Simulating Real-World Signal Environments

Dr. Alexander Mitelman, Cambridge Silicon Radio

SCPNT Symposium, Stanford, CA, USA
10 November 2010

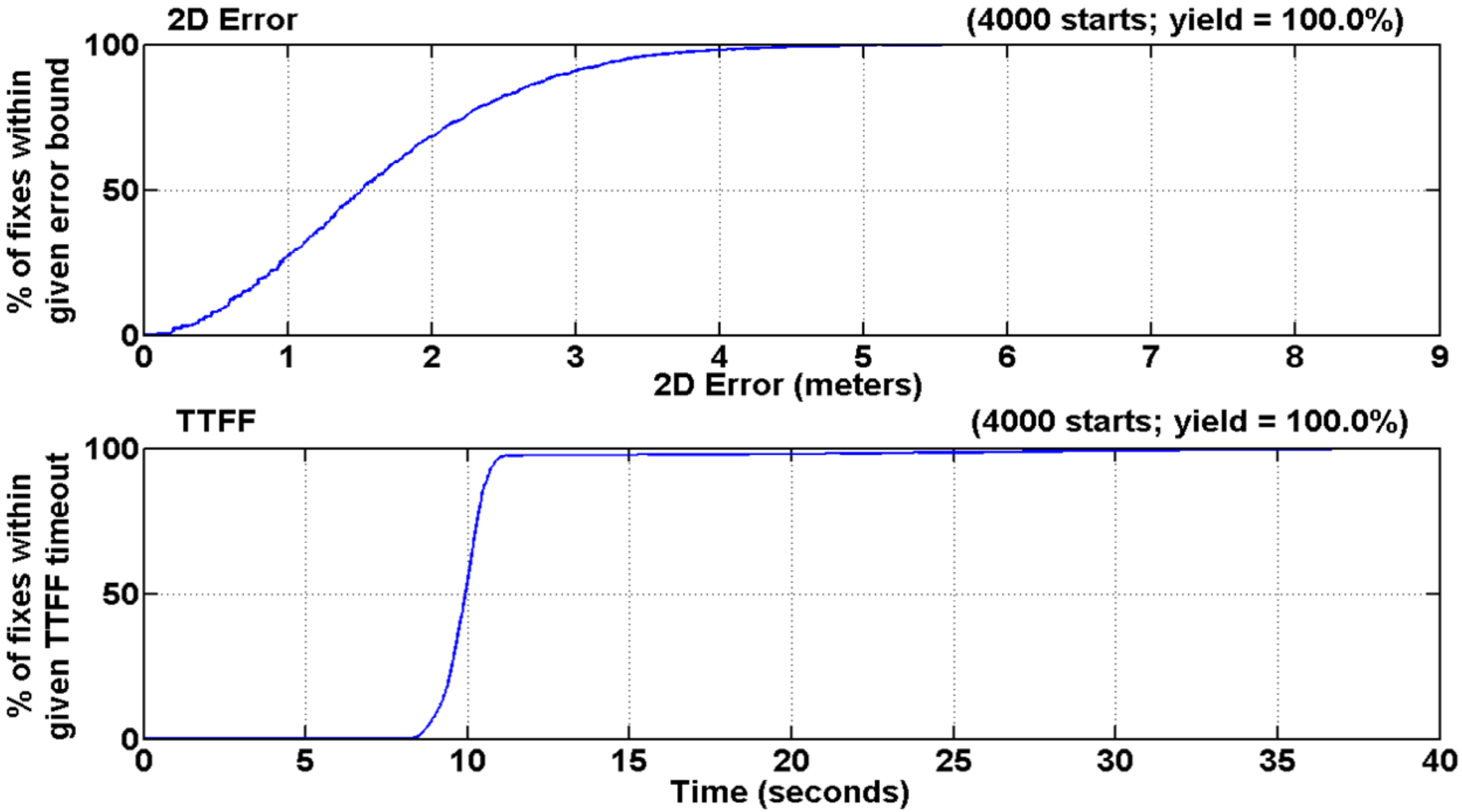
Presentation Outline

- Motivation
  - What to measure, and why?
  - Existing test methods and drawbacks
- Novel testing approach: fluctuating signals
  - Raw histogram data collection
  - Figures of merit, mathematical tools
  - Differences from conventional methods
- Implementation and results

Motivation

- Object of measurement?
  - Accuracy (2D/3D error: max / mean / 95th percentile) [acq. or nav.]
  - Time to first fix (seconds) [acq. only]
- Goals
  - Predictivity via meaningful metrics ("user experience isn't everything; it's the only thing")
    - Reasonable reproduction of real-world performance in the lab
    - Element of randomness to harness full power of Monte Carlo-type simulations
  - Standardization-friendly alternative to existing approaches
    - Requires no additional equipment
    - Minimal storage requirements (*c.f.* RF replay: 10Msps @ 16b, 100 reference files @ 20min → reference set ~ **2.4TB**)

