Efficient Monitoring of Constellation Clock Offset Faults in Advanced RAIM and Related Systems

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ABSTRACT

We describe a novel method to mitigate constellation clock offset faults in ARAIM. The method, which can be seamlessly integrated in current ARAIM user algorithms, is simple, efficient, and results in significant performance improvements compared to the state of the art, provided that the clock offset fault dominates the probability of constellation-wide fault budget.

INTRODUCTION

The standardization of Advanced RAIM for aviation, an extension of RAIM that will integrate up to four core constellations (GPS, Galileo, BeiDou, and GLONASS), and signals in two frequencies (L1 and L5), is well underway [1, 2]. These standards will be used for decades to come and are very slow to update. It is therefore desirable that they make room for future improvements, so that users can get the full potential of the GNSS core constellations as they improve their Integrity Support Data (ISD). Of the classes of faults now accounted by the constellation wide fault probability and rate (P_{const} and R_{const}), there are some that GNSS providers may be able to reduce, and others that are better left for the users to mitigate.

The purpose of this paper is to show that an important class of constellation wide faults could be mitigated by the user more efficiently than grouping it with an arbitrary constellation wide fault.

In ARAIM, if there is a user algorithm that can mitigate a given fault, then it can be mitigated with solution separation [3]. In fact, solution separation algorithms are, in a certain (and precise) way, optimal, in that they offer the smallest probability of missed detection for a fixed probability of false alert for an all-in-view least squares estimator. For all other estimators, they provide a very simple and conceptually simple approach to fault detection. The idea at the core of solution separation is to protect against each fault by comparing the user position solution (for example an all-in-view solution) to a position solution that is tolerant to the fault, or, in other words, computed with a fault tolerant filter. Even if a different algorithm is used, the achievable protection level is directly dependent on the strength of the best fault tolerant filter [4]. The computation of the Protection Level in ARAIM can therefore be reduced to the computation of the user error standard deviation for each of the fault tolerant filters corresponding to the monitored faults [5, 6].

One of the faults driving the performance of Advanced RAIM is the constellation wide fault [7]. For example, the coverage of Horizontal RAIM with GPS and Galileo using the default ISD is mostly driven by the probability of a constellation wide fault in Galileo. This probability (P_{const}), which is set at $2\text{x}10^{-4}$, forces the algorithm to monitor a hypothetical fault affecting all Galileo satellites. This fault mode covers any possible fault in all or a subset of the Galileo satellites. This is extremely conservative because some of the most likely constellation-wide fault mechanisms result in faults with a very specific structure. For example, an erroneous Earth Orientation Predicted Parameter will result in a position error that is a translation in a known direction. This constraint on the parameters of the fault can greatly increase the geometry strength of the fault tolerant filter [8].

Another possible mechanism, and the focus of this work, is an update in the clock reference that is only applied to a few satellites in the constellation. This type of fault would result in the use of two distinct clock offsets within one constellation. This would be akin to the use of two different clock offsets for two different core constellations (which is already accounted for in ARAIM). If we were to apply the principles of solution separation to this class of faults, each fault corresponding to a partition of satellites would be monitored using the corresponding fault tolerant filter. Each one of these filters is usually very strong, at least compared to the filter tolerant to an arbitrary constellation wide fault. The trouble is that there are many such partitions. For example, in a geometry with 14 satellites in view from the constellation that may be affected by the fault, we would need to compute the standard deviation of 2^{13}-1 = 8191 filters. While it may not be impossible to compute such number of covariances, it is unpractical, at least for current aviation receivers. Partly for this reason, the Advanced RAIM standards do not contemplate the distinction of constellation-wide faults between these very structured faults and all other faults, even though the clock offset fault may be the most likely mechanism for a constellation-wide fault.

In this work, we show that a user ARAIM algorithm can protect against all such clock offset faults, without the need to list and compute every single corresponding fault tolerant filter. This is achieved by, first, using a sum of square residuals detection test and, second, by computing an upper bound on the variance of the fault tolerant filter errors. Rather than simply grouping the fault with the general constellation-wide fault, the upper bound exploits the very specific structure of the fault. As in the methods proposed for very large probability of faults [5, 6], the method replaces thousands of terms in the PL equation by one single term. The computation of this term is orders of magnitude faster than computing all the terms, and it results in very significant performance improvements (if it is assumed that it is the leading cause of constellation wide faults).

