artifacts are also due to the variety of resolutions contributing to the mosaic. We develop a weighting function to emphasize the higher resolution datasets, such as the SAR data, but some of the gaps in SAR swaths are completed by compressed scatterometry mode data with resolutions that are almost a factor of 30 poorer. Thus, in these cases, the seams of the different modes are apparent. Increased coverage over the course of the Solstice mission will replace some of the low-resolution coverage with higher-resolution results, thereby removing some of the seams in the map.

The global mosaic includes data collected through the T71 Titan pass (7-Jul-2010), and covers more than 99.9% of the surface at real aperture resolutions between 6 km and 300 km. This mosaic represents the most complete surface map of Titan produced to-date. The optical instrument observations have thus far been limited to latitudes below about 60° N due to the polar winter darkness, but the northern latitudes will be increasingly visible as the summer solstice progresses in the north. By the end of the Solstice Mission, the ISS instrument should have obtained a nearly global optical map at resolutions near 4 km, and the VIMS instrument should have obtained a comparable spectroscopic global map at resolutions near 50 km.
The optical and radar mosaics provide complementary information about the surface. We find that the optical and 2 cm-λ radar reflectivities are often positively correlated (see Figure 6.4), with occasional exceptions. Differences between the datasets indicate either changes in the surface composition with depth (2 cm-λ penetrates down to decimeters, whereas the sub-5 µm optical wavelengths are almost entirely superficial), differences in apparent physical scale, or a composition that is more attenuating at longer wavelengths. In the next chapter, we analyze specific Titan surface features with respect to their radar backscatter characteristics, as well as their inter-instrument differences.
Chapter 7

Ontario Lacus Wave and Depth Constraints

Cassini RADAR altimetry data collected on its 49th flyby of Titan (T49; 21-December-2008) over Titan’s largest south polar lake, Ontario Lacus, uncovered evidence for a smooth, specularly reflecting surface, the first truly specular detection on Titan. Histograms of the raw radar lake echoes demonstrate the uniqueness of this dataset: instead of the Gaussian distributed amplitudes representative of other areas on Titan, the Ontario Lacus amplitude histograms are distinctly U-shaped, the defining characteristic of a sinusoidal signal. This signifies a perfect mirror-like reflection of the transmitted signal (a pulsed linearly-frequency-varying sinusoid, also called a chirped waveform), which is possible only when the surface is extremely smooth in comparison to the 2.2 cm incident wavelength.

In this chapter, we review the scattering theory for smooth surfaces and define a model that constrains the rms surface heights based on the strength of the specular returns. The strengths of the lake echoes in this experiment were much larger than expected, severely saturating the receiver. Consequently, the measured echo strengths are lower bounds, and, as we will show, the derived rms surface heights are upper bounds. We develop a method to partially correct the echoes for the distortion incurred and thus tighten the bounds on signal strength and rms heights.
Following the T49 altimetry experiment, we acquired radar data of Ontario Lacus at higher incidence angles (greater than 20°) on the 58th (T58; 8-July-2009) and 65th flybys of Titan (T65; 12-January-2010). The T58 data consist entirely of SAR mode data, whereas the T65 Ontario Lacus data comprise both SAR mode and scatterometry mode data. Further SAR mode data of the northern tip of the lake were acquired on T57 (22-June-2009), but these data suffer from scalloping artifacts due to the observational geometry and are not useful for our analysis here.

The SAR images reveal systematic structure within the Ontario Lacus boundary: notably, the lake signal decreases with increasing distance from the shoreline. We interpret these data, in both their imaging and real aperture forms, with a two-layer scattering model to constrain the depths of the lake. This analysis yields near-complete bathymetry maps from the imaging data, and hence volume estimates, as well as three depth profiles from the main beam real aperture data: one profile along the length of the lake, one along its deepest width, and one along its narrow waist. The bathymetry maps are complete except for holes over the deepest sections where the signal level is below the noise floor. The real aperture data do not suffer the same noise limitations as the imaging data and thus their depth profiles are complete and valid all the way across, at the expense of resolution. Furthermore, because the real aperture data are processed independently from the imaging-mode data, the depth profiles affirm the accuracy of the bathymetry maps, and vice versa.

The structure of this chapter is as follows: we begin by introducing the Ontario Lacus feature and the T49 altimetry experiment. We next describe our process for correcting the T49 signal saturation through histogram modeling, which requires understanding the receiver transfer function. We use these corrections to determine a lower bound on the normalized radar cross section, which we then use to evaluate coherent scattering models and constrain the rms surface height of the surface. We consider the implications the smoothness of the surface has on the liquid properties and wind conditions. Finally, we conclude with an evaluation of the T58 and T65 data and what they contribute to our understanding of the shape and smoothness of
7.1 **Introduction to Ontario Lacus**

Ontario Lacus, with its smooth boundary and dark uniform interior, was the first promising candidate for liquids on Titan’s surface (McEwen *et al*., 2005). The feature is 175 km by 70 km, 18,700 km² in area, and is centered near 72° S, 175° W (Hayes *et al*., 2011; Turtle *et al*., 2009; Wall *et al*., 2010). It was discovered by the Cassini Imaging Science Subsystem (ISS) team in June of 2005. Observations of the interior of Ontario Lacus using the Cassini Visual and Infrared Mapping Spectrometer (VIMS) provide additional evidence supporting its liquid nature: spectral absorption lines suggest constituent liquid ethane and other low-molecular-mass hydrocarbons (Brown *et al*., 2008), and VIMS images reveal annuli that suggest a time-variable shoreline (Barnes *et al*., 2009). Brown *et al*. interpret the very low VIMS 5 µm albedo to imply that the lake interior is extremely smooth and quiescent, and free of scattering centers larger than a few micrometers in size.

Hundreds more lakes of variable sizes have been discovered by the Cassini RADAR and ISS instrument teams in both the north and south polar regions since the discovery of Ontario Lacus (Lopes *et al*., 2007a; Stofan *et al*., 2007; Turtle *et al*., 2009). Some radar-observed lakes are so dark that there is no detectable signal above the noise floor, while others are as bright as, and sometimes brighter than, the surrounding terrain, with only their morphology signaling their lake-like character (Hayes *et al*., 2008). Stofan *et al*. (2007) interpret the dark lakes as liquid whose smooth surface at the radar’s 2.2 cm wavelength acts like a mirror, reflecting the transmitted energy away from the radar (images are observed at incidence angles between 10° and 40°, well away from nadir). The bright “lakes” are likely dry, empty basins resulting from evaporation or drainage into a porous subsurface over time (Hayes *et al*., 2008).

Changes in lake state have not been directly observed in the north, but there is abundant evidence for lake surface change in the south polar region. South-polar ISS
images acquired 11 months apart, in 2004 and 2005, reveal the sudden appearance of
dark surface features near 80° S, 120° W, not far from Ontario Lacus (Turtle et al.,
2009). A large cloud outburst occurring between the two observations (Schaller et al.,
2006) led Turtle et al. to conclude that the observed surface darkening likely demon-
strates the ponding of hydrocarbon rain. Moreover, the dearth of lake candidates
in radar images in the south more than 2.5 years later (since Dec. 2007), implies
that many newly formed lakes may have quickly evaporated or percolated into the
subsurface (Turtle et al., 2009). Indeed, Hayes et al. (2011) document evidence in
SAR imagery for decreasing lake levels in small lacustrine features in two south polar
regions over a 1-1.5 year baseline. Furthermore, by comparing the 2005 ISS images
of Ontario Lacus with the 2009 SAR images, Hayes et al. (2011) determine that the
lake’s shoreline has receded by ∼1 meter per year.

7.2 Roughness Constraints from T49 Altimetry

The first unambiguous detection of a radar echo from a candidate liquid surface oc-
curred as Cassini flew directly over Ontario Lacus at approximately 1900 km altitude
during the 49th Titan flyby. The radar, in its nadir-looking altimeter configuration,
captured the specular return that had previously eluded the radar imagery of Titan
lakes (and that had also eluded searches for sunglint reflections at visible and infrared
wavelengths, (Fussner, 2006; West et al., 2005)). The altimetric height profile across
Ontario shows the entire optically-dark area to be flat and the slopes leading into the
lake to be shallow (the measured slopes are ∼10^{-3}, or ∼0.06°).

Of the 468 echoes in the T49 altimetry observation (all at incidence angles less
than 0.14°), 72 have distinctive U-shaped histograms, revealing that the surface re-
fects the radar signal (a linearly-frequency-varying sinusoid) in a mirror-like manner
that preserves its sinusoidal character. For this to happen, the surface must be so
smooth that the observable scattering centers are essentially confined to a small area
at the sub-radar point on par with the first Fresnel zone (diameter ∼100 m, 1% of the
Effective beam diameter). This surface scattering scenario is illustrated in Figure 7.1b. In contrast, a typical Titan surface echo comprises many independent signals scattered from patches of wavelength-scale roughness over the entire beam-illuminated area. These effectively add incoherently, and tend via the central limit theorem to a Gaussian distribution regardless of the transmitted signal waveform and the surface scattering statistics (Figure 7.1a).

The T49 echo amplitude histogram transitions from Gaussian to that of a sinusoid as the radar track moves over the lake (Figure 7.2). The sinusoid histograms stand...
Figure 7.2: We compute the voltage amplitude histogram (from the signal-only portion of the received echo) for each of the 468 altimetry echoes (bursts) in T49. Each echo histogram consists of 256 bins, ranging from $-145.2\,\text{dV}$ to $+145.2\,\text{dV}$ (see Section 3.5.2). Here, we stack the histograms vertically to form an image. The colorscale represents the number of counts in the histogram voltage amplitude bins and is clipped at 1000 for enhanced contrast. In this image, we see the histogram of a Gaussian signal fade into that of a sinusoidal signal and then back out into that of a Gaussian signal as the spacecraft moves over the surface (see Figure 7.3 for sample plots of the histograms).

out because they are quantized to ten amplitude levels (see discussion at the end of Section 7.2.1.1). The 58 strongest sinusoidal echoes correspond to bursts 105 to 162, at along-track distances between 152 km and 241 km, or surface coordinates between $(73.6^\circ\text{S}, 178.9^\circ\text{W})$ and $(73.0^\circ\text{S}, 185.1^\circ\text{W})$, concurrent with the most-saturated echo signals and the darkest part of the ISS Ontario Lacus image. The strongest sinusoidal echoes are more clipped and thus have lower histogram centers and more counts in the extreme bins (bins $\pm145.2$, which are hardly visible in the figure since they are at the edge of the image). The strongest sinusoidal echoes are bordered by weaker sinusoidal echoes, marking a distinct transition region. At the same time, there is evidence for weak sinusoid histogram signatures as early as echo 59 (an along-track
distance of 84 km) and as late as echo 211 (an along-track distance of 321 km). The
more muted specular echoes located nearby indicate the presence of possible puddles,
or a damp surface that is a mixture of flat and rougher surfaces, or even smooth, dry
patches.

A flat surface focuses radar waves more coherently than the typical wavelength-
scale rough surface. As a result, the lake echo returns are much brighter than antici-
pated and saturate the receiver in this experiment. We develop a method to correct for
some of this saturation distortion. Our method uses histogram modeling to estimate
the original peak signal amplitude at the input of the receiver from the saturated,
distorted sinusoid we record.

7.2.1 Histogram Modeling

The sampling of a sinusoidal signal yields a distinctive histogram with a low, flat
center that curves up at the extreme bins (see Figure 7.3). Components of a radar
receiver are designed to properly scale and record the Gaussian signals that are typical
of radar surface echoes, and thus may not properly record sinusoidal signals. This,
along with the unanticipated signal intensities, causes the Ontario Lacus echoes to be
severely clipped, and, consequently, the measured signal amplitudes are much lower
than the actual values.

The original peak amplitude of a distorted sinusoid can be estimated by analyzing
the shape of the received histogram. Clipping increases the number of counts in
the extreme bins, while simultaneously increasing the degree of flatness of the center
bins in the histogram. If the receiver transfer function is well-known a priori, the
saturation effect is deterministic and the original signal level can be recovered quite
accurately. The transfer function for the Cassini radar system is not well-documented,
so we model the receiver response to higher-than-expected signal levels and estimate
the input signal strength by matching theoretical echo histograms to those measured.

We compute the theoretical echo histograms by simulating the transmitted signal,
applying a digitizer transfer function model, and compressing the signal with the
Figure 7.3: Echo histograms of Ontario Lacus (red) and typical Titan surface (black). The mirror-like lake surface preserves the characteristics of the transmitted signal and returns the histogram of a sinusoid (saddle-shaped), while echoes from rougher terrain around the lake exhibit Gaussian histograms. The lake histogram of T49 altimetry burst echo 104 shows the ten-level quantization effect that we use to quickly identify specular returns. The lake histogram is asymmetric due to a receiver mean offset that is exaggerated by the high-level of saturation. The histograms are computed from the signal-only portion of the echo.

The Cassini RADAR digitizer maps a continuous input voltage signal to 256 codes (8 bits), ranging from -127.5 to +127.5 digitized volts ($dV$). The relationship between the input ($x$) and output ($y$) values is called the transfer function. An ideal digitizer has a linear transfer function with unity gain and will clip any signal outside of its full scale range to its maximum representation.

The T49 Ontario Lacus echoes saturate the radar receiver so severely that additional nonlinearities are introduced, and a linear model fails to reproduce the observed
Figure 7.4: Histogram matching for T49 burst 2000, which occurs just on the edge of Ontario Lacus. The histogram of the echo voltage amplitudes is colored black. For clarity, the discrete histogram bins are connected rather than displayed as a stem plot. (A) The best-fit result using a linear transfer function model (2 parameters) is not able to simultaneously match the curvature and the level of the histogram. (B) The best-fit result using the nonlinear transfer function model described in the text reproduces the model very well (the sum of the squared errors (sse) is an order of magnitude better), but requires 6 parameters. Note that the estimated peak amplitude is significantly higher for the nonlinear model.

... echo histogram accurately (Figure 7.4a). Using a nonlinear clipping model, we are able to match the observed histograms to within 0.1% - 0.6% (rms percent error; see Figure 7.4b). Our nonlinear model is described by two primary parameters (Eq. 7.1). The parameter \( p \) represents transfer function nonlinearity: small \( p \) values yield transfer functions that are more curved, and as \( p \) increases, it approaches the linear model. The parameter \( K \) determines the transfer function saturation point. \( K \) can be greater than or equal to 127.5 \( dV \), but we subsequently clip the output values to stay within the 8-bit range.

\[
y = \frac{x}{\left(1 + \left(\frac{x}{K}\right)^p\right)^\frac{1}{p}} \quad (7.1)
\]

In addition to saturating and distorting the receiver response, the higher-than-anticipated signal levels from Ontario Lacus also dramatically increase the DC offset error, or the difference between the measured mean of the signal and the expected
**Figure 7.5:** The best-fit transfer function models for various amplitudes. Echoes outside of the lake boundary have low apparent peak amplitudes (between 137 and 260) and linear transfer functions, whereas echoes within the lake boundary have high apparent peak amplitudes (between 765 and 1097) and extremely nonlinear, asymmetric transfer functions. Increasing amplitude moves the curve to the right (increased offset) and increases function curvature.

mean (zero). For typical Titan surface data, the offset error is near -0.6 \( dV \), suggesting a small DC bias voltage in the receiver when the signals are at normal levels. However, over Ontario Lacus, the offset error jumps to greater than -21 \( dV \), providing further evidence that the signal is saturating the receiver components and distorting their response characteristics. The offset causes a shift in the digitizer transfer function, adding an extra parameter to the model. Moreover, the offset will cause the negative and positive data to saturate differently, and, consequently, the transfer function will be asymmetrical, with distinct \( p \) and \( K \) parameters for the positive \((p_1, K_1)\) and negative \((p_2, K_2)\) sections. Thus, we require five parameters to properly describe the nonlinear digitizer transfer function. The best-fit digitizer model parameters for the Titan T49 altimetry specular echoes are estimated by matching the data histogram using minimum-least-squares. For each specular echo, we calculate the five digitizer parameters, as well as the peak amplitude of the specular sinusoidal signal (e.g. Figure 7.4b).

We find that the best-fit transfer function varies with the peak signal level: as
Table 7.1: Mean digitizer model parameters matched to the set of specular echo histograms from outside the lake boundary and the set from inside the lake boundary.

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{peak}}$</th>
<th>DC offset</th>
<th>$p1$</th>
<th>$p2$</th>
<th>$K1$</th>
<th>$K2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside Boundary</td>
<td>168.4</td>
<td>7.8</td>
<td>28.0</td>
<td>53.1</td>
<td>225.9</td>
<td>207.1</td>
</tr>
<tr>
<td>Inside Boundary</td>
<td>927.3</td>
<td>187.7</td>
<td>0.9</td>
<td>6.4</td>
<td>398.7</td>
<td>142.3</td>
</tr>
</tbody>
</table>

As the signal level increases, the offset error and degree of nonlinearity also increase (see Figure 7.5). Table 7.1 demonstrates the effect of signal level on the receiver model parameters. The table characterizes the best-fit mean parameters for two cases: low-amplitude specular echoes that occur outside of the lake boundary and high-amplitude specular echoes from within the lake boundary.

The specular echoes that occur outside of the apparent lake boundaries have lower estimated peak amplitudes (e.g. between 137 and 260). Consequently, their transfer functions maintain a very linear nature (Figure 7.5) and their $p$ values are relatively large (Table 7.1). In the low-amplitude linear regime, it is mainly the DC offset parameter that is controlling the response output, with the offset error steadily increasing as the amplitude increases. However, over the lake, the estimated peak amplitudes grow very large (between 765 and 1097), and the transfer functions are decidedly nonlinear and asymmetric (again see Figure 7.5).

We use data from engineering tests with an intentionally-saturated receiver to calibrate our digitizer model. During the T56 flyby, the transmitted chirp signal was rerouted directly back into the receiver, and the attenuation was varied to control the level of saturation. In this way, we sampled the receiver response to input peak amplitudes from $73 \, dV$ to $582 \, dV$ in 2 dB steps. We find that our modeled amplitudes correctly track the input amplitudes up to about $150 \, dV$, and above this they linearly overestimate the amplitude until they level out near $850 \, dV$ (Figure 7.6a). We correct the linear overestimation (output amplitudes 150-850 are mapped linearly to the corresponding input values), but we can only determine a lower bound for the larger
signals, since these map ambiguously to multiple input amplitudes (these values are clamped to a corrected value of 245, which serves as the lower bound estimate). The initial model amplitude estimates for the T49 specular data are shown in Figure 7.6b (gray), together with their corrected and bound amplitudes (black).

Subsequent to 8-bit digitization, the received altimetry signal is further compressed to 4 bits using a block adaptive quantization algorithm (Kwok and Johnson (1989), see Section 3.5), which is optimized for Gaussian signals. We examine the effect of block adaptive quantization on sinusoidal signals (vs. Gaussian signals) and find that distortions are much less than those due to signal saturation, so we need not model these effects. A convenient consequence of this algorithm is that a sinusoidal signal will encompass only 10 of the 16 possible quantization levels, offering a simple metric for detecting specular echoes. A severely-clipped Gaussian signal can mimic this behavior, but the Cassini dataset has no such high-intensity Gaussian echoes.
7.2.2 Specular Reflection Theory

The very presence of a mirror-like specular return requires that the surface be smooth at the scale of the illuminating wavelength ($\lambda = 2.2$ cm). The signal will be strongest for a completely smooth surface, when the surface scatterers all add in phase, constructively interfering. As the surface roughness increases, the path length differences to the surface scatterers will cause destructive interference, and the reflectivity will decrease exponentially. We express the reflected signal amplitude $S$ as

$$S \propto \int_{-\infty}^{\infty} e^{-j\omega t} e^{-j2k\delta} \frac{e^{-\delta^2/2\sigma_h^2}}{\sigma_h \sqrt{2\pi}} d\delta = e^{-j\omega t} \left(e^{-8\left(\pi\sigma_h/\lambda\right)^2}\right)$$

where the first term in the integral represents the transmitted sinusoidal signal of frequency $\omega$, the second represents the phase deviation due to a surface height $\delta$, $k$ is the transmit signal wavenumber, and the third term is the probability distribution function of the surface heights, assuming a zero-mean Gaussian distribution with standard deviation of $\sigma_h$. We simulate the scattering response for a surface of increasing roughness and derive results similar to the analytic theory (Figure 7.7). The exponential fall-off of the specular signal strength for a surface with a roughness scale much less than the wavelength has also been well-documented in the literature (Fung and Eom, 1983).

We bound the surface roughness of Ontario Lacus, and the nearby locations that have specular signatures, by comparing the surface radar cross sections (RCS, or $\sigma$) to modeled values. The theoretical RCS for near-nadir observations of a slightly roughened specular surface is

$$\sigma = \frac{\rho\pi a^2 R^2}{(a + R)^2} e^{-\left(4\pi\sigma_h/\lambda\right)^2}$$

(7.3)
Figure 7.7: The measured echo amplitude $S$ falls off exponentially with increasing roughness, as described in Eq. 7.2 (plotted in black). We simulate the reflected signal strength for increasing surface roughness, and the results (plotted in gray) match the analytical theory.

where we have combined Eq. 7.2 (in power form) with the theoretical RCS for a smooth spherical surface (Fjeldbo, 1964; Ruck et al., 1970). We evaluate Eq. 7.3 for the rms surface height ($\sigma_h$) using our T49 specular echo RCS measurements ($\sigma$), the spacecraft distance values ($R$), and the radius of Titan ($a = 2575$ km). We consider different candidate surface materials, each characterized by their surface dielectric constant through the Fresnel power reflection coefficient at normal incidence ($\rho$).

We measure the T49 specular echo RCS values following the processing and calibration technique of Chapter 3. This technique corrects for attenuation changes in the receiver, which jumps in several 2 dB increments throughout the observation. However, we find that the received signal over Ontario Lacus is saturated to such a degree that it is no longer affected by changes in observation geometry or changes in the receiver attenuation. Thus, we modify the processor parameters to match the observed data, holding the range, area, and attenuation constant for highly saturated echoes (those with amplitudes near or above 850 $dV$). We choose values for range, area, and attenuation that yield the most conservative bound on the RCS.
7.2. ROUGHNESS CONSTRAINTS FROM T49 ALTIMETRY

7.2.3 Wave Height Results and Implications

The RCS estimates for the T49 altimetry observation are shown in Figure 7.8a, plotted as normalized radar cross section (NRCS, or $\sigma^0$), after scaling by the beam illuminated area. The brightest, most specular echoes, which come from the central part of Ontario Lacus, lie between (73.6° S, 178.9° W) and (73.0° S, 185.1° W), as computed using the Titan spin state of Stiles et al. (2008). The brightest radar return appears to come directly from the Ontario Lacus dark area imaged in 2005 by the ISS instrument and imaged in 2009 by the RADAR instrument (Figure 7.8b).

