Abstract—This paper applies recent advances in communicating over the interference channel while treating the interference as noise. Two maximization utilities are considered: maximizing the sum rates and maximizing the minimum rate. The channel model is based on a 2-sector interference channel within a single cell. The utilities maximization is presented from the perspective of the interference channel’s achievable rates region. Maximizing the minimum rate parts from the traditional problem formulation of casting it as a pure power control problem. Instead, the problem is presented as maximization over the convex hull of the rates region, therefore employing both power control and time-sharing strategies. For maximizing the minimum rate, it is shown that optimal communication over a 2-sector channel is not homogenous, and an interplay of different time-sharing types and/or power control is required to achieve optimality. Systems results are presented together with the gains achieved by using the proposed techniques over the traditional paradigms used in communicating over the 2-sector channel. Finally, insights and the contextualization of the aforementioned techniques within an information theoretic perspective are discussed.

I. INTRODUCTION

The capacity region of a 2-user communication channel has been an open problem for about 30 years [1], [2]. Information-theoretic bounds through achievable rates region have been proposed, most famously with the Han-Kobayashi bounds [3]. Recent results about the 2-user interference channel to within one bit of capacity have been shown in [4]. The achievable rates region of the interference channel treating the interference as noise has been found in [5]. The treatment of interference as noise is especially of interest when low complexity transceivers are desired, such scenarios are encountered in cellular communications, ad hoc and sensor networks. There is no cooperation at the transmit side nor at the receive side, and no multi-user decoding is used.

The achievable rates region was found to be the convex hull of the union of n regions; where each region is outer-bounded by a hyper-surface frontier of dimension n – 1. Each hyper-surface is characterized by having one of the transmitters transmitting at full power, and the other users sweeping their full range of transmit powers. For the case of 2-user interference channel, the achievable rates region is translated as the union of two regions \( R_1 \) and \( R_2 \). Each region \( R_i \) is outer-bounded by a log-defined line \( \Phi_i \), which is characterized by having \( P_i = P_{\text{max}} \) and \( P_{\tilde{j}} \neq i \) sweeps its full power range from 0 to \( P_{\text{max}} \). Explicitly, the achievable rates region for the 2-user interference channel is:

\[
\text{Convex Hull}\{R_1 \cup R_2\},
\]

where \( R_i = R(\Phi_i) \) is the rates region outer-bounded by \( \Phi_i \). The \( \Phi_i \) lines could be convex, concave, or exhibiting a non-stationary inflection point, see Figs. 2-4.

Maximizing the sum rates by analyzing the objective function have been found in [6]. The same result was also presented in [7] as a corollary from the achievable rates region theorem in [5]. In this case, specifically for the 2-user interference channel, the maximizing sum rates power tuple is either \( \{(0, P_{\text{max}}), (P_{\text{max}}, 0), (P_{\text{max}}, P_{\text{max}})\} \), where \( P_{\text{max}} \) is the maximum transmit power for each transmitter.

For maximizing the minimum rate, it is a bit more involving as the convexity over \( \Phi_1 \) and \( \Phi_2 \) can take different shapes and this needs to be taken into account. This can lead to different types of time-sharing: between ‘corner’ points using binary power control (using either zero or full power, see points A, B, C in Figs. 2-4), or between points using non-binary power control, depending on the convexity behavior.

A system model of a 2-sector interference channel is presented in Section II. Maximizing the sum rates is treated in Section III, and treatment of maximizing the minimum rate follows in Section IV. In Sections III and IV, each starts with introducing the problem formulation and then proceeds discussing the technique proposed in finding the solution. Rates regions are shown for different scenarios of users’ location, annotated with the solution point for maximum sum.

Fig. 1: 2-sector interference channel
rates and maximum minimum rate. This is followed by a grid-level illustration of when each different solution scheme is used based on the users’ geographical location. Insights and contextualization in information theory of the proposed approach are discussed in Section V.

II. SYSTEM MODEL: DOWNLINK 2–SECTOR INTERFERENCE CHANNEL

A 2–sector interference channel is presented in Fig. 1. Universal spectrum reuse is used, and each sector employs a directional antenna. Therefore in downlink communication, the base station (BS) antenna of sector 1 will cause co-channel interference to the user in sector 2, and vice-versa. The system is presented for two sectors. Whereas the same fundamental concepts can be utilized to devise a scheme for 3–sector communication, 2–sector is presented for simpler illustration.