PROTECTING AGAINST FAULTS WITHOUT EXPLICIT FAULT TOLERANT SOLUTION

In this section, we review basic elements of autonomous fault monitoring that we will need to describe the contribution of this paper.

We start with a key result on fault monitoring that forms the basis of this paper, and fault monitoring in general. We consider a standard measurement nominal error model

$$y = Gx + \varepsilon \tag{1}$$

where $\varepsilon \sim N\left(0, W^{-1}\right)$, where W is the inverse of the measurement noise covariance.

The occurrence of a fault can often be modeled with the addition of an unknown state as

$$y = Gx + Fb + \varepsilon \tag{2}$$

where matrix F, which dictates the structure of the fault, is known and the state b is unknown.

In solution separation, this fault is monitored by comparing the all-in-view solution to the solution that is tolerant to the fault. That is, the test checks that

$$\left| e^T \left(\hat{x}^{(0)} - \hat{x}^{(F)} \right) \right| \leq T_F \tag{3}$$

where

$\hat{x}^{(0)}$ is the all-in-view solution

$\hat{x}^{(F)}$ is the fault tolerant solution

e is the unit vector for the coordinate of interest (for the vertical coordinate, it is [0 0 1 0])

T_F is the detection threshold

For a given test statistic τ and detection threshold T, we define $P_{md}(H_F)$ as the probability of loss of integrity due to fault hypothesis H_F

$$P_{md}(H_F) = P\left(\tau \le T \; \& \left|e^T\left(\hat{x}^{(0)} - x\right)\right| > L \mid H_F\right) \tag{4}$$

For the solution separation test, we have the following key upper bound

$$P_{md}(H_F) \le P\left(e^T\left(\hat{x}^{(F)} - x\right) > L - T_F\right) + P\left(e^T\left(\hat{x}^{(F)} - x\right) > L\right) \tag{5}$$

This means that the probability of not bounding the error by L when applying the solution separation test is bounded by the right-hand term.

Under hypothesis H_F, the right-hand side can be computed using the statistics of the nominal mode, because the estimator is fault tolerant. We therefore have

$$P_{md}(H_F) \le Q\left(\frac{L - T_F}{\sigma_{(F)}}\right) + Q\left(\frac{L}{\sigma_{(0)}}\right) \tag{6}$$

where

Q is the complementary CDF of a normal distribution.

$\sigma_{(0)}$ is the standard deviation of the Gaussian distribution bounding the all in view solution error

$\sigma_{(F)}$ is the standard deviation of the Gaussian distribution bounding the fault tolerant solution error

The expressions for $\sigma_{(0)}$ and $\sigma_{(F)}$ are well-known and can be found, for example, in [3]. The upper bound (6) is the basis of the protection level equation described in [3].

We also have the following well-known inequality [3], [5]

$$\left|\frac{e^T\left(\hat{x}^{(F)} - \hat{x}^{(0)}\right)}{\sigma_{ss,F}}\right| \le \sqrt{y^T P y} \tag{7}$$

where $P = W - WG\left(G^T WG\right)^{-1} G^T W$

This inequality enables the evaluation of the solution separation test without the need to compute the fault tolerant solution, because

$$\sqrt{y^T P y} \leq K_{FA} \Rightarrow \left| e^T \left(\hat{x}^{(F)} - \hat{x}^{(0)} \right) \right| \leq K_{FA} \sigma_{ss,F} \tag{8}$$

where $\sigma_{ss,F} = std\left(e^T \left(\hat{x}^{(0)} - \hat{x}^{(F)} \right)\right)$(when assuming the Gaussian model)

Therefore, we have

$$P\left(\sqrt{y^T P y} \leq K_{FA} \ \& \left| e^T \left(\hat{x}^{(0)} - x \right) \right| > L \mid H_F \right) \leq Q\left(\frac{L - K_{FA}\sigma_{ss,F}}{\sigma_{(F)}} \right) + Q\left(\frac{L}{\sigma_{(0)}} \right) \tag{9}$$

