

Analysis of Synchronized Ionosphere Anomaly Wave Front Impacts on Multiple Satellites

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OBJECTIVE:

Investigate actual satellite geometries to see if, in practice, there exist geometries which could suffer from the theoretical condition of undetectable fronts for the ionosphere monitoring method introduced in [1].

INVESTIGATION:

- (1) Generate satellite geometries for three airports (Memphis, LA, NY) based on the 24-satellite almanac on July 1, 1993 given by [2].
- (2) Search for ionosphere-pierce-point (IPP) pairs and triplets whose geometries have the potential to experience the undetectable condition.

RESULTS:

Found several IPP pairs that satisfy this particular condition, but no triplets .

CONCLUSIONS:

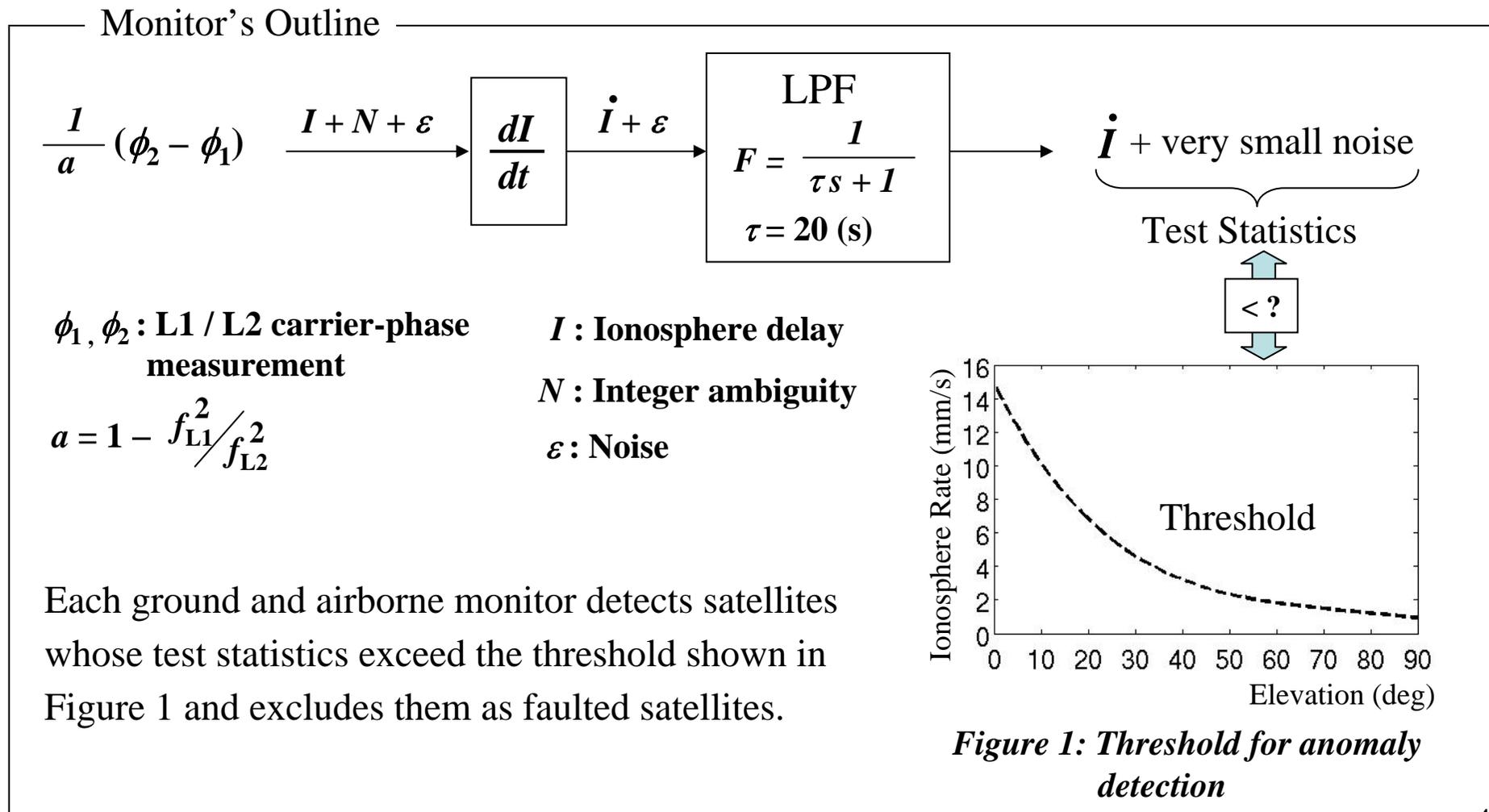
In practice, the undetectable condition for fronts affecting two satellites could be satisfied, while the condition for those affecting three satellites is almost impossible to be satisfied.

Therefore, the ionosphere monitoring method could (rarely) miss detecting fronts affecting two satellites but practically always detects those affecting more than two satellites.

Ionosphere monitoring method

Outline of the ionosphere monitoring method:

Ground and airborne monitors independently estimate instantaneous rates of change of ionosphere delays to detect satellites whose signals are most probably affected by ionosphere anomalies.