Figure 7.8c shows upper bounds on surface roughness for both liquid and solid hydrocarbons, materials consistent with the low-albedo observations from RADAR, VIMS, and ISS. For liquid hydrocarbons, with dielectric constant ($\epsilon$) between 1.6 and 1.9, the rms surface heights must be less than 2.8 mm to reproduce the observed specular signal strength over the lake. For solid hydrocarbons ($\epsilon = 2.0 - 2.4$), the rms heights are constrained to be less than 3.1 mm. We note that the saturation limits us to lower bounds on signal strength, thus the actual echo intensities may be several times larger than we conservatively estimate, so that the surface roughness may in fact be much less than the 3 mm rms we report here.

The smoothest natural solid surfaces on Earth of which we are aware are playas at ~6 mm rms at the meter scale (Shepard et al., 2001), although local (across ~30 cm) 1 mm rms dry lakebed surfaces have also been observed (Archer and Wade, 1998). The roughness of Earth playas is often dominated by cm-scale halite and sulfate evaporite crystals or polygonal patterns associated with clay shrinkage – it may well be that Titan materials dry in a smoother, plastic fashion that retains the smoothness and flatness of the lake. Nonetheless, the simplest and hence most likely interpretation of our smoothness constraint is that the lake is liquid-filled.

We observe a distinct transition in the specular echo histograms at the eastern and western edges of the lake. In this 5-10 km wide fringe, the echoes possess sinusoid histograms similar to the lake center but with noticeably lower echo strength. This region must be smooth over the entire beam footprint since we do not see evidence of
Figure 7.8: The beam-averaged normalized radar cross sections (NRCS, or $\sigma^0$), partially corrected for saturation effects, are (A) plotted against the along-track distance and (B) mapped over the Cassini RADAR T57/T58 mosaic of Ontario Lacus. The NRCS values over the lake are lower bounds (due to saturation distortion) and are normalized for the beam footprint area, not the actual scattering area, for consistency over all echoes. (A) and (B) are roughly vertically aligned. (C) The rms surface height upper bounds derived from the specular NRCS measurements are plotted as a function of the surface composition: liquid hydrocarbons have $\epsilon = 1.6$-1.9, and solid hydrocarbons have $\epsilon = 2.0$-2.4. (D) The heights for $\epsilon = 1.9$ are mapped. (D) also reveals the locations of the most significant specular echoes, showing that there is at least one significant specular return outside of the lake boundary (at longitude 175° W, or along track distance 100 km). Note that in (C), the markers correspond to the 72 identified sinusoidal echoes and are connected by lines for clarity only. All echoes between ~240 km and 150 km are specular and mark the main part of the lake.
a Gaussian component in the echo histograms, but it must be slightly rougher than the central part of the lake. This may correspond to the ∼10 km “shelf” or “beach” identified in VIMS observations, but this is difficult to confirm due to differences in coordinate systems. The radar beam footprint in these areas is ∼9 km, and the active scattering area is the size of the Fresnel zone radius (∼100 m), so we are capable of resolving the VIMS feature, assuming that the width has not significantly changed in the year between our observations. Barnes et al. (2009) favor interpreting the annular area as exposed lake-bottom sediments, similar to mudflats. If we are detecting the same feature, our analysis implies that these sediments would need to be smooth on the order of 3.2 mm or less. Another possibility consistent with our data is that the region is very shallow liquid, where the signal decrease may result from interference with echoes reflected from the lake bottom.

We also observe specular signatures similar to the lake-edge echoes well removed from the lake boundary, the strongest being about 50 km past the eastern lake boundary (see Figure 7.8d). Barnes et al. note the presence of a small dark spot a few hundred km south of Ontario Lacus that is similar in albedo to the “shelf” unit, which they suggest may be a dried pond (2009). We may be observing similar evidence for past liquids or even current ponds in the area surrounding Ontario Lacus.

Assuming Ontario Lacus is filled with liquids, constraints on the surface roughness provide insight into Titan’s south polar wind characteristics and material properties. Ghafoor et al. (2000) predict that wave scales on Titan lakes will be 7 times larger than those on Earth for a given wind speed, and consequently, the lake surfaces could be extremely rough (Tokano, 2005). However, the Ghafoor model only accounts for the different gravity on Titan, while the air density and the liquid properties (density, surface tension, and viscosity) will also have effects, as demonstrated in wave-generation experiments with liquid hydrocarbons in the NASA Mars Wind Tunnel (Lorenz et al., 2005). The Lorenz et al. (2005) results suggest that the higher air density of Titan (∼4.5 times that of Earth’s at sea level) should allow cm-scale capillary waves to form very easily on Titan. In fact, Lorenz et al. (2010) derive a wind speed threshold for
capillary wave generation on Titan by evaluating terrestrial studies and applying a straightforward correction for the higher air density. They determine that a minimum wind speed of 0.5-1 ms\(^{-1}\) is required to form perceptible waves on Titan’s surface (in contrast to the 1-2 ms\(^{-1}\) threshold speed on Earth). This capillary wave generation threshold assumes water-like liquid properties on Titan.

If the Titan lake fluids are less dense or have higher surface tension than water, waves will be easier to generate and the corresponding threshold would be lower (Lorenz et al., 2005). Laboratory data suggests that candidate materials will indeed be less dense but will also have lower surface tension (Lorenz et al., 2010), and it is not known how the two contrasting properties will affect the wave generation threshold. On the other hand, the effect of the liquid’s viscosity on wave suppression is well known from wind tunnel data. Kahma and Donelan (1988) demonstrate the dampening effect for water waves: as the temperature of the water decreases from 35°C to 5°C, the viscosity increases by a factor of ~3 and the wave height is subsequently reduced by a factor of 5. Thus, we can expect that for a higher viscosity material, a greater wind speed is needed to generate winds of perceptible heights. Yet, liquid material on Titan composed of pure methane or methane and nitrogen should have a viscosity much lower than that of water, and thus should be able to maintain higher wave amplitudes (or require lower wind thresholds). But the viscosity will increase by a factor of 5 if the fluid is ethane-rich and by up to a factor of 10 if heavy hydrocarbons are dissolved into the fluid (Lorenz et al., 2010). And if solid material, such as tholin haze, is suspended in the liquid, the viscosity will increase even further. Laboratory data from Halder et al. (1997) show that as the volume fraction of suspended particles approaches 45-50%, a fluid will reach its gelling point and will no longer be able to flow. All of these viscosity-increasing scenarios are likely at Ontario Lacus: ethane is likely to be prevalent in south polar lakes due to the transport of methane towards northern latitudes from seasonal and longer-term differences in incident solar radiation (Aharonson et al., 2009), higher hydrocarbons from atmospheric photochemistry are expected to be deposited in the lakes (Cordier et al., 2009), and very fine-grained
solid material with low sedimentation velocity might also be substantial (Lorenz et al., 2010). Thus, the wave generation threshold may be much greater than 0.5-1 ms$^{-1}$ due to these viscosity effects.

The Huygens Probe Descent Imager/Spectral Radiometer measured surface winds of 0.3-1 ms$^{-1}$ at the probe’s landing site (10.4°S, 192.4°W) (Tomasko et al., 2005), but no other direct observations of wind speeds exist. Global models of Titan’s atmosphere such as the TitanWRF global circulation model (GCM) of Titan’s atmosphere suggest that winds near the surface at the high polar latitudes can vary between near zero and 2 ms$^{-1}$ over the course of Titan’s year (29.5 Earth years), where the peak in wind speed occurs near early summer and the minimum occurs near early winter (Lorenz et al., 2010). At the time of the T49 observation, occurring in late southern summer, the TitanWRF GCM predicts that wind speeds varied between 0.1 and 0.4 ms$^{-1}$, a speed that is not large enough to generate perceptible waves according to the Lorenz et al. (2010) threshold. The TitanWRF GCM predicts that winds will continue to be low at south polar latitudes through all future observations.

In summary, our tight constraint on wave heights admits at least three possible explanations. First, the near-surface winds near Titan’s south pole were extremely weak at the time of the observation, too weak to exceed the required wave generation threshold. Second, the liquid of Ontario Lacus has a high viscosity, perhaps implying much suspended material, which suppresses wave generation. Or third, our understanding of wind-wave generation under Titan conditions is inadequate. We summarize the results and observations presented in this section in Wye et al. (2009).

### 7.2.4 Future RADAR Altimetry Observations of Lakes

While the specular echoes from Titan’s Ontario Lacus were much brighter than anticipated during our observation, and consequently saturated the received signal, we were able to constrain the surface smoothness to be less than 3 mm rms height. The T60 flyby (9-Aug-2009) promised further low-angle radar observations over Ontario Lacus and was specifically designed to receive the full-scale specular echo without
saturation. These data would have enabled us to solve for the surface smoothness more accurately. Unfortunately, the entire collection of T60 Titan data were lost when the Goldstone Deep Space Network’s Antenna Logic Controller at DSS-14 failed just prior to download. The usual redundant playback was not scheduled for this data sequence because of the proximity to the high-priority Saturn equinox crossing (11-Aug-2009), where Cassini instruments had the unique opportunity to observe thermal changes and topographical oddities as the rings were illuminated edge-on by the sun.

The only other opportunity to repeat an altimetry lake experiment will be on the T91 flyby (23-May-2013) over the northern sea Ligeia Mare; no south polar near-nadir lake observations are scheduled. Titan GCM models suggest that the wind speeds have been low ($\sim 0.5 \text{ ms}^{-1}$) in past observations of the northern lakes, but it is expected that the wind speeds will pick up as the northern summer progresses. The increasing wind speeds, coupled with a possibly lower wave generation threshold (if the northern liquids have a lower viscosity than southern liquids from the greater abundance of methane, then they will also have a lower wave generation threshold) may result in a substantial roughening of the northern lake surfaces that should be increasingly detectable in future observations. Thus, the T91 altimetry observation of Ligeia Mare may measure a much rougher surface than we calculated from the T49 observation of Ontario Lacus.

With the sun advancing towards the northern lakes, the Cassini optical instruments will encounter more opportunities to look for sun glints. Indeed, the VIMS instrument has already detected glints from the edge of Kraken Mare and Jingpo Lacus at northern latitudes (Stephan et al., 2010), indicating that the surfaces of those lakes are smooth and free of scatterers with respect to the 5 $\mu$m wavelength. Barnes et al. (2011a) develop a quantitative model for analyzing the photometric lightcurve generated by the specularly reflected light flux and find that the surface slopes of Jingpo Lacus are likely less than 0.05°. The data from Kraken Mare show rapid flux changes that require finer sampling in time before they can be properly modeled and understood.
7.3 Depth Constraints from T58 and T65

Following the T49 altimetry experiment, we acquired radar data of Ontario Lacus at higher incidence angles (greater than $20^\circ$) on the 57th flyby (T57; 22-June-2009), the 58th flyby (T58; 8-July-2009) and the 65th flybys of Titan (T65; 12-January-2010). The T57 and T58 data consist entirely of synthetic aperture radar (SAR) imaging data, whereas the T65 Ontario Lacus data comprise both SAR mode and scatterometry mode data. The T57 image covers only the northern tip of the lake, while the T58 image covers the rest of the lake, minus the southernmost tip. Together, the two swaths form a nearly complete radar mosaic of Ontario Lacus. The T65 image, acquired six months later, covers the entire lake in one pass by observing parallel to the lake's long-axis. Following the T65 SAR acquisition, the RADAR instrument was instructed to turn its antenna around and sweep the beam across the darkest part of the lake in scatterometer mode. The longer integration times and the smaller scatterometer bandwidth reduce the noise in the data and increase the likelihood of detecting an echo within the area of the lake that appears dark and noisy on the T57 and T58 flybys. The hypothesis was that, if this area was very deep, deep enough to attenuate a bottom reflection, then the presence of an echo would indicate the presence of another source of scattering within the lake, be it from wind-induced roughness on the surface, or volume scattering within the liquid medium. In this section, we explore the information contained within the Ontario Lacus T58 and T65 data passes (we do not consider the T57 data here because of scalloping artifacts that compromise the quality of the data). We consider two sets of data: the imaging SAR data processed by the Cassini RADAR Team at JPL, and the real aperture radar (RAR) data that we reduce using our processor described in Chapter 3. The latter contains the total beam-averaged backscatter measurements for data collected in both SAR mode and scatterometer mode.

The SAR images of Ontario Lacus (Figure 7.9) reveal systematic structure within
the Ontario Lacus boundary: notably, the lake signal decreases with increasing distance from the shoreline, as might be expected if there is a deepening liquid layer that is attenuating the reflection from a roughened bottom. Hayes et al. (2010) observe that the T57 and T58 SAR imaging magnitudes appear to decrease exponentially near the shoreline, as if the lake bed sloped at a constant gradient. They interpret these data with a two-layer scattering model to deduce the near-shore slopes at 13 locations around the lake perimeter. We apply a similar model here to directly convert the radar imaging backscatter measurements to depth measurements across all

Figure 7.9: Summary of the synthetic aperture radar observations of Ontario Lacus on the T58 and T65 flybys. We downsample the original images (256 pixel/degree) using a mean filter that is 8 pixels tall and 8 pixels wide. The color of the images maps to the logarithmic backscatter values. Dark blue within the swath represents the absence of valid data, or “holes” due to noise. The lines along the center of the swaths are imaging artifacts.
of Ontario Lacus, thereby obtaining near-complete bathymetry maps, as well as estimates of the liquid volume. However, noise in the images at the deepest part of the lake prevents us from estimating the maximum depths, resulting in “holes” in the bathymetry maps. We reduce the extent of the holes, i.e. reduce the noise, by downsampling the original backscatter image with an 8 pixel by 8 pixel mean filter, as shown in Figure 7.9.

Real aperture processing of the imaging data increases the SNR (signal to noise ratio) over the SAR-processed results, and thus the derived depth profiles are complete and valid throughout the lake, at the expense of resolution (e.g., the 8 km SAR beam footprint will slightly smear out the actual depth profile, and the >15 km scatterometer beam footprint will do so to a greater extent). Furthermore, because the real aperture data processing is independent of the image-mode processing of the imaging data, the depth profiles affirm the accuracy of the bathymetry maps, and vice versa. Thus, we supplement the bathymetry maps with three depth profiles: a profile along the length of Ontario Lacus (obtained from the real aperture form of the T65 central beam SAR data), a profile across the narrow waist of Ontario Lacus (obtained from the real aperture form of the T58 central beam SAR data), and a depth profile across the darkest width of Ontario Lacus (obtained from the real aperture form of the T65 scatterometry data). The two T65 profiles intersect at the darkest, and likely deepest, region of the lake, and represent an accurate measurement of the maximum depth of the lake. We summarize the real aperture observations in Figure 7.10.

We assume that the variation in backscatter across the lake is due largely to changes in depth, but we also allow for scatter from small-scale waves on the surface of the lake. We relate the lake surface backscatter magnitudes to wave heights using small perturbation models, as we describe in the last sub-section of this chapter. In the next sub-section, we describe the scattering model that we use to constrain the depths and surface scatter of Ontario Lacus.
Figure 7.10: Summary of the real aperture radar observations of Ontario Lacus for the T58 and T65 flybys, in SAR mode (central beam only) and scatterometry modes. The incidence angle (θᵢ) and polarization angle (θₚ) of each observation are indicated at the top, as well as the look direction (red arrow). The upper row of the figure illustrates the central beam footprint size, where each footprint is outlined in red, and the darkest footprint (smallest signal) along the track is outlined in cyan. The lower row of the figure illustrates the backscatter variation along the track, where the colorbar maps to the logarithmic backscatter values.

7.3.1 Two-Layer Scattering Model

A backscatter reflection from a lake will exist if: A) there is small-scale structure on the surface of the lake (e.g. waves or floating material), B) there are suspended scatterers within the lake volume (e.g. sediments), or C) the attenuating liquid layer is thin enough for the signal to penetrate and reflect off of a rough lake bed (see Figure 7.11). For this analysis, we consider a simple two-layer model that incorporates the backscatter from the lake surface (A) and the lake bed (C), but we do not currently
incorporate backscatter from the lake volume. Furthermore, we assume that the scattering contributions are uniform across the lake, i.e. the lake bottom and surface roughness do not change with position, and the loss tangent stays constant across the volume.

If $\sigma^0_S$ represents the backscattered signal from the surface of the lake, and $\sigma^0_B$ represents the backscattered signal from the bottom of the lake, then our received backscattered signal will be

$$\sigma^0_R(\theta_i) = \sigma^0_S(\theta_i) + \sigma^0_B(\theta_i) T^2 \exp(-Cd), \quad (7.4)$$

where $T$ is the Fresnel power transmission coefficient, which depends on the incidence.

Figure 7.11: Possible sources of scattering within a lake include (A) surface scattering from the atmosphere-liquid interface, (B) volume scattering from suspended material, or (C) attenuated reflections from the lake bottom. Our simple two-layer model (D), describes the received echo as a combination of surface scattering and lake bed scattering. We ignore volume scattering contributions for model simplicity.
angle into and out of the liquid medium, the polarization angle of the signal, and the dielectric constant of the liquid medium. For our observations, $T$ is around 99% for parallel polarizations (i.e., for T65 scatterometry, which occurs near the Brewster angle by design) and 97% for perpendicular polarizations (i.e., for SAR), where we assume that the liquid has a dielectric constant of 1.75 (as discussed below) and the atmosphere has a dielectric constant of 1. The exponential attenuation function in Eq. 7.4 accounts for the absorption within the liquid medium and depends on the loss tangent of the liquid ($\tan \delta$), the real part of the dielectric constant ($\epsilon$), the angle of incidence within the liquid ($\theta_{iL}$) after accounting for refraction using Snell’s law, and the average depth ($d$) of the liquid medium over the resolution cell of the radar. The power attenuation coefficient in a lossy medium is defined from electromagnetic wave theory as

$$\alpha = \frac{4 \pi}{\lambda} \sqrt{\frac{\epsilon}{2}} \left( \sqrt{1 + \tan^2 \delta} - 1 \right)^{1/2}$$

$$\approx \frac{4 \pi}{\lambda} \sqrt{\frac{\epsilon}{2}} \tan \delta$$

$$\approx \frac{2 \pi}{\lambda} \sqrt{\epsilon \tan \delta},$$

where we have simplified the expression using the binomial approximation and recognizing that we have a low-loss dielectric, where $\tan \delta << 1$. We relate the standard definition for the attenuation coefficient $\alpha$ to the effective attenuation coefficient $C$ needed in Eq. 7.4 by first noting that we require an additional factor of two for two-way transmission. We further require a factor to translate attenuation along the propagation direction to attenuation along the vertical depth direction, a factor that depends only on the incidence angle within the liquid. We then have

$$C = \frac{2 \alpha}{\cos \theta_{iL}}$$

$$= \frac{4 \pi}{\lambda} \sqrt{\frac{\epsilon \tan \delta}{\cos \theta_{iL}}}.$$
This leads us to the final form of our two-layer model:

\[
\sigma_R^0(\theta_i) = \sigma_S^0(\theta_i) + \sigma_B^0(\theta_i L) T^2 \exp \left( -\frac{4\pi}{\lambda} \sqrt{\epsilon \tan \delta} \cos \theta_i L \right),
\] (7.7)

Within our model, we know only \(\sigma_R^0(\theta_i)\) and \(\theta_i L\) absolutely, but we have reasonable estimates for \(\epsilon (\epsilon = 1.75)\) and \(\tan \delta (\tan \delta = 9.2 \times 10^{-4})\) from other analyses (Hayes et al., 2010; Paillou et al., 2008). We plot the attenuation response of the liquid medium for these parameters in Figure 7.12. The two-way penetration depth, where the power falls to 37\% (or \(e^{-1}\)) of its unattenuated value, is about 1.2 meters. We estimate that the signal will fall below the noise level of the SAR mode RADAR at depths of 8-10 meters. In other words, if we detect an echo return over the lake in the SAR image, and if we presume that the echo originated entirely from reflections off the lake bottom, then we can constrain the average lake depth to be less than 10 meters over the resolution cell of that echo. The scatterometer mode of RADAR has a lower noise floor, and thus can observe slightly deeper maximum depths.
The inference of depth from echo strength is complicated by the possibility that signal may be scattered from other interfaces besides the lake bottom (i.e. the surface interface between the atmosphere and the liquid layer in our two-layer model), and is also complicated by the fact that the apparent brightness of the lake bottom is unknown. Eq. 7.7 requires that we make some assumptions about the surface scattering contributions ($\sigma_0^S$) and the apparent lake bed brightness ($\sigma_0^B$) to retrieve the average depth values. We next consider the limiting values that each parameter is likely to assume.

We consider two limiting values for the lake surface backscatter: either $\sigma_0^S = 0$, or $\sigma_0^S = -30$ dB. The first case, or the null case, applies if there are no wavelength-scale waves over the entire surface of the lake. In this model scenario, the received backscatter signal must be entirely due to an attenuated lake bed reflection. For a given lake bed brightness level (fixed $\sigma_0^B$), the null surface scattering case leads us to a lower limit on lake depths; i.e. the output of the attenuation function is as large as it can possibly be, and thus the contributing depth is as small as it can be. On the other hand, the maximum surface scatter case will occur in our model if the echo is due solely to reflections off the liquid-atmosphere interface, i.e. the lake bed contribution has been completely attenuated. The $-30$ dB value equates to the smallest backscatter value that we measure over the lake (note: we can be sure this signal is truly an echo, and not noise, by analyzing its frequency content and showing that the signal corresponds to the transmitted spectrum). If we assume that the surface scattering term is uniform, then the maximum value of $\sigma_0^S$ cannot be larger than the smallest signal observed. In other words, if the region with $\sigma_0^R = -30$ dB occurs over the deepest part of the lake, where we cannot see through to the bottom, then our model requires that $\sigma_0^S = -30$ dB. We discuss the implications of this value on wave heights at the end of this section. And if $\sigma_0^S$ is assumed steady across the lake’s surface, larger values of $\sigma_0^R$ require detectable contributions from the lake bed, i.e. shallower depths. For a given lake bed brightness level (fixed $\sigma_0^B$), the maximum surface scattering case leads us to an upper limit on lake depths; i.e. the output of
the attenuation function is as small as it can possibly be, and thus the contributing depth is as large as it can be.

With these reasonable constraints on $\sigma^0_S (0 \leq \sigma^0_S \leq 0.001)$, let us now consider likely values for the apparent brightness of the lake bed, $\sigma^0_B$. We wish to know how much of the incident signal is reflected off the lake bed (the scattering level) as well as how the scattering response varies with incidence angle (the scattering shape). We begin by considering the scattering response from the class of empty lakes on Titan. Hayes et al. (2011) fit backscatter models to the empty lake data and find that they are best fit with a Gaussian quasispecular model plus a diffuse model ($\epsilon = 2.7, \theta_{\text{rms}} = 12.9, A = 0.55, n = 1.2$) or a Hagfors quasispecular model plus a diffuse model ($\epsilon = 5.3, \theta_{\text{rms}} = 12.6, A = 0.41, n = 1.2$) (see Chapter 4 for backscatter model descriptions). Hayes et al. (2011) find that the Gaussian composite model fits slightly better than the Hagfors composite model, thus this is the model that we consider in our analysis, although for our range of incidence angles ($19 < \theta_i < 45$) the difference between the two is minimal. The Gaussian composite model yields an estimate of the expected level and shape of the empty lake bed scattering response (plotted as the light-gray dashed line in Figure 7.13), but what happens to the level and the shape of the scattering response when the lake bed is covered by liquid hydrocarbons?

