Correlation Estimation in SAR Interferograms
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Background
Knowing the relative accumulation and loss rates of ice sheets in the Earth’s polar regions is critical for understanding global climate change. Satellite radar interferometry (SAR) is a remote sensing technique that generates maps of ice accumulation by measuring the correlation of the radar echoes. Here, we present an algorithm that obtains more accurate estimates of radar correlation compared to previous remote sensing measurements. It then follows that we can compute more accurate estimates of snow accumulation.

What is correlation?
Correlation is a statistical measure indicating how related the two radar signals are. Values run between zero and one where zero indicates no relation and one indicates an exact match-up in the two signals. The equation on the right is the mathematical definition of correlation where $c_1$ and $c_2$ are two signals. In practice, we estimate local correlation by averaging over a small box of pixels in the interferogram.

What is an interferogram?
Interferograms, like the two depicted at the right, are formed by cross-correlating the complex-numbered radar images of a region on the ground for two separate satellite passes. Each complex element of an image matrix is represented as a pixel where the magnitude is mapped to a given colormap color. Because phase depends on topography and ice motion, as well as accumulation rate, phase images tend to have colored bands called fringes.

The Problem
The presence of interferometric fringes lowers the estimated correlation below true values. The diagram on the right depicts eight complex entries in vector form. As depicted by cartoon (a), an area with quickly varying phases will have an average signal of zero or nearly zero, implying a low calculated correlation. Our goal is to remove the quickly varying fringe patterns caused by ice motion and topography to obtain a slowly varying fringe pattern caused by snow accumulation. In doing so, we will obtain higher and more accurate correlation estimations as depicted by cartoon (b) of the same diagram.

Our Solution
To remove the correlating effects of fringes, we use a local Fourier filter to estimate fringe rates and then subtract the fringe component of the signal before calculating correlation. We refer to this technique as defringing or phase-flattening. In addition, this algorithm creates its own bias which we remove in a second step.

The Methods
Removing phase fringes
High fringe rates caused by local motion must be removed to estimate correlation accurately. The following depicts phase-flattening over a 48x48 pixel sample of the larger image:

Original phase matrix

$A = e^{i 2 \pi f_x x + i 2 \pi f_y y}$

We subdivide the full image into 48x48 pixel boxes, to be calculated individually. In the equation above, $A$ represents one 48x48 block of the original phase matrix where $f_x$ and $f_y$ are the fringe rates in the $x$ and $y$ directions and $e$ is the residual phase.

Interpolated FFT

$F = [F(x,y)]$

We estimate the average fringe rate in each box by using Fourier techniques. We interpolate the FFT of each box by a factor of 8 to better estimate the exact fringe rate. The peak location in the transform gives the frequency of the fringe rates in the $x$ and $y$ directions. We also determine the phase of the fringe at the maximum point.

Estimated Phase Rate

$E = \frac{\left| F\left(x, y + \frac{1}{2}\right) - F\left(x, y - \frac{1}{2}\right) \right|}{2\pi}$

In the above equation, $F(x,y)$ is the fringe rates across each block of the estimated by the position of the FFT maximum and $\frac{1}{2}$ accounts for the center of the FFT peak. Note that the color run in opposite directions in the phase matrix because the exponent is negative. There are noticeable discrepancies in the upper-right corner where the original matrix was noisy.

Phase-flattened matrix

$|F\left(x, y + \frac{1}{2}\right) - F\left(x, y - \frac{1}{2}\right)|$

Then $A \cdot B = 1$.

The corrected data results from the cross multiplication of the original and the estimated matrices. If the estimated fringe rate exactly equals the real fringe rates of the original matrix, the result is an 8x8 box with no phase variation. We Renaissance 8x8 boxes to form complete phase-flattened image.

Problems with defringing

Removing phase fringes produces a lot about defring fringes. The box must be small enough to detect very small fringe phases and yet large enough so that the algorithm will not compute phase locally.

• The algorithm has difficulty phase-flattening areas where fringes gradually change rate. The result will not be a uniformly phase-flattened area as a residual fringe pattern will remain below peaks that have an 8x8 FFT box size deals with the same uniformly changing phase. The smallest box size phase-flattens with less residual change.

Algorithmic Bias Correction

Applying the phase-flattening algorithm, a FFT box cannot cause the proper to defringe noisy areas that do not have actual fringes. Because of this, it over-estimates correlations in areas that are supposed to have low correlations. The Bias Correction Curve, depicted below, helps to visualize the algorithmic bias.

Ideally, we would like the measured correlations to exactly equal true correlations as represented by the solid black line in the plot above. The dotted blue line represents the calculated correlations after applying the defringing algorithm with an 8x8 box FFT box. For true correlations above 0.4, we are able to approximate correlations very well. However, for true correlations below 0.4, the calculated correlations are higher than actual values. This is what is meant by algorithmic bias. If the shape of the bias curve is known, the bias can be subtracted in the final estimation step.

How do we obtain the Bias Correction Curve?

We use the correlation method to obtain the defring fringes box shown in the plot above. We then use the estimated fringe rate to form a polynomial fit of the bias curve and the solid black true correlation curve to remove bias from the correlation images shown at the right.

What does this bias look like in the actual correlation images?

The top row figures are correlation images of simulated noisy signals with set true correlation values 0.9, 0.9, 0.4, and 0.9. The bottom row figures are correlation images of Ball FFT box defring versions of their corresponding top row figures. At high correlations, the difference is unnoticeable. However, as true correlations drop, the defringing algorithm causes greater correlation misestimation.

Steps in the simulation algorithm

1. Generate noisy matrix with a set true correlation value $\rho$.
2. Apply phase fringe gradient removal algorithm for FFT box size 8x8.
3. Calculate mean-correlation for each of the resulting matrices.
4. Repeat process for true correlation values between 0.0 and 0.4 in steps of 0.01.
5. Plot mean calculated correlation vs. true correlation to form bias curve.
6. Fit curve to a high-order polynomial. An 16th order polynomial was used in the plot above.
7. Calculate the difference between the polynomial fits to the bias curve and the true correlation.
8. Subtract the difference from correlation images to correct for biases due to the phase-flattening algorithm.

Problems with bias correction

From the bias correction curve depicted above, the bias is well corrected for true correlation values of 0.9 and above. However, as the measured correlation flattens out for true correlation values below 0.9, the algorithmic bias is hard to correct for because a small variation in measured bias becomes a large variation in estimated true correlation. Thus, the blue line representing the Bias Corrected Correlation in the above plot appears to be scattered for true correlation values below 0.9 instead of perfectly fitting the solid black true correlation curve as they do for true correlation values above 0.9.

Results

We apply the defringing technique described under the Methods section. In a 904x2516 pixel interferogram from Antarctica as shown in the figures below. The area covers approximately 45x125km from the location indicated by the red box on the globe to the right. Note that the following images are reduced by a factor of twelve and resized to better accommodate the size of the poster.

Correlation Images

(a) Original (b) Defrung (c) Bias-corrected

As can be seen by comparing correlation images (a) and (b), the presence of interferometric fringes lowers measured correlations below actual values. To increase measured correlations, we apply an algorithm to remove the fringes with a local Fourier filter. The defringed image (b) is noticeably more correlated, especially in ice streams.

The algorithm creates its own bias where it underestimated correlations in areas with low actual values. We removed this bias with a method also described on the left, resulting in the defrung and bias-corrected correlation image, (c). The mountainous areas of the bias-corrected image now have pieces with zero correlation while still maintaining high correlations in the ice streams.