In addition, $\sigma_{ss,F}$ and σ_F are linked by the equation

$$\sigma_{ss,F} = \sqrt{{\sigma_F}^2 - {\sigma_{(0)}}^2} \tag{10}$$

We therefore have

$$P_{md}\left(H_F\right) \leq Q\left(\frac{L - K_{FA}\sqrt{{\sigma_F}^2 - {\sigma_{(0)}}^2}}{\sigma_{(F)}} \right) + Q\left(\frac{L}{\sigma_{(0)}} \right) \tag{11}$$

Note that it can be shown that the exact computation of the loss of integrity risk is also a function of $\sigma_{(F)}$ [3].

The important point here is that to compute an upper bound on the probability of loss of integrity for a given fault mode, we only need an upper bound on $\sigma_{(F)}$. This is the subject of the next section.

UPPER BOUND OF THE ERROR COVARIANCE WITH CLOCK OFFSET

The clock offset fault can be modeled as indicated, where the matrix *F* is now a vector, which we will note *f*, with ones in the measurements affected by the clock offset and zeros everywhere else. An example could be

$$f = \begin{bmatrix} 1 & 0 & 1 & 1 & \cdots & 0 & 1 \end{bmatrix}^T \tag{12}$$

Because adding an offset to a subset of satellites is strictly equivalent to adding an offset to the complementary set, there are 2^{n-1}-1 possible fault vectors f (we don't need to consider the case where no satellites are affected). For 14 satellites in view, this would amount to 8190 vectors.

The measurement model is as in (1) but with $F = f$. The all-in-view covariance and estimator is given by

$$C = \left(G^T W G\right)^{-1} \tag{13}$$

$$S = C \cdot G^T W \tag{14}$$

The all-in-view sigma in the coordinate of interest e is given by

$$\sigma_{(0)} = \sqrt{e^T C e} \tag{15}$$

The covariance of the errors in the fault tolerant estimator (that is, the one that estimates the clock offset states) can be computed by computing the error covariance of the estimate corresponding to the augmented observation equation

$$C_{aug} = \left(H^T W H\right)^{-1} \tag{16}$$

where $H = \begin{bmatrix} G & f \end{bmatrix}$

After replacing H with its expression above, we have

$$C_{aug} = \begin{bmatrix} G^T W G & G^T W f \\ f^T W G & f^T W f \end{bmatrix}^{-1} \tag{17}$$

We note $C_{(f)} = \begin{bmatrix} I_q & 0 \end{bmatrix} C_{aug} \begin{bmatrix} I_q & 0 \end{bmatrix}^T$ where q is the number of original states (position plus original clock states).

Using the matrix inversion formula, the covariance of the original states after adding the clock offset states representing the fault, $C_{(f)}$, is given by

$$C_{(f)} = C + \frac{Sf \cdot (Sf)^T}{f^T P f} \tag{18}$$

where the matrix P was defined above (Equation (7)).

The sigma of the coordinates is therefore given by

$$\sigma_{(f)}^2 = e^T \cdot C_f \cdot e = \sigma_{(0)}{}^2 + \frac{\left(s^T f\right)^2}{f^T P f} \tag{19}$$

where $s = S^T e$

As mentioned earlier, all we need to compute the protection level is an upper bound of $\sigma^2_{(f)}$, which is equivalent to finding an upper bound of

$$\max \frac{\left(s^T f\right)^2}{f^T P f} \tag{20}$$

with f subject to

$f_i = 0$ or 1 for i corresponding to indices in the faulted constellation

$f_i = 0$ for all other indices

Maximum

It is possible to compute an exact upper bound by computing $\sigma^2_{(f)}$ for all possible subsets of satellites affected by the offset by using Equation (20) for each possible vector f. There are 2^{n-1}-1 of them. While this number seems prohibitive for current aviation receivers, it might not be in the future. For the availability simulations shown in the results section, it was entirely possible to compute the exact upper bound.