The apparent brightness of the lake bed scattering will depend largely on the dielectric contrast between the lake bed and the liquid medium. For instance, at $20^\circ$ incidence, the Gaussian-composite-modeled empty lake backscatter is around $-2$ dB. But this value occurs at the interface between solid Titan and the atmosphere ($\epsilon = 1$), whereas we are interested in the interface between solid Titan and the overlying liquid material. The smaller dielectric contrast between the lake bed and the liquid medium will reduce the lake bed backscatter by a factor equivalent to the ratio of the Fresnel reflection coefficients at zero incidence. Solid surfaces in Titan’s polar regions are thought to comprise a mixture of water ice ($\epsilon = 3.1$) and solid hydrocarbon tholin ($\epsilon = 2.0 - 2.4$) (Lorenz et al., 2003; Lunine, 1993; Paillou et al.,
2008; Thompson and Squyres, 1990), whereas the composition of the liquids will be a mixture of nitrogen, methane, ethane, and higher hydrocarbons (\(\epsilon = 1.6 - 1.9\)) (Lorenz et al., 2010). The solid terrain may also be porous and saturated with liquid hydrocarbons, further reducing the effective dielectric contrast. A high dielectric contrast will yield a brighter bottom reflection than a low dielectric contrast, and will thus indicate greater liquid depths (i.e. a larger value for \(\sigma_B^0\) in Eq. 7.7 would require the output of the attenuation function be smaller to match the data, and thus \(d\) must be larger).

We now consider the limiting cases for the dielectric contrast values. The maximum dielectric contrast occurs for a solid surface composed of impermeable water ice (\(\epsilon \approx 3.1\)) covered by low dielectric constant liquid hydrocarbons (\(\epsilon \approx 1.6\)). In
this scenario, the dielectric contrast is 1.9, and the normal-incidence Fresnel reflection coefficient is 0.027. By contrast, impermeable water ice that is exposed to the atmosphere has a dielectric contrast of 3.1 and a normal-incidence Fresnel reflection coefficient of 0.076. As a result, the reduced dielectric contrast decreases the reflected signal level to 35% of the dry empty lake value (at 20° incidence, the lake bed backscatter level drops to around −6.5 dB instead of −2 dB). The lake bed backscatter model corresponding to the maximum dielectric contrast case is plotted as the medium-gray dashed line in Figure 7.13. At the other extreme, the minimum dielectric contrast occurs for a solid surface composed of low dielectric constant solid hydrocarbons, or equivalently saturated porous water ice, where $\epsilon \approx 2.0$, covered by a high dielectric constant liquid hydrocarbon mixture, where $\epsilon \approx 1.9$. In this scenario, the dielectric contrast is less than 1.1. The Fresnel reflection coefficient is now reduced from 0.03 for the atmosphere interface, to 0.0007 for the liquid interface. This results in a reflected signal that is only 2% of its dry empty lake value (at 20° incidence, the backscatter level drops to around −18 dB). The lake bed backscatter model corresponding to the minimum dielectric contrast case is plotted as the black dashed line in Figure 7.13. The maximum dielectric contrast provides upper limits on lake depth, and the minimum dielectric contrast provides lower limits on lake depth.

The limiting cases for dielectric contrast, i.e. the apparent lake bed brightness ($\sigma^0_B$), combine with the limiting cases for surface scattering ($\sigma^0_S$), to yield the bounding cases for depth retrieval, as summarized in Table 7.2. We recognize that the

<table>
<thead>
<tr>
<th></th>
<th>Surface Scatter (at 20°)</th>
<th>Wave Height (mm)</th>
<th>Bed Scatter (at 20°)</th>
<th>Dielectric Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Bound</td>
<td>$\sigma^0_S = 0$</td>
<td>0</td>
<td>$\sigma^0_B \approx -18$ dB</td>
<td>1.1</td>
</tr>
<tr>
<td>Upper Bound (1)</td>
<td>$\sigma^0_S = -30$ dB</td>
<td>0.5</td>
<td>$\sigma^0_B \approx -6.5$ dB</td>
<td>1.9</td>
</tr>
<tr>
<td>Upper Bound (2)</td>
<td>$\sigma^0_S = -30$ dB</td>
<td>0.5</td>
<td>$\sigma^0_B \approx -2.0$ dB</td>
<td>unmodified</td>
</tr>
</tbody>
</table>
unmodified empty lake bed backscatter model, where we do not account for dark-
ening from decreased dielectric contrast, will also yield an upper bound on depths.
Thus, we consider both scenarios; the maximum dielectric contrast case represents the
more realistic upper bound estimate (Upper Bound 1), and the unmodified dielectric
contrast case represents a more conservative upper bound estimate (Upper Bound 2).

7.3.2 Depth Retrieval

With the bounding values of $\sigma_S^0$ and $\sigma_B^0$ in hand, let us now consider the evaluation
of lake depths from the backscatter measurements. Rearranging Eq. 7.7, we obtain
the expression

$$d = \frac{-\lambda \cos \theta_{iL}}{4\pi \sqrt{\epsilon} \tan \delta} \log \left( \frac{\sigma_R^0(\theta_i) - \sigma_S^0(\theta_i)}{T^2 \sigma_B^0(\theta_{iL})} \right),$$

where $\sigma_R^0(\theta_i)$ represents the received T58 and T65 imaging backscatter measurements
at each pixel.

Eq. 7.8 applies only to the SAR data, where the size of each pixel is such that the
observation parameters remain relatively constant across the pixel, and $d$ thus pro-
vides a good estimate of the average depth over the pixel area. For the scatterometry
data, where the beam footprint is around 15 km, we recognize that the depth may
change significantly within the illuminated area, so that the measured backscatter
value is expressed as the gain-weighted integral of the attenuated bottom reflection
over many different depths. In this case, Eq. 7.4 becomes

$$\sigma_R^0(\theta_i) = \sigma_S^0(\theta_i) + \frac{\int_{\text{beam}} g(dA)^2 \sigma_B^0(dA) T(dA)^2 \exp(-C(dA) T(dA))dA}{\int_{\text{beam}} g(dA)^2 dA},$$

where $g$ represents the normalized gain of the antenna pattern. Eq. 7.9 is simpler to
evaluate for the T65 scatterometry observation because that scan is oriented roughly
perpendicular to the shoreline, such that depth effectively changes only in one dimen-
son. The integral can then be carried out along an assumed depth profile, such that
dA becomes $dx$, where $x$ is the distance from the shoreline. To solve Eq. 7.9 for the T65 scatterometry data, we forward-model to solve for the depth profile that best matches the observed backscatter response as a function of distance from shore. We find that we only need to apply this approach to the T65 scatterometry data; the T58 and T65 real aperture data are accurately evaluated with the same inverse modeling approach used for the T58 and T65 imaging data (Eq. 7.8).

### 7.3.3 Ontario Lacus Depth Maps

We invert the T58 and T65 imaging measurements using Eq. 7.8 for each of the three bounding cases described in Table 7.2, where the lake bed backscatter models are based on the best-fit Gaussian composite model from Hayes et al. (2011). The resulting bathymetry images are shown in Figure 7.14. A comparison of the T58 and T65 images suggests differences in depth between the two observations. For a more quantitative comparison, we evaluate depth parameters for the area that is common between the two observations. Table 7.3 describes the mean depths, the 1st and 99th percentiles, and the volume estimates for the observations’ bounding cases, and Figure 7.15 plots the corresponding depth histograms. It is clear from these results that the depths observed during T65 appear shallower than those observed during T58. In fact, the apparent volume of Ontario Lacus during T65 (January, 2010) is about 6-8% less than what it was during T58 (July, 2009), and the mean depth is about 0.34 meters less. If we interpret these changes as evidence of evaporation, then we derive an evaporation rate of $\sim$0.7 meters per year, a rate that is consistent with the $\sim$1 m/year rate measured by Hayes et al. (2011). However, the T58 and T65 observations occur around the Titan equinox, when we expect the rate of methane evaporation at the southernmost latitudes to be significantly smaller (Graves et al., 2011). Can we attribute the differences in apparent depths to differences in viewing geometry rather than actual physical changes on the surface?

To answer this question, we compare the T58 and T65 imaging measurements and their observation geometries in Figure 7.16. There are strong systematic backscatter
**Figure 7.14:** Ontario Lacus bathymetry maps derived using the Gaussian-based lake bed model for each of the bounding cases described in Table 7.2. Large differences in the derived depths between the two observations are apparent.

**Table 7.3:** Summary of Ontario Lacus Bathymetry Results*

<table>
<thead>
<tr>
<th>Lake Bed Model Shape</th>
<th>Bounding Case</th>
<th>Min Depth (m)</th>
<th>Max Depth (m)</th>
<th>Mean Depth (m)</th>
<th>Volume (km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gaussian</strong></td>
<td>Lower Bound</td>
<td>(0, 0)</td>
<td>(4.91, 4.19)</td>
<td>(0.67, 0.51)</td>
<td>(10.04, 7.65)</td>
</tr>
<tr>
<td></td>
<td>Upper Bound (1)</td>
<td>(1.48, 1.40)</td>
<td>(8.77, 8.51)</td>
<td>(3.98, 3.64)</td>
<td>(60.06, 54.81)</td>
</tr>
<tr>
<td></td>
<td>Upper Bound (2)</td>
<td>(2.83, 2.78)</td>
<td>(10.15, 9.90)</td>
<td>(5.35, 5.02)</td>
<td>(80.68, 75.71)</td>
</tr>
<tr>
<td><strong>Empirical</strong></td>
<td>Lower Bound</td>
<td>(0, 0)</td>
<td>(4.94, 4.79)</td>
<td>(0.56, 0.49)</td>
<td>(8.49, 7.38)</td>
</tr>
<tr>
<td></td>
<td>Upper Bound (1)</td>
<td>(0.92, 1.14)</td>
<td>(8.52, 8.41)</td>
<td>(3.53, 3.43)</td>
<td>(53.22, 51.64)</td>
</tr>
<tr>
<td></td>
<td>Upper Bound (2)</td>
<td>(2.75, 2.98)</td>
<td>(10.37, 10.28)</td>
<td>(5.35, 5.27)</td>
<td>(80.71, 79.50)</td>
</tr>
</tbody>
</table>

* The table results are ordered by observation, (T58, T65), and are computed from the T58-T65 intersecting region only (to provide an unbiased comparison between the pair of results). Min Depth and Max Depth equate to the 1st and the 99th percentile, respectively. The volume estimate is the mean value (from the joint region), extrapolated over the entire lake surface.
7.3. DEPTH CONSTRAINTS FROM T58 AND T65

Figure 7.15: Depth Histograms for the three bounding cases using the Gaussian lake bed model (left column) and the empirically-derived lake bed model (right column). The T65 depths are plotted in red on top of the T58 depths in blue. The figure illustrates the tendency for shallower depths in T65 when using the Gaussian-based lake bed model, whereas this bias is essentially removed when using the empirical model.
Figure 7.16: Comparison of T58 and T65 Ontario Lacus Observations. The backscatter measurements from the two observations are shown in the upper left panels, with the backscatter difference image in the upper right panel. The incidence angle geometries from the two observations are plotted in the lower left panels, with the incidence angle difference image in the lower right panel. The two difference images show the same gradient along the diagonal long axis of the lake, suggesting that differences in backscatter are due largely to differences in viewing geometry. In the text, we explore whether these trends carry over into the derived depth maps, in spite of our efforts to correctly account for the viewing geometry in our two-layer scattering model.

The backscatter measurements from the two observations are shown in the upper left panels, with the backscatter difference image in the upper right panel. The incidence angle geometries from the two observations are plotted in the lower left panels, with the incidence angle difference image in the lower right panel. The two difference images show the same gradient along the diagonal long axis of the lake, suggesting that differences in backscatter are due largely to differences in viewing geometry. In the text, we explore whether these trends carry over into the derived depth maps, in spite of our efforts to correctly account for the viewing geometry in our two-layer scattering model.

differences between the two observations, with the heel of the lake appearing much brighter in T65 than in T58, whereas the backscatter levels of the opposite end of the lake appear more similar to each other. We plot the incidence angle difference image and observe a linear gradient along the diagonal axis of the lake, the same trend observed in the backscatter difference image. A plot directly pairing the backscatter difference values against the incidence angle difference values reveals the extent of the strong linear correlation (see Figure 7.17A). The best-fit polynomial (shown in red) quantifies the dependence and suggests that we see negligible backscatter difference at similar incidence angles, thereby confirming the trend observed in the difference
7.3. DEPTH CONSTRAINTS FROM T58 AND T65

**Figure 7.17:** (A) The observed dependence of backscatter difference on incidence angle difference between T65 and T58 (for the overlapping region only). The y-intercept is close to zero, suggesting negligible backscatter difference between the two observations for similar incidence angles (however, zero incidence angle difference occurs over the darker part of the lake). We observe that the dependence on viewing angle carries over to the derived depths (C), in spite of accounting for the different incidence angles in the two-layer scattering model. The best-fit linear polynomial and corresponding parameters are shown in red, where the dashed green line marks the zero point. The strong dependence of depth on angle suggests that the shape of the Gaussian lake bed model may be in error. We empirically derive a steeper backscatter model for the lake bed and show that the average dependence of depth on viewing geometry can be removed (D; the slope of the polynomial goes towards zero). The empirically-derived lake bed backscatter models are shown in gray-levels for each of the dielectric contrast cases in (B), with the corresponding Gaussian lake bed models plotted in color in the background. (C) and (D) are shown for the Upper Bound (1) case only, but similar results can be derived for the other bounding cases. Note that we evaluate the lake bed models at the incidence angle within the liquid medium, after accounting for refraction with Snell’s law.
images. We could interpret these results as evidence that there have not been significant surface changes between the two observations, but we note that as the incidence angle difference approaches zero, we are also approaching the deeper end of the lake, where the signal-to-noise ratio is poorer and signal differences are thus much harder to detect.

We observe a similar trend in the derived depths (Figure 7.17C); the depths appear to converge for zero difference in incidence angle (which happens to occur near the deep end of the lake), but diverges in a linear fashion (towards larger T58 depths) as the magnitude of the incidence angle difference increases (which happens to occur near the shallow end of the lake). This suggests that the shape of the lake bed backscatter model assumed in our two-layer scattering model may be incorrect. We reevaluate the shape of the lake bed model empirically, using forward-modeling to find the backscatter slope needed to level off the difference in the derived depths with incidence angle (Figure 7.17D). We discover that we need a much steeper backscatter response (0.5 dB of backscatter change per degree incidence angle, versus the 0.1 dB per degree suggested by the Gaussian empty lake bed model from Hayes et al. (2011)). The empirically-derived backscatter models are plotted in Figure 7.17B for the different dielectric contrast scenarios. If we reapply our depth model equation (Eq. 7.8) to the bounding cases of Table 7.2 using the empirically-derived lake bed models, then we find that the depth differences between T58 and T65 diminish (see Table 7.3 and Figure 7.15). The bathymetry maps derived for the bounding cases are shown in Figure 7.18 and demonstrate that the two observations are much more similar when we use the steeper backscatter response.

Before we analyze the depth results further, we consider the implications of the steepness of the empirically-derived lake bed backscatter model. While our empirical model appears to slightly overshoot the correction of depth dependence on viewing geometry (hence the slightly negative slope of the best-fit polynomial in Figure 7.17D), the steepness of the backscatter function is not that different from what is actually required. A backscatter slope of this magnitude is not encountered anywhere else.
7.3. DEPTH CONSTRAINTS FROM T58 AND T65

Bathymetry Maps derived using the scaled Empirical Lake Bed Models

Figure 7.18: Ontario Lacus bathymetry maps derived using the empirically-derived lake bed backscatter model for each of the bounding cases described in Table 7.2. The empirical model removes the large-scale average depth differences between the two observations, but small-scale differences still exist.

on Titan’s surface; the steepest slopes we have previously observed at this range of incidence angles occur over Titan’s cryovolcanic terrain (see Section 5.7). But even for the cryovolcanic terrain, the backscatter slope is not greater than 0.23 dB/degree, or about half of the slope required for Ontario Lacus’ lake bed. We find that we can reproduce the empirical model’s behavior with a small and focused diffuse component ($A = 0.1, n = 3$) and a rough Gaussian component ($\epsilon = 3, s = 15$). While these model parameters are not a unique solution, the relative combination of the parameters (a small diffuse component together with a rough quasispecular component) does appear to be required. These model parameters are not unphysical, and are in fact similar to the the cryovolcanic terrain model; the cryovolcanic diffuse term magnitude would need to be about 75% smaller to approximate the backscatter behavior that
we observe for Ontario Lacus’ lake bed. This is a surprising and interesting result for two reasons: 1) Hayes et al. (2011) find that the empty lakes have a large diffuse component relative to surrounding terrain \((A = 0.55, n = 1.22)\), and (2) Barnes et al. (2011b) find that many of the empty lakes identified by RADAR have the same 5 µm-λ bright VIMS spectral signature as seen on putative cryovolcanic terrain (e.g. Hotei Regio and Tui Regio), a signature that they interpret as evidence for ice-free organic deposits that form when the liquid methane solvent evaporates. We find it peculiar that the closest analog to the observed Ontario Lacus’ lake bed backscatter response is a feature that has been spectrally linked back to the lakes (albeit empty lakes). Yet, we note that, while the cryovolcanic and the evaporitic empty lake deposits look similar to each other using VIMS, they look very different from each other using RADAR (mostly in their diffuse components). We do not have an explanation to connect these observations, but instead merely note their coincidence.

### 7.3.4 Ontario Lacus Depth Profiles

To validate the bathymetry maps derived from the SAR images, we turn our attention to the independently process real aperture data. We process the central beam T58 SAR and T65 SAR lake data using the real aperture processor described in Chapter 3. We then apply Eq. 7.9 to invert the beam-averaged normalized radar cross section values to give depth estimates. We repeat the depth retrieval for each of the bounding cases of Table 7.2 considering only the empirically-derived lake bed backscatter function. The retrieved real aperture depth profiles are plotted in pink (T58) and red (T65) in Figure 7.19. Their tracks over the lake are shown in the same colors over the T65 SAR image. We next slice the bathymetry maps along the same tracks to compare the depth profiles implied by the SAR images to the depth profiles derived from the real aperture data. The bathymetry map profiles are shown in gray tones behind the colored real aperture profiles. In general, the real aperture depth profiles reproduce the large-scale trends implied by the bathymetry maps, although at a slightly elevated level. The subtle bias towards shallower depths may be an outcome
Figure 7.19: Depth profiles for each of the bounding cases, derived assuming the empirical lake bed backscatter model. The real aperture depth profile results are shown in color and their corresponding bathymetry map slices are shown in gray tones. The tracks of each profile are color-coded over the T65 SAR backscatter image (note that the size of circles along the track do not have any physical correspondence to beam footprint size). The letters in the SAR image mark the noteworthy lake features identified by Wall et al. (2010), following their same labeling scheme. The reference point for along-track distance (0 km from the shore) is on the eastern side of the lake for the T65 SCAT and T58 SAR tracks (near C), and on the southern side of the lake for the T65 SAR track (near G).

of the different calibration procedures utilized.

We also process the T65 scatterometry lake data (those with more than 99% of their beam footprint located within the lake boundary) with the real aperture processor, but due to the larger beam footprint size (>15 km), we need to integrate the antenna gain pattern over the depth profile, as in Eq. 7.8. We use a forward modeling
approach to determine the depth profiles required to match the observed backscatter data for each of the bounding cases. The best-fit scattering depth responses, as a function of distance from the shore along the track, are plotted in Figure 7.20 against the measured backscatter response. The corresponding real aperture depth profiles for the T65 scatterometry data are shown in yellow in Figure 7.19, with the corresponding T65 bathymetry slices plotted behind in gray tones. Here, the real aperture depth profiles are very coarse, but they do tend toward the average profile suggested by the bathymetry maps, especially away from the shore (note that radar bursts closest to the shore have been filtered out to avoid contamination with on-shore backscatter; as a result, we force the scatterometry-derived depth profiles to converge to zero at the shoreline, contrary to the imaging-derived depth profiles).

The T65 scatterometry depth profiles suggest slightly deeper maximum depths than their SAR counterparts over the same region. We attribute this to two factors:
the improved sensitivity of the scatterometer receiver, and the more accurately determined calibration scale factor. The T65 scatterometry experiment was interspersed with several instances of receive-only data, which made it possible to accurately measure the system noise power over the lake. We found that the noise power is about 6% larger over the lake center than it is for other terrain on Titan, consistent with the higher brightness temperatures measured by the radiometer (Wall et al., 2010). We incorporate this improved noise measurement into our calibration procedure for the T65 scatterometry data (see Section 3.4 for details on the calibration), but we lack a similarly accurate noise measurement to fine-tune the SAR calibration. The different calibration inputs may account for the majority of the discrepancies between the datasets.

While the T65 scatterometry depth profiles suggest maximum depths that are $\sim 1$-2 meters deeper than the real aperture and imaging depth results, we emphasize that this does not offer a strict upper bound on the maximum depth of the lake. In addition to the possibility of deeper features hiding within the poor beam-smeared resolution, the RADAR instrument is simply not sensitive to changes in depth beyond $\sim 9$-10 meters.

Aside from providing an independent validation of the lake depths, the real aperture depth profiles are valuable for interpreting the geophysical nature of the lake. The bathymetry maps and their depth profiles are noisy, in spite of the mean filtering applied to the imaging data at the outset. The source of the fluctuation may be speckle noise within the image or variation in the properties of the lake, which we assume to be uniform in our model. In any case, the smoothed depth profiles derived from the real aperture data better depict the shape of the lake bed, making it easier to understand their limnological implications, as we discuss in the next section.
7.3.5 Discussion of Depth Results

The depth and volume estimates reported in Table 7.3 are only valid over the T58/T65 joint region. We focus on the intersecting data in order to provide an unbiased comparison between the pair of results. The volume estimates in the table are calculated by extrapolating the mean depth value (from the joint region) over the entire lake surface. The true lake volume estimates will be larger than those reported in Table 7.3 because the section of the lake that is missing from the joint region, just beyond the T58 border, is deeper than the rest of the lake. The total lake volume estimates are more accurately computed from the entire lake region (e.g. T65) using the empirical lake bed model. These volume estimates are 8.1 km$^2$, 53.8 km$^2$, and 81.8 km$^2$ for the Lower Bound, Upper Bound (1), and Upper Bound (2) cases respectively.