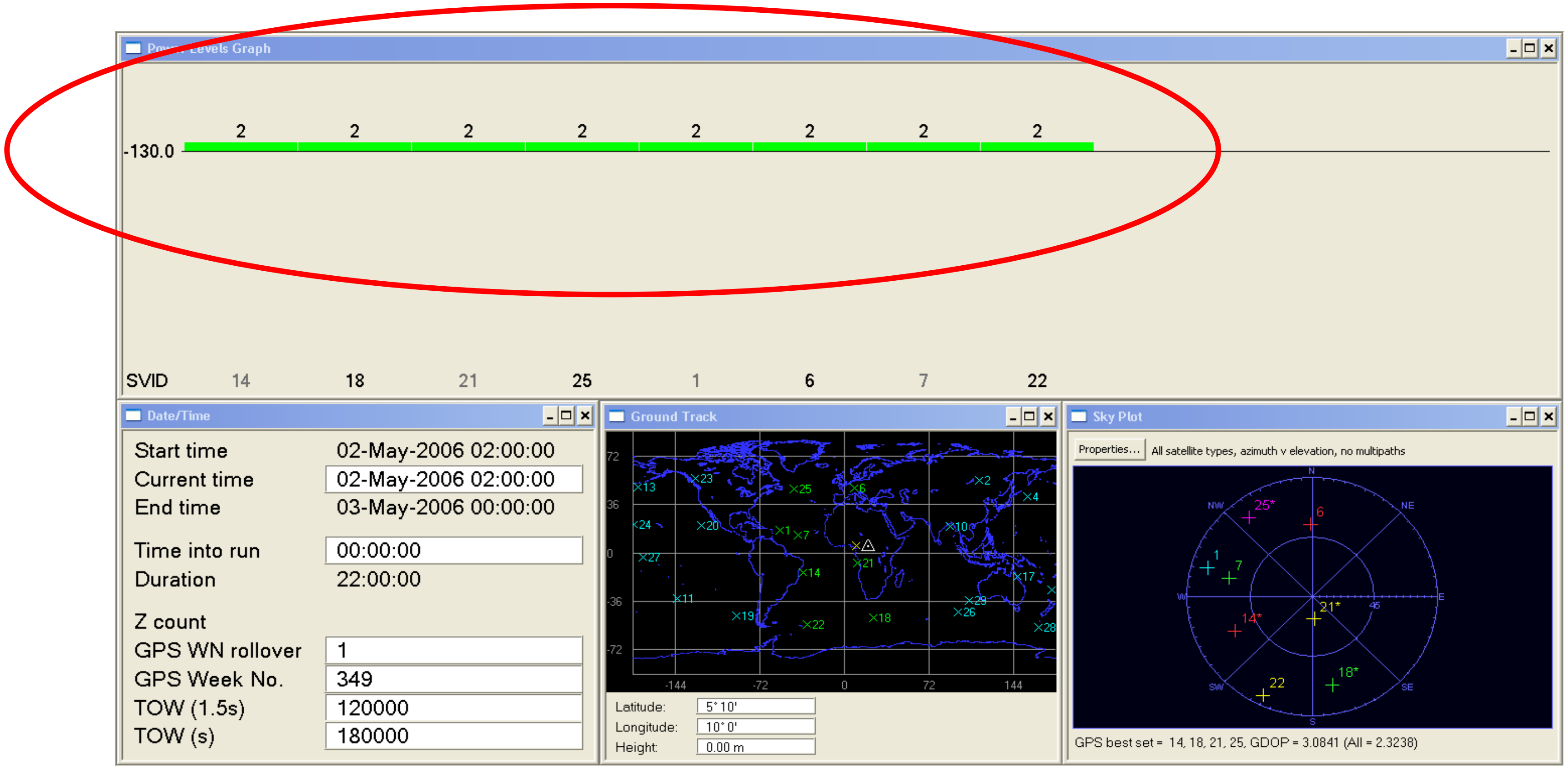
Representing 2D Error and Time-to-First-Fix



- 3GPP specs implicitly consider *cumulative distribution functions* (CDFs).
- Requirements formulated as 67th or 95th percentile bounds on 2D error and, for acquisition (as shown here), time-to-first fix (TTFF).