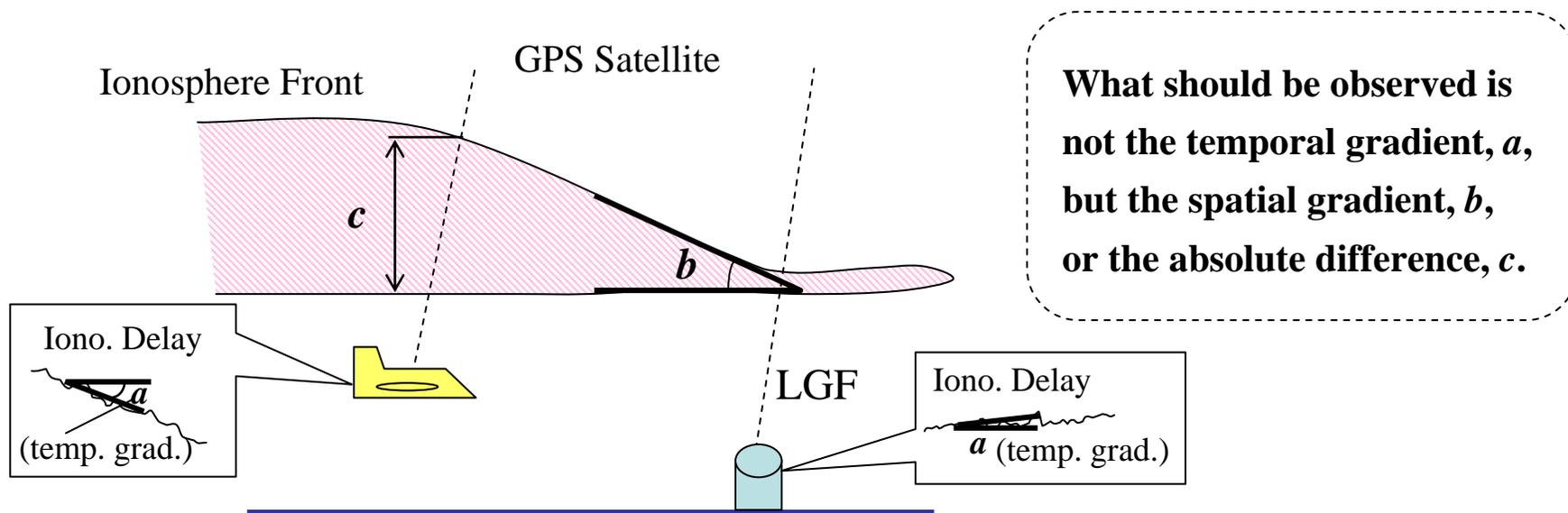


Fundamental problem:

A fundamental problem of this method is that it observes temporal gradients, \dot{I} , rather than spatial gradients or absolute differences of ionosphere delays between the user and LAAS Ground Facility (LGF).

If an ionosphere front that induces a large differential range error looks stationary from the monitor's point of view, it is very difficult to detect.

What does "looks stationary" mean? (see following slides)



What does “looks stationary” mean? (1):

Suppose an ionosphere front with a gradient of α (mm/km) is moving with a velocity of V_{front} (km/s).

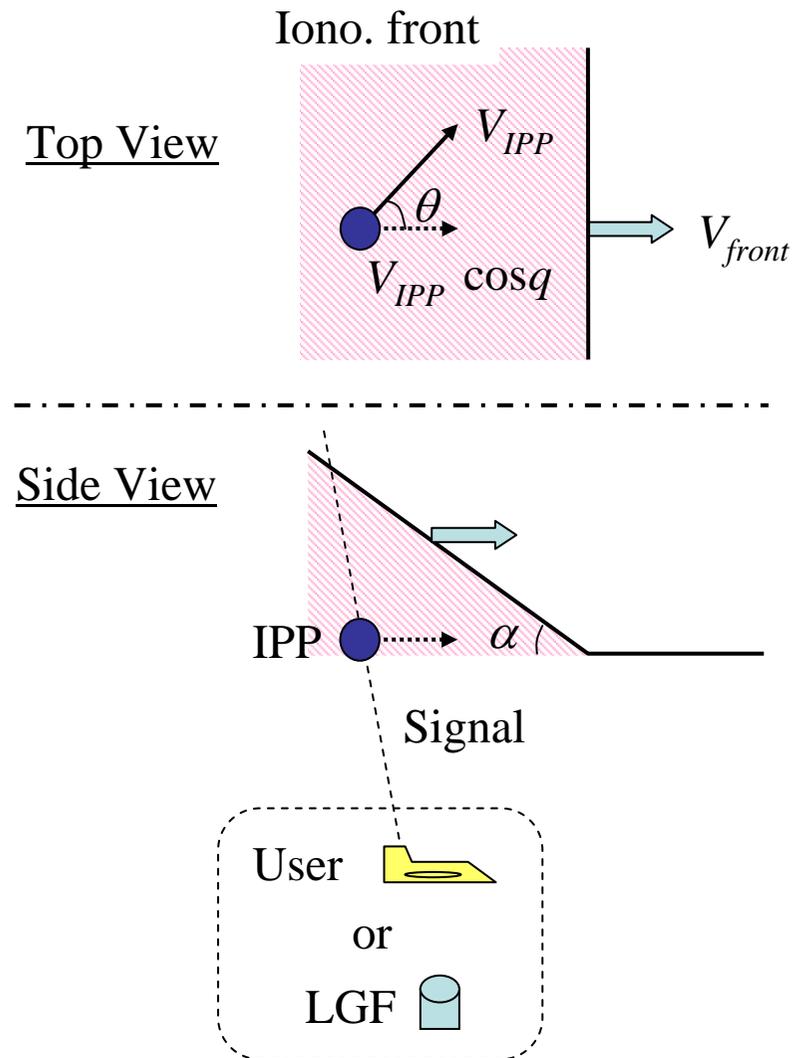
Suppose also that the IPP of the signal affected by the front is moving with a velocity of V_{IPP} (km/s), and that an angle θ exists between the front movement and the IPP movement directions.

The ionosphere delay on the signal varies with the rate of change \dot{I} (mm/s), which is given as follows.

$$\dot{I} = \alpha (V_{IPP} \cos\theta - V_{front}) \quad (1)$$

$dV_{front/IPP}$

The relative velocity between the front and the IPP



What does “looks stationary” mean? (2):

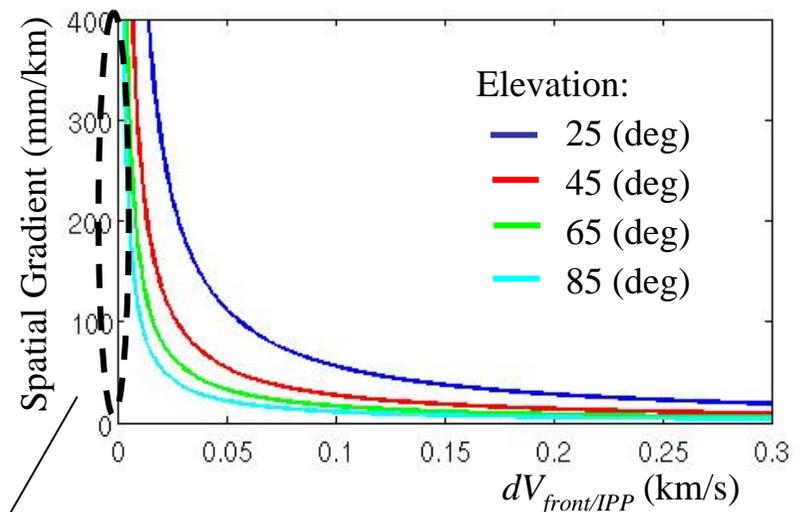
As equation (1) shows, if the relative velocity between the front and the IPP is zero ($dV_{front/IPP} = 0$), then the monitor observes an ionosphere rate of zero ($\dot{I} = 0$) regardless of the gradient of the front, namely, it “looks stationary”.