Assuming the extreme Upper Bound (2) parameters, we constrain the mean depths of Ontario Lacus to be less than 5.5 meters and the volume to be less than 82 km$^3$, with the deepest region of the lake likely being not much greater than 10 meters. We believe that the Upper Bound (1) case yields more realistic constraints, given the physical parameters assumed, and this case implies that the mean depths are less than 3.6 meters, the deepest region is less than 9 meters, and the volume is less than 54 km$^3$. The depth results reported here assume a loss tangent near $\sim 10^{-3}$. If the effective loss tangent of the liquid material is closer to $\sim 10^{-4}$, the depth results will be ten times deeper than what we report. The shallow depths derived for Ontario Lacus are comparable to terrestrial lakes of similar size and geological setting, such as the Great Salt Lake in Utah.

While the measurements suggest that the depths of Ontario Lacus are not much greater than several times the two-way absorption length (1.2 m for the assumed dielectric properties), the subsurface reflection in Titan’s large northern lakes quickly extinguishes as the radar beam moves away from the shore, suggesting the northern lake floors occur at much greater depths. Jingpo Lacus and Ligeia Mare are the only sizeable northern lakes where we find detectable backscatter variation away from the shore. Data from the primary and extended missions reveal that all other sizable
northern lakes fail to produce a backscatter reflection, appearing uniformly dark to the radar, with the exception of regions very close to the lake shores. Assuming dielectric properties similar to those assumed for Ontario Lacus, we calculate volumes greater than 512 km\(^3\) for Ligeia Mare and greater than 22 km\(^3\) for Jingpo Lacus, but it is very likely that lake compositions in the north differ from those in the south due to seasonal effects. We expect the seasonal composition difference to be largely in the relative amounts of methane and ethane, which is not likely to change the loss tangent significantly, but will affect the real part of the dielectric constant.

The empirical lake bed backscatter model removes much of the large-scale depth difference between T58 and T65, as demonstrated by the depth histograms of Figure 7.15. Yet, the tabulated results in Table 7.3 still hint at a subtle change in depth between the two observations (T58 appears 0.1 meters deeper than T65, suggesting an evaporation rate of \(\sim 0.2\) m/year). Furthermore, the bathymetry maps of Figure 7.18 show the presence of small-scale differences in depth. It is possible that the shape of the lake bed scattering function could be further tweaked to eliminate some of the small-scale differences in depth, but because we do not expect the large-scale features (or the lake volume estimates) to be significantly affected, we do not further the effort here. It is also possible that our assumptions of uniformity are incorrect; perhaps the loss tangent value changes with depth, or the lake bed scattering function is not the same everywhere across the lake floor, or the wave roughness (if any) changes with distance from the shoreline. The presence of any of these heterogeneities would introduce artificial structure in the bathymetry map, and it would also not be properly corrected for viewing geometry changes, thereby introducing small-scale changes between the two observations. There also remains the possibility that subtle changes on the surface did in fact occur in the \(\sim 6\) months between the two observations. We cannot distinguish between these different possibilities here.

The shapes of the intersecting orthogonal depth profiles have implications for the lake geology. They appear to be consistent with interpretations of the T57 and T58 imaging data made by Wall et al. (2010). We note the relevant lake features
identified by Wall et al. (2010) in Figure 7.19, following the same labeling scheme. They describe (A) as “deeply incised bays that resemble drowned river valleys”, (C) as a smooth beach, (G) as a patchy bright surface that may be semi-solid or very shallow liquid, (H) as flooded river valleys, and (J) as a fluvially-fed delta lobe. From the depth profiles along the T65 SCAT track and T58 central-beam SAR track, we see that the eastern side of the lake (near the beach marked as ‘C’) falls off much more steeply than the opposite side (near the depositional area marked as ‘J’ and ‘H’). Furthermore, we find that the bathymetric shape is more convex near the beach and more concave near the deltas, as might be expected. The T65 central-beam SAR profile along the diagonal length of the lake emphasizes the contrast between the two ends of the lake. The heel, starting at ‘G’, has a more gradual, concave shape, consistent with a shallow depositional area, and the opposite side, near ‘A’, is much steeper, consistent with the flooded-valley morphological interpretation.

7.3.5.1 Comparison to Other Analyses

Hayes et al. (2010) present an alternative depth analysis of the Ontario Lacus SAR data. They model the near-shore exponential falloff of the backscatter data with a two-layer model to derive slopes for 13 locations around the lake perimeter. Upon extending their slopes into the lake, they find depths as large as 10 m at distances less than 10 km from the shoreline. Our results, on the other hand, suggest that depths of 10 m can only be reached at the center of the lake in the extreme upper bound scenario. A comparison of our results suggests that the slopes derived by Hayes et al. (2010) cannot extend further than a couple of kilometers from the shoreline (depending on the bounding case) before shallowing out, but otherwise their slopes are consistent with our derived upper bound depths. We demonstrate this agreement in Figure 7.21 for the region labeled ‘F’ by Hayes et al. (2010), but the conclusions will be the same for other regions around the perimeter. Region ‘F’ occurs just north of the start of the T65 SCAT track, which we label ‘C’ Figure 7.19.

In the left panel of Figure 7.21, we show that the backscatter data used in our
two different analyses are the same for region ‘F’. Hayes et al. (2010) use long (∼15 km) and narrow (∼1 km) bins oriented parallel to the shoreline to average the data and reduce speckle noise without affecting the expected exponential decay in the near-shore region. We are more concerned with average depth changes than near-shore depth sensitivity, so we use square bins with similar areas (65 pixels versus their ∼100 pixels) to average the same data. As expected, we find that the two approaches show the same average backscatter levels away from the shore. Hayes et al. (2010) calculate a near-shore slope of $2.7 \times 10^{-3}$ for the exponential backscatter decay observed at region ‘F’. We plot the depths implied by this slope in red in the right panel of Figure 7.21, and we plot our derived depths for the same region in black (T58) and gray (T65) for each of the bounding cases. The depth profiles that we show are slices from our bathymetry maps at an orientation perpendicular to the shore at ‘F’. We note that there may be slight differences in the shoreline definition.
between the two approaches, and further note that Hayes et al. (2010) use different surface and lake bed scattering parameters, which they fit separately for each region. In spite of these differences, we find that the slope calculated by Hayes et al. (2010) at ‘F’ is consistent with our upper bound depth profiles up to ∼2 km from the shoreline for the Upper Bound (1) case and ∼3 km from the shoreline for the Upper Bound (2) case, but the Lower Bound case is not consistent with the derived slope. We note that our results do not affect the evaporation rates calculated by Hayes et al. (2011) since our results do not change the near-shore slopes measured by Hayes et al. (2010) within a couple kilometers of the shoreline.

We also compare the depth outputs of this analysis to those of another double layer model applied to the SAR data. This model uses a Bayesian inversion algorithm to compute mean values and PDFs of single parameter estimates, including wind speeds and optical thickness (Notarnicola et al., 2009). Ventura et al. (2011) convert the derived optical thickness maps to depths map using the same dielectric properties measured by Hayes et al. (2010) and compare their results to those that we present here. The depth results implied by the Bayesian analysis are shallower than the results that we present here for similar assumed conditions. We are still exploring possible explanations for why the depth results differ between the two approaches.

7.3.6 Off-Nadir Wave Height Modeling

We allow for scatter from small-scale waves on the surface of Ontario Lacus in our analysis. We apply small perturbation models to the observed minimum backscatter levels and find that the rms wave height has to be less than 1 mm, with a correlation length near 0.73 cm. These results are consistent with our analysis of the T49 altimetry echoes presented earlier in this chapter and summarized in Wye et al. (2009). The low wave activity suggests that the wind speed has not picked up in the interval between the T49 observation in December 2008 and the T65 observation in January 2010.
7.4 Conclusion

In this chapter, we have analyzed the altimetry, SAR, and scatterometry data acquired by Cassini RADAR over Ontario Lacus to constrain the wave heights and depths of the lake. The nadir reflections measured by the RADAR altimeter are modeled with specular reflection theory to derive an upper bound of 3 mm for the surface rms wave heights over the 100 meter-wide Fresnel zone. The upper bound is highly conservative due to the saturated nature of the data. The off-nadir imaging and real aperture backscatter measured by the RADAR SAR and scatterometer show brightness values consistent with less than 1 mm rms surface heights, according to small perturbation theory. The extreme smoothness of the lake surface suggests either that winds are not strong enough to surpass the wave generation threshold, or that the material properties of the liquid are not well understood.

The off-nadir imaging and real aperture data show variations in the lake brightness with distance from the shoreline, which we interpret as evidence of sub-surface bottom reflections. We model the backscatter variation with a uniform two layer model accounting for the possibility of wind-generated roughness on the surface of the lake. We derive depth maps across the entire lake and orthogonal depth profiles across the length and width of the lake. The depth maps provide volume estimates and the depth profiles describe the shape of the lake bed. We find that the depth values depend largely on the strength of the lake bed backscatter, which in turn depends predominantly on the dielectric contrast with the overlying liquid. We bound the depth results by assuming likely values for the lake bed backscatter strength. The presence of small-scale roughness on the surface of the lake permits larger depth values over the deepest region, but it does not significantly affect the shallower depths derived over the rest of the lake.

We find that the depths of Ontario Lacus appear extremely shallow. Our most probable bounding case implies that the mean depths are less than 3.6 meters, the deepest region is less than 9 meters, and the volume is less than 54 km$^3$. These shallow
depths are comparable to terrestrial lakes of similar size and geological setting, such as the Great Salt Lake in Utah. The depth results reported here assume a loss tangent near $\sim 10^{-3}$. If the effective loss tangent of the liquid material is closer to $\sim 10^{-4}$, the depth results will be ten times deeper than what we report.

We observe changes in the measured backscatter between the T58 and T65 observations, a difference that is correlated with differences in the viewing angle. The two viewing geometries are orthogonal to each other and help us constrain the shape of the lake bed backscatter response. The derived shape is steeper at $20^\circ$-$30^\circ$ incidence by more than a factor of two compared to the backscatter shapes of other terrains on Titan. This suggests a more focused and less diffusive scattering mechanism at the liquid-bed interface. The derived backscatter function also helps to eliminate most of the systematic differences between the T58 and T65 depth results. It is difficult to attribute residual differences to the occurrence of actual physical change on the surface.
Chapter 8

Radar Observations of Saturn’s Icy Satellites

8.1 Introduction

The Cassini spacecraft orbits Saturn along a variety of elliptical paths that are controlled primarily by encounters with the giant moon Titan. These carefully-designed encounters steer the spacecraft through the Saturnian system, permitting regular visits to many of Saturn’s other moons, the so-called icy satellites. We characterize the properties of these moons in Table 8.1. Some of these visits are targeted flybys, where the spacecraft passes by a specific moon at a predetermined distance, and other visits are non-targeted, where the moon just happens to be near enough to Cassini’s flight path to be easily observed. Targeted flybys might bring the spacecraft very close to moon, sometimes as close as 25 km from the surface, whereas non-targeted flybys might be as distant as several hundreds of thousands of kilometers. All of the icy satellite observations collected by Cassini’s RADAR instrument over the primary and extended missions are listed in Table C.1. These are active mode observations, with coincident passive radiometry collected while the instrument waits for the return echoes. Dozens more passive-only radiometry observations were also collected over the course of the mission.

The typical icy satellite radar observation occurs at large distances, usually about
Table 8.1: Major satellites of Saturn studied by Cassini RADAR and their properties, ordered by increasing orbital distance from Saturn.

<table>
<thead>
<tr>
<th>Body Name</th>
<th>Mean Radius (km)</th>
<th>Mass (10^{19}) kg</th>
<th>Density g cm(^{-3})</th>
<th>Orbital Radius (km)</th>
<th>Orbital Period (days)</th>
</tr>
</thead>
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<tr>
<td>Mimas</td>
<td>198.3</td>
<td>3.7</td>
<td>1.15</td>
<td>185,000</td>
<td>0.9</td>
</tr>
<tr>
<td>Enceladus</td>
<td>252.1</td>
<td>10.8</td>
<td>1.61</td>
<td>238,000</td>
<td>1.4</td>
</tr>
<tr>
<td>Tethys</td>
<td>533.0</td>
<td>61.7</td>
<td>0.97</td>
<td>295,000</td>
<td>1.9</td>
</tr>
<tr>
<td>Dione</td>
<td>561.7</td>
<td>109.5</td>
<td>1.48</td>
<td>377,000</td>
<td>2.7</td>
</tr>
<tr>
<td>Rhea</td>
<td>764.3</td>
<td>230.7</td>
<td>1.23</td>
<td>527,108</td>
<td>4.5</td>
</tr>
<tr>
<td>Titan</td>
<td>2575.5</td>
<td>13452.0</td>
<td>1.88</td>
<td>1,222,000</td>
<td>16</td>
</tr>
<tr>
<td>Hyperion</td>
<td>133.0</td>
<td>0.6</td>
<td>0.57</td>
<td>1,481,009</td>
<td>21.3</td>
</tr>
<tr>
<td>Iapetus</td>
<td>735.6</td>
<td>180.6</td>
<td>1.08</td>
<td>3,560,000</td>
<td>79</td>
</tr>
<tr>
<td>Phoebe</td>
<td>106.6</td>
<td>0.8</td>
<td>1.63</td>
<td>12,955,759</td>
<td>550.6</td>
</tr>
</tbody>
</table>

100,000 km, but sometimes as far as 400,000 km. In these cases, the signal can be very weak. For all icy satellite observations, the RADAR operates in the scatterometer receiver mode, where the narrow receiver bandwidth of 117 kHz helps to minimize the thermal noise. To improve the signal-to-noise ratio (SNR), the radar transmits a pulsed monochromatic tone that can be easily bandpass filtered in the frequency domain to eliminate more of the background noise and obtain a more accurate measurement of the signal power. The noise variance is further reduced by averaging together the spectra of the many echoes collected from the same spot on the surface. Subsequent application of the radar equation produces a measurement of the radar cross section, and further analysis determines the 2 cm-\(\lambda\) radar albedo and backscatter properties of the visible area (Ostro et al. (2006, 2010); see also Section 4.6).

At distances approaching 100,000 km, the signal power is often strong enough to be redistributed into time delay (range) bins and Doppler bins for enhanced spatial resolution. The radar is commanded to transmit a chirped pulse, a linearly swept frequency modulated sinusoid, instead of a tone pulse, to enable pulse compression
for finer range resolution. Dozens of pulses are transmitted to sample the surface and enable Fourier analysis of the Doppler dispersion created by the rotating target. Of the 91 observations listed in Table C.1, 11 are distant observations with chirped transmissions that can be range-Doppler processed (Table 8.3). We form preliminary range-Doppler images of six of these in Section 8.3.

A few of the radar observations occur on targeted flybys at distances smaller than 45,000 km. At these altitudes, the RADAR instrument is close enough to implement the same high-altitude synthetic aperture radar (Distant-SAR or D-SAR) imaging technique that is used for distant imaging on Titan (West et al., 2009). Like the Titan D-SAR data, the icy satellite D-SAR data is collected using the scatterometer receiver mode, which affords longer echo window times and thus better Doppler resolutions, as well as a smaller noise-power bandwidth and thus better SNR than other receiver modes. The radar transmissions are chirped for pulse compression, and the data are quantized from 8 to 2 bits using a block adaptive quantization algorithm (8-2 BAQ) to increase the data capacity and, subsequently, the number of looks (see Section 3.5.1). The Cassini RADAR project applies the D-SAR processor on three of the targeted icy satellite flybys, forming images with resolutions of a few kilometers.

In this chapter, we process the D-SAR mode data in the real aperture sense. We measure the backscatter response of the observed surfaces and estimate the scattering parameters using traditional backscatter models. Where the beam footprint is small enough to substantially resolve the visible surface (i.e. where the projected antenna pattern covers 1-5% of the visible surface), we form real aperture radar (RAR) images. The RAR images complement the D-SAR images by emphasizing large-scale albedo changes and also filling in the D-SAR imaging blind spots. We use our backscatter analysis to correct the RAR and D-SAR images for incidence angle effects.

We develop two techniques to model the real aperture backscatter returns collected during D-SAR observations: one is optimized for observations centered over the same location on the target surface, called a *stare*, while the other is optimized for observations that scan the beam over the surface. Future work will apply the
stare modeling technique to the other 88 icy satellite observations, those collected in standard scatterometry mode, to obtain backscatter parameters that can be compared with those obtained by Ostro et al. (2006, 2010) and further complete their list, which stops with the Mimas observation Mi64 in April 2008.

The radar observations remaining in the Cassini Solstice mission, the extended-extended mission phase, include a flyby of Enceladus (En156 in November 2011) that will collect SAR, D-SAR, and distant scatterometry data, two observations of Dione from $\sim 50,000$ km and $\sim 200,000$ km away (Di163 and Di177, both in 2012), an observation of Rhea from $\sim 50,000$ km distance (Rh177 in 2012), and a final observation of Enceladus (En250 in 2016). These data promise to complement the radar coverage already obtained for these three moons, improving the analysis of the distribution of 2 cm-$\lambda$ albedo over their surfaces.

### 8.1.1 Icy Satellite Observation Naming Convention

There have been 34 icy satellite flybys containing active RADAR observations. These encounters occur for the eight significant icy moons of Saturn described in Table 8.1. In addition, six Titan flybys contain distant RADAR observations and are included in our icy satellite data collection because of the similarities in processing requirements. Together, these 40 encounters contain 91 observations, where we define an observation to be a collection of data centered on one location of the target surface, or a stare. In two cases, we extend the definition of an observation to include a collection of raster scanned data.

We identify each observation by the targeted moon’s abbreviated name and the formal orbit revolution number. For example, an observation of Enceladus on the 61st revolution of Cassini around Saturn is denoted En61 (note: the revolution 61 is actually the 62nd orbit, as the initial tour sequencing was modified after launch to correct a communications problem discovered with the Huygens’ probe - as a result, the revolution identifiers start a, b, c, 3, 4, and then continue to ascend numerically). Multiple observations might occur in a flyby when RADAR is assigned observation
times at different points in the orbital path, or multiple observations might occur when the antenna boresight is pointed at different spots on the targeted surface. In the first case, the observations are distinguished by hyphenating their names with their observation count index; e.g. the first observation on the En61 encounter is En61-1, and the second observation is En61-2. In the second case, when observations occur over the same section of the orbit, but where the antenna beam is directed to stare at different locations on the surface, the observation identifiers are appended with an ‘s’ and their “stare” count index. For example, the observational design for many of the Dione and Rhea flybys, when the radar antenna beam is often small enough to resolve subsections of the visible target, involves a stare at the sub-spacecraft point as well as stares at the four visible corners of the target disk. Each such encounter is thus decomposed into five individual observations, e.g. Rh18s1, Rh18s2, Rh18s3, Rh18s4, and Rh18s5. The multiple observations are listed separately in Table C.1 to indicate the different regions observed on each moon.

8.2 Real Aperture Processing of Icy Moons

Of the 91 icy satellite observations collected during the primary and extended Cassini missions, three are close enough to the target, and have the correct geometry, for D-SAR mode high-resolution (on the order of a few km) imaging: Ia49-3, En120s2, and Rh127s2. Because the SNR of the individual burst echoes is large, we can process these data in the real aperture sense, measuring a normalized radar cross section (NRCS or $\sigma^0$) for each burst footprint (on the order of 100 km wide). We use these results to model the surfaces’ backscatter characteristics.

En120s2 is similar to the other icy satellite observations in that the beam covers a large fraction of the visible surface (>20%) and is designed to stare at a single region. The other two observations, Ia49-3 and Rh127s2, are unique in that the spacecraft is so close to the target that the beam footprint is just a small fraction of the visible target surface area (1-5%). Consequently, the Ia49-3 and Rh127s2 D-SAR
images are constructed by steering the beam over the visible hemisphere in a raster scanning pattern. The raster scanning pattern provides full incidence angle diversity and yields complete backscatter responses that can be readily modeled. En120s2 also has some incidence angle spread because of the large flyby velocities that shift the effective look angle over the course of the stare. However, the En120s2 spread is limited to roughly ten degrees and so requires additional analysis to verify the backscatter model. The analysis technique developed for En120s2 is appropriate for all icy satellite stare observations and will be applied in future work. We analyze each of these three observations in turn, beginning with En120s2.

### 8.2.1 Enceladus (En120s2): November 2009

The seventh flyby of Enceladus occurred during Cassini’s 120th orbit around Saturn; we denote this observation En120. This was the first direct flyby through Enceladus’ plumes, with a closest-approach altitude of 103 km directly over the south polar region. Several instruments took turns observing the surface and the plumes of Enceladus during this observation, and RADAR was designated an observation slot from $\sim 100,000$ km altitude to $\sim 30,000$ km. This time was divided into three segments: above 77,000 km the instrument collected engineering calibration data, between 77,000 km and 72,000 km the antenna stared at the sub-spacecraft point near the equator in scatterometer mode (En120s1), and between 57,000 km and 30,000 km the antenna pointed off-nadir at a point around 25° south latitude and 320°-328° west longitude in D-SAR mode (En120s2), slowly drifting equator-ward for the last 2000 km. The image corresponding to the D-SAR segment, processed by the RADAR engineering team at JPL, is shown in Figure 8.1, with an optical image of the same area shown in the rightmost panel.

We process all of the collected En120 active data in the real aperture sense, measuring a normalized radar cross section (NRCS, or $\sigma^0$) for each burst. Because the ratio of observation distance to target diameter is especially large, a factor of ten larger than similar Titan observations, the curvature of the surface is more severe
Figure 8.1: The processed En120s2 D-SAR image is pictured on the left and the corresponding optical ISS mosaic is pictured on the right. The optical mosaic was released February 2010 by NASA / JPL / Space Science Institute. The D-SAR image magnitudes represent the NRCS corrected to 32° incidence; the backscatter model used for the correction is detailed in the text. Black magnitudes map to a $\sigma^0$ value of 0.2 and white magnitudes map to a $\sigma^0$ of 4.5. Several geologic features are identified in the D-SAR image: the craters Ma’aruf and Sabur are the two largest of the three visible craters, the trench Anbar Fossae cuts vertically from $\sim$10° S to $\sim$20° S near the 320° W parallel, and the parallel grooves Cashmere Sulci run latitudinally around 50° S and 320° W. The two images appear to be well correlated in brightness, except for a region just west of Cashmere Sulci and south of Ma’aruf, which appears darker to the 2 cm radar wavelength. The area adjacent to this dark-radar region, just east of it, also appears anti-correlated, but in the opposite sense: it is brighter at 2 cm-$\lambda$ and darker at optical wavelengths.

within the antenna beam. As a result, the local incidence angle changes rapidly over the illuminated area and we must carefully consider this change relative to the antenna power pattern. In Titan flyby observations, the slow variation of viewing geometry within the illuminated area allows us to localize the echo signal to an effective area centered around the boresight location, at a single incidence angle. The effective area is calculated such that the antenna gain is constant over the area and
zero without, as described in Section 3.1. This approximation greatly simplifies the processor computation requirements. The burst \( \sigma^0 \) derived from the effective area approximation accurately represents the average \( \sigma^0 \) value over the beam-illuminated area as long as the viewing geometry stays roughly constant across the beam; i.e., the set of illuminated incidence angles and spacecraft range values are well-approximated by their boresight values. This is true at the 1-2\% level for Titan observations that are pointed below \( \sim 65^\circ \) incidence; above this, the curvature of the illuminated surface becomes sufficiently large to increase the spread in the viewing geometry parameters beyond the single-angle approximation. For the En120 observation, the illuminated surface curvature is extreme in all bursts. The radius of Enceladus is only 252 km, so the half-power beam diameter varies from 100\% to 40\% of the target diameter as the spacecraft moves from 77,000 km to 30,000 km altitude. As a result, we cannot accurately apply the effective area approximation; we must map the full antenna pattern to the surface.