The system has the following characteristics: a) the antenna 3–dB beamwidth is 120°, b) the transmit antenna front to back ratio is 15dB, c) the path loss exponent is 4.5, d) the cell radius is 1Km, e) the bandwidth is 10MHz, and f) the white noise power density is $-174$dBm/Hz. Based on the users’ locations in each sector, the path loss and the directional antenna gains are computed in constructing the interference channel gain matrix.

In Fig.1, the directional antenna in sector 1 communicates with user 1; user 1 receives also interfering signal from the directional antenna in sector 2. Assuming the sectors at the base station do not cooperate, the situation is modeled as a 2–user interference channel. For the purpose of illustration the location of user 1 in sector 1 is fixed, and three positions of user 2 in sector 2 are considered. The three positions are labeled respectively as (a), (b), and (c), with the corresponding polar coordinates: 300°±125°, 500°±125°, and 700°±125°, with distance in meters (see Fig.1).

The underlying equations for $R_1$ and $R_2$ in function of the users transmit powers $P_1$ and $P_2$ are

$$R_1(P_1, P_2) = \log_2 \left(1 + \frac{aP_1}{1 + bP_2} \right),$$
$$R_2(P_1, P_2) = \log_2 \left(1 + \frac{cP_2}{1 + dP_1} \right).$$

Where $a, b, c, d$ are channel gains normalized by noise power:

$$a = |g_{1,1}|^2/\sigma_n^2, \quad b = |g_{1,2}|^2/\sigma_n^2, \quad c = |g_{2,1}|^2/\sigma_n^2, \quad d = |g_{2,2}|^2/\sigma_n^2,$$

with $g_{i,j}$ denoting the flat channel gain at receiver $i$ from transmitter $j$. For each of the three cases, the log-defined lines $\Phi_i$ are evaluated to trace the rates region, achieved through power control, see Figs. 2-4. The equations of $\Phi_i$ are [5]:

$$\Phi_1 = R_2(r_1) = \log_2 \left(1 + \frac{c}{b} \frac{(P_{\text{max}}^1 - (2^{r_1} - 1))}{(2^{r_1} - 1)(1 + dP_{\text{max}}^2)} \right),$$
$$\Phi_2 = R_2(r_1) = \log_2 \left(1 + \frac{d}{a} \frac{P_{\text{max}}^2}{(1 + bP_{\text{max}}^2)(2^{r_1} - 1)} \right).$$

The range for $r_1$ in $\Phi_1$ is from $R_1(P_{\text{max}}^1, P_{\text{max}}^2) \leq r_1 \leq R_1(P_{\text{max}}^1, 0)$, see Eq. 2. And $r_1$ in $\Phi_2$ takes the complementary range of $0 \leq r_1 \leq R_1(P_{\text{max}}^1, P_{\text{max}}^2)$.

Fig. 2: Case (a): (sector-edge, sector-edge) communication

Fig. 3: Case (b): (sector-edge, center) communication

Fig. 4: Case (c): (sector-edge, far) communication

III. SUM RATES MAXIMIZATION

A. Problem Formulation

The sum rates maximization problem is formulated as

$$\max_{P_1, P_2} R_1 + R_2$$
subject to
$$P_i \leq P_{\text{max}}, \quad i = 1, 2. (3)$$
The objective function is non-concave in \([P_1, P_2]^T\). It could be solved using numerical techniques such as sequential geometric programming [8], [9]. The solution have been found in [6] to be one of the three power tuples \(\{(0, P_{\text{max}}), (P_{\text{max}}, 0), (P_{\text{max}}, P_{\text{max}})\}\). By designating \(\Phi(p_1, p_2)\) a point in the rates region with x-y coordinates, \(R_1(p_1, p_2)\) and \(R_2(p_1, p_2)\), respectively. The solution point is therefore one of the three points A (\(\Phi(0, P_{\text{max}})\)), or B (\(\Phi(P_{\text{max}}, P_{\text{max}})\)), or C (\(\Phi(P_{\text{max}}, 0)\)). In [7], it is found that if \(\Phi, s\) are concave or have inflection point, then point B is the solution; otherwise whenever it is convex the maximizing sum rates solution is either A, B, or C.