This document says that, when $dV_{front/IPP}$ is zero, the front “moves with” or “is synchronized with” the associated IPP. Clearly, it is very difficult to detect such fronts.

The difficulty can also be explained by using the threshold for anomaly detection.

Equation (1) transforms the threshold for the ionosphere rates shown in Figure 1 into a threshold in the domain of spatial gradients, which is shown in Figure 2.

Because fronts below the threshold are unlikely to be detected, Figure 2 shows that fronts moving with the IPP are very difficult for the monitor to detect.



This region represents the fronts moving with IPPs, and they exist below the thresholds.

Figure 2: Threshold of gradients as a function of $dV_{front/IPP}$

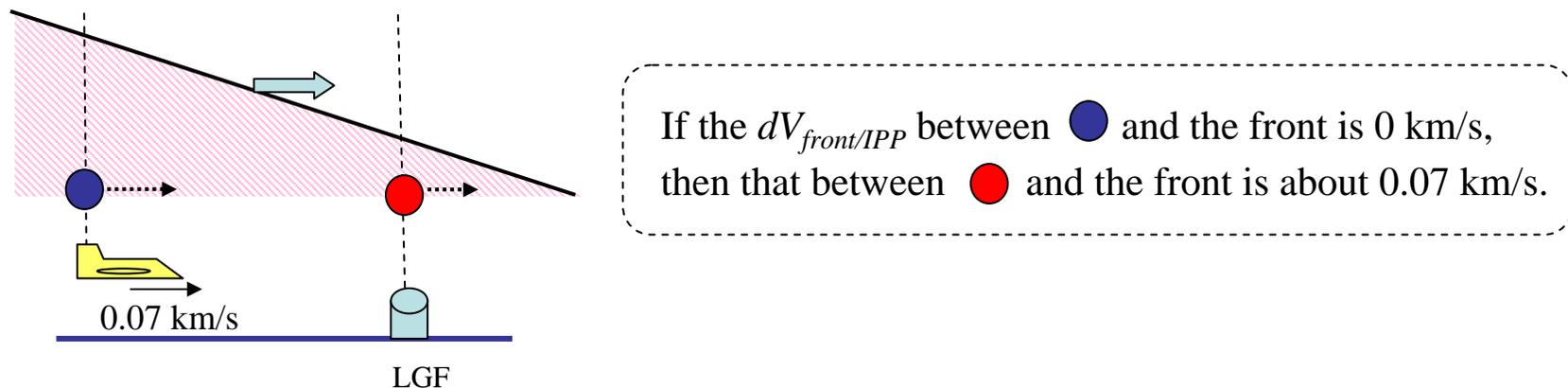
Use of the airborne monitor:

Implementing the monitor in both the user and the LGF helps compensate for the system's weakness against “synchronized” fronts.

The user IPP of the user has a different velocity from the associated IPP for the LGF, and this difference approximates the velocity of the user relative to the ground, which is about 0.07 km/s at the decision point.

Hence, if a front is synchronized with the user IPP, the front must have a relative velocity of 0.07 km/s with respect to the LGF IPP, and vice versa.

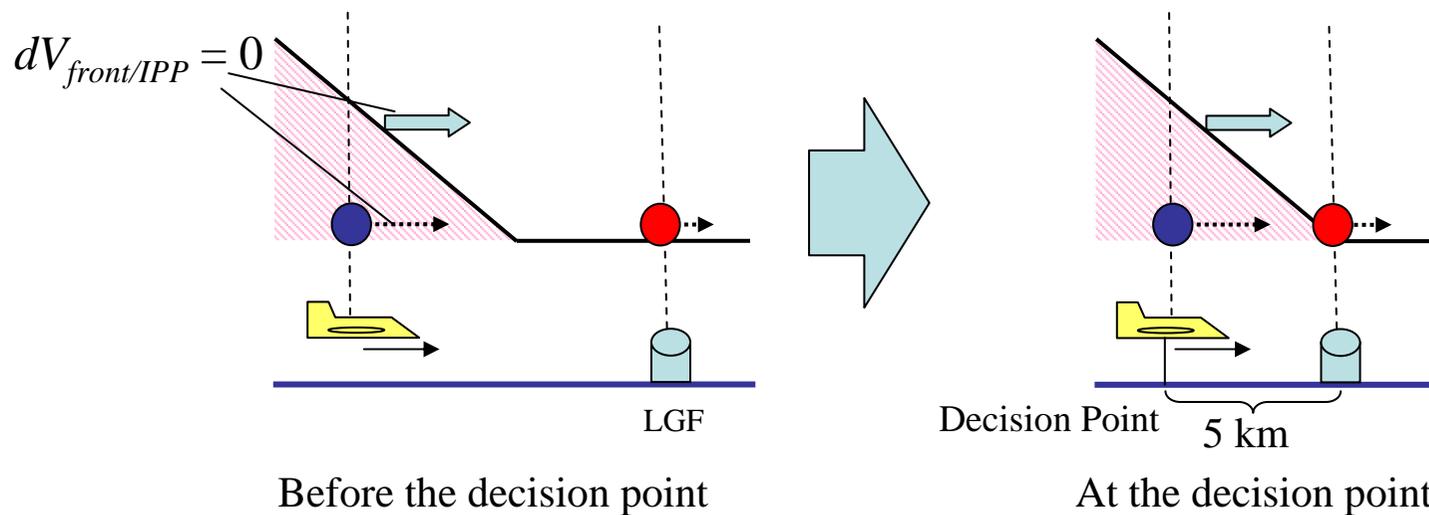
As shown in Figure 2, $dV_{front/IPP}$ of 0.07 km/s is high enough for the monitor to detect most threatening fronts.