The real aperture processor that we use for the En120 data, as well as for the other icy satellite flyby data, is based on the following form of the radar equation:

\[
P_s = \frac{P_t G_t^2 \lambda^2}{(4\pi)^3} \sigma_{\text{avg}}^0 \int_{A_{\text{res}}} g(dA)^2 dA R(dA)^4.
\]  

where \( g \) is the gain of the antenna pattern at surface element \( dA \), normalized by the peak gain \( G_t \), and \( R \) is the range to that element. Eq. 8.1 is similar to Eq. 3.7 introduced in Section 3.1, but here we keep the range parameter \( R \) within the integral. The apparent beam-averaged NRCS, \( \sigma_{\text{avg}}^0 \), which we defined earlier in Eq. 3.6 and repeat in Eq. 8.2 for convenience, is the quantity desired for each radar burst:

\[
\sigma_{\text{avg}}^0 = \frac{\int_{A_{\text{res}}} g(dA)^2 \sigma^0(dA) dA}{\int_{A_{\text{res}}} g(dA)^2 dA R(dA)^4}.
\]

To solve for \( \sigma_{\text{avg}}^0 \), we need to first evaluate the integral in Eq. 8.1. We do this
8.2. REAL APERTURE PROCESSING OF ICY MOONS

numerically by creating a Cartesian grid across the target surface. We divide the full range of latitude and longitude into 0.5° bins and project the three parameters that form the integrand of Eq. 8.1 onto this grid (0.5° is sufficiently small to represent a constant parameter value, but not so small to make the computations unwieldy). The projected antenna gain $g$ is interpolated from the average beam profile, which we have calculated from 10 slices rotated uniformly about the center of the measured 2-D antenna pattern (see Appendix A). Thus, for each surface element, we know the antenna gain $g$, the surface area $dA$, and the range $R$. The product of these three parameters for a specific surface element determines the weight of that element, that is, how much it contributes to $\sigma_{\text{avg}}^0$. As an example, we superimpose the maps of these weights over ISS optical imagery (Figure 8.2) for the first and last burst of the En120s2 D-SAR stare segment (57,000 km and 32,000 km altitude). The peak weight stays within a few degrees longitude of the stare spot as the spacecraft travels 25,000 km closer. Meanwhile, the weights become increasingly localized around this spot such that the $\sigma_{\text{avg}}^0$ values measured for the end bursts better represent the reflectivity of the stare area and are less influenced by the surrounding areas.

We evaluate the integral in Eq. 8.1 by summing the weights across all of the surface elements in the grid; $\sigma_{\text{avg}}^0$ readily results. The measured $\sigma_{\text{avg}}^0$ can be significantly different than the $\sigma^0$ value expected at the given boresight incidence angle. As already mentioned, this is because the observation parameters vary widely over the illuminated surface and the combined weights emphasize reflections away from the boresight direction. This presents a complication when we wish to model and correct the variation of backscatter with incidence angle. For proper backscatter analysis, we need to know the effective incidence angle associated with the $\sigma_{\text{avg}}^0$ measurement. The effective incidence angle, $\theta_{\text{eff}}$, may be envisioned similarly to $\sigma_{\text{avg}}^0$ in Eq. 8.2, where we would replace the $\sigma^0$ of each surface element with the element’s incidence angle and calculate the weighted average incidence angle. However, there is an additional weighting that we need to consider, and that is the mean backscatter response of the surface, which is in fact the unknown function that we wish to characterize.
Figure 8.2: The upper panel is the total weight map \( (g^2 dA/R^4) \) for the first burst in the distant-SAR (D-SAR) segment of EN120s2 (57,000 km altitude). The lower panel is the total weight map for the last burst in the D-SAR stare segment (32,000 km). Both weight maps are superimposed over the ISS optical map released in February, 2010. The weights are normalized by their respective maximum values for ease of display; the max weight of the first D-SAR burst is 4.2e-25, and the max weight of the last D-SAR burst is 4.2e-24. Maximum weights are colored red and zero weights are dark blue. The uncolored area is not visible to the radar instrument.

8.2.1.1 Backscatter Modeling: Method 1

Most icy satellite surfaces scatter radiation back towards the radar diffusely, according to a cosine power law: \( a \cos(\theta_i)^n \) (Ostro, 1993; Ostro et al., 2010). This backscatter function describes how the scattered power is distributed on average over our gridded illuminated surface. For example, we show in the upper panel of Figure 8.3 the incidence angle variation over the visible surface for the first burst in the EN120s2 segment. Let us suppose that this surface is uniform and scatters according to the
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Figure 8.3: The upper panel is the visible incidence angle grid for the first burst in En120s2 (57,000 km altitude). The lower panel is the corresponding backscatter distribution for a model surface that follows the diffuse cosine power law, with $a = 2$ and $n = 1.5$. Both weight maps are superimposed over the ISS optical map released in February, 2010.

diffuse law with parameters $a = 2$ and $n = 1.5$, where these values are chosen arbitrarily. In this model, the Enceladus surface would appear as shown in the lower panel of Figure 8.3, where we have evaluated the assumed scattering model at each surface element.

The average backscatter measured for this model surface is the weighted sum of the distribution of backscatter shown in the lower panel of Figure 8.3, where the weights are determined by the observation parameters described previously:

$$
\sigma_{avg}^0 = \frac{\int_{A_{res}} g(dA)^2 a \cos(\theta_i(dA))^n dA}{\frac{1}{R(dA)^4} \int_{A_{res}} g(dA)'^2 dA} \frac{1}{\frac{1}{R(dA)^4} \int_{A_{res}} R(dA)^4}.
$$

(8.3)
Eq. 8.3 is the same as in Eq. 8.2, but we have replaced the $\sigma^0$ of each surface element with the solution of the scattering model evaluated at that element’s incidence angle. The parameters $a$ and $n$ represent the average backscatter response of the entire illuminated surface; the scattering parameters will actually vary about $a$ and $n$ for each surface element in the grid. For the model example, $\sigma_{\text{avg}}^0 = 1.12$, whereas the $\sigma^0$ value of the boresight surface element is actually closer to 1.74. The average measurement is lower than the boresight value due to the greater contribution of surface elements at higher incidence angles.

From the above discussion, we see that the variation of incidence angle over the beam contributes an additional factor in the overall average backscatter measurement, and this factor needs to be explicitly considered when evaluating the measurement’s corresponding effective incidence angle. We define the effective incidence angle of the average backscatter measurement as the angle that satisfies the mean backscatter function of the illuminated surface for the true value of the average backscatter measurement, i.e. the exact average value that would be measured if the signal was noise-free:

$$\theta_{\text{eff}} = \arccos \left( \left( \frac{\sigma_{\text{avg-true}}^0}{a} \right)^{1/n} \right),$$

(8.4)

where $\sigma_{\text{avg-true}}^0$ represents the true value of the average backscatter measurement, and $\sigma_{\text{avg}}^0$ is its stochastic estimate. In the model example presented in Figure 8.3, $\sigma_{\text{avg-true}}^0 = \sigma_{\text{avg}}^0$, and $\theta_{\text{eff}}$ then evaluates to 47.3°. This is the incidence angle that will reproduce the observed average backscatter value of 1.12 according to the assumed scattering model. The incidence angle along the boresight axis is actually 24.5°, thus the weighting from the observation parameters may indeed significantly change the expected result. Further consider the observation geometry of the last burst in En120s2. The boresight incidence angle of the last burst, 24.9°, remains similar to the first En120s2 burst, and the beam is still roughly centered on the same targeted stare location: (25.3° S, 323.2° W) versus (25.1° S, 319.8° W) for the first and last D-SAR bursts respectively. However, because the footprint size is smaller in the last D-SAR
burst, the total weighting is concentrated over a smaller range of incidence angles and the measured average backscatter value can be expected to converge towards the boresight value. We measure an average backscatter value of 1.42 and an effective incidence angle of 37.3° for the last D-SAR burst.

If the scattering parameters \(a\) and \(n\) are known, and the average backscatter value can also be measured exactly, the calculation of \(\theta_{\text{eff}}\) is straightforward. For the real Enceladus surface, \(a\) and \(n\) can be readily estimated, but the inherent error in the average backscatter measurement prevents the direct inversion used in Eq. 8.4. Rather than evaluate \(\theta_{\text{eff}}\) for each burst one at a time, we consider the average backscatter behavior over all bursts in the observation. This strategy involves an iteration over different \((a, n)\) pairs. For a particular \((a, n)\) pair, we compute the expected \(\theta_{\text{eff}}\) for each burst by calculating \(\sigma_{\text{avg-true}}^0\) from the burst’s gridded map of incidence angles and applying Eq. 8.4. We then create a backscatter plot \((\sigma_{\text{avg}}^0 \text{ vs. } \theta_{\text{eff}})\) over all of the bursts and analyze the average backscatter response. We solve for the best-fit model parameters and compare them to the assumed parameters. We repeat this procedure for all physically possible pairs of \(a\) and \(n\), using increments of 0.01 for each, until the apparent average backscatter response exhibits the same scattering behavior as was initially assumed. The assumed amplitude \(a\) actually cancels out in Eq. 8.4 (because \(a\) does not vary with \(dA\) it can be separated from the integral in Eq. 8.3, thus \(\sigma_{\text{avg-true}}^0\) equals \(a\) times the integral over \(\cos^n(\theta_1)\)), so it is only the \(n\) parameter behavior that we need to match.

The \((a, n)\) fitting results are plotted against the assumed-\(n\) values in Figure 8.4A: the fitted-\(n\) are in black and plotted according to the left y-axis, and the fitted-\(a\) are in red and plotted according to the right y-axis. Each \((a, n)\) pair maps to an assumed-\(n\) value, which is mapped to the x-axis. The dashed line has a slope of unity, according to the left y-axis, and represents the line that the measured-\(n\) would follow if it perfectly matched the assumed-\(n\) at every point. We fit a line to the fitted-\(n\) versus the assumed-\(n\) pairs and find where the line converges with the assumed-\(n\) line. The two lines converge at \(n=1.23\) and we choose this value as our final \(n\) result,
Comparison of $E_{120s2}$ Model Fit Results to Initial Assumption

Converges at: $n=1.23$

Parameter relationship: $a = 0.46n + 2.94$

Best-Fit Solution: $(a=3.51, n=1.23)$

Figure 8.4: Determination of backscatter model parameters $a$ and $n$. The effective incidence angle of a burst measurement depends on how the surface reflectivity changes with incidence angle over the beam footprint. We assume the surface scatters according to a diffuse cosine power law and vary the model parameters until the measured backscatter response matches the assumed backscatter behavior. The angle dependence is really a function only of the parameter $n$, so this is the parameter that is matched. We model the relationship between the measured $n$ and $a$ parameters with a first-order polynomial to find the value of $a$ that pairs with the matched $n$ value.

as well as the set of $\theta_{\text{eff}}$ that go with this value. To determine an accurate value for the $a$ parameter that pairs with the matched $n$ value, we fit a line to the $(a, n)$ fitting results and evaluate the line equation at the finalized $n$ value. By evaluating the fitted line equation, we minimize the scatter inherent to the fitting procedure. This technique is illustrated in Figure 8.4B and yields $a = 3.51$. We report the 95% confidence intervals for the fitting parameters in our table of results (Table 8.2). The confidence intervals are calculated from the fit residuals and Jacobian matrix using MATLAB’s `confint()` function.

The corresponding backscatter response and model curve for the best-fit scattering
Figure 8.5: The backscatter data from the modeled D-SAR stare region are plotted in gray together with the best-fit En120s2 diffuse model. The non-stare En120 data are plotted in black, with the En120s1 scatterometry data occurring at larger effective incidence than the stare data and the En120 drift data (after En120s2, the antenna beam drifts back towards the equator) occurring at smaller effective incidence than the stare data. Including the non-stare data in the backscatter analysis would yield higher $a$ and $n$ parameter values, as described in the text. That model would describe an average backscatter response, somewhere between the true equatorial region’s response and the true stare region’s response. The model we solve for here represents solely the stare region.

parameters ($a = 3.51$, $n = 1.23$) are shown in Figure 8.5. The D-SAR stare points are plotted in gray and represent the backscatter response from the stare region centered around (25° S, 323° W); these are the data that we model in the above discussion. The black points are the non-stare D-SAR points, which occur at lower effective incidence and closer to the equator. Also plotted in black, at higher effective incidence, are the scatterometry points, which also lie closer to the equator. The backscatter from the area north of the stare region (25° S, 323° W) appears to have a different, and brighter, backscatter response. Indeed, if we include the non-stare points in our model.
analysis, we measure $a = 3.70$ and $n = 1.46$. This total backscatter model describes an average of the terrain north of the stare region and the stare region itself. We choose to focus on the stare area points by themselves to better characterize the stare region rather than characterize an average of two regions.

The set of $\theta_{\text{eff}}$ derived for $n = 1.23$ are stored with their bursts’ average backscatter measurements, $\sigma^0_{\text{avg}}$, so that the measurements can be properly analyzed and also corrected for incidence angle effects. For example, the D-SAR image pictured in Figure 8.1 represents the backscatter after normalizing by the best-fit diffuse scattering model, such that incidence angle effects are minimized.

We apply our albedo equation to the model results (see Section 4.6) and obtain an SL-2 (same-sense linear polarization at 2 cm-$\lambda$) albedo of $3.14 \pm 0.08$ and a total albedo of TP-2 = $4.77 \pm 0.40$. These radar albedo measurements are extremely high, much higher than the radar albedo measurements of other bright solar system bodies. We discuss the implication of these results in Section 8.2.4.

### 8.2.1.2 Backscatter Modeling: Method 2

The backscatter values from the En120s2 stare points are spread over a small range of effective incidence angle. To verify that the derived model solution is not affected by this limited spread, we also estimate $a$ and $n$ with a separate, yet similar approach. Iterating over the same set of ($a$, $n$) pairs, we again solve for the expected “true” average backscatter using Eq. 8.3. Substituting this solution into the right side of Eq. 8.1 yields the receive power expected for a surface that follows the assumed scattering model, $P_s|_{(a,n)}$. We compare this estimate with the actual measurement of received power $P_s$ and solve for the model parameters that minimize the difference error:

$$\min \left( |P_s - P_s|_{(a,n)}| \right).$$  

(8.5)

This is equivalent to minimizing the difference between the measured average NRCS and the expected average NRCS: $\min(|\sigma^0_{\text{avg}} - \sigma^0_{\text{avg-true}}|)$. Because of the uncertainty
inherent to $P_s$, or $\sigma_{\text{avg}}^0$, we have the same situation as before: we must solve for the best fit average parameters over a set of bursts rather than one burst at a time. Thus, we accumulate the absolute difference errors for all bursts in the observation and find the model parameters that minimize the sum of the errors. The parameter pairs that yield the smallest cumulative errors are presented in two ways in Figure 8.6. The pair solutions are colored by their normalized error in the upper panel, whereas the lower panel portraits the same information over three separate axes. The relationship between the model parameter pairs is well-described by the first-order polynomial $a = 0.84n + 2.47$, and the cumulative model error $E_{\text{cum}}$ is best described by the quadratic polynomial $E_{\text{cum}} = 0.31a^2 - 2.17a + 4.43$. The minimum error solution of the quadratic polynomial occurs for $a = 3.51$ and $n = 1.23$, the same results derived with the backscatter curve modeling technique. In general, for the other 88 icy satellite observations, we do not have enough spread in effective incidence angle to consider using the first method, where we model the measured backscatter curve, so we are limited to using the second method, where we minimize the error between expected and measured receive powers assuming different backscatter parameters. In future work, we will apply the second method to the rest of the icy satellite observations to complete the backscatter analysis of the eight moons.

The two different methods produce the same results as long as the modeled data are derived from the same stare region. For example, if we consider the non-stare points in our backscatter curve method, we derived $(a = 3.70, n = 1.46)$. If we consider the non-stare points in the second method, we derive $(a = 3.67, n = 1.43)$. In this case, the parameter pair solution from the second method is slightly smaller than that measured from the first method, and is in fact near the latter’s lower confidence interval bound. We attribute this difference to the individual weighting procedures used in each fitting method. In the first method, the model is fit to the mean backscatter response, not the individual data points (i.e. the backscatter values are averaged over $0.5^\circ$ angle bins, the same approach we use for Titan modeling in Section 4.5). This approach ensures that all angle data are considered equally, rather
Figure 8.6: The parameter pairs that yield the smallest cumulative errors using the second modeling method are presented here in two ways. In A, the pair solutions are colored by their normalized error in the upper panel and illustrate the linear relationship between the two parameters $a$ and $n$. In B, the pair solutions are plotted along parallel abscissas ($a$ on the bottom, $n$ on the top) against the normalized error along the ordinate. The latter plot illustrates the quadratic dependence of the error on the model parameters. We model the quadratic relationship and evaluate the polynomial at its minimum to find the best-fit parameter pair.

than emphasizing the angles where the bulk of the data exists, e.g. 3951 bursts exist between the $37^\circ$ to $47^\circ$ range of angles for the En120s2 stare, whereas the other 1160 bursts occur at lower and higher angles from regions closer to the equator. The second method, alternatively, weights each measurement by its individual error value. Thus, the results of the second method will be dominated by the surface backscatter characteristics where most bursts are concentrated, and the results of the first method will better represent the average backscatter characteristics of the different terrains. For a mixed set of data, the first method would better characterize the average scattering behavior.
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8.2.1.3 En120s2 Calibration

The above results all assume a calibration system temperature, $T_{\text{sys}}$, of 850 K. This number relies on a receive temperature value near 809 K, as characterized in Section 3.4, and an antenna temperature near 40 K. The antenna temperature depends on the ratio of target area to cold sky area illuminated by the antenna beam, an amount that varies with the spacecraft distance. The $\sim 40$ K antenna temperature is only achieved at distances when the target fills the beam. At farther distances in the En120 observation, the antenna temperature is closer to 30 K. To appreciate the effects of the system temperature value on the results, we repeat the above analysis for $T_{\text{sys}} = 840$K. The lower system temperature reduces the $\sigma^0_{\text{avg}}$ measurements by 1.2%, and the model parameter $a$ by about the same amount, i.e. the first-order polynomial fit to the $T_{\text{sys}} = 840$K ($a$, $n$) pair solutions is a simple downward shift of 1.2% from the $T_{\text{sys}} = 850$ K parameter pair line fit. The model parameter $n$ is unaffected by the change in $T_{\text{sys}}$, since it is the level of the backscatter response, not the shape, that is modified.

We keep the calibration system temperature constant for all bursts, in spite of the variation in the measured antenna temperature, for the primary reason that we only have only one measurement of the noise level $P_{n,\text{dW}}$. We measure $P_{n,\text{dW}}$ through on-target receive-only measurements at the beginning of the En120 observation, finding $P_{n,\text{dW}} = 365.97\text{dW}$. As described in Chapter 3, the noise level measurement is used to determine the signal-only power level $P_{s,\text{dW}}$ from the total receive power measured $P_{r,\text{dW}}$ (see Eq. 3.12), but it is also used to calculate a calibration scale factor from the ratio of $T_{\text{sys}}$ to $P_{n,\text{dW}}$ (see Eq. 3.43). Since we only have the single $P_{n,\text{dW}}$ measurement, we elect also to keep $T_{\text{sys}}$ constant near its full value. This is the same procedure we use for Titan observations, and the effects on the final results are minimal, as discussed in the previous paragraph.
CHAPTER 8. RADAR OBSERVATIONS OF SATURN'S ICY SATELLITES

8.2.2 Iapetus (Ia49-3): September 2007

Iapetus is much further from Saturn than most of the other icy satellites, almost three times as far as Titan. The large distance, combined with its highly inclined orbit, creates fewer opportunities for observation by Cassini’s instruments. Ia49 is the only close targeted flyby of Iapetus in the Cassini mission, approaching the mysterious moon within 1640 km (compare to the 2004 flyby Ia0b, which did not get closer than 123,000 km). With a slow relative velocity of 2.4 km/s, the Ia49 flyby presents much more time for data collection than the usual icy satellite flyby. As such, RADAR was assigned four time slots at different points in the orbit. The first three time slots occur on the inbound approach, when the dark leading hemisphere of Iapetus, an area known as Cassini Regio, is visible. Ia49-1 occurs at a mean distance of 244,000 km and involves a simple stare at the sub-spacecraft point centered within Cassini Regio. Ia49-2 occurs at a closer distance, around 100,000 km, where the beam spot size is around 20% of the visible surface area, and incorporates stares at the four corners of Cassini Regio, as well as the sub-spacecraft point. The parallel to Ia49-2 on outbound is Ia49-4, when the bright trailing hemisphere is visible. The most interesting of the four Ia49 observations is Ia49-3, which occurs near 20,000 km, with an antenna spot size only 1-2% of the target surface area. Ia49-3 is the closest observation of all the RADAR icy satellite observations.

Ia49-3 occurs on the inbound leg of the flyby with a sub-spacecraft point centered at (10° N, 72° W). To increase surface coverage, the RADAR instrument scans the antenna beam longitudinally in a east-to-west raster pattern over the entire visible disk, an area confined to latitudes between 60° S and 75° N and longitudes between 0° W and 150° W. This pattern is illustrated in Figure 8.7A. The spacecraft altitude decreases from 23,600 km during the eastern-most acquisition to 16,900 km during the western-most acquisition, altitudes that are ideal for D-SAR imaging. The D-SAR image, processed by the RADAR engineering team at JPL, is shown in Figure 8.8, with an optical image of the same area shown in the rightmost panel. The D-SAR image reveals the dark hemisphere of Iapetus at resolutions close to 2 km by 6 km.
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Figure 8.7: The raster scan pattern is shown in the upper panel, where a point is plotted at the boresight center of each burst, and each point is colored according to its burst index. Thus, the first bursts scan the east side of the map, and the last bursts scan the west side. The altitude changes by about 6700 km over the 4832 D-SAR bursts. The lower panel shows the real aperture image that we form from the D-SAR data, where we have corrected for incidence effects using our backscatter analysis, as discussed in the text.