Conventional Method 1: "Apples to Apples" Testing [1]

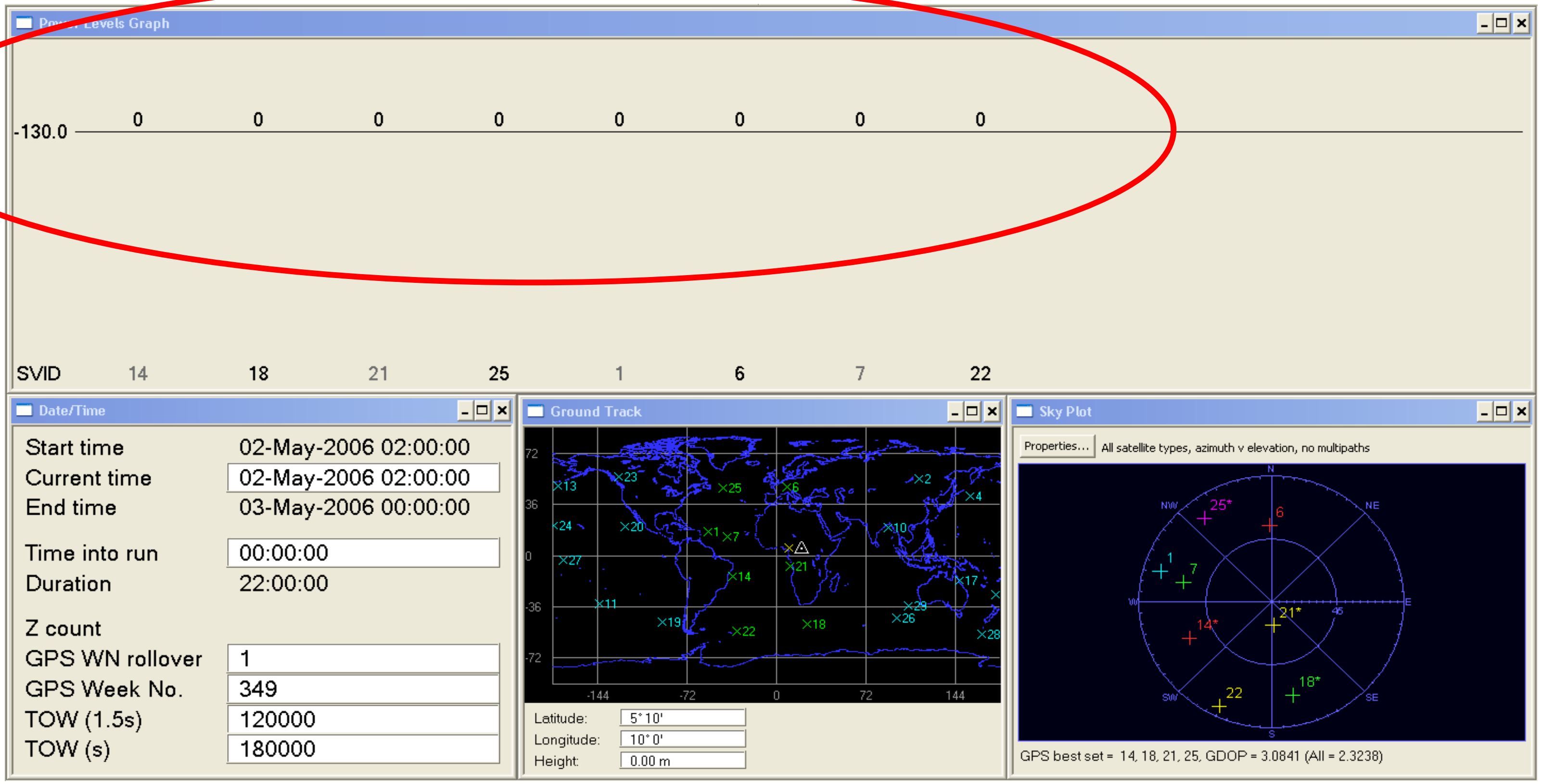
- Step down power levels on all SVs simultaneously, observe lowest level at which behaviors of interest (navigation, [re-]acquisition, etc.) are possible:



[1] "Apples to Apples: A Standardized Testing Methodology For High Sensitivity GNSS Receivers," ION GNSS-2007, Fort Worth, TX, September 2007.

Conventional Method 1: "Apples to Apples" Testing [1]