Undetectable front affecting one satellite:

Undetectable fronts are those that move with the IPP of the user (or the LGF) and hit the IPP of the LGF (or the user) just as the user passes over the decision point. (In this work, the decision point is assumed to be 5 km from the LGF.)

The figure below illustrates a front that satisfies this condition. The airborne monitor cannot detect the front because of the synchronization. The ground monitor cannot detect the front either, because its IPP does not “catch up to” the front before the user passes over the decision point.



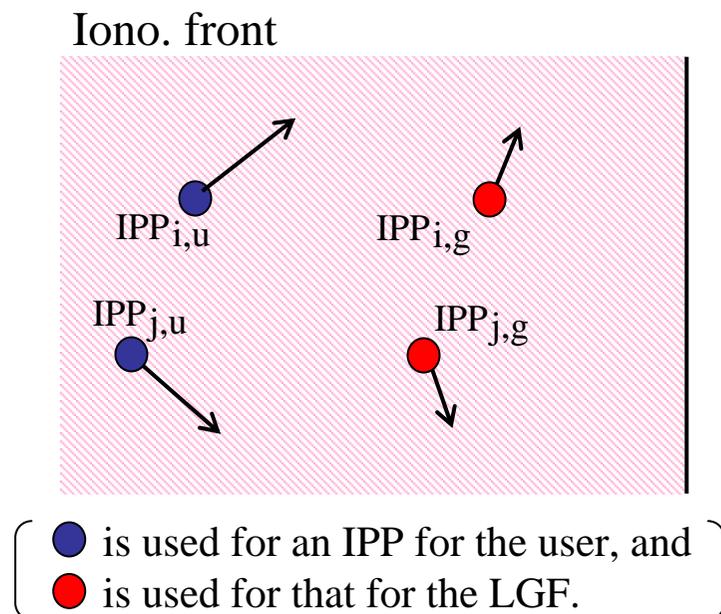
Note: This figure illustrates the front moving with the user’s IPP. However, the same condition is applicable for the front moving with the LGF’s IPP.

Undetectable front affecting two satellites (1):

The undetectable condition described in the previous slide can be easily expanded to the case where a front is simultaneously affecting two satellites.

To explain this condition, the symbol $IPP_{i,u}$ is used to designate the IPP created by the signal from satellite i to the user (for the LGF, $IPP_{i,g}$ is used).

Suppose that an ionosphere front affects two satellites, i and j . In this case, there are four signals (hence four IPPs) from which the monitors have a chance to detect the front.



In such situations, the “critical” fronts are those which move with both of the IPPs associated with the user, $IPP_{i,u}$ and $IPP_{j,u}$.

Note:

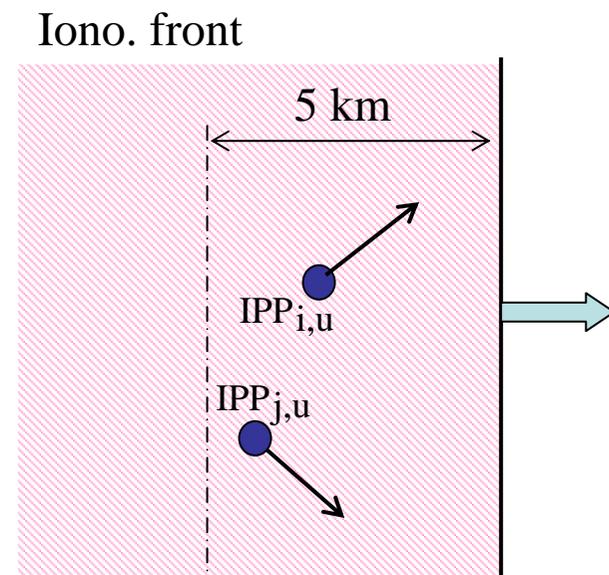
The fronts moving with IPPs of the LGF ($IPP_{i,g}$ and $IPP_{j,g}$) are also critical, and all discussions below are also applicable to them.

Undetectable front affecting two satellites (2):

If the front moves with both $IPP_{i,u}$ and $IPP_{j,u}$, and if the front does not hit $IPP_{i,g}$ and $IPP_{j,g}$ before the user passes over the decision point, the airborne and ground monitors cannot detect it.

Such a situation occurs when the following conditions are satisfied.

- (1) The front is synchronized with $IPP_{i,u}$ and $IPP_{j,u}$.
- (2) Both IPPs are within 5 km (the user-to-LGF separation) of the leading edge of the front.



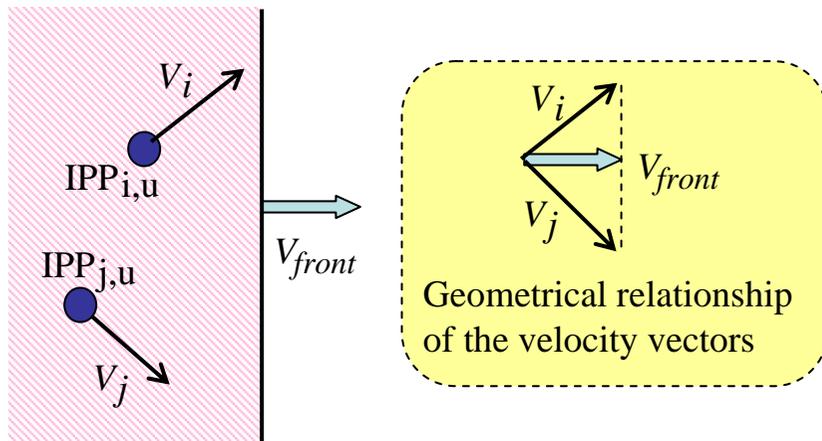
The following slides explain these conditions in detail.