While the entire visible disk is scanned, only a portion of the scan is favorable for imaging, the portion where the angle between the iso-range and iso-Doppler lines is greater than about 30° (West et al., 2007). Since the spacecraft is moving roughly from east to west during the observation, there is a strip along the equator for which this criterion is not met and there is subsequently a gap in the image.

Dominating the D-SAR image is the dark albedo region Cassini Regio (CR). Cassini Regio is centered on the apex of orbital motion (90° W) at latitudes less than ~45° and is one of the optically darkest surfaces known in the solar system, with an optical albedo between 0.03 and 0.05. Cassini Regio is about 10x optically darker than the bright trailing hemisphere, the largest such asymmetry in the solar
Figure 8.8: The processed Ia49-3 D-SAR image is pictured on the left and the corresponding Cassini optical ISS mosaic is pictured on the right. The ISS mosaic was released October 2008 by NASA / JPL / Space Science Institute. The D-SAR image is corrected for incidence angle effects using the model described in the text. The black gap across the diagonal of the image follows the spacecraft nadir track and represents geometry that cannot be imaged in D-SAR mode. Black magnitudes map to a $\sigma^0$ value of 0 and white magnitudes map to a $\sigma^0$ of 0.8. The bright polar terrain is part of the bright albedo feature named Roncevaux Terra, a feature that covers the northern half of the trailing hemisphere. The bright polar terrain is purposefully saturated in the D-SAR image shown here to improve the contrast of the dark region called Cassini Regio. Several geologic features are identifiable within Cassini Regio: the big crater Falsaron is visible just north of center, and the northern part of the neighboring big crater to the east, Turgis, is also visible. Several smaller craters are notable: Dapamort is the crater near the center of Falsaron, Valdebron is the bright crater on the western edge of Falsaron, and Climborin and Margaris are consecutively westerward of Valdebron.

system (Squyres et al., 1984). The bright terrain has an optical albedo between 0.5 and 0.6 and is divided into two sections based on coloring: Roncevaux Terra (RT) is the bright terrain at northern latitudes, and Saragossa Terra (ST) is the bright terrain at southern latitudes, where ST has a slightly reddish color and RT is more whitish. Optical images from both Voyager and Cassini reveal that the albedo dichotomy is
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not evenly divided between the hemispheres; the bright material of the trailing hemisphere stretches over the poles, such that the dark leading hemisphere is reminiscent of an inverted oreo cookie (Porco et al., 2005). We analyze the backscatter from CR and the polar part of RT separately.

We measure the real aperture average backscatter \( \sigma_{avg}^0 \) for the D-SAR bursts in Iap49-3 following the approach described for En120s2: we numerically evaluate the integral in Eq. 8.1 by creating a 0.5° by 0.5° Cartesian grid across the target surface and projecting the three parameters that form the integrand of Eq. 8.1 onto this grid. The product of the three parameters, antenna gain \( g \), the surface area \( dA \), and the range \( R \), at a specific 0.5° by 0.5° surface element determines the weight of the element, or its relative contribution towards \( \sigma_{avg}^0 \). As an example, we superimpose the maps of these total weights over ISS optical imagery (Figure 8.9) for the first and last burst of the Ia49-3 D-SAR stare segment (23,890 km and 17,300 km altitude).

To appreciate the nearness of the Ia49-3 observation and the improved resolution of the real aperture footprint, compare the sizes of these total weight footprints to those pictured for En120s2 in Figure 8.2.

Similar to En120s2, we evaluate the integral in Eq. 8.1 by summing the weights across all of the surface elements in the grid; \( \sigma_{avg}^0 \) then readily results. As discussed previously, the observation parameters can vary widely over the illuminated surface and the combined weights can pull the emphasis away from the boresight direction, such that the incidence angle of the boresight axis is not the correct incidence angle to associate with \( \sigma_{avg}^0 \). For proper backscatter analysis, we measure the effective incidence angle \( \theta_{i,\text{eff}} \) for each burst by following the technique described in Section 8.2.1.1.

The raster scanning pattern of Ia49-3 generates much greater incidence angle diversity than En120s2, increasing the accuracy of the backscatter curve modeling method described in Section 8.2.1.1. In fact, the power matching method described in Section 8.2.1.2 will not work as well for scanned observations as it does for stare observations because of the nonuniform distribution of bursts over the surface and the subsequent unequal weightings applied. Future work might improve the application
CHAPTER 8. RADAR OBSERVATIONS OF SATURN’S ICY SATELLITES

Figure 8.9: The upper panel is the total weight map \( g^2 dA/R^4 \) for the first burst in the distant-SAR (D-SAR) segment of Ia49-3 (23,600 km altitude). The lower panel is the total weight map for the last burst in the D-SAR stare segment (16,900 km). Both weight maps are superimposed over the ISS optical map released in May 2008. The uncolored area is not visible to the radar instrument. The weights are normalized by their respective maximum values for ease of display; the max weight of the first D-SAR burst is 1.2e-22, and the max weight of the last D-SAR burst is 4.2e-22. Maximum weights are colored red and zero weights are dark blue.

of the second method to scanned observations by deriving the correct individual burst weights.

Before applying the backscatter model of Section 8.2.1.1, we consider that the dark terrain of CR scatters much differently than that of the polar RT terrain. Further, we note the big craters Falsaron and Turgis appear brighter than the surrounding CR terrain in the D-SAR image. We separate the data over CR from the data over RT and the large crater terrain to improve the accuracy of the CR model. Because CR comprises the majority of the covered area, it is important to model this terrain well so that we can properly correct the D-SAR image, as well as the real aperture image
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Figure 8.10: The real aperture radar (RAR) image of Ia49-3 is shown on the left and the corresponding ISS optical image is shown on the right. Superimposed on both are the Cassini Regio (CR) and Roncevaux Terra (RT) outlines that we use to filter out the feature data for our backscatter analysis.

that we form from our backscatter measurements. The boundary that defines the set of CR bursts under study is outlined in black in Figure 8.10. We also wish to analyze the backscatter from the polar RT region to quantify its brightness relative to CR; the RT boundary is outlined in white in Figure 8.10.

When we apply the model method of Section 8.2.1.1 to CR, we retrieve model parameters $a = 0.41 \pm 0.01$ and $n = 1.81 \pm 0.04$ (Table 8.2). Figure 8.11 shows the method results. We repeat the procedure for the RT polar terrain and measure $a = 0.55 \pm 0.03$ and $n = 1.08 \pm 0.09$. The backscatter plots of the two regions are pictured together with their backscatter model curves in Figure 8.12, where the diffuse CR model is shown in blue and the diffuse RT model is in red. This plot illustrates that the diffuse model fails to describe an observed increase in CR backscatter at
incidence angles less than $\sim 20^\circ$. This is the only icy moon other than Titan whose backscatter indicates the presence of a small quasispecular term (about 3% of the total radar albedo). This quasispecular response has not been previously detected in distant radar spectra of this moon (Black et al., 2004; Ostro et al., 2006), perhaps due to limited frequency resolution.

We introduce the Hagfors quasispecular model to describe the increase in low-angle CR backscatter, although the exponential quasispecular model works equally well. The quasispecular models suggest that the effective dielectric constant of the CR material is low ($\epsilon = 1.49 \pm 0.16$ for Hagfors' law, and $\epsilon = 1.39 \pm 0.17$ for the
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Figure 8.12: The backscatter data for Cassini Regio (CR) are shown as black dots and the backscatter data for the polar region of Roncevaux Terra (RT) are shown as red dots. CR and RT are defined in Figure 8.10. The best-fit diffuse backscatter model for CR is the blue line and the best-fit diffuse backscatter model for RT is the red line. An additional quasispecular model is required to match the increase in CR backscatter at low angles; we show the best-fit Hagfors model in gray.

exponential law), while the facet-scale roughness is moderate \(s = 10.6^\circ \pm 2.4^\circ\) for Hagfors, and \(s = 17.5^\circ \pm 4.5^\circ\) for the exponential). Note that the lack of calibrated data below \(5^\circ\) limits the interpretation of the CR model results. Lower angle data do exist, in the form of nine altimetry mode bursts collected at the center of the Ia49-3 scan, but the absence of coincident noise measurements prevents us from calibrating the altimetry data well. If we apply the same calibration constant that we use for Titan altimetry data, which should be a reasonable estimate of the true calibration constant considering the similar brightness temperatures of the two bodies, then we observe the altimetry backscatter to rise sharply towards nadir, with \(\sigma_0^{\text{avg}}\) equal to about 0.8 at \(1^\circ\) incidence. To model the nadir increase, we require the superposition
of an additional quasispecular law, where the first quasispecular law captures the sharp peak in backscatter below 5° and the second quasispecular law captures the gradual rise from 25° to 5° incidence. As discussed in Section 4.4, that the data necessitate a combination of quasispecular models to accurately describe its variations with incidence angle implies that a single assumed slope distribution is inadequate, i.e. a different slope distribution is required to describe the shallower facets than is needed to describe the rougher facets. The superposition of multiple slope distributions within the radar resolution cell may be the result of different processes acting to shape the surface at different scales.

Quasispecular backscatter, as described in Section 4.2, originates from large-scale facets that are tilted towards the radar according to a slope distribution. The presence of a small quasispecular term in the Iapetus CR backscatter response indicates that the surface interface is partially detectable to the radar; i.e. it is smooth enough and compact enough to reflect 2 cm-λ radiation. As a result, not all of the transmitted energy penetrates through to the volume, corroborating the interpretation of the dark CR terrain as a poorer volume scattering medium. However, volume scattering still appears to dominate the CR backscatter response, making Iapetus’ CR surface similar to many of the detected Titan surfaces.

As detailed in Chapter 5, Titan features have varying degrees of quasispecular reflections that contend with large volume scattering components. We compare the modeled Titan feature backscatter responses to that of CR in Figure 8.13. Iapetus’ CR has a diffuse backscatter shape that mimics that of the brightest (Xanadu) and the darkest (dune) terrains on Titan, but CR’s brightness level exists between the two Titan extremes (Xanadu has a SL-2 radar albedo of 0.82, although its bright core has an albedo greater than unity, and the Titan dune terrain have a SL-2 radar albedo of 0.16; CR has an SL-2 albedo of 0.29, as discussed next). The CR quasispecular component pictured appears small relative to those of the Titan features, but because we are missing well-calibrated very low angle CR data, we cannot determine whether this is a valid interpretation; yet, note that a preliminary fit to the near-nadir CR
altimetry data suggests that it has a shape similar to that of Xanadu. Based solely on the shape of their backscatter responses, Cassini Regio appears like an attenuated version of Titan’s Xanadu terrain. The similarities suggest the presence of similar scattering mechanisms on the different moons: perhaps CR has Xanadu-like terrain underneath a dark attenuating layer, or perhaps CR is composed of a material that is more absorptive at 2.2 cm wavelengths but is otherwise structurally similar to Xanadu. These are only theories, which we cannot further investigate without acquiring more data.

Applying our albedo equations (Section 4.6) to the CR model results yields a same-sense linear polarization albedo of $SL_2=0.29 \pm 0.01$ and a total-power albedo of $TP_2=0.44 \pm 0.04$. For the polar region of RT, we obtain $SL_2=0.53 \pm 0.05$ and $TP_2=0.81 \pm 0.07$, which is only 1.8x brighter than CR. The 2 cm-$\lambda$ albedo dichotomy is substantially less than the 10x optical asymmetry, but is still greater than the 1.3x asymmetry observed at 13 cm-$\lambda$ (Black et al., 2004). The decrease in albedo asymmetry with increasing wavelength suggests that the dark layer may be only a
few centimeters thin (Black et al., 2004; Ostro et al., 2010). These results are further discussed in Section 8.2.4.

We create a real aperture radar (RAR) image of the Ia49-3 region by mapping our $\sigma_{\text{avg}}^0$ measurements to their respective beam footprints (using the circular effective beamwidth, see Appendix A). We first normalize the measured $\sigma_{\text{avg}}^0$ by the best-fit CR backscatter model and then scale by the model’s value at 32° incidence to show the $\sigma_{\text{avg}}^0$ corrected to 32° incidence, with any angle effects removed in the process. This result is projected over the ISS optical map in the lower panel of Figure 8.7, and is also pictured in Figure 8.10. We use the same approach to correct the D-SAR image for incidence angle effects, with the result pictured in Figure 8.8. The RAR image fills in the D-SAR image’s equatorial gap and also better emphasizes large-scale albedo differences across the CR terrain. For instance, the crater Falsaron, north of center, stands out more clearly in RAR as being a brighter surface than the rest of CR. Similarly, the neighboring big crater to the east, Turgis, is not visible in the D-SAR image, but shows some brightening effects near its eastern rim in the RAR image. There is also an area just south of the equator in the western half of the RAR image, within the D-SAR image’s blind spot, that appears slightly darker than the rest of CR.

8.2.2.1 Ia49-3 Calibration

In addition to D-SAR and altimetry data, there are several small segments of standard scatterometry collected during the Ia49-3 observation. The D-SAR data uses the 8-2 BAQ algorithm to increase the efficiency of the data collection, but this has the undesirable effect of precluding the use of noise-only data at the margins of the received burst echo for calibration (see Section 3.5). The scatterometry bursts, which are quantized to 8 bits straight and thus avoid the pitfalls of BAQ, are inserted into the scan during the slow moving turn around points, where there is collection time to spare. In this manner, the scatterometry bursts provide the noise floor estimate (we measure $P_{\text{ndW}} = 370 \pm 7dW$) necessary for calibrating the D-SAR data. The
calibration system temperature that we use is the sum of the previously measured 809 K receiver temperature (for the 8.4 dB attenuation setting, see Section 3.4) and the measured Ia49-3 antenna temperature of 80 K (Janssen and Le Gall, pers. comm., April 2011).

8.2.3 Rhea (Rh127s2): March 2010

On the 128th orbit around Saturn (Rev127), the Cassini spacecraft performed its closest targeted flyby of Rhea, Saturn’s second largest moon, approaching within 100 km of the surface. The RADAR observation begins at 65,100 km altitude with 688 scatterometry bursts (Rh127s1) staring at the sub-spacecraft point (0° N, 164° W). At 55,000 km altitude, RADAR switches to D-SAR mode (Rh127s2), sweeping the antenna beam almost latitudinally across the visible anti-Saturn hemisphere, an area confined to latitudes between 64° N and 56° S and longitudes between 107° W and 224° W. The raster pattern continues, from north to south and than back up to the north, until the spacecraft reaches an altitude of 25,000 km, obtaining 6579 burst echoes along the way. This raster scan is illustrated in the upper panel of Figure 8.14, where blue denotes the first D-SAR burst and red denotes the last D-SAR burst. The D-SAR image is processed by the RADAR engineering team at JPL and is shown in Figure 8.15, with an optical image of the same area shown in the rightmost panel.

We process the D-SAR data in its real aperture sense, measuring the average backscatter \( \sigma^0_{\text{avg}} \) of each burst together with the corresponding effective incidence \( \theta_{\text{eff}} \) angle as we do in Section 8.2.1. The total weight grids used in the calculations of the first and the last burst of the Rh127s2 D-SAR stare segment (55,000 km and 25,000 km altitude) are superimposed over ISS optical imagery (Figure 8.16). The footprint sizes of the ending bursts are comparable to the footprint sizes in Ia49-3 (see Figure 8.9); compare the sizes of these total weight footprints to those pictured for En120s2 in Figure 8.2.

As discussed with the Ia49-3 observation, the raster scanning pattern generates greater incidence angle diversity, improving the accuracy of the backscatter curve
Figure 8.14: The raster scan pattern is shown in the upper panel, where a point is plotted at the boresight center of each burst, and each point is colored according to its burst index. The first burst is colored blue, and the last burst is colored red. The scan moves from north to south and then back up to the north, obtaining overlapping coverage of the visible anti-Saturn disk. The altitude changes by about 29,800 km, more than 50% of the starting distance, over the 6,579 D-SAR bursts. The lower panel shows the real aperture image that we form from the D-SAR data, where we have corrected for incidence effects using our backscatter analysis, as discussed in the text.

The modeling method described in Section 8.2.1.1. The power matching method described in Section 8.2.1.2 does not work as well for the scanned observations as it does for stare observations because of the nonuniform distribution of bursts over the surface and the subsequent unequal weightings applied.

When we apply the model method of Section 8.2.1.1 to Rh127s2, we retrieve model parameters $a = 2.14 \pm 0.01$ and $n = 1.35 \pm 0.02$ (Table 8.2). Figure 8.17 shows the method results. The backscatter plot of the the Rh127s2 data is pictured together with its backscatter model curve in Figure 8.18. Applying our albedo equations (Section 4.6) to the Rh127s2 results yields a same-sense linear polarization albedo of
Figure 8.15: The processed Rh127s2 D-SAR image is pictured on the left and the corresponding Cassini optical ISS mosaic is pictured on the right. The optical mosaic, released May 2009, was created by Steve Albers at NOAA by reprojecting Cassini ISS images produced by NASA / JPL / Space Science Institute over Voyager images. The D-SAR image magnitudes represent the NRCS corrected to 32° incidence; the backscatter model used for the correction is detailed in the text. Black magnitudes map to a $\sigma^0$ value of 0.2 and white magnitudes map to a $\sigma^0$ of 4.5. The black gap across the diagonal of the image follows the spacecraft nadir track and represents geometry that cannot be imaged in D-SAR mode. The only geologic features readily identified in the D-SAR image are craters: Tirawa is the large crater north of center, and it is surrounded by many smaller craters. While most of the features visible at 2 cm-λ are readily matched to their optical counterparts, the brightness variations are difficult to compare, largely due to the smearing within the D-SAR image.

SL-$2=1.82 \pm 0.03$ and a total-power albedo of TP-$2=2.77 \pm 0.24$. These results are further discussed in Section 8.2.4.

We create a real aperture radar (RAR) image of the Rh127s2 region by mapping our $\sigma^0_{\text{avg}}$ measurements to their respective beam footprints (using the circular effective beamwidth). The RAR image is corrected to 32° incidence using the backscatter model shown in Figure 8.18. The correction procedure involves normalizing the
Figure 8.16: The upper panel is the total weight map ($g^2 dA/R^4$) for the first burst in the distant-SAR (D-SAR) segment of Rh127s2 (54,850 km altitude). The lower panel is the total weight map for the last burst in the D-SAR stare segment (25,100 km). Both weight maps are superimposed over the ISS optical map released in February 2010. The weights are normalized by their respective maximum values for ease of display; the max weight of the first D-SAR burst is 2.9e-24, and the max weight of the last D-SAR burst is 1.1e-22. Maximum weights are colored red and zero weights are dark blue. The uncolored area is not visible to the radar instrument.

backscatter values by the predicted model values, and the residual is scaled by the model solution evaluated at $32^\circ$. The angle-corrected result is projected over the ISS optical map in the lower panel of Figure 8.14. We use the same model to correct the D-SAR image for incidence angle effects, with the result pictured in Figure 8.15. The RAR image fills in the D-SAR image's equatorial gap and also better emphasizes large-scale albedo differences across visible region. For instance, the old cratered terrain visible optically on the eastern side of the imaged area appears about 50% brighter to RADAR than the terrain near the center of the RAR image, suggesting a much rougher surface at 2 cm-$\lambda$. Interestingly, the western side of the RAR image
Comparison of Rh127s2 Model Fit Results to Initial Assumption

Converges at: $n = 1.35$

Parameter relationship: $a = 0.21n + 1.85$

Best-Fit Solution: $(a=2.14, n=1.35)$

Figure 8.17: Determination of backscatter model parameters $a$ and $n$. The effective incidence angle of a burst measurement depends on how the surface reflectivity changes with incidence angle over the beam footprint. We assume the surface scatters according to a diffuse cosine power law and vary the model parameters until the measured backscatter response matches the assumed backscatter behavior. The angle dependence is really a function only of the parameter $n$, so this is the parameter that is matched. We model the relationship between the measured $n$ and $a$ parameters with a first-order polynomial to find the value of $a$ that pairs with the matched $n$ value.

also shows some brightening effects in the RAR image, even though this is an area that appears darker at optical wavelengths. Perhaps there is a change in composition at decimeter depths in the western area that enables enhanced volume scattering. On the other hand, the radar brightness decreases in the north, and appears consistent with a subtle darkening at optical wavelengths. While the large crater Tirawa, north of center, is clearly pictured in the D-SAR image, the RAR image emphasizes the lack of brightness contrast with its surrounds. The similarly sized crater Mimaldi, just south-west of Tirawa, occurs in the D-SAR imaging blind spot and is not visible at high-resolution, and, like Tirawa, it also does not demonstrate enough brightness
contrast to appear in the RAR image. Apparently, at decimeter depths, the terrain beneath the large craters is not much different than the nearby anti-Saturn subsurface.

### 8.2.3.1 Rh127s2 Calibration

The Rh127s2 data are entirely 8-2 BAQ, thus the noise-only data within the receive window are biased by echo signal and are unusable. We can instead measure the noise power by considering the 688 scatterometry bursts (Rh127s1) that stare at the sub-spacecraft point at the beginning of the RADAR sequence. The leading and trailing noise-only regions, i.e. the \( \sim 3 \) pulse intervals that occur before and after the pulse echo train, are receive-only data that measure the noise very accurately. This data is uncorrupted by BAQ effects and is distinct from the echo signal due to the relatively high SNR. Using this approach, we measure \( P_{n,dW} = 377dW \). The antenna temperatures measured during the Rh127s1 and Rh127s2 observations average to
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around 41 K, yielding a total system temperature of 850 K.

8.2.4 Discussion of Results

We summarize the backscatter model results of the three close RADAR flybys in Table 8.2. Portions of the studied areas were observed on more distant Cassini RADAR observations, as presented in Fig. 3 of Ostro et al. (2010). The albedo values that we compute for the three regions are systematically higher, by a factor of two, than those computed in Ostro et al. (2010). This appears to result from a difference in albedo definition, where the Ostro et al. (2010) albedo values are normalized to surface area rather than projected area. Future work will apply the modeling technique presented here to all RADAR icy satellite observations to supply a consistently processed data set.