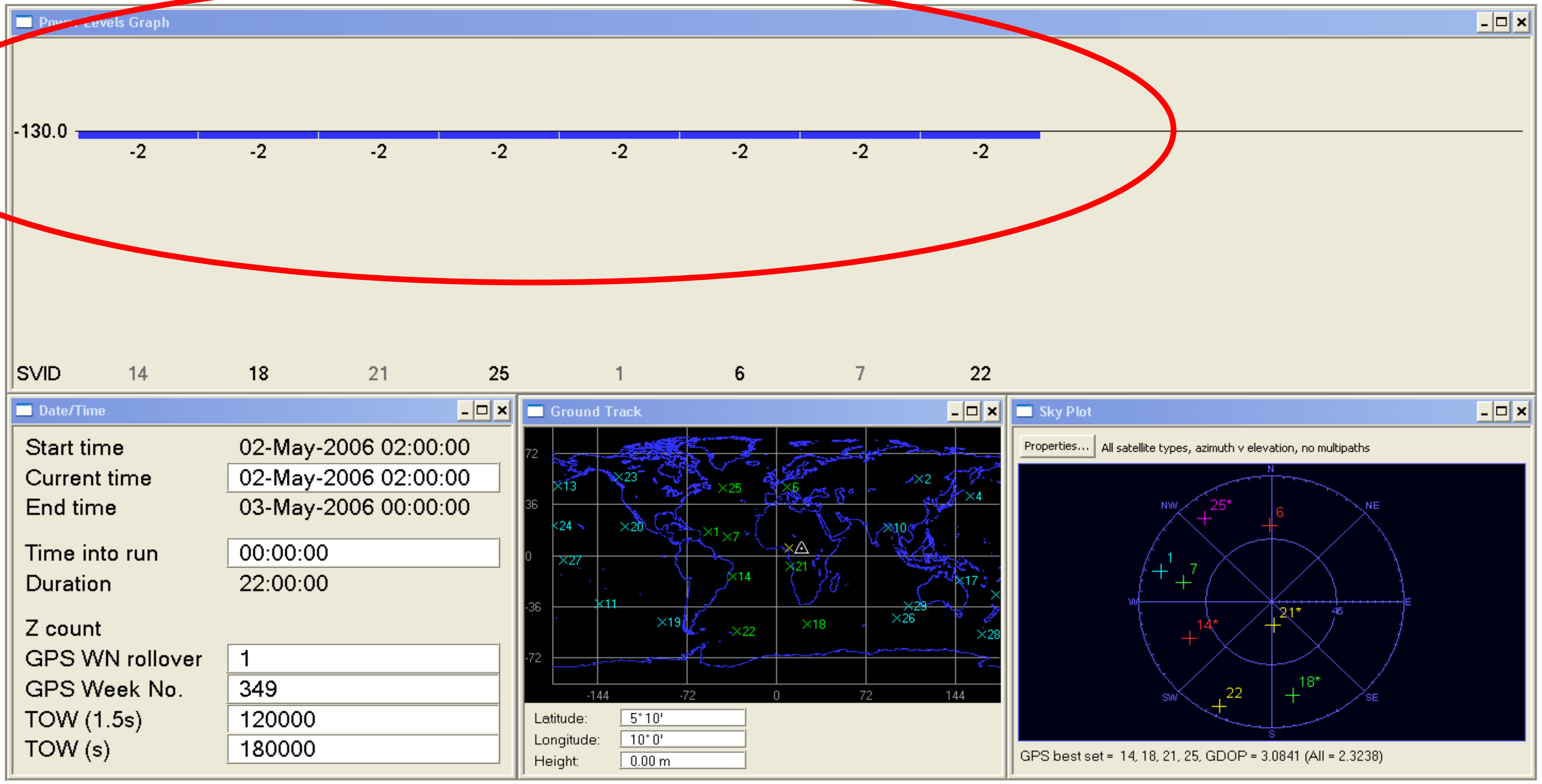
- Step down power levels on all SVs simultaneously, observe lowest level at which behaviors of interest (navigation, [re-]acquisition, etc.) are possible:



[1] "Apples to Apples: A Standardized Testing Methodology For High Sensitivity GNSS Receivers," ION GNSS-2007, Fort Worth, TX, September 2007.

Conventional Method 1: "Apples to Apples" Testing [1]