Upper bound

Another possible approach is to compute an upper bound. We will first compute an upper bound for a fixed number of clock offsets *k*. That is, we have the additional constraint

$$[\underline{1}]^T f = k \tag{21}$$

To find an upper bound of the above expression constrained by (21), we compute an upper bound of the denominator and a lower bound on the denominator.

For the denominator, we compute

$$\max \left| s^T f \right| = \max_{|J|=k} \left| \sum_{i \in J} s_i \right| \tag{22}$$

This can be computed very fast by sorting the elements of *s* in ascending order and taking the maximum of the absolute value of the first *k* terms or the last *k* terms.

$$\max \left| s^T f \right| = \max \left\{ \left| \sum_{i=1}^{k} s_{[i]} \right|, \left| \sum_{i=n_{wf}-k+1}^{n_{wf}} s_{[i]} \right| \right\} \tag{23}$$

For the denominator, we want to find a lower bound of

$$\max f^T P f \tag{24}$$
$$\text{subject to } f_i = 0 \text{ or } 1 \text{ for } i \in J \text{ , } f_i = 0 \text{ for } i \notin J \text{ and } \left[\underline{1}\right]^T f = k \text{ .}$$

Computing the above expression requires the solution of a constrained quadratic binary optimization. This is a well-known problem. In particular, it is known to be NP hard. For our purposes, it means that an exact solution is a combinatorial problem. In our application, this is not strictly out of reach, since we “only” need to evaluate thousands of points. However, compared to the current computational demands for RAIM or even ARAIM, it is not practical.

We compute a lower bound as follows. First, we will only consider the portion of P that is affected by f by defining

$$\tilde{P} = P\left(J, J\right) \tag{25}$$

We then compute the singular value decomposition of P

$$\tilde{P} = UDU^T \tag{26}$$

where U is orthogonal and D is diagonal. Without loss of generality, we redefine f as the component of f corresponding to the faulted constellation.

We have

$$f^T \tilde{P} f = f^T UDU^T f = \sum_{i=1}^{n} d_i \left(u_i^T f\right)^2 \tag{27}$$

Let us now order the singular values are in ascending order. We have, for any integer p:

$$d_{\min} \sum_{i=p+1}^{n} \left(u_i^T f\right)^2 \leq \sum_{i=p+1}^{n} d_i \left(u_i^T f\right)^2 = \sum_{i=p+1}^{n} d_i \left(u_i^T f\right)^2 \leq f^T \tilde{P} f \tag{28}$$

where $d_{\min} = \min_{i \geq p+1} d_i$

Since U is orthogonal, we have

$$|f|^2 = \sum_{i=1}^{p}\left(u_i^T f\right)^2 + \sum_{i=p+1}^{n}\left(u_i^T f\right)^2 \tag{29}$$

Combining (26) and (27), we obtain

$$d_{\min}\left(|f|^2 - \sum_{i=1}^{p}\left(u_i^T f\right)^2\right) \le f^T \tilde{P} f \tag{30}$$

For p such that d_{min}>0, the null space of $\tilde{P}$ is included in the span of the vectors u_i for i=1 to p. We already know one vector in the null space of $\tilde{P}$ because $\tilde{P}\underline{1} = 0$

We note

$$\tilde{U} = \begin{bmatrix} u_1 & \cdots & u_p \end{bmatrix} \tag{31}$$

So that

$$\sum_{i=1}^{p}\left(u_i^T f\right)^2 = \left|\tilde{U}^T f\right| \tag{32}$$

We now rotate the basis of $\tilde{U}$ so that $\frac{\underline{1}}{|\underline{1}|}$ is one of the vectors defining the basis. That is, we want to find a p by p rotation matrix R such that

$$\tilde{U} \cdot R = \begin{bmatrix} \underline{1} & \bar{U} \end{bmatrix} \tag{33}$$