On all of the icy satellites, the 2 cm-\(\lambda\) radar echoes result primarily from volume scattering. Only Iapetus seems to require a specular term to match the data. We see a large range of radar brightness in the albedo values presented in Table 8.2. The strength of volume scattering brightness is sensitive to the ice purity and regolith maturity. A “mature” surface is old enough for a process like meteoroid bombardment to have created density heterogeneities of a given scale at depth. When combined with a low-loss material, the wavelength-scale heterogeneities act as embedded scattering centers, enabling coherent backscattering. Following Ostro et al. (2010), the upper

<table>
<thead>
<tr>
<th>Obs ID</th>
<th>Beam Coverage</th>
<th>Longitude (deg)</th>
<th>Latitude (deg)</th>
<th>Model Parameters</th>
<th>Radar Albedo*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(a)</td>
<td>(n)</td>
</tr>
<tr>
<td>En120</td>
<td>17-45%</td>
<td>324 ± 5</td>
<td>-25 ± 1</td>
<td>3.51 ± 0.04</td>
<td>1.23 ± 0.03</td>
</tr>
<tr>
<td>Ia49-3</td>
<td>1-2%</td>
<td>82 ± 61</td>
<td>10 ± 58</td>
<td>0.41 ± 0.01</td>
<td>1.81 ± 0.04</td>
</tr>
<tr>
<td>Rh127</td>
<td>1-7%</td>
<td>162 ± 59</td>
<td>9 ± 60</td>
<td>2.14 ± 0.01</td>
<td>1.35 ± 0.02</td>
</tr>
</tbody>
</table>

* Note that \(\hat{\sigma}^0_{\text{SL-2}}\) denotes the measured same-sense linear polarization 2 cm-\(\lambda\) albedo, while \(\hat{\sigma}^0_{\text{TP-2}}\) denotes the inferred total-power 2 cm-\(\lambda\) albedo (see Section 4.6)
regoliths of many of the Saturnian icy satellites are “old enough” that large variations in 2 cm-\(\lambda\) radar brightness likely reflect variations in composition, or electrical loss. However, differences between brightness at 2 cm-\(\lambda\) and brightness at longer wavelengths can reflect differences in the maturity or the composition at the sensed depths (depths of 10 to 20 wavelengths according to Black et al. (2001)). Ostro et al. (2010) and Black et al. (2007) compare some of the results collected at Cassini’s 2 cm-\(\lambda\) and Arecibo’s 13 cm-\(\lambda\). The ground-based systems typically use circularly polarized signals, whereas Cassini RADAR uses a linearly polarized signal, thus inter-wavelength comparison is done using the total power albedo values (the sum of the albedo values measured in two orthogonal polarizations). The 2 cm-\(\lambda\) total power albedo (TP-2) values listed in Table 8.2 are inferred using the relationship presented in Eq. 4.31, where we assume that the 2 cm-\(\lambda\) opposite-sense linear polarization albedo (OL-2) is about half of the 2 cm-\(\lambda\) same-sense linear polarization albedo (SL-2), similar to Ostro et al. (2010).

We consider in turn the TP-2 values that we measure for the trailing side of Enceladus, the leading side of Iapetus, and the anti-Saturn side of Rhea. That Enceladus’ surface is bright is not surprising, given its proximity to Saturn’s icy E ring, but the TP-2 > 4 result is difficult to explain with traditional backscatter models. A perfect reflector will have a TP-2 albedo equal to unity, while coherent volume scattering allows for TP-2 up to a value of two. Enceladus’ large apparent albedo requires something akin to ordered structure to enhance the reflection in the backscatter direction, the same explanation presented for the anomalously bright areas on Titan (Janssen et al. (2011); Le Gall et al. (2010)). With its geometric albedo of \(~1.4\), Enceladus is the brightest of Saturn’s satellites at optical wavelengths as well (Verbiscer et al., 2007). The larger-than-unity geometric albedo is often attributed to enhanced opposition effects, such as that produced by coherent backscatter (Buratti et al., 2009). The 13 cm-\(\lambda\) total power albedo (TP-13) measured by Black et al. (2007) over Enceladus’ trailing hemisphere is less than unity, which Ostro et al. (2010) interpret as evidence for a young surface, where only the top few decimeters have matured into
a coherently backscattering regolith. All observations point to a very clean, low-loss icy composition.

The anti-Saturn hemisphere of Rhea is also anomalously bright at 2 cm-\(\lambda\), however with a TP-2 < 3, the backscatter is more easily attributed to a coherent backscattering mechanism, although it still requires some additional focusing in the backscatter direction. Ostro et al. (2010) and Black et al. (2007) observe albedo values for Rhea that are similar to each other, and, assuming their definitions of radar albedo are self-consistent, they conclude that Rhea is old, well-mixed, and uniformly clean. The optical geometric albedo is high, but not larger than unity (\(\sim 0.95\), Verbiscer et al. (2007)), and a sizable opposition spike is observed (Buratti et al., 2009).

Iapetus is darker all around than the bright satellites closer to Saturn. Even its bright trailing hemisphere, whose TP-2 we measure to be close to 0.8, is dark in comparison to the likes of Enceladus and Rhea. With a TP-2\(\sim 0.44\), Iapetus’ dark leading hemisphere is similar in brightness to much of the terrain on Titan, although it is about 50% brighter than the equatorial dunes, Titan’s darkest terrain. The 1.8x hemispheric asymmetry in 2 cm-\(\lambda\) brightness is significantly less than the dichotomy measured at optical wavelengths (10x, Squyres et al. (1984)), but still greater than that measured at 13 cm-\(\lambda\) (1.3x, Black et al. (2004)). Ostro et al. (2010) attribute this trend to the dark contaminant being shallow, i.e. present to depths of at least one to several decimeters, and possibly not any deeper. This is consistent with the hypothesis that the dark material was formed by the thermal redistribution of volatiles, a shallow phenomenon (Spencer and Denk, 2010). Further, high-resolution optical images show evidence of small crater impacts apparently punching through the dark layer to a brighter surface below (e.g. the ISS image labeled 1_N1568127660.118 in the Planetary Data System’s Image Atlas maintained by NASA). To explain the smaller TP-13 values, (\(\sim 0.15\) on average, a value similar to that of main-belt asteroids), Ostro et al. (2010) suggests the presence of a lossy contaminant that is greater at meter depths than at decimeter depths, or a contaminant that has a greater intrinsic electrical loss at the larger wavelength.
We compare the icy satellite backscatter curves to those of Titan’s features (see Chapter 5) in Figure 8.19. We see that Enceladus and Rhea are far brighter than Titan’s brightest terrain (Xanadu), while Iapetus exists between the brightest and darkest terrains of Titan. All of the features have similarly-shaped diffuse scattering responses, with the exception of Titan’s cryovolcanic terrain, which represents one of the more unusual scattering surfaces on Titan (see Section 5.7). Further data is required for a more comprehensive comparison of the 2 cm-\(\lambda\) Saturnian surfaces.

We form real aperture images of the icy satellites when the beam footprint is small enough (on the order of 100 km) to resolve the surface. The RAR images emphasize large-scale brightness correlations with optically-identified surface features. The images are equivalent in resolution to the scatterometry images that we produce for Titan. In the next section, we demonstrate the first step in producing high-resolution range-Doppler images (tens of km resolution) for other distant icy satellite observations, similar to the D-SAR images produced above.
8.3 Distant Range-Doppler Observations

Three icy satellite RADAR observations were described in the previous section, those for which the radar was close enough to the target for the antenna beam to resolve subsections of the visible target disk. In these cases, the signal was large enough for pulse compression and Doppler frequency analysis, and high-resolution D-SAR images, with resolutions of several kilometers, were formed using the processor developed at JPL for Titan D-SAR imaging observations. Here, we extend the range-Doppler processing to six other icy satellite chirped observations. We produce the preliminary first step images, in range-Doppler space. Future work will map the range-Doppler bins to the actual surface coordinates. While the inherent north-south ambiguity cannot be avoided, the images contribute to our understanding of how the radar brightness is distributed over the illuminated surface.

Each time step in a distant icy satellite radar return corresponds to a different area on the surface. For a nadir-staring observation, the earliest return is from the very center of the disk and the farthest return comes from the limb. If we can distinguish the echo in time delay, or range, we can map the echo power into annular rings on the surface. The slant range resolution of a pulse-limited radar echo is proportional to the pulse length $\tau$ (the resolution is $c\tau/2$, where $c$ is the speed of propagation). For a 300 $\mu$s pulse, a design parameter typical of an icy satellite observation, the slant range resolution is 45 km. A 45 km slant range resolution equates to roughly 17 range bins for a 764 km-radius moon like Rhea. For the 252 km-radius Enceladus, we would measure roughly 6 range bins. We can increase the range resolution significantly by range compressing the echo signal. Range compression enhances the distribution of echo power in time delay (range) by encoding the transmitted pulse using chirp modulation and applying a matched filter to reduce the apparent impulse response and maximize the signal-to-noise ratio. With range compression, the apparent pulse length is the inverse of the signal bandwidth $B$, and the slant range resolution is then $c/2B$. The scatterometer receiver bandwidth is 117 kHz, and the transmitted chirp
signal is nearly this wide, permitting slant range resolutions near 1.4 km, a significant improvement over the uncompressed range resolutions.

We can further separate the energy within each range bin by considering the Doppler dispersion created by the rotating target and the spacecraft motion. Because different parts of a rotating target have different velocities relative to the radar, they will contribute echo powers at different frequencies. A nadir pointed radar above the equator will map the frequency bins to vertical strips on the surface. In this manner, the echo power is isolated to two points within a particular range ring, one north of the equator, and the other south of the equator. We transmit multiple pulses to sample the surface (the sample rate is the pulse repetition frequency, or PRF) and use FFT methods to resolve Doppler bins.

The frequency resolution on the surface will be the surface length with a Doppler dispersion equivalent to the PRF, or the limb-to-limb Doppler spread if it is smaller, divided by the number of frequency bins, which can be up to the number of pulses received. We estimate the frequency resolution on the surface, \( \delta f_{\text{surf}} \), by first computing the limb-to-limb Doppler spread, \( \Delta F_{\text{L2L}} \), from the target rotation and the spacecraft motion and then calculating the ratio of the PRF to the Doppler spread. This ratio multiplied by the visible target length (roughly half of the target circumference, \( \pi R_t \)) gives the surface length equivalent to a PRF in Doppler dispersion. Dividing the computed surface length by the number of received pulses \( N_{\text{pul}} \) provides a rough estimate of the surface resolution of each frequency bin (Eq. 8.6). The results are given in Table 8.3 for all of the chirped observations.

\[
\delta f_{\text{surf}} = \frac{\min(\text{PRF}, \Delta F_{\text{L2L}})}{\pi R_t N_{\text{pul}}} \tag{8.6}
\]

We process the chirped icy satellite observations marked with an asterisk in Table 8.3 into range-Doppler images as follows: 1) for each of the \( \sim 1000 \) radar bursts, we match filter the echo with a digital version of the transmitted chirped signal, 2) we apply an FFT to each range bin, where the size of the FFT is the number of pulses received, 3) we accumulate the results over all the bursts in range-Doppler space to
8.3. DISTANT RANGE-DOPPLER OBSERVATIONS

<table>
<thead>
<tr>
<th>Obs ID</th>
<th>Doppler Spread $\Delta F_{L2L}$ (Hz)</th>
<th>PRF (Hz)</th>
<th>No. Pulses $N_{pul}$</th>
<th>Tx Npul $\delta f_{surf}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mi126</td>
<td>4637</td>
<td>2717</td>
<td>185</td>
<td>2</td>
</tr>
<tr>
<td>En4*</td>
<td>1558</td>
<td>940</td>
<td>40</td>
<td>12</td>
</tr>
<tr>
<td>En61-2</td>
<td>1292</td>
<td>801</td>
<td>58</td>
<td>8</td>
</tr>
<tr>
<td>Te48</td>
<td>2280</td>
<td>1506</td>
<td>80</td>
<td>14</td>
</tr>
<tr>
<td>Di16-1*</td>
<td>1461</td>
<td>1302</td>
<td>75</td>
<td>21</td>
</tr>
<tr>
<td>Di50-5</td>
<td>906</td>
<td>2500</td>
<td>80</td>
<td>22</td>
</tr>
<tr>
<td>Rh18-1*</td>
<td>1250</td>
<td>1302</td>
<td>75</td>
<td>32</td>
</tr>
<tr>
<td>Rh22-1*</td>
<td>6970</td>
<td>2717</td>
<td>165</td>
<td>6</td>
</tr>
<tr>
<td>Hy15*</td>
<td>307</td>
<td>434</td>
<td>30</td>
<td>14</td>
</tr>
<tr>
<td>Ia0b-1*</td>
<td>817</td>
<td>801</td>
<td>40</td>
<td>57</td>
</tr>
<tr>
<td>Ia0c-1</td>
<td>499</td>
<td>601</td>
<td>40</td>
<td>58</td>
</tr>
</tbody>
</table>

decrease the noise variance. The results are shown in Figure 8.20.

To visualize the range-Doppler images presented in Figure 8.20, first find the bright leading arc (the faint arcs and lines in Di16-1, Rh18-1, and Rh22-1 are due to range and Doppler ambiguities). In all cases except Ia0b and Hy15, the leading bright edge maps roughly to the target equator, and the peak of the leading edge maps to the sub-spacecraft point. Then, moving down in range delay from the bright edge, along a particular Doppler frequency bin, imagine moving north (or south) from the equator and mapping the bin’s corresponding signal power along the way. This picture gives a rough idea of how the signal is distributed over the illuminated area (with north and south being averaged together because they contribute the same Doppler frequencies at the same range, and their echoes cannot be disentangled). Figure 8.20 thus shows clear structure beyond the leading-edge bright return, with some shapes looking very crater-like.
Figure 8.20: The range-Doppler images for six of the eleven chirped distant icy satellite observations. Range and Doppler ambiguities are present as faint arcs in Di16-1, Rh18-1, and Rh22-1. Beyond the bright leading arc, structures such as craters begin to become visible. These features will be more clear when the range-Doppler images are properly projected to their moons’ surfaces. The locations of each observation are listed in Table C.1
Future work will extend the range-Doppler processing to the other chirped observations (those listed in Table 8.3 without an asterisk) and will map the range-Doppler images to their respective surfaces, ignoring the inherent north-south ambiguity.

8.4 Conclusion

This chapter focuses on the real aperture processing of three close icy satellite observations of the trailing hemisphere of Enceladus, the leading hemisphere of Iapetus, and the anti-Saturn hemisphere of Rhea: En120s2, Ia49-3, and Rh127s2. We process the data into normalized radar cross section (NRCS) measurements by projecting the observation parameters onto the visible surface and solving the area-extensive radar equation. Care is taken to also measure the effective incidence angle of each NRCS measurement. When paired together, the NRCS and incidence angle values provide the backscatter response of the illuminated surface. We model the backscatter response with a diffuse cosine power law to estimate the backscatter parameters \( a \) and \( n \), where \( a \) is a measure of the reflection strength and \( n \) is a measure of the directivity of the backscatter power pattern. Isotropic surfaces will have \( n = 1 \), whereas Lambertian surfaces have \( n = 2 \). Our three measurements span the range from \( a = 0.41 \) for Iapetus’ dark leading hemisphere to \( a = 3.51 \) for Enceladus’ bright trailing hemisphere, representing some of the darkest and the brightest surfaces in the Saturnian system. Perhaps coincidentally, the measured \( n \) values seem correlated with the \( a \) measurements; Iapetus appears more Lambertian \((n = 1.81)\) and Enceladus looks more isotropic \((1.23)\). The Rhea measurements are in between the two extremes, but are closer to the bright end of the spectrum \((a = 2.14, n = 1.35)\).

The Iapetus observation exhibits a small quasispecular rise at angles less than 20°, a backscatter response similar to those measured for features on Titan. We fit a Hagfors and an exponential quasispecular model to the low-angle backscatter, and the results imply a low effective dielectric constant, possibly indicative of porous solid hydrocarbons or porous carbon dioxide ice, and moderate large-scale \((\gg \lambda)\)
facet slopes. This is the first detection of a quasispecular component on an icy moon other than Titan, indicating the presence (albeit small) of a surface scattering mechanism in addition to the more prominent diffuse volume scattering mechanism that tends to dominate the backscatter of icy surfaces. The quasispecular detection might have evaded previous distant spectral observations of Iapetus’ dark side due to the combination of limited resolution and the subtle rise of the observed quasispecular component. It is possible that Enceladus and Rhea also have a small quasispecular component, but we do not have enough low angle diversity to determine this with our current set of measurements.

Future work will focus on applying the backscatter modeling method we developed for single-stare observations, e.g. En120s2, to the rest of the Cassini icy satellite observations. This will produce an extensive set of consistently processed measurements that we can use to understand the distribution of 2 cm-λ backscattering characteristics across a moon’s surface, as well as between the moons.
Chapter 9

Summary and Conclusions

One of the main objectives of this work was to construct a real aperture processor capable of measuring the normalized radar cross section (NRCS) for all of the data collected by the RADAR instrument. The developed processor incorporates detailed corrections for the variety of viewing geometries and receiver configurations encountered during RADAR operation. We carefully measure and characterize the thermal noise behavior of the receiver to calibrate the backscatter data to an absolute scale. These data are stored in the Planetary Data System (PDS) for use by members of the scientific community.

In this work, we apply the processor to all data collected from the surface of Titan through the T71 flyby (7-July-2010). We produce a near-global real aperture radar (RAR) backscatter map covering more than 99.9% of Titan’s surface. The global backscatter map is useful for analyzing the RADAR data in conjunction with other remote sensing data, such as the visible and near-infrared data collected by the ISS and VIMS instruments. Furthermore, the global RAR map fills in the gaps within the high-resolution RADAR mosaic so that the full extent of some features can be measured (e.g. Xanadu).

The calibrated data combine to complement each other in viewing angle, enabling the retrieval of complete backscatter functions for various locations on Titan’s surface. We discover that the form of the backscatter function varies widely with location, attesting to the observed heterogeneity of the surface. The backscatter response
defies traditional scattering models by requiring an additional term to describe the near-nadir peak. A composite model based on the linear superposition of classical quasispecular models and an empirical diffuse model properly describes the scattering behavior. We measure the backscatter functions for specific features on Titan and apply the composite scattering model to determine how the surface properties vary across the surface. We find that the 2 cm-\(\lambda\) same-sense linear polarization albedo \(\hat{\sigma}_{\text{SL-2}}\) varies from 0.16 for the dark dunes to 0.82 for the bright Xanadu province, although there are areas on the surface with reflectivity levels outside of this range (e.g. the dark lakes and seas and the bright core of Xanadu). The composite modeling process further reveals the extent of diffuse scattering relative to quasispecular scattering. Most of the Titan features we analyze show more than 80% of the echo power originating from diffuse scattering mechanism (all features except the cryovolcanic terrains, notably). Such a strong diffuse component is a good indicator of the presence of volume scattering.

We also apply the real aperture processor to targeted flyby data collected from the surfaces of Enceladus, Iapetus, and Rhea. We form real aperture maps for the latter two moons and discover regions that are anticorrelated with optical maps. We measure the backscatter response for the visible hemispheres of the icy moons and apply the empirical diffuse backscatter model to properly determine the radar albedo. The dark side of Iapetus further requires a quasispecular component to describe the scattering response – the first detection of a quasispecular component on an icy moon other than Titan, indicating the presence of a surface scattering mechanism in addition to the more prominent diffuse volume scattering mechanism that tends to dominate the backscatter of icy surfaces. The scattering models suggest that the radar albedo of the dark side of Iapetus \((\hat{\sigma}_{\text{SL-2}} = 0.29)\) is similar to the mean radar albedo of Titan \((\hat{\sigma}_{\text{SL-2}} = 0.26)\). The albedo of the bright polar terrain of Iapetus \((\hat{\sigma}_{\text{SL-2}} = 0.53)\) is larger than that derived for the bright hummocky terrain on Titan \((\hat{\sigma}_{\text{SL-2}} = 0.42)\), but not quite as bright as the mean value derived for Xanadu \((\hat{\sigma}_{\text{SL-2}} = 0.82)\). However, the radar albedo values calculated for Rhea \((\hat{\sigma}_{\text{SL-2}} = 1.82)\)
and Enceladus ($\sigma_{\text{SL-2}} = 3.14$) are much brighter than any of the other analyzed surfaces and demand further investigation to explain such anomalously high brightness levels.

The largest southern lake on Titan, Ontario Lacus, presents opportunities for further radar analysis beyond simply mapping its perimeter and analyzing its geomorphology. We measure and model altimetry data over Ontario Lacus to constrain the surface smoothness, concluding that there is little, if any, small-scale wave activity on the lake surface; the lake is mirror-smooth. This quantitative analysis has implications for the wind speeds at the time of the observation, as well as the liquid material properties. We further measure and model the off-nadir radar data over Ontario Lacus to constrain the lake depths. We constrain the maximum depth of the lake to be less than 9 meters, the average depth of the lake to be less than 4 meters, and the volume to be less than 54 km$^3$. Such shallow depths are not unheard of for a large lake; several lakes on Earth are of comparable size and depth. These terrestrial analogs may aid our understanding of the nature and origin of Ontario Lacus.

Areas of further investigation related to this work include:

- Investigating a more physical form of the diffuse model, beyond the empirical cosine power law that is commonly utilized. The diffuse model form would likely incorporate significant volume scattering contributions, but would need an additional focusing mechanism to explain the extremely high backscatter values observed on the Saturnian satellite surfaces, e.g. Enceladus.

- Exploring the effect that a significant diffuse scattering component has on the inferred quasispecular surface parameters.

- Further consideration of the discrepancy between the dielectric constant inferred from the quasispecular backscatter models and the dielectric constant inferred from the radiometry models.

- Improved modeling of near-nadir radar echoes to better understand the low angle peak observed in the backscatter data. We use an additional quasispecular
model to describe the sharp increase in backscatter near zero incidence, but
further investigation is needed to understand and characterize the source of this
scattering phenomena.

• Measuring and modeling feature backscatter functions at a more localized level.
   With the acquisition of more Titan data over the coming years, and the possibil-
   ity for developing a higher resolution scatterometer processor (Wye and Zebker,
   2006; Zebker et al., 2011), multi-angle backscatter from smaller areas on the
   surface can be analyzed and modeled. This study would help to classify the
   surface by parameter value at a finer level.

• Further investigation of the surface properties of candidate materials at Titan
   conditions in a controlled environment. For example, what is the expected
   porosity of the solid surface, and does this improve the accuracy of the inferred
dielectric constant parameter? What are the physical implications of a measured
rms slope of 10°? Can we better characterize the material properties (e.g.
viscosity, surface tension, and density) of liquid compositions to help explain
the absence of measurable waves? Answering these questions would improve
our interpretation of the modeled scattering results from Titan’s surface.

• Combining the radar data with other sensing data (e.g. VIMS, ISS, and CIRS)
   to further the geophysical characterization of the surface. Much effort has al-
   ready been invested in this area (e.g. Le Mouélic et al., 2008; Soderblom et al.,
   2007, 2009; Tosi et al., 2010), but more comprehensive development is still
   needed.

