Precoding and Interpolation for Spatial Multiplexing MIMO-OFDM with Limited Feedback

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Outline

- Introduction - Closed-loop MIMO communication
- Quantized precoding for spatial multiplexing
- Precoding and interpolation for MIMO-OFDM
- Simulation results
- Conclusions and ongoing work
Introduction

- Closed loop vs. open loop MIMO
  - Increased throughput
  - Intelligently share resources among multiple users
  - Simplify decoding algorithms
  - More easily obtain diversity

- Main challenge
  - Ensuring the transmitter is informed about the channel
**Linear Precoding w/ Spatial Mux**

- Transmit $M < M_t$ streams
  - Transmit precoder $F$ determined based on $H$
  - Linear receiver applied to effective channel $HF$
    \[ \hat{s} = \text{decision}(Gy) \]
  - Transmitter must be informed about $H$ (or $F$)

\[ y = HF s + v \]
Background

- Prior work on precoding
  - MMSE precoding [Sampath et. al.][Scaglione et. al]
  - ML based precoding [Berder et. al]
  - Precoding for MIMO-OFDM [Zhou et. al.]
- Antenna subset selection (like precoding)
  - Early work [Gore et. al.][Molisch et. al.]
  - With linear receivers [Heath et. al.][Narasimhan]
- Limited feedback precoding
  - Quantized precoding [Love & Heath]
  - Multi-mode antenna selection [Heath & Love]
Precoding in MIMO-OFDM w/ SM

Per tone model: \[ y(k) = H(k)F(k)s(k) + v(k) \]
Problem Summary

- Precoding in MIMO-OFDM requires $\{F(n)\}^{N-1}_{n=0}$
  - Feedback requirements $\propto$ Number of subcarriers

- How can we limit the feedback for each matrix?
  - Leverage limited feedback precoding [Love & Heath 2003]

- How can we reduce the number of vectors fed back?
  - Exploit correlation between precoding matrices
  - Send back fraction of vectors and use “smart” interpolation
Limited Feedback Precoding

Consider unitary precoders
\\[ F(k)^H F(k) = \frac{1}{M} I_M \]

Restrict \( F(k) \) to lie in finite codebook

\[ F = \{ F_1, F_2, \ldots, F_C \} \]
Selection of Codewords

- Choose based on desired performance metric
  - Example: Mean squared error
    \[ MSE(F(k), H(k)) = \frac{E_s}{M_s} \{ I_{M_s} + \frac{E_s}{M_s N_0} F^H(k) H^H(k) H(k) F(k) \}^{-1} \]
  - Example: Mutual information
    \[ C(F(k), H(k)) = \log_2 \det \left[ I_{M_s} + \frac{E_s}{M_s N_0} F^H(k) H^H(k) H(k) F(k) \right] \]
- Quantized precoder requires finite search
  \[ F(k) = \arg \min_{F_i \in \mathcal{F}} \det (MSE(F_i, H(k))) \quad F(k) = \arg \min_{F_i \in \mathcal{F}} \text{tr} (MSE(F_i, H(k))) \]
  \[ F(k) = \arg \max_{F_i \in \mathcal{F}} C(F_i, H(k)) \]
Selection of Codebook

- Use Grassmannian precoding [Love & Heath]
  - Codebooks are optimal packings in $G(M_t, M)$
  - Distance measure depends on precoding criteria
    - Projection 2-norm for MMSE w/ trace
    - Fubini-Study distance for capacity, MMSE w/ determinant
  - Minimizes bound on avg distor. in Rayleigh channels

$$\mathcal{F} = \{F_1, F_2, \ldots, F_C\}$$

Set of subspaces

Codebooks available at
http://www.ece.utexas.edu/~rheath/research/mimo/lf/
Reducing Feedback w/ Clustering

- Exploit coherence bandwidth of channel
  - Use every $K^{th}$ precoding matrix
  - Use same precoding matrix per cluster (ex $K=5$)

- Disadvantages
  - Performance degradation at cluster boundary
Interpolation of Precoding Mtxs

- Subsampling & interpolation

![Diagram showing interpolation of precoding matrices with subcarriers and feedback points labeled as $F(0)$, $F(K)$, $F(2K)$, etc.]