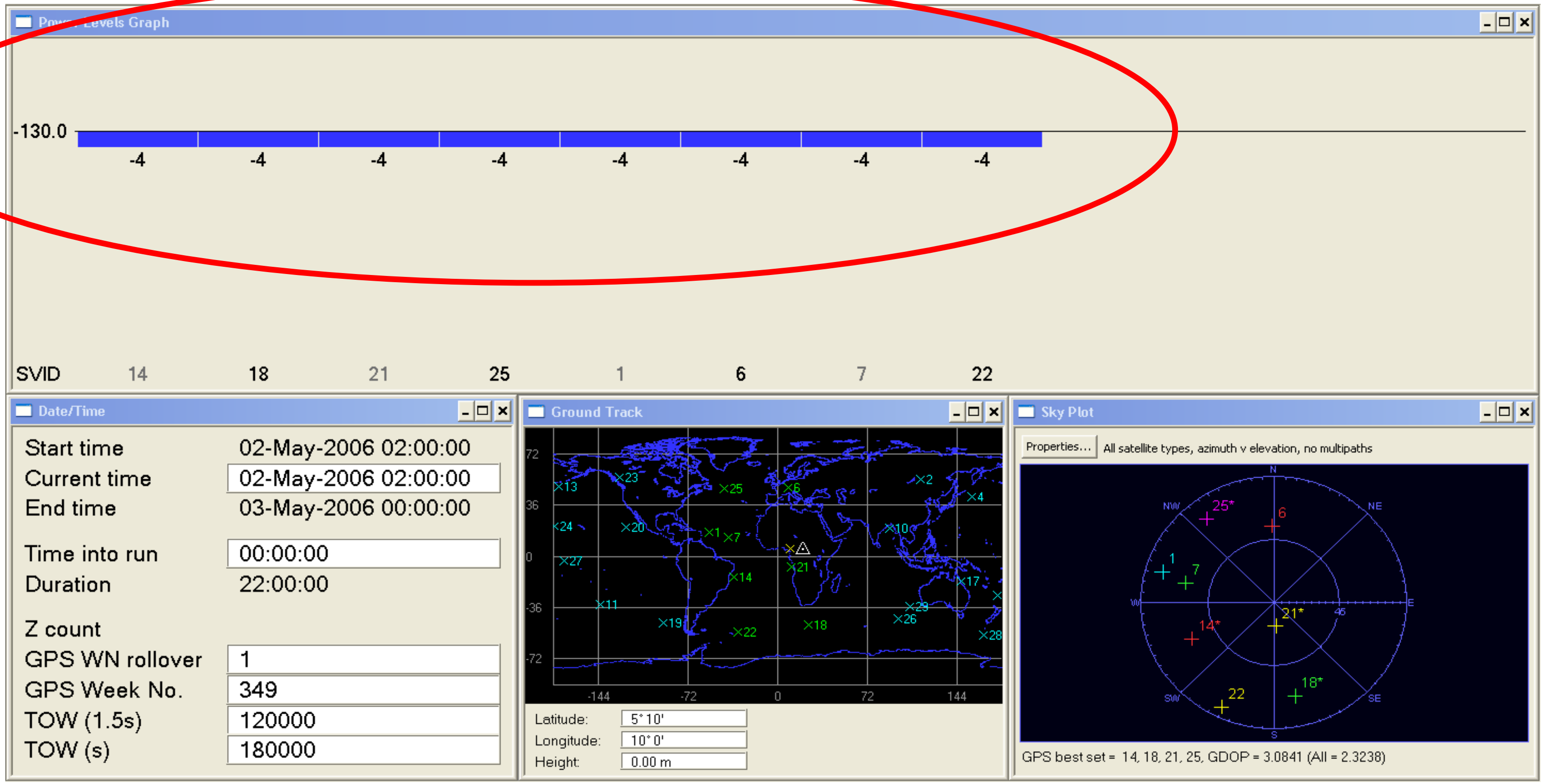
- Step down power levels on all SVs simultaneously, observe lowest level at which behaviors of interest (navigation, [re-]acquisition, etc.) are possible:



[1] "Apples to Apples: A Standardized Testing Methodology For High Sensitivity GNSS Receivers," ION GNSS-2007, Fort Worth, TX, September 2007.

Conventional Method 1: “Apples to Apples” Testing [1]

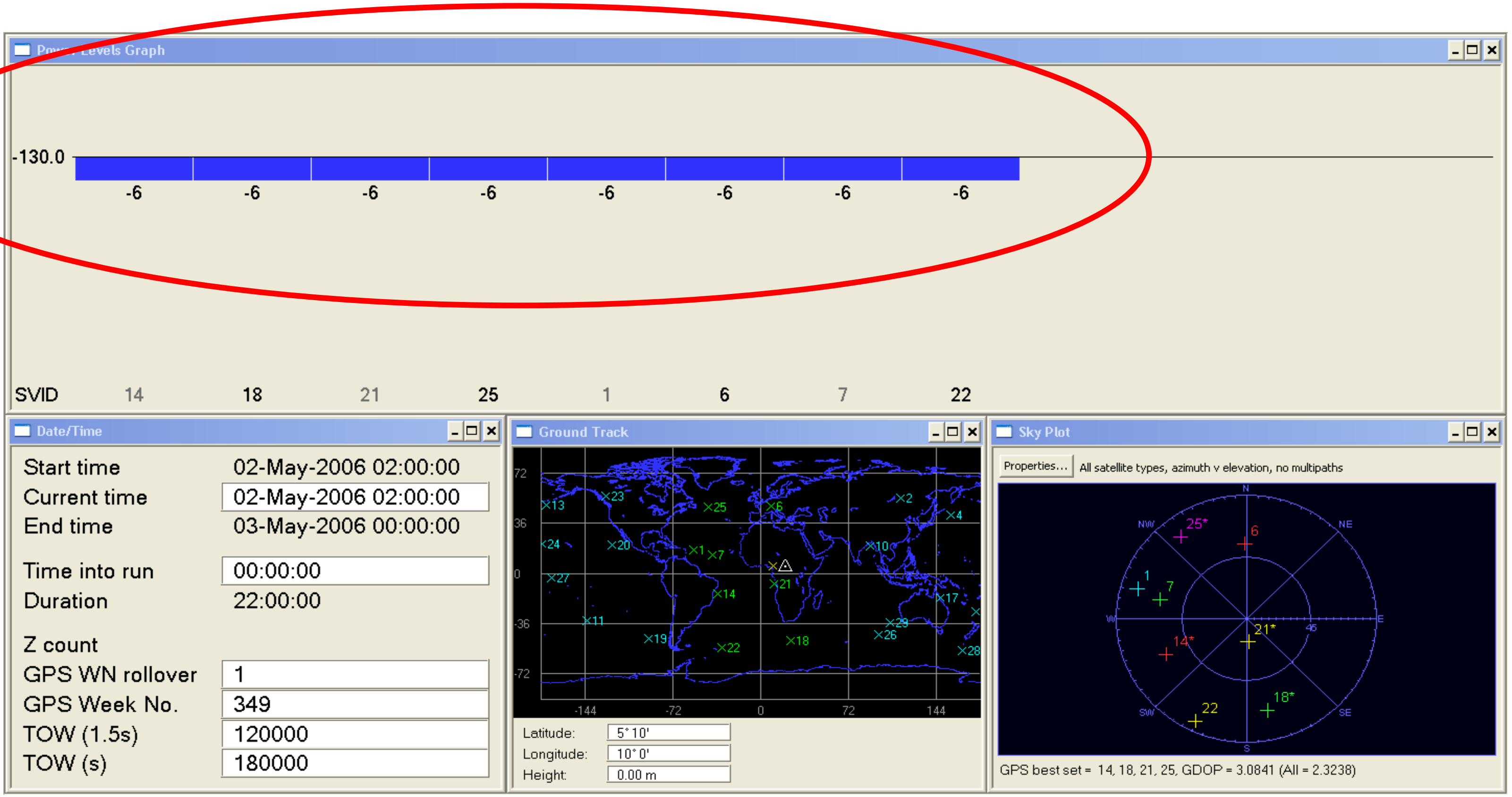
- Step down power levels on all SVs simultaneously, observe lowest level at which behaviors of interest (navigation, [re-]acquisition, etc.) are possible:



[1] “Apples to Apples: A Standardized Testing Methodology For High Sensitivity GNSS Receivers,” ION GNSS-2007, Fort Worth, TX, September 2007.

Conventional Method 1: "Apples to Apples" Testing [1]

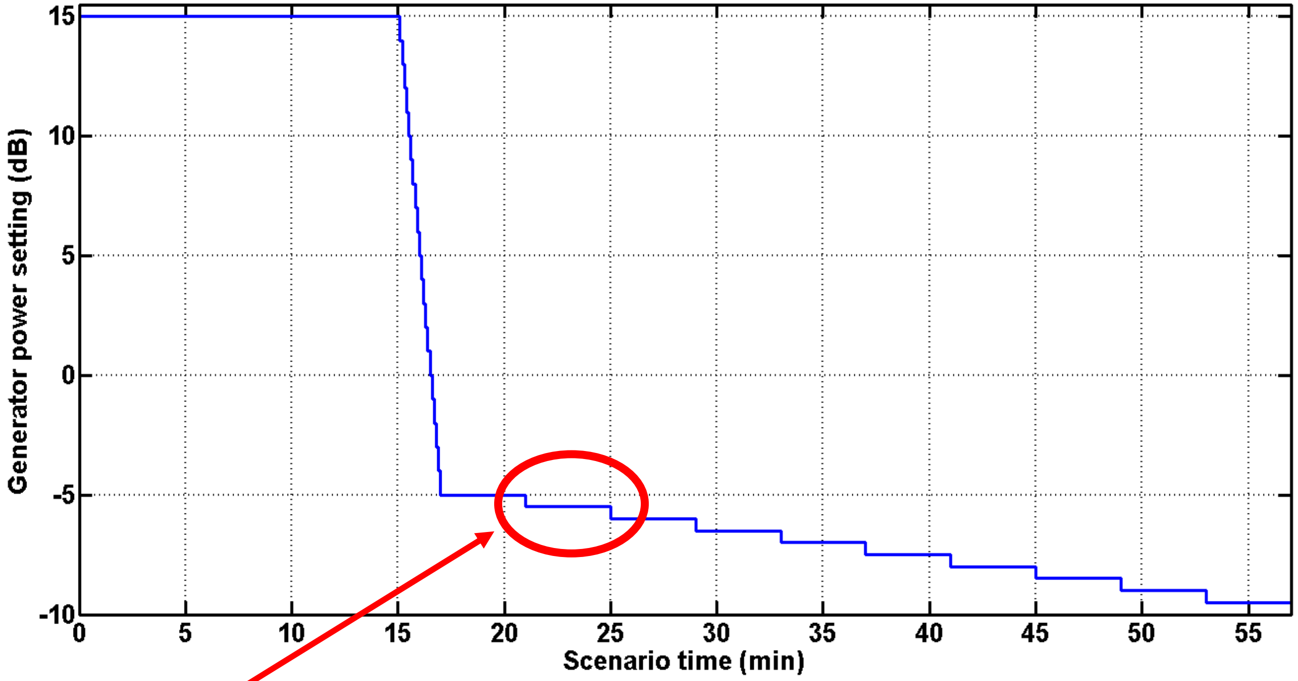
- Step down power levels on all SVs simultaneously, observe lowest level at which behaviors of interest (navigation, [re-]acquisition, etc.) are possible:



[1] "Apples to Apples: A Standardized Testing Methodology For High Sensitivity GNSS Receivers," ION GNSS-2007, Fort Worth, TX, September 2007.