B. Rates Region and Solution Points Versus Users’ Location

Using \((\text{user}_1 \text{ location}, \text{user}_2 \text{ location})\) notation, this involves: case (a) (sector-edge, sector-edge) users in Fig.2, case (b) (sector-edge, center) users in Fig.3, and case (c) (sector-edge, far) users in Fig.4. Analyzing each case, it follows:

- Case (a), Fig.2: When both users are on sector edges, it usually leads to interference-limited regime. In this instance, the rates region log-defined lines \(\Phi_2\) and \(\Phi_1\) are both convex. The strong user, user 2, is favored. Hence, point A \(\Phi(0, P_{\text{max}})\) is the solution point for the maximum sum rates. It is a greedy scheme, and therefore in this case, user 1 is always silent.
- Case (b), Fig.3: The rates region exhibits an inflection point on the log-defined line \(\Phi_1\), and \(\Phi_2\) is concave and bounds a convex region. In this case, the maximum sum rates solution is always point B \(\Phi(P_{\text{max}}, P_{\text{max}})\).
- Case (c), Fig.4: The rates region exhibits concave \(\Phi_2\) and convex \(\Phi_1\). Therefore, the solution is either B or C. For this instance, point B \(\Phi(P_{\text{max}}, P_{\text{max}})\) is optimum.

C. Numerical Results: Generalizing over a Grid

Fig.5 generalizes over a grid location of user 2 in sector 2. The location of user 1 in sector 1 is fixed and the location of user 2 is varied over sector 2. For every “drop” of user 2, the channel gain matrix values change (explicitly the channel gains \(c\) and \(d\) change, while gains \(a\) and \(b\) remain the same). Hence for every user “drop”, a new rates region is evaluated. The convexity characterization of the rates region affects the solution. Fig.5 shows that transmitting to both users happens mainly when user 2 is facing the main directional beam of sector 2. A close user 2 is favored, sector 2 transmits at full power and sector 1 is silent. Whereas for a far user 2, user 1 is favored vice-versa.

D. Comparison with Traditional Paradigm

In communication systems such as GSM, where there is no inter-sector power control, the solution to maximizing the sum rates would be always point B, transmitting simultaneously at full power. Numerical results are presented for different user 1 locations in sector 1, see Fig.7. Three distance levels were selected: 200m, 500m, and 700m; and three angular directions are selected: 50°, 0°, and −50°, with respect to the main beam of sector 1 antenna.

For users close to the BS, transmitting to only one user is optimal in maximizing sum rates. Transmitting to two users simultaneously is accommodated generally when the users are both facing the main sectors’ beams and at a certain distance away from the BS. For sector-edge users, see top row in Fig.7, it is mostly preferred to transmit to only one user at a time; with gains that can exceed 500%!

IV. MINIMUM RATE MAXIMIZATION

A. Problem Formulation

The dual of the greedy scheme of maximizing the sum rates is the fairness scheme of maximizing the minimum rate. The power control maximizing minimum rate problem is formulated as

\[
\max_{P_1, P_2} \min_i R_i \\
\text{subject to } P_i \leq P_{\text{max}}, \quad i = 1, 2
\]

The problem is non-concave in the power arguments. However with a change of variables from \(P_i\) to \(\exp(\alpha_i)\), it becomes a Geometric Programming (GP) problem [8], [10] and solvable in convex optimization. In the context of the rates region in Figs. 2-4, this is illustrated as the dashed straight line with a slope of +1 starting from the origin and intersecting with \(\Phi_i\).s.
A revisited maximum minimum rate problem formulation suggested in this paper is:

$$\max_{P_1, P_2; \alpha_1, \alpha_2} \min C = \text{Convex Hull}\{R_i\}$$

subject to $P_i \leq P_{\text{max}}$ $i = 1, 2$. (5)

Where $\alpha_i$s are time-sharing coefficients, in the event that a time-sharing scheme is optimal. For 2-user interference channel, the time-sharing scheme is a line; therefore connecting two points, and thus each point requires a time-sharing coefficient coupled with it to define the operating point along that line. For the $n$-user interference channel, there are at maximum $n$ non-zero time-sharing coefficients; instead of 2-dimensional lines, it would be $n$-dimensional hyper-surfaces.

B. Rates Region and Solution Points Versus Users’ Location

The description of the three cases is presented herein:

- Case (a), Fig.2: The convex hull of the rates region constitutes of time-sharing between point A and point C. Therefore power control schemes should be avoided and time-sharing should be employed. The solution point is the intersection of the A-C time sharing line and the dotted line with the slope of $+1$ starting from the origin. The coordinates of A and C are known, and the intersection point of these two lines can be easily found. In the A-C time-sharing strategy, if user $i$ needs to be silent and user $j$ should transmit, then hypothetically if extra spectrum is available, user $i$ can transmit using a frequency spectrum that is orthogonal to the frequency spectrum of user $j$. In this case we depart from the universal reuse of 1, to a reuse of 2.