- Interpolate(?) precoding matrices
Interpolation Challenges 1/2

- Problem: Must respect orthogonality constraints
  - Recall columns of $F(k)$ should be orthogonal

$$F(k)^H F(k) = \frac{1}{M} I_M$$

$\Rightarrow$ Noneuclidean Interpolation

- Proposed solution: (inspired by SLERP [Watson ‘83])
  - Simple 1st order linear interpolation

$$Z(lK + k) = (1 - c_k)F(lK + 1) + c_kF((l + 1)K + 1)$$

$$\hat{F}(lK + k; \theta) = Z(lK + k)\{Z^H (lK + k)Z(lK + k)\}^{-\frac{1}{2}}$$

Enforces orthogonal column constraint

$$c_k = (k - 1)/K$$
Interpolation Challenges 2/2

- **Problem:** Nonuniqueness of precoders
  - Performance invariant to right x by orthogonal matrix
  - Example: \( \text{MSE}(F(k), H(k)) = \text{MSE}(F(k)Q, H(k)) \)
  - Nonuniqueness results in interpolation problems

- **Proposed solution:** “derotated interpolation”

\[
Z(lK + k) = (1 - c_k)F(lK + 1) + c_kF((l + 1)K + 1)Q_l
\]

\( Q_l \) is a M x M unitary matrix
Derotation Optimization

- Optimize rotation over a finite set
  \[ Q = \{ Q_1, Q_2, \ldots, Q_{2^P} \} \]
  - Enables limited feedback implementation
  - Use a “uniform” set of unitary matrices

- Example: MMSE with trace solves

\[
Q_l = \arg \min_{\tilde{Q} \in Q} \max_{k=0\ldots K-1} \text{tr} \left( MSE \left( \hat{F}(lK + k; \tilde{Q}) \right) \right)
\]
Proposed Interpolation Algorithm

- Step 1: Quantize \( \{ F(lK) \}_{l=0}^{\lceil N/K \rceil - 1} \)
- Step 2: Optimize \( \{ Q_l \}_{l=0}^{\lceil N/K \rceil - 1} \)

- Feedback bits required
  - Precoding matrices \( \lceil N/K \rceil \log_2 |\mathcal{F}| \)
  - Derotation matrices \( \lceil N/K \rceil \log_2 |\mathcal{Q}| \)
IEEE 802.11n Example Calculation

- Parameters
  - 4 TX antennas
  - 2 RX antennas
  - 2 streams
  - 64 tones
  - K=8 (take every 8th precoding mtx)

- Use 6-bit codebook [Love & Heath]

- Use 2-bit codebook \( \left\{ \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \pm \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \right\} \)

- Feedback required 8*6 + 8*2 = 64 bits
Quantized Beamforming/Interpolation

4TX, 2RX, 1 streams
N=64, P=2, K=8, L=6
Rayleigh channel
MMSE receiver
No coding
6 bit precoder codebook $F$
2 bits per phase

Feedback per channel
Ideal: $6N=384$ bits quantization
Selection diversity: $2N=128$ bits
Proposed: $6N/K+2N/K=64$ bits
Clustered: $6N/K=48$ bits
Orthogonal STBC = 0 bits
Quantized Beamforming/Interpolation

4TX, 2RX, 1 streams
N=64, P=2, K=8, L=6
Rayleigh channel
MMSE receiver
Rate 1/2 CC w/ interleaving
6 bit precoder codebook $F$
2 bits per phase

Feedback per channel
Ideal: 6N=384 bits quantization
Selection diversity: 2N=128 bit
Proposed: 6N/K+2N/K=\textbf{64 bits}
Clustered: 6N/K = 48 bits
Orthogonal STBC = 0 bits
Quantized Precoding/Interpolation

4TX, 2RX, 2 streams
N=64, P=2, K=8, L=6
Rayleigh channel
MMSE receiver
No coding
6 bit precoder codebook $F$
2 bit rotation codebook $Q$

Feedback per channel
384 bits quantization
166 bits antenna selection
64 bits proposed
48 bits clustered
Mutual Information Comparison

4TX, 2RX, 2 streams
N=64, P=2, K=8, L=6
Rayleigh channel
MMSE receiver
No coding
6 bit precoder codebook \( F \)
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Feedback per channel
384 bits quantization
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Conclusions

- Limited feedback precoding for MIMO-OFDM
  - Quantize and decimate precoders
  - Use smart interpolation to fill in gaps
  - Provides good diversity and capacity performance

- On-going work
  - Performance in correlated channels
  - Performance with coding and interleaving
  - Better clustering techniques
  - Extensions to multi-mode precoding
Further Reading

Overview


Narrowband Limited Feedback Techniques


Limited Feedback for MIMO-OFDM