The first condition for undetectable fronts:

Although $IPP_{i,u}$ and $IPP_{j,u}$ have independent velocities, V_i and V_j , a velocity, V_{ij} , with which the front moves with both IPPs always exists. Geometrically, such a velocity is given as follows.

$$\begin{cases} e_{ij} \in \text{span}(V_i, V_j) \\ \langle e_{ij} \cdot (V_i - V_j) \rangle = 0 \\ \langle e_{ij} \cdot V_i \rangle \geq 0 \end{cases} \quad \text{Eq. (2)}$$

$$V_{ij} = \langle e_{ij} \cdot V_i \rangle e_{ij}$$



Here, e_{ij} is the unit vector of V_{ij} .

The first two equations decide the direction of V_{ij} , and the third gives the sign of it. The fourth equation decides the length of the vector.

If the front moves with velocity ($V_{front} = V_{ij}$), then it is synchronized with $IPP_{i,u}$ and $IPP_{j,u}$.

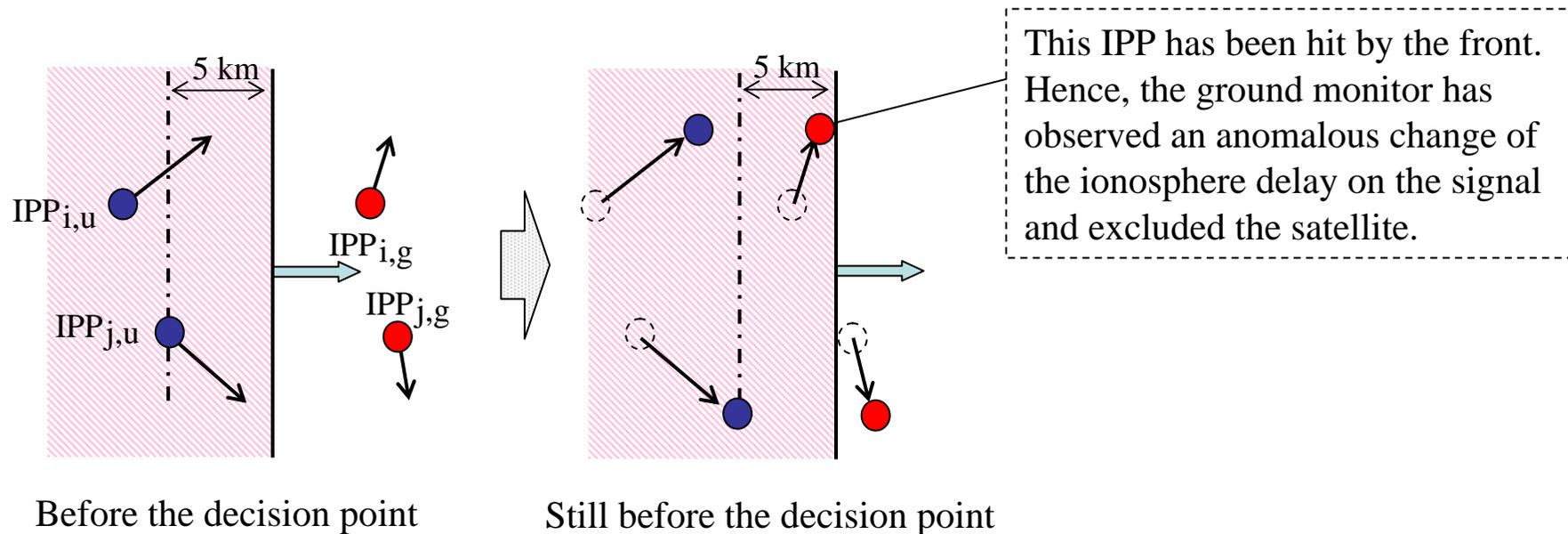
Note:

Here, velocities are expressed in the Cartesian coordinates, and actual movement of ionosphere fronts may be more precisely modeled in the spherical coordinates. However, equation (2) is accurate enough for our purpose while avoiding Undue complexity.

The second condition for undetectable fronts:

The second condition dictates that the front does not reach the associated IPPs for the LGF ($IFF_{i,g}$ and $IPP_{j,g}$) before the user passes over the decision point.

In other words, if either of the user IPPs, say $IPP_{i,u}$, is farther than 5 km from the leading edge of the front, the associated IPP for the LGF, $IPP_{i,g}$, will be hit by the front before the decision point, and the ground receiver can detect the front and exclude satellite i (neglecting the LGF time to detection and exclusion).



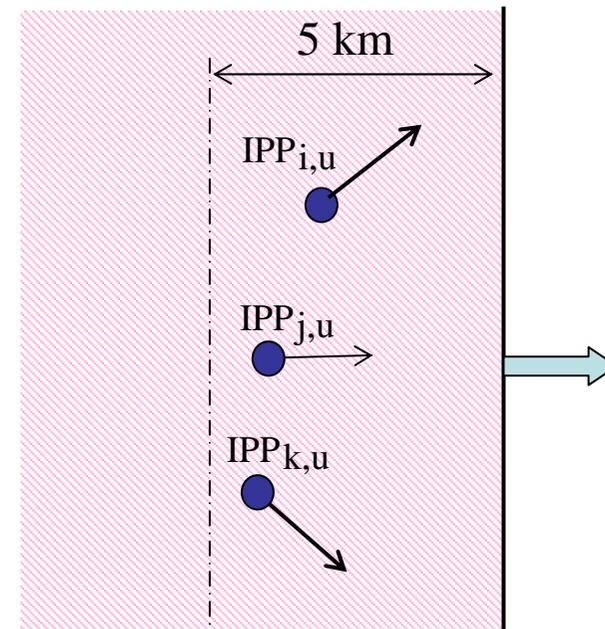
Undetectable front affecting three satellites:

The condition for undetectable fronts that affect three satellites (i , j , and k) is a natural expansion of the two-satellite case, which is:

- (1) The front synchronizes with all three IPPs: $IPP_{i,u}$, $IPP_{j,u}$, and $IPP_{k,u}$
- (2) All three IPPs exist within 5 km (the user-to-LGF separation) from the leading edge of the front.

Note about condition (1)

For the two-satellite case, a front that moves with the two IPPs always exists, as discussed on slide 12. However, for the three-satellite case, a front moving with three IPPs does not necessarily exist. Hence, the undetectable condition for the three-satellite case is much more restrictive than is the analogous condition for the two-satellite case.



Investigation of actual satellite geometries

Investigation of actual satellite geometries:

As shown on previous slides, it is easy to construct theoretical conditions for undetectable fronts that affect multiple satellites.