• Furthering the icy satellite radar processing effort to create a consistent and
   complete collection of radar properties across the surfaces of the moons. Over-
   lapping the measurements and utilizing enhanced resolution processing tech-
   niques (i.e. range Doppler) may help to localize the inferred scattering param-
   eters.
Appendices
Appendix A

RADAR Effective Antenna Beamwidth

The Cassini RADAR consists of five beams, where the antenna patterns of the beams are illustrated in Figure A.2. The central main beam, also called beam 3, is aligned with the spacecraft Z-axis. The outer beams are offset from the Z-axis by 1° to 2° in the Y-axis direction. Beams 1 and 5 are rotated ±2.2° about the spacecraft X-axis so that they lie in the Y-Z plane. Beams 2 and 4 are first rotated by 1.2° about the Y-axis and then ±0.85° about the X-axis so that they are slightly offset from the Y-Z plane. The five beams together form an elongated pattern that is stretched in the Y direction of the spacecraft coordinate system (c.f. Fig. 9, West et al., 2009).

The transmitted radar energy is spread over the area illuminated by the antenna beam, and each scattering point within the beam is weighted by the antenna’s power pattern. To reduce the complexity of the real aperture radar processor, we consider the illuminated scattering points to lie within an effective beam of constant power, where the power is set to the peak power gain of the central beam ($G_t=50.7$ dB) within the effective beam and is zero without. The definition of the effective beamwidth is illustrated in Figure A.2. We solve for the width of this effective beam by equating the total power of the true beam with that of the effective beam. One could also choose the average main beam antenna gain as the constant power that defines the effective beamwidth; the effective beamwidth would be accordingly larger. Furthermore, one could also change the shape of the beam. As long as the total power within the effective beam equals the total power within the true beam, the selection choice does
Figure A.1: The five antenna beam patterns used by the Cassini RADAR instrument. Beam 3, the central or main beam, is used for the scatterometry, altimetry, and D-SAR mode data. The outer beams are only used in SAR mode. The beams are roughly placed in their relative position to each other; i.e. Beam 1 and Beam 5 are offset from Beam 3 along the Y-axis of the spacecraft. Beam 2 and Beam 4 are offset in both the Y and the X directions. Beam 3 aligned with the spacecraft Z-axis. This configuration allows the beams to sweep over a wide area when forming the SAR image swath.
In this appendix, we describe our calculation of the effective beamwidth of the central antenna beam, $\theta_{b\text{-eff}}$. We use $\theta_{b\text{-eff}}$ to compute the effective beam-illuminated area (Eq. 3.16) that is needed for the real aperture radar processor. When the data originates from one of the outer beams, as it does in SAR mode, we change the weight of $\theta_{b\text{-eff}}$ to reflect the different total power contained by the outer beam, but we do not change the width of $\theta_{b\text{-eff}}$.

If we integrate the true antenna pattern over the illuminated surface to find the total receive power expected from the radar equation, then we have

$$P_{r\text{-pattern}} = \frac{P_t \sigma_0 \lambda^2 G_t^2}{4\pi (4\pi R^2)^2} \int_{-X/2}^{X/2} \int_{-Y/2}^{Y/2} g^2(x, y) \, dx \, dy$$

where $g$ is the normalized antenna pattern, $G_t$ is the peak power gain, and $X$ and $Y$ represent the extent of the beam pattern on the surface.
We find an analytical expression for the total power expected from a circularly-
shaped beam with constant gain equal to the peak of the antenna pattern ($G_t = 50.7$ dB),

\[
P_{\text{r,circ}} = \frac{P_t \sigma_0 \lambda^2 G_t^2}{4\pi (4\pi R^2)^2} \frac{D/2}{r} \int_0^{D/2} r dr = \frac{P_t \sigma_0 \lambda^2 G_t^2}{4\pi (4\pi R^2)^2} \frac{2\pi}{2} \left( \frac{D_{\text{circ}}^2}{8} \right)
\]  
(A.2)

and an expression for the total power expected from a rectangular approximation of
the circular beam of the same constant gain

\[
P_{\text{r,rect}} = \frac{P_t \sigma_0 \lambda^2 G_t^2}{4\pi (4\pi R^2)^2} \int_{-D/2}^{D/2} \int dxdy = \frac{P_t \sigma_0 \lambda^2 G_t^2}{4\pi (4\pi R^2)^2} \left( \frac{D_{\text{rect}}^2}{D_{\text{circ}}^2} \right)
\]  
(A.3)

where $D_{\text{circ}}$ and $D_{\text{rect}}$ represent, respectively, the extent of the circular and rectangular
effective beams on the surface, i.e. they are proportional to the product of the distance
and the respective effective beamwidths for which we are solving.

Equating Eq. A.1 with Eq. A.2 and noting that we have an antenna pattern
measured at angular bins of $0.01^\circ$, so that $dxdy = 0.01 \times 0.01 = 10^{-4}$ degrees, we
calculate the circular effective beamwidth:

\[
\theta_{\text{circ}} = \sqrt{\frac{8}{2\pi} \times 10^{-4} \times \sum \sum \left( \frac{\text{beam\_pattern}}{G_t} \right)^2} = 0.3223^\circ
\]  
(A.4)

And we evaluate the rectangular effective beamwidth by equating Eq. A.1 with
Eq. A.3:

\[
\theta_{\text{rect}} = \sqrt{10^{-4} \times \sum \sum \left( \frac{\text{beam\_pattern}}{G_t} \right)^2} = 0.2856^\circ
\]  
(A.5)

In the above calculations, $\text{beam\_pattern}$ refers to the pattern of the central antenna beam.

While we have derived both the circular and the rectangular effective beamwidths
for completeness, we note that the rectangular effective beamwidth is the one used
in the real aperture reduction. We use the rectangular effective beam in the real
aperture processor because of the need to evaluate the azimuth resolution separately from the range resolution for the purposes of computing illuminated area (Eq. 3.16) in the radar equation.

The 0.29° effective beamwidth result applies only for the central beam, or beam 3, of the RADAR instrument. To apply the result to the other beams, we solve Eq. A.5 using the outer beam patterns and then ratio the result to 0.29°. The square of this ratio yields the ratio of the power contained in the outer beams relative to that in the central beam (about 27% to 34%). We then scale the denominator of the real aperture radar equation (Eq. 3.36) by this ratio of powers. Thus, we are maintaining the same size for the effective beamwidth, or the same illuminated area, but are simply changing the gain of the effective beam to reflect the true power contained within the outer beams.

We find that the effective beam approximation is accurate at the 1-2% level for Titan geometries as long as the incidence angle is less than about 65°-75°. Above this, the curvature of the illuminated surface can cause a large spread in the viewing geometry parameters so that the true beam pattern integral must be considered. In the case of the icy satellites, the observation is generally large relative to the diameter of the target, so that the curvature is significant for all incidence angles and the proper beam pattern must be considered (see Chapter 8).
Appendix B

Compressed Scatterometry Mode

The compressed scatterometry mode data uses a technique called magnitude profiling, where the data from the pulses in a burst are accumulated on board the spacecraft. This approach reduces the required data rate for long-distance scatterometer experiments. At distances much larger than 40,000 km, long integration times are needed to produce a detectable signal, but this requirement would quickly use up the available data volume. Furthermore, the data need not be processed in a phase coherent fashion, so returning every sample of data is unnecessary. Accumulating the echo profiles on board the spacecraft reduces the data volume by a factor proportional to the half of the number of pulse intervals in the burst, making the long-distance experiment more feasible. The magnitude profiling technique utilized by the RADAR instrument is described in JPL Memo 334RW-2001-004, which we summarize here.

Instead of transmitting the entire receive window, composed of $K_{rxw}$ pulse repetition intervals, only data from a single pulse repetition interval (PRI) is returned to Earth. This PRI contains the sum of the absolute magnitudes of the signals at the corresponding positions over all $K_{rxw}$ PRI. In other words, if $M_i$ is the total magnitude at sample index $i$ within the downlinked PRI, then it is computed as follows

$$M_i = \sum_{j=1}^{K_{prx}} |V_dV [i + N_{pri}(j - 1)]|,$$  \hspace{1cm} (B.1)
where \( N_{\text{pri}} \) is the number of samples within one PRI, and \( V_{dV}[i + N_{\text{pri}}(j - 1)] \) represents the received digitized voltage sample at sample index \( i \) within PRI \( j \).

In the real aperture radar equation (Eq. 3.36), we need to know the total signal energy, or the total signal power. The PRI magnitude profile can be related to the power profile by a simple multiplicative factor using folded normal distribution properties. The received signal is expected to be normally distributed, with a zero mean, because it is the sum of many independent scattered signals. The average magnitude of a normally distributed signal is related to the standard deviation as follows:

\[
E(|x|) = \sqrt{\frac{2}{\pi}} \sigma \tag{B.2}
\]

and the variance, or the square of the standard deviation relates to the average power:

\[
E(x^2) = \sigma^2 \tag{B.3}
\]

Thus, we solve for the total received signal energy \( E_{r,d,j} \) (or the mean sample power scaled by the total number of received samples, \( K_{\text{prx}}N_{\text{pri}} \)) from the received magnitude profile as follows:

\[
E_{r,d,j} = K_{\text{prx}} \sum_{i=1}^{N_{\text{pri}}} \left( \frac{M_i}{K_{\text{prx}} \sqrt{\frac{\pi}{2}}} \right)^2 \tag{B.4}
\]

Collecting the profile over one PRI, rather than summing the entire burst into a single number, maintains the ability to locate the echo signal within the PRI. In this manner, the signal energy estimate can be improved by only considering the portion of the PRI where the echo signal exists. Further, the noise energy \( E_{n,d,j} \) can be calculated from the segments around the echo signal. \( E_{n,d,j} \) can also be calculated from receive-only bursts. Once the noise energy is known, we can calculate the total signal energy \( E_{s,d,j} \) from Eq. 3.12.
We find that the power levels of the compressed data measurements are underestimated by about 5-10%. This may be due to a small DC bias in the received signal (which is assumed to be zero-mean), or it may be due to leading and trailing noise-only PRI that are folded into the echo profile. We are able to correct for some of the resulting low bias by carefully calculating the compressed scatterometry noise power \( P_{n,dW} = 352.42 \text{ dW} \) and using this value in our calibration procedure instead of the values calculated in Section 3.3, but a small amount of bias may still remain. All compressed scatterometry data occur at the same attenuator setting (8.4 dB), which simplifies the calibration step.
Appendix C

Tabulated Observations of Icy Moons

The active RADAR observations of Saturn’s icy satellites through the primary and extended missions include data from 34 flybys of eight moons. Additionally, six Titan flybys include distant RADAR observations. We tag each observation with the targeted moon’s abbreviated name and the orbit revolution number. For example, an observation of Enceladus on the 61st orbit of Cassini around Saturn will be denoted En61. An individual flyby may contain several observations. When these observations occur at different sections of the orbit, the observation names are hyphenated with their count index; e.g. the first observation on the En61 flyby is En61-1, and the second observation is En61-2. Sometimes the observations occur at the same section of the orbit, but, when the antenna beam is small enough relative to the apparent target size, the antenna beam may be directed to stare at different locations on the surface. In this case, the observations are appended with an ‘s’ and the stare count index. For example, the En120 flyby contains stare data centered around (1° S, 314° W) followed by stare data centered around (25° S, 324° W), so we denote the first observation En120s1, and the second observation En120s2. In some encounters, we have both types of observations. The Ia49 flyby contains RADAR observations on four different sections of the orbit (at four different times), and the second and fourth observations were designed to steer the radar antenna beam from the sub-radar location, the center of the visible disk, to each of the disk’s four visible corners. As a result, the second and fourth observations each comprise five additional observations:
Ia49-2s1, Ia49-2s2, Ia49-2s3, Ia49-2s4, and Ia49-2s5, and similar for Ia49-4.

In Table C.1, the 40 encounters are decomposed into their specific observations, where each observation is centered on a different surface location. The observations are identified by their orbital revolution number, their observation index and their stare index, as described above. The observations are listed sequentially in order of their moon’s mean distance from Saturn. For each observation, the mean distance and stare coordinates (the antenna boresight coordinates) are given together with their range of values, e.g. Mi47 occurs at distances uniformly spaced between 216 and 226km, so the value is tabulated as $221 \pm 5$. The longitude is in west coordinates. The relative size of the antenna beam on the target surface is calculated by weighting the visible surface area by the projected antenna pattern. This beam coverage value is given as a percentage and will vary with the boresight pointing direction in addition to distance. For example, if the radar is so far from the target that the target disk fills the main beam, the beam coverage will be close to 100%. If the radar is closer to the target, then the beam will begin to resolve the target disk and the calculated beam coverage will be correspondingly smaller. When the beam is pointed off-nadir, such as at the corner of the disk, the beam coverage will also be smaller than when it is pointed at the disk center.

Table C.1: RADAR observations of Saturn’s Icy Moons

<table>
<thead>
<tr>
<th>Obs ID</th>
<th>Date</th>
<th>Distance (1000 km)</th>
<th>Beam Coverage</th>
<th>Longitude (deg)</th>
<th>Latitude (deg)</th>
</tr>
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<tbody>
<tr>
<td>Mi47</td>
<td>27-Jun-07</td>
<td>221 ± 5</td>
<td>97%</td>
<td>58 ± 3</td>
<td>-2 ± 3</td>
</tr>
<tr>
<td>Mi53</td>
<td>03-Dec-07</td>
<td>178 ± 10</td>
<td>95%</td>
<td>108 ± 1</td>
<td>18 ± 1</td>
</tr>
<tr>
<td>Mi64</td>
<td>11-Apr-08</td>
<td>110 ± 2</td>
<td>87%</td>
<td>151 ± 5</td>
<td>-43 ± 5</td>
</tr>
<tr>
<td>Mi126</td>
<td>13-Feb-2010</td>
<td>41 ± 2</td>
<td>38-44%</td>
<td>273 ± 2</td>
<td>-5 ± 1</td>
</tr>
<tr>
<td>En3</td>
<td>17-Feb-2005</td>
<td>165 ± 12</td>
<td>76-80%</td>
<td>211 ± 4</td>
<td>30 ± 3</td>
</tr>
<tr>
<td>En4</td>
<td>09-Mar-2005</td>
<td>84 ± 7</td>
<td>65-72%</td>
<td>70 ± 3</td>
<td>-12 ± 2</td>
</tr>
<tr>
<td>En28</td>
<td>09-Sep-2006</td>
<td>163 ± 3</td>
<td>90%</td>
<td>187 ± 3</td>
<td>59 ± 2</td>
</tr>
<tr>
<td>En32</td>
<td>08-Nov-2006</td>
<td>91 ± 1</td>
<td>74%</td>
<td>242 ± 1</td>
<td>-29 ± 3</td>
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</tbody>
</table>

Continued on next page
<table>
<thead>
<tr>
<th>Obs ID</th>
<th>Date</th>
<th>Distance (1000 km)</th>
<th>Beam Coverage</th>
<th>Longitude (deg)</th>
<th>Latitude (deg)</th>
</tr>
</thead>
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<td>En50</td>
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<td>119 ± 1</td>
<td>84%</td>
<td>121 ± 1</td>
<td>17 ± 1</td>
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<td>En61-1</td>
<td>12-Mar-2008</td>
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<td>88-90%</td>
<td>101 ± 4</td>
<td>68 ± 1</td>
</tr>
<tr>
<td>En61-2</td>
<td>12-Mar-2008</td>
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<td>64-69%</td>
<td>327 ± 2</td>
<td>-69 ± 1</td>
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<tr>
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<td>311 ± 9</td>
<td>97%</td>
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<td>53 ± 3</td>
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<tr>
<td>En120s1</td>
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<td>En120s2</td>
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<td>324 ± 5</td>
<td>-25 ± 1</td>
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<tr>
<td>Te15</td>
<td>24-Sep-2005</td>
<td>120 ± 2</td>
<td>46-48%</td>
<td>207 ± 1</td>
<td>0 ± 1</td>
</tr>
<tr>
<td>Te21</td>
<td>25-Feb-2006</td>
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<td>82%</td>
<td>250 ± 0</td>
<td>-1 ± 0</td>
</tr>
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<td>32%</td>
<td>107 ± 1</td>
<td>2 ± 1</td>
</tr>
<tr>
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<td>36-39%</td>
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<td>-1 ± 0</td>
</tr>
<tr>
<td>Di16s2</td>
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<td>115 ± 1</td>
<td>39-40%</td>
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<td>19 ± 0</td>
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<td>Di16s3</td>
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<td>120 ± 1</td>
<td>41-42%</td>
<td>36 ± 1</td>
<td>-3 ± 1</td>
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<tr>
<td>Di16s4</td>
<td>11-Oct-2005</td>
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<td>44-45%</td>
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<td>1 ± 1</td>
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<tr>
<td>Di16s5</td>
<td>11-Oct-2005</td>
<td>129 ± 1</td>
<td>46-47%</td>
<td>15 ± 1</td>
<td>-19 ± 1</td>
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<td>Di27</td>
<td>16-Aug-2006</td>
<td>169 ± 1</td>
<td>65%</td>
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<td>-34 ± 1</td>
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<td>Di33s1</td>
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<td>17 ± 1</td>
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<td>Di33s2</td>
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<td>43 ± 1</td>
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<td>22%</td>
<td>319 ± 1</td>
<td>55 ± 0</td>
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<td>Di50s1</td>
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<td>30%</td>
<td>242 ± 0</td>
<td>-12 ± 0</td>
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<td>Di50s2</td>
<td>30-Sep-2007</td>
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<td>54-55%</td>
<td>62 ± 6</td>
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<td>27-Nov-2005</td>
<td>117 ± 5</td>
<td>21-25%</td>
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<td>-23 ± 1</td>
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<tr>
<td>Obs ID</td>
<td>Date</td>
<td>Distance (1000 km)</td>
<td>Beam Coverage</td>
<td>Longitude (deg)</td>
<td>Latitude (deg)</td>
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<tr>
<td>Rh18s5</td>
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<td>155 ± 2</td>
<td>38-39%</td>
<td>0 ± 1</td>
<td>0 ± 0</td>
</tr>
<tr>
<td>Rh22s1</td>
<td>21-Mar-2006</td>
<td>93 ± 2</td>
<td>13-14%</td>
<td>107 ± 3</td>
<td>1 ± 0</td>
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<td>17%</td>
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<td>-35 ± 1</td>
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<td>Rh22s3</td>
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<td>17-18%</td>
<td>76 ± 1</td>
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<td>18%</td>
<td>100 ± 1</td>
<td>36 ± 1</td>
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<tr>
<td>Rh22s5</td>
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<td>18%</td>
<td>145 ± 1</td>
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<td>Rh22s6</td>
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<td>Rh27</td>
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<td>182 ± 2</td>
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<td>27 ± 1</td>
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<td>27-May-2007</td>
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<td>Rh47s1</td>
<td>28-Jun-2007</td>
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<td>-2 ± 1</td>
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<td>154 ± 1</td>
<td>37-38%</td>
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<td>20 ± 1</td>
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<td>38%</td>
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<tr>
<td>Rh49s1</td>
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<td>106 ± 1</td>
<td>18-19%</td>
<td>347 ± 0</td>
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<td>19-20%</td>
<td>348 ± 0</td>
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<td>18%</td>
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<td>16-17%</td>
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<td>Rh127s1</td>
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<td>63 ± 3</td>
<td>6-7%</td>
<td>164 ± 0</td>
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<td>Rh127s2</td>
<td>02-Mar-2010</td>
<td>39 ± 15</td>
<td>1-7%</td>
<td>162 ± 59</td>
<td>9 ± 60</td>
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<tr>
<td>Ti0a-1</td>
<td>26-Oct-2004</td>
<td>731 ± 7</td>
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<td>141 ± 1</td>
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<tr>
<td>Ti6</td>
<td>18-Apr-2005</td>
<td>641 ± 4</td>
<td>53-54%</td>
<td>236 ± 1</td>
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<tr>
<td>Ti23</td>
<td>29-Apr-2006</td>
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<td>45%</td>
<td>348 ± 0</td>
<td>0 ± 0</td>
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<tr>
<td>Ti37-1</td>
<td>12-Jan-2007</td>
<td>573 ± 3</td>
<td>46-47%</td>
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<td>88-90%</td>
<td>173 ± 3</td>
<td>46 ± 1</td>
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<tr>
<td>Hy39</td>
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<td>196 ± 1</td>
<td>98%</td>
<td>133 ± 3</td>
<td>-12 ± 4</td>
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<tr>
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<td>Date</td>
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<td>Beam Coverage</td>
<td>Longitude (deg)</td>
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<tr>
<td>Ia0b-1</td>
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<td>151 ± 0</td>
<td>41%</td>
<td>67 ± 1</td>
<td>39 ± 1</td>
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<td>150 ± 0</td>
<td>39%</td>
<td>42 ± 1</td>
<td>52 ± 0</td>
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<td>150 ± 0</td>
<td>38%</td>
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</tr>
<tr>
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<td>38%</td>
<td>82 ± 1</td>
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<td>49 ± 0</td>
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<td>01-Jan-2005</td>
<td>198 ± 5</td>
<td>57-60%</td>
<td>297 ± 2</td>
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</tr>
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<td>88%</td>
<td>359 ± 4</td>
<td>37 ± 1</td>
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<td>70%</td>
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<td>11 ± 1</td>
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<td>40 ± 0</td>
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<td>22%</td>
<td>57 ± 0</td>
<td>36 ± 0</td>
</tr>
<tr>
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<td>09-Sep-2007</td>
<td>105 ± 0</td>
<td>21%</td>
<td>91 ± 0</td>
<td>17 ± 0</td>
</tr>
<tr>
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<td>09-Sep-2007</td>
<td>100 ± 1</td>
<td>17-18%</td>
<td>65 ± 0</td>
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<tr>
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<td>10-Sep-2007</td>
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<td>1-2%</td>
<td>82 ± 61</td>
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<td>22%</td>
<td>247 ± 0</td>
<td>-10 ± 0</td>
</tr>
<tr>
<td>Ia49-4s2</td>
<td>11-Sep-2007</td>
<td>111 ± 0</td>
<td>23%</td>
<td>275 ± 1</td>
<td>-17 ± 0</td>
</tr>
<tr>
<td>Ia49-4s3</td>
<td>11-Sep-2007</td>
<td>112 ± 0</td>
<td>24%</td>
<td>238 ± 0</td>
<td>-37 ± 0</td>
</tr>
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<td>Ia49-4s4</td>
<td>11-Sep-2007</td>
<td>113 ± 0</td>
<td>24%</td>
<td>220 ± 1</td>
<td>-2 ± 1</td>
</tr>
<tr>
<td>Ia49-4s5</td>
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<td>113 ± 0</td>
<td>24%</td>
<td>255 ± 1</td>
<td>16 ± 0</td>
</tr>
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<td>Ph00s1</td>
<td>11-Jun-2004</td>
<td>93 ± 1</td>
<td>95%</td>
<td>246 ± 3</td>
<td>-22 ± 2</td>
</tr>
<tr>
<td>Ph00s2</td>
<td>11-Jun-2004</td>
<td>56 ± 3</td>
<td>85-88%</td>
<td>326 ± 6</td>
<td>25 ± 1</td>
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</table>
Bibliography


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