- Case (b), Fig.3: The solution point lies on the frontier $\Phi_1$ which is characterized by having $P_1 = P_{\text{max}}$. In this case the solution point is achieved through power control on the variable $P_2$. The max min GP problem over the Shannon capacity equations in Eq.(4) will hence lead the optimum solution, as no convex time-sharing hull is needed.

The solution point involves power control, therefore the reuse factor is 1.

- Case (c), Fig.4: The maximum minimum rate solution is on the time-sharing line B-C.

The solution point involves time-sharing between B and C. Point B commands a reuse factor of 1, while C allows a reuse factor of 2. Based on the duty cycle of time-sharing between point C with time-sharing coefficient $\alpha_1$, and point B with time-sharing coefficient $\alpha_2$, $(\alpha_1 + \alpha_2 = 1)$, then the effective fractional reuse for the solution point is: $\alpha_1 \times 2 + \alpha_2 \times 1$. In this figure example the fractional reuse value is 1.21.

C. Numerical Results: Generalizing over a Grid

The color-coded maximum minimum rate solution is relatively more involved, and it is illustrated in Fig.6. Note that the maximum min rate problem using solely power control scheme
follows Eq. (4), whereas the revisited maximum minimum rate over the rates region convex hull, see Eq. (5), allows time-sharing to be used in conjunction with power control. The outcome is manifested in Fig. 6. The color-coding policies employed are explained as follows:

- 'PC-P1': Power control on $P_1$ with $P_2 = P_{\text{max}}$.
- 'PC-P2': Power control on $P_2$ with $P_1 = P_{\text{max}}$.
- 'TS-DB': Time-sharing between point D and point B. Point D is the non-stationary inflection point on the $\Phi_{BC} = \Phi_1$, with $P_1 = P_{\text{max}}$.
- 'TS-BE': Time-sharing between point B and point E. Point E is the non-stationary inflection point on the $\Phi_{AB} = \Phi_2$, with $P_2 = P_{\text{max}}$.
- 'TS-AB': Time-sharing between point A and point B. In this case, $\Phi_{AB}$ is convex. As point B commands a reuse of 1, and point A allows for reuse of 2, this time-sharing scheme results in fractional reuse. Note that the fractional reuse value is also color-coded in the plot. The RGB (Red, Green, Blue) color composition has been set to $[\alpha_1, \alpha_2, \alpha_3] = [\alpha \Phi(P_{\text{max}}, 0), \alpha \Phi(P_{\text{max}}, P_{\text{max}}), \alpha \Phi(0, P_{\text{max}})] = [\alpha C, \alpha B, \alpha A]$. Therefore for this 'TS-AB' case, the closer the solution point is to point A, the closer the marker color is to Blue; and the closer the solution point is to point B, the closer the marker color is to Green. As discussed earlier, from the alphas, at maximum two are nonzero (as it is time-sharing between two points). For this 'TS-AB', $\alpha_C$ is zero.
- 'TS-BC': Similarly to 'TS-AB', this case denotes time-sharing between point B and point C. Using the color-coding policy, the closer the solution point to B, the closer the marker color is to Blue; and the closer the solution point is to C, the closer the marker color is to Red.
- 'TS-AC': Time-sharing between point A and point C, using the same color-coding policy.

Remark: In fact, ignoring the cases where the $\Phi_s$ exhibit inflection point leads to marginal loss in the rates region. Considering only the schemes of 'TS-AB', 'TS-BC', and 'TS-AC', i.e. forcing time-sharing between corner points (formed by binary power control: zero or full power) in the rates region, leads to what is described as a crystallized rates region [11].

The cases where the log-defined lines $\Phi_i$ exhibit inflection point are described and included here for completeness purposes.

D. Comparison with Traditional Paradigm

Fig. 8 addresses the maximum minimum rate for the same user cases as in Subsection III-D. The gains from optimizing over the convex hull are presented versus the traditional scheme of only employing power control in formulating the maximum minimum rate problem.

Focusing on the sector-edge users, see the top row sub-figures in Fig. 8, time-sharing is used extensively as the communication is more interference-limited. The gains of time-sharing in such situations can be quite significant.

For the center row in Fig. 8, user 1 has a good channel gain as it is facing directly the sector antenna’s beam; in this case, power control is a good strategy.