To this purpose, we compute the first column of R by using

$$\tilde{U} \cdot r_1 = \frac{\underline{1}}{|\underline{1}|} \tag{34}$$

Therefore

$$r_1 = \tilde{U}^T \frac{\underline{1}}{|\underline{1}|} \tag{35}$$

We complete r_1 into a basis, for example by computing

$$\tilde{R} = Ker\left(r_1^T\right) \tag{36}$$

We define

$$\bar{U} = \tilde{U} \cdot \tilde{R} \qquad (37)$$

We have (by construction)

$$\bar{U}^T \frac{\underline{1}}{|\underline{1}|} = \tilde{R}^T \cdot \tilde{U}^T \tilde{U} \cdot r_1 = \tilde{R}^T r_1 = 0 \qquad (38)$$

$$\left|\tilde{U}^T f\right|^2 = \left|R\left[\underline{1} \quad \bar{U}\right]^T f\right|^2 = \left(\frac{\underline{1}}{|\underline{1}|}^T f\right)^2 + \left|\bar{U}^T f\right|^2 \qquad (39)$$

This implies

Going back to Equation (30) and (32), our goal is to find an upper bound of $\left|\tilde{U}^T f\right|^2$

The first term in the right-hand side of (3) can be directly computed because for any vector *f* meeting Equation (21)

$$\frac{\underline{1}}{|\underline{1}|}^T f = \frac{k}{\sqrt{n_{wf}}} \qquad (40)$$

We now only need to find an upper bound of $\left|\bar{U}^T f\right|^2$.

For most geometries, it turns out that p=2 yields $d_{min}>0$, so that $\bar{U}$ is a vector. In this case, we compute the upper bound as for the denominator (see equations above), that is

$$\max\left|\bar{U}^T f\right| = \max\left\{\left|\sum_{i=1}^{k} \bar{U}_{[i]}\right|, \left|\sum_{i=n_{wf}-k+1}^{n_{wf}} \bar{U}_{[i]}\right|\right\} \qquad (41)$$

We end up with the lower bound on the denominator

$$d_{\min}\left(k - \frac{k^2}{n_{wf}} - \max\left\{\left|\sum_{i=1}^{k} \bar{U}_{[i]}\right|, \left|\sum_{i=n_{wf}-k+1}^{n_{wf}} \bar{U}_{[i]}\right|\right\}\right) \le f^T \tilde{P} f \qquad (42)$$

For the cases where $d_{min} = 0$, we use the upper bound on the fault tolerant filter provided by the filter tolerant to the constellation wide fault.

This upper bound can now be used in the term (11) that will account for the clock offset fault mode in the protection level equation.

SIMULATION RESULTS

We evaluate this approach in a Horizontal ARAIM scenario using our publicly available MAAST for ARAIM simulation tool [9]. The general settings for the simulation are as follows:

- GPS 23 – Galileo 23 (this is a depleted scenario)
- Dual frequency L1-L5

- Default ISD (Table 1). The GPS IURA and nominal URA were set at 2.4 m for the simulations.
- FDE availability (as in [9])
- 24 hours, every 5 minutes on a 10 by 10 user grid

Table 1. *ISD for H-ARAIM scenario based on draft ARAIM SARPS*

	GPS	Galileo
$P_{const,default}$	1×10^{-8}	2×10^{-4}
$P_{sat,default}$	1×10^{-5}	3×10^{-5}
$R_{const,\ default}$	1×10^{-8}/h	1×10^{-4}/h
$R_{sat,\ default}$	1×10^{-5}/h	2×10^{-5}/h
$MFD_{const,\ default}$	1 hour	ILB
$MFD_{sat,\ default}$	1 hour	ILB
$\sigma_{URA,default,\ dual\ frequency}$ [m]	IAURA	6
$\sigma_{URE,default,\ dual\ frequency}$ [m]	Nominal URA	4
$b_{nom,\ default}$ [m]	0	0

Figure 1 shows the histogram corresponding to the distribution of $\sigma_{ss,(f)}$ for one geometry. The maximum are 1.3 m and 3.2 m for the East and North coordinates respectively. The bounds obtained using the upper bound developed above are 2.8 m and 5.9 m, which is conservative, but much less than the σ_{ss} corresponding to the unconstrained constellation-wide fault, which are 7.1 m and 32 m.