However, being able to construct theoretical conditions does not mean that such situations can actually occur.

To confirm if such special conditions could happen in practice, actual satellite geometries for three airports (Memphis, LA, NY) Have been investigated.

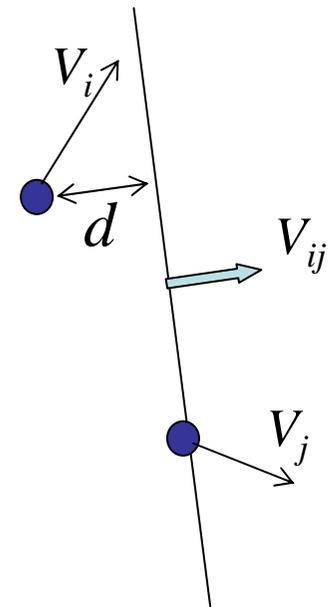
Specifically, GPS satellite geometries for these airports were generated with 10-minute time intervals based on the 24-satellite almanac on July 1, 1993 [2], and particular IPP pairs and triplets were searched for as described in the following slides.

Investigation for the two-satellite case:

To confirm if the undetectable condition for the two-satellite case (see slide 11) would occur in practice, the satellite geometries are investigated as follows.

- (1) Pick an arbitrary IPP pair from the satellite geometry, and compute the velocity of the front that moves with the pair using equation 1 (on slide 6).
- (2) Place the leading edge of the front having this velocity on one of these two IPPs and measure the distance, d , from this edge to the other IPP.
- (3) Do (1) and (2) for all independent IPP pairs.

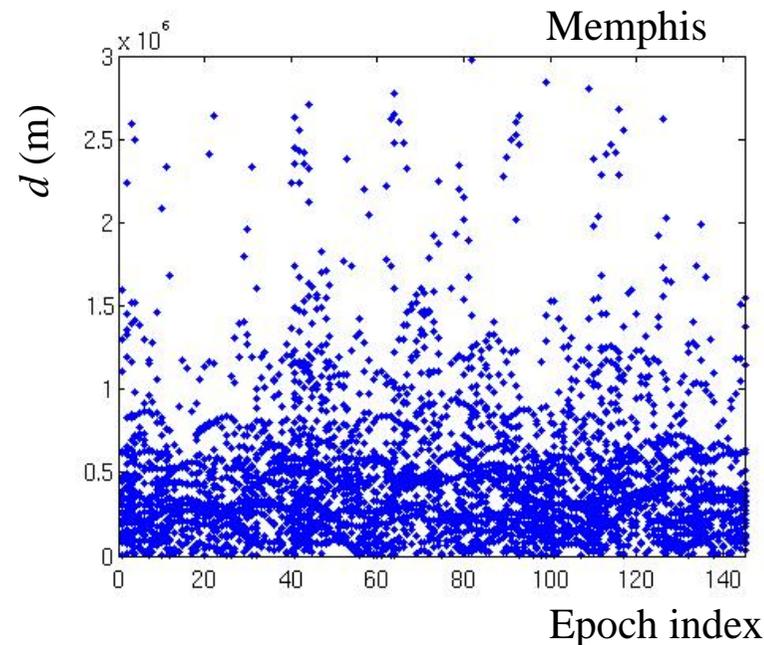
If d is less than 5 km (the user-to-LGF separation), the IPP pair can be considered to have the potential to experience an undetectable front.



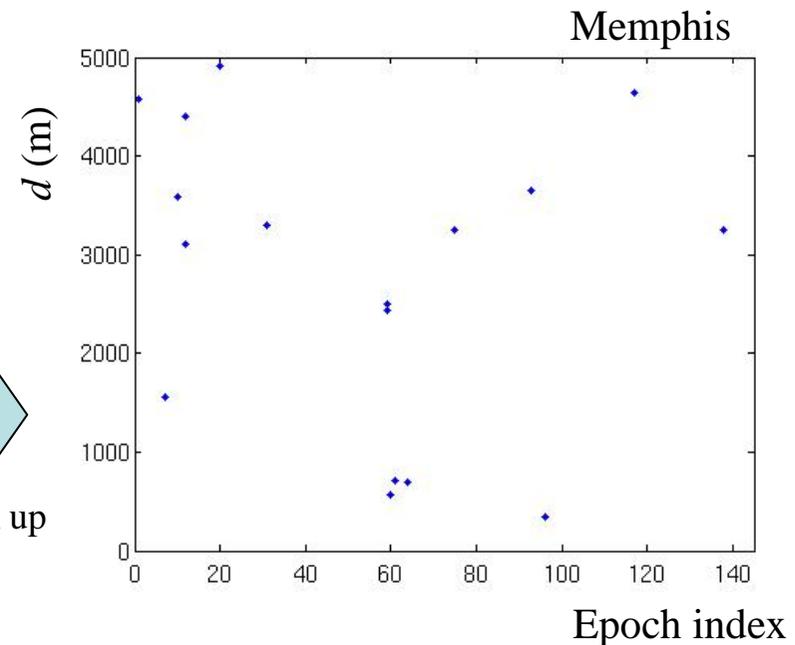
Leading edge of the hypothetical front

Results of the investigation (1):

Overall, 48 IPP pairs with d values less than 5 km were found in the geometries at the three airports, and 10 of them had d values less than 1 km.



Zoom up



Distribution of d (m) as a function of epoch index at Memphis. There are 144 epochs in 24 hours, And the geometry at each epoch has multiple IPP pairs (hence multiple d values).

Zoom-in to the region where $d < 5$ km. These data points are counted as IPP pairs that have the potential to experience an undetectable front.