For the bottom row, user 1 is in the back lobe of sector 2; the communication is more noise-limited and hence power control is a good strategy. Note that in reality, sector 3 would cause interference to sector 1. In our analysis we discounted this case, and we only treated the system as a 2-user interference channel between sector 1 and sector 2. Therefore in practice and by symmetry, the situation in the bottom row of sub-figures would be similar to the top row sub-figures.

V. INSIGHTS AND CONTEXTUALIZATION IN INFORMATION THEORY

Fig. 9: Insights over communication strategies

Earlier communication systems were noise-limited, and in that regard the rates region frontiers ($\Phi_s$) are concave, see Fig. 9. This had championed power control as the favorite communication scheme to trace the operating point on the rates region frontier. As more base stations got deployed and as the cell size became smaller, to accommodate a growing number of users and to allow more spatial reuse of the spectrum within a geographical area, the interference level started to become increasingly the limiting factor of the communication rates. Thus, interference is considered the latest barrier in wireless communication [12], [13]. Fig. 9 shows clearly that when the system is interference-limited then power control is no longer optimal and dimension-orthogonalization should occur; in this case it takes the form of time-sharing. The concept of dimension-orthogonalization is important to escape the hindering effect of interference. The dimension-orthogonalization gives rise to the degrees of freedom in the interference channel. It has been shown recently, through interference alignment, that the degrees of freedom in the interference channel scales linearly in the number of users [14]–[16].

The interference channel capacity to within one bit has been found in [4], from which we reproduce the illustrative figure, Fig. 10, to examine the assumptions in this paper. The work in [4] assumes infinite interference to noise ratio (INR $\to \infty$) and infinite signal to noise ratio (SNR $\to \infty$), while keeping a deterministic $\log(\text{INR})/\log(\text{SNR})$ ratio. The channel under study is 2-user symmetric, and as a consequence the users indices are dropped. The x-axis is the $\log(\text{INR})/\log(\text{SNR})$ ratio, which is varied from 0 onwards.
We note some points of interest:

- If $\log(INR)/\log(SNR) \geq 2$, then the interference is strong. And from information theoretic point of view, it is decoded and removed [17]. Therefore $C/C_{\text{no interference}}$ is again 1, signifying that the strong interference is equivalent to no interference in capacity terms.

- The green dotted line is the capacity bound within one-bit where the interference is treated as noise. In the range of $0 \leq \log(INR)/\log(SNR) \leq 0.5$, treating the interference as noise is the information theoretic optimum strategy. This is reminiscent to noise-limited communication, where the interference is not strong; the achievable rate $R$ is equal to the capacity $C$.

- At $\log(INR)/\log(SNR) = 0.5$, see Fig.10, this represents the switching point from treating the interference as noise to using dimension-orthogonalization such as TDM or FDM. In terms of $a$, $b$ and $P_{\text{max}}$, this translates as: $b \geq \sqrt{aP_{\text{max}}}$, With no infinite SNR nor infinite INR assumption, that switching point is found to be [5]:

$\log(INR)/\log(SNR) = \frac{1 + \sqrt{aP_{\text{max}}}}{P_{\text{max}}}$.

Fig.10: The approach of power control and time-sharing

![Fig. 10: The approach of power control and time-sharing](image)

The paper applied recent advances in communicating over the interference channel while treating the interference as noise for the context of a 2-sector interference channel. It treated the sum rates and minimum rate maximization from the perspective of the achievable rates region. The maximum minimum rate problem formulation was revisited and presented a maximization over the convex hull of the rates region, thus permitting time-sharing to be involved. Numerical results for the 2-sector communication showed perceived gain which accentuates for sector edge users. In addition, insights behind the shift from power control to the importance of using time-sharing were presented when the underlying communication system is interference-limited. The proposed scheme of using power control and time-sharing has been also justified as attractive in the light of recent information theoretic result on the capacity of the interference channel.

VI. CONCLUSION

The paper applied recent advances in communicating over the interference channel while treating the interference as noise for the context of a 2-sector interference channel. It treated the sum rates and minimum rate maximization from the perspective of the achievable rates region. The maximum minimum rate problem formulation was revisited and presented a maximization over the convex hull of the rates region, thus permitting time-sharing to be involved. Numerical results for the 2-sector communication showed perceived gain which accentuates for sector edge users. In addition, insights behind the shift from power control to the importance of using time-sharing were presented when the underlying communication system is interference-limited. The proposed scheme of using power control and time-sharing has been also justified as attractive in the light of recent information theoretic result on the capacity of the interference channel.

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