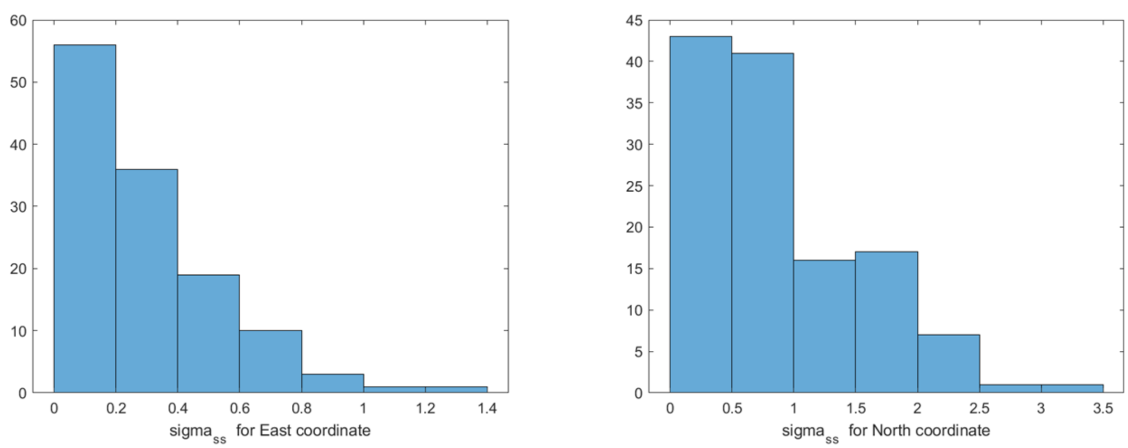

Figure 1. Distribution of $\sigma_{ss,(f)}$ for one example geometry for the East and North coordinates

Figure 2 shows the HPLs for one user location over 24 hrs. The new formulation (combined with the restricted definition of the constellation-wide fault) removes the HPL peaks due to the weak GPS geometries.

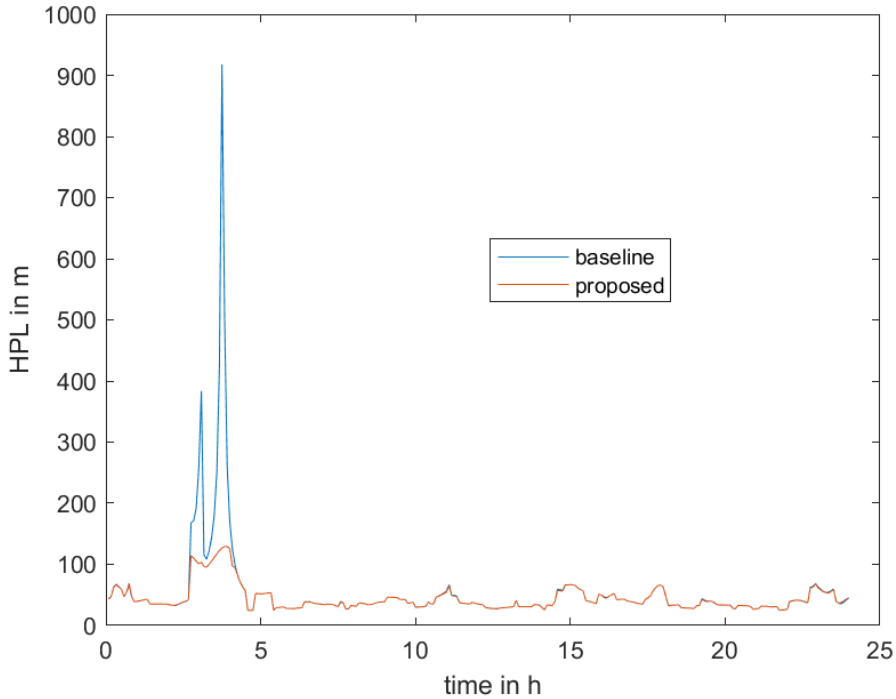

Figure 2. *HPL for baseline and proposed method for one user location*

Figure 3 shows the coverage map for the baseline case. In this case, the user receiver monitors the Galileo constellation-wide fault by comparing the all-in-view solution to the GPS only solution. For this reason, anytime there is a weakness in the GPS only geometry, the PLs are very large. In this scenario, HPLs of 556 m are not achieved with high availability in large portions of the globe.