Results of the investigation (2):

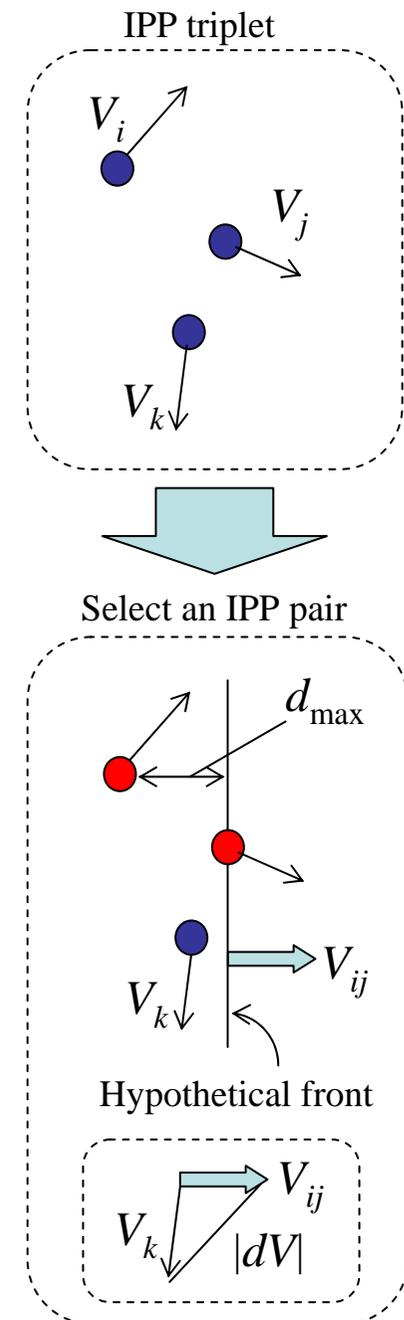
Number of IPP pairs with small d values for each airport

Airport	Number of pairs with d less than 5 km	Number of pairs with d less than 1 km
Memphis	17	4
LA	17	3
NY	14	3

Investigation for the three-satellite case:

To confirm if an undetectable condition for the three-satellite case (see slide 14) could happen, the satellite geometries are investigated as follows.

- (1) Pick an arbitrary IPP triplet, i, j , and k .
- (2) For these IPPs, select an arbitrary IPP pair, say i and j , and compute the velocity of the front which moves with this pair, V_{ij} .
- (3) Compute the absolute difference between this velocity, V_{ij} , and the velocity of the other IPP, V_k . Call this difference $|dV|$.
- (4) Place the leading edge of the front having V_{ij} on one IPP so that all three IPPs are located on one side of the edge; then compute the maximum distance between this edge and the IPPs. Call this distance d_{\max} .
- (5) Do (2) ~ (4) for all three IPP pairs in the triplet.
- (6) Do (1) ~ (4) for all independent IPP triplets.

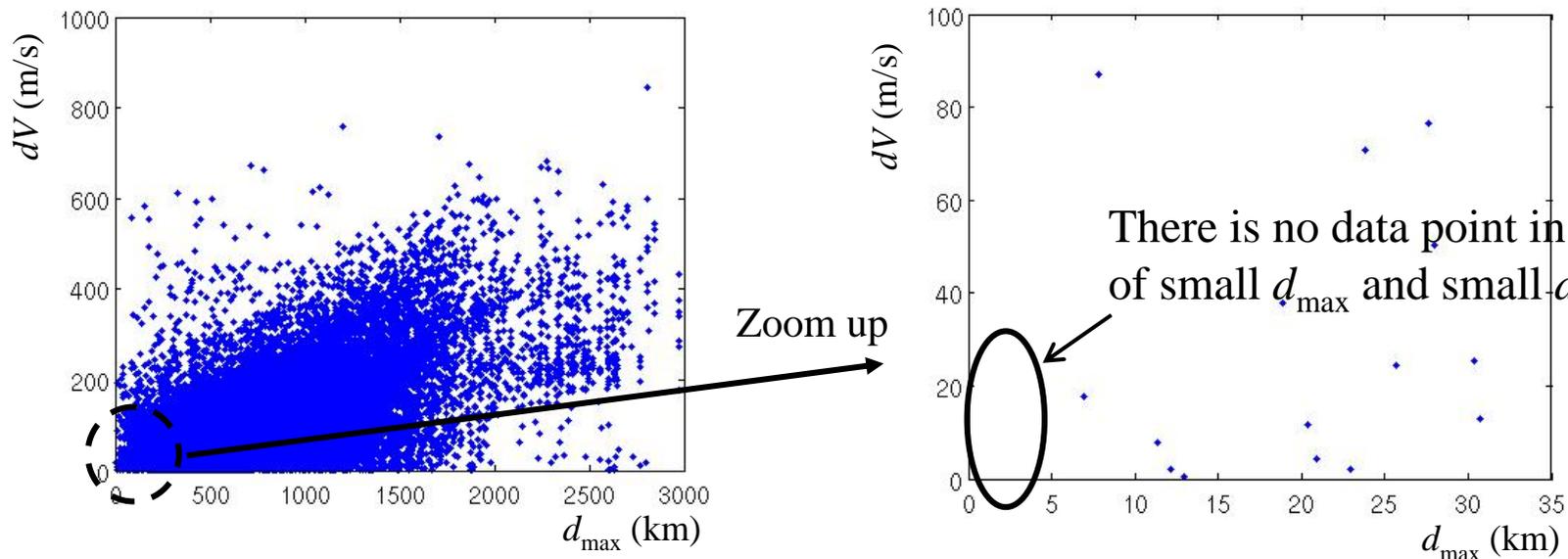
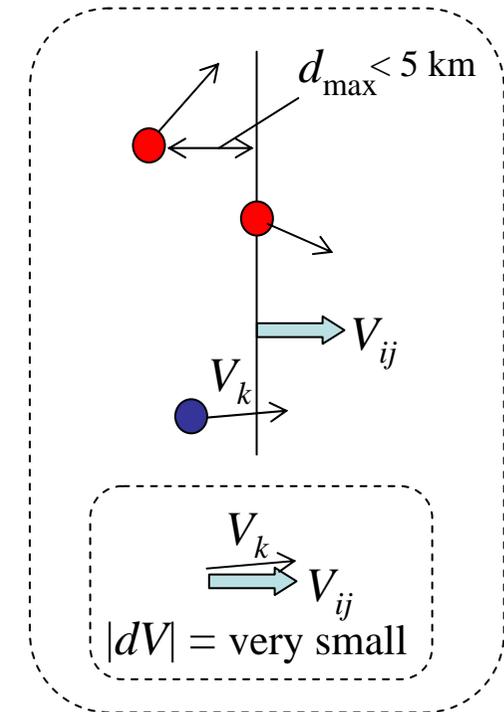


Results of the investigation (1):

If dV is small enough (say less than 30 m/s), the front moving with V_{ij} can be considered to move with all three IPPs.