Conventional Method 1: “Apples to Apples” Testing [1]

- Tracking sensitivity

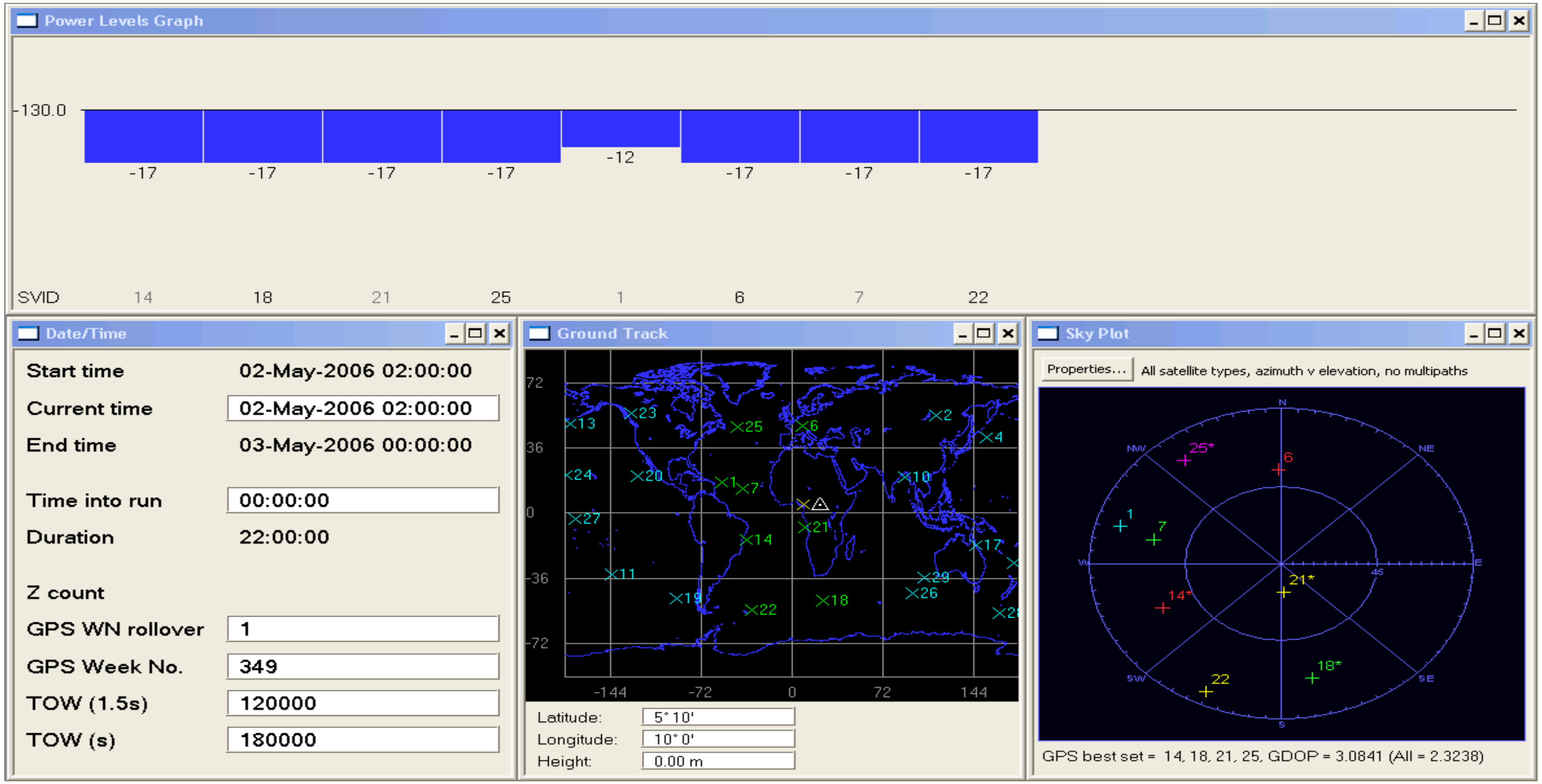


- At each time step (typically 3-4 minutes), all SV signals are at the same power
- Analogous tests also defined for acquisition and reacquisition
- Well-controlled and repeatable, but how useful are these tests practically?