Figure 4 shows the coverage map for the case where we have assumed that the Galileo P_{const} main contributor is the set of clock offset faults that is covered by the method based on taking the maximum of $\sigma^2_{(f)}$. The constellation-wide fault mode is protected using the sum of square residuals and the bound on the integrity risk contribution using Equation (11) and the bound on the North and East components for $\sigma_{(F)}$ obtained computing the maximum using (19) and (20).

Figure 5 shows the coverage map for the case where we have assumed that the Galileo P_{const} main contributor is the set of clock offset faults that is covered by the method based on the upper bound described above. The constellation-wide fault mode is protected using the sum of square residuals and the bound on the integrity risk contribution using Equation (11) and the bound on the North and East components for $\sigma_{(F)}$ obtained using the bounds (23) and (42). We now have almost complete coverage of HPLs below 556 m.

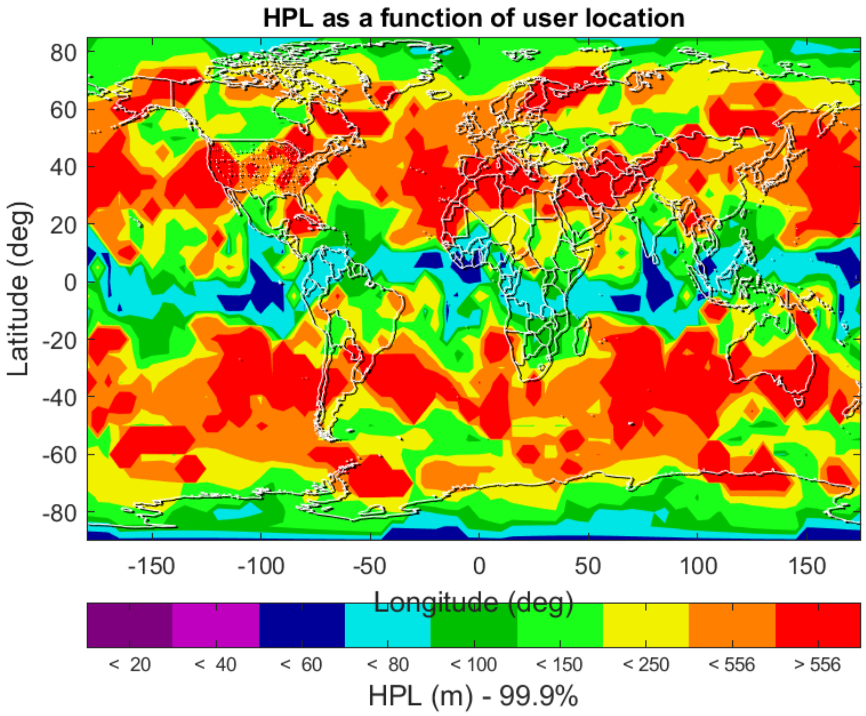

Figure 3. *99.9% HPL for a GPS 23 – Galileo 23 H-ARAIM scenario with the default ISD*