If d_{\max} is also less than 5 km, the IPP triplets can be considered to have the potential to experience an undetectable front.

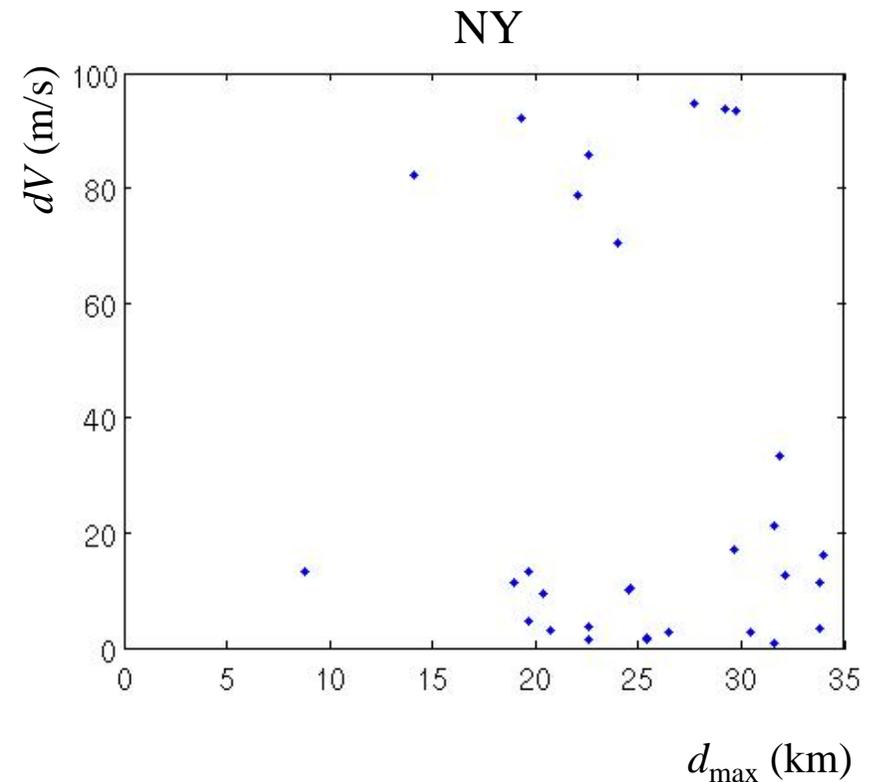
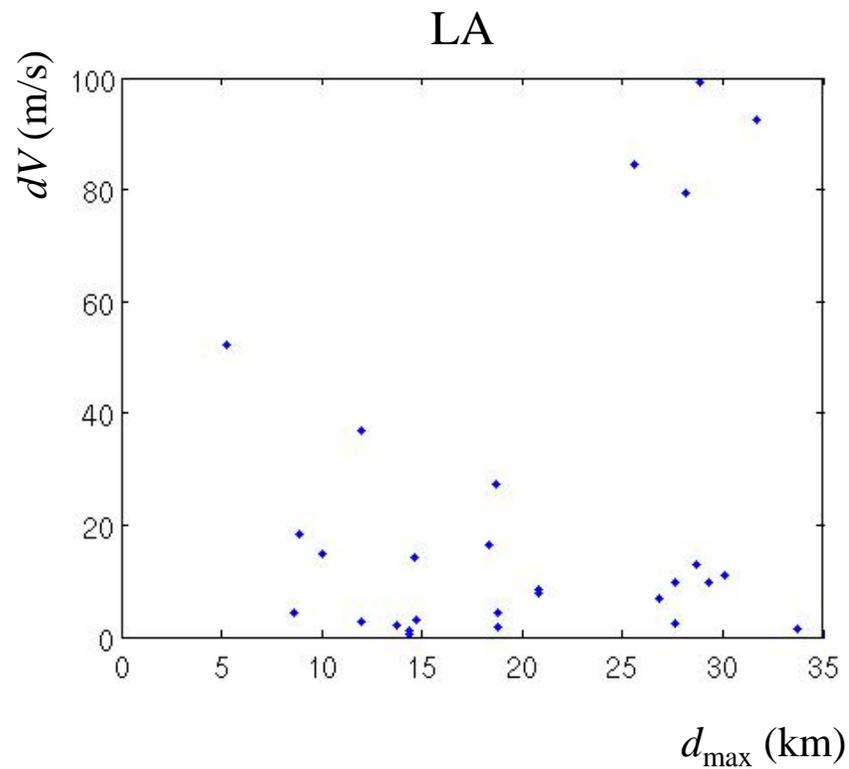
Hence, IPP triplets having dV less than 30 m/s and d_{\max} less than 5 km were searched for in the satellite geometries of the three Airports shown on slide 19, and no such triplet was found.



Scatter plots of d_{\max} and dV for Memphis

Results of the investigation (2):

Scatter plots of d_{\max} and dV for LA and NY
(only for the region of small values)



Conclusions

Conclusions:

Because of the discovery of IPP pairs having the potential to be affected by undetectable fronts in actual satellite geometries, the threat that the monitoring method misses detecting fronts affecting two satellites cannot be neglected.

On the other hand, the absence of IPP triplets that could satisfy the undetectable condition indicates that this condition is extremely improbable to be satisfied in practice.

Although these investigations do not constitute a formal integrity proof, it can be concluded that the undetectable condition for the front affecting three satellites will not be satisfied in practice, and hence the monitoring method always detects the front if it covers more than two satellites.

References:

[1] H. Konno, S. Pullen, et al. “Ionosphere Monitoring Methodology for Hybrid Dual-Frequency LAAS,” *Proceedings of ION GNSS 2006*, Fort Worth, TX, September 26-29, 2006.

<http://waas.stanford.edu/~wwu/papers/gps/PDF/KonnoIONGNSS06.pdf>

[2] *Global Positioning System Standard Positioning Service Performance Standard*. Washington, D.C., U.S. Department of Defense, October 2001.

<http://www.navcen.uscg.gov/gps/geninfo/2001SPSPerformanceStandardFINAL.pdf>