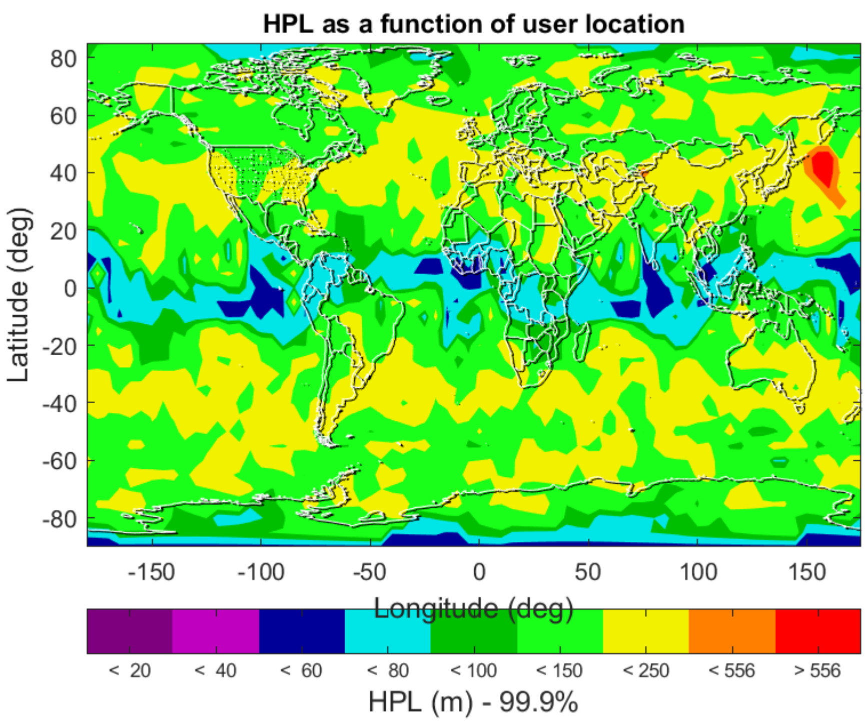

Figure 4. *99.9% HPL for a GPS 23 – Galileo 23 H-ARAIM scenario with the proposed restriction of the Galileo constellation-wide fault and the proposed method with an explicit maximum (Equations (19) and (20))to protect against them*

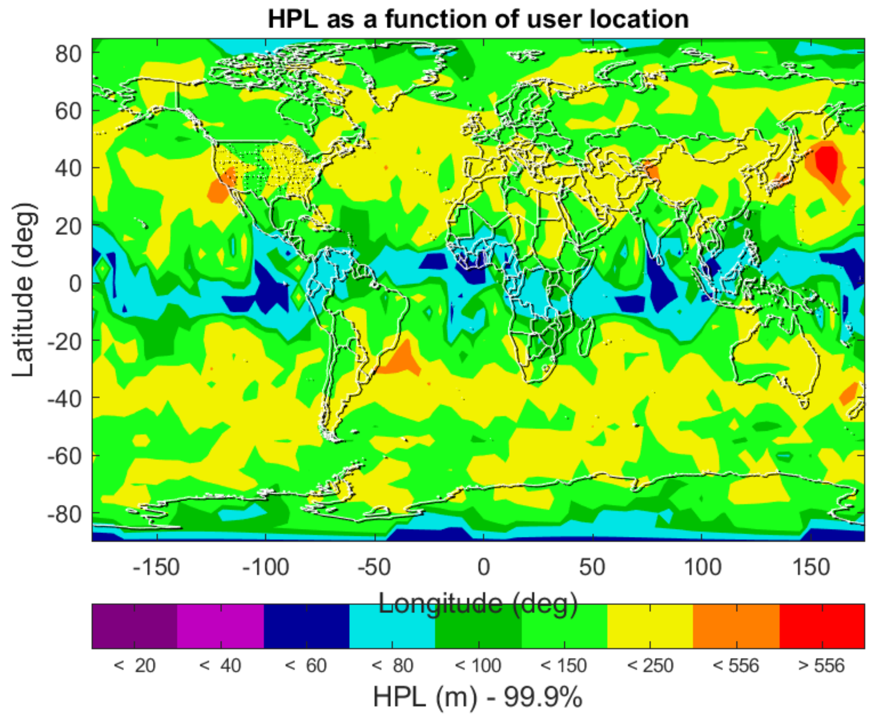

***Figure 5**. 99.9% HPL for a GPS 23 – Galileo 23 H-ARAIM scenario with the proposed restriction of the Galileo constellation-wide fault and the upper bound proposed above (Equations (22)-(42)) to protect against them*

SUMMARY

This paper describes a novel method to mitigate constellation clock offset faults in ARAIM. The method, which can be seamlessly integrated in current ARAIM user algorithms, is simple, efficient, and results in significant performance improvements compared to the current baseline approach, provided that the clock offset fault dominates the probability of constellation-wide fault budget. H-ARAIM availability simulations for depleted constellations show that significant performance improvements could be obtained.

ACKNOWLEDGEMENTS

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