[1] “Apples to Apples: A Standardized Testing Methodology For High Sensitivity GNSS Receivers,” ION GNSS-2007, Fort Worth, TX, September 2007.

Conventional Method 2: 3GPP A-GPS Testing [1] [2]

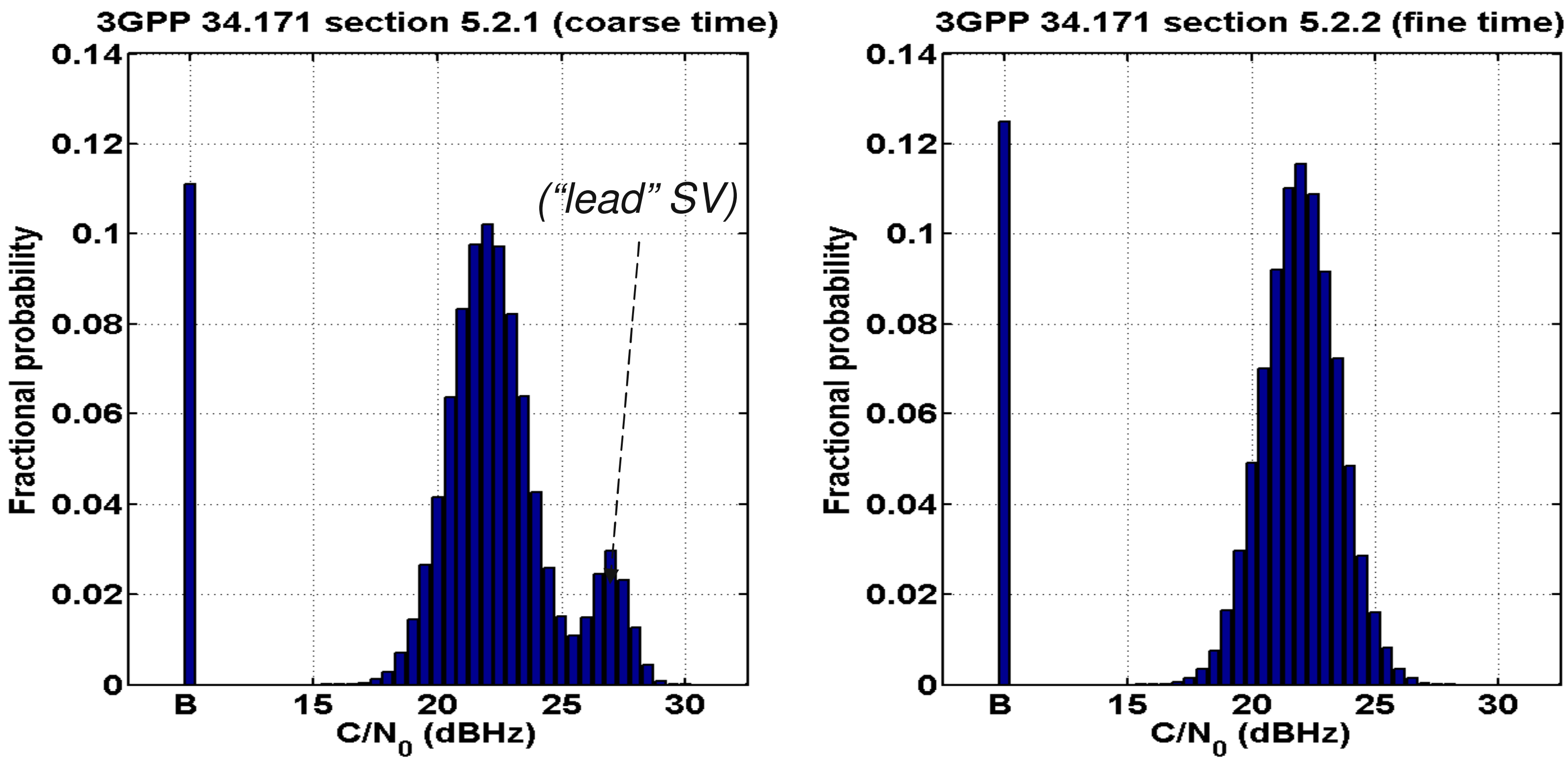
- Primary goals are acquisition (TTFF and accuracy), for push-to-fix-style use cases (coarse/fine time, "lead SV"):



[1] "UMTS Requirements for support of A-GPS," 3GPP TS 25.171, Release 7.
[2] "UMTS Terminal conformance specification, A-GPS," 3GPP TS 34.171, Release 7.

Proposed Alternative Method: C/N_0 Histograms (1 of 2)

- Tests just described assume *fixed, static signal levels*. For example, consider histograms of receiver-reported C/N_0 for 3GPP 5.2.1 and 5.2.2:



<u>Key question</u>: *How well do these cases represent real-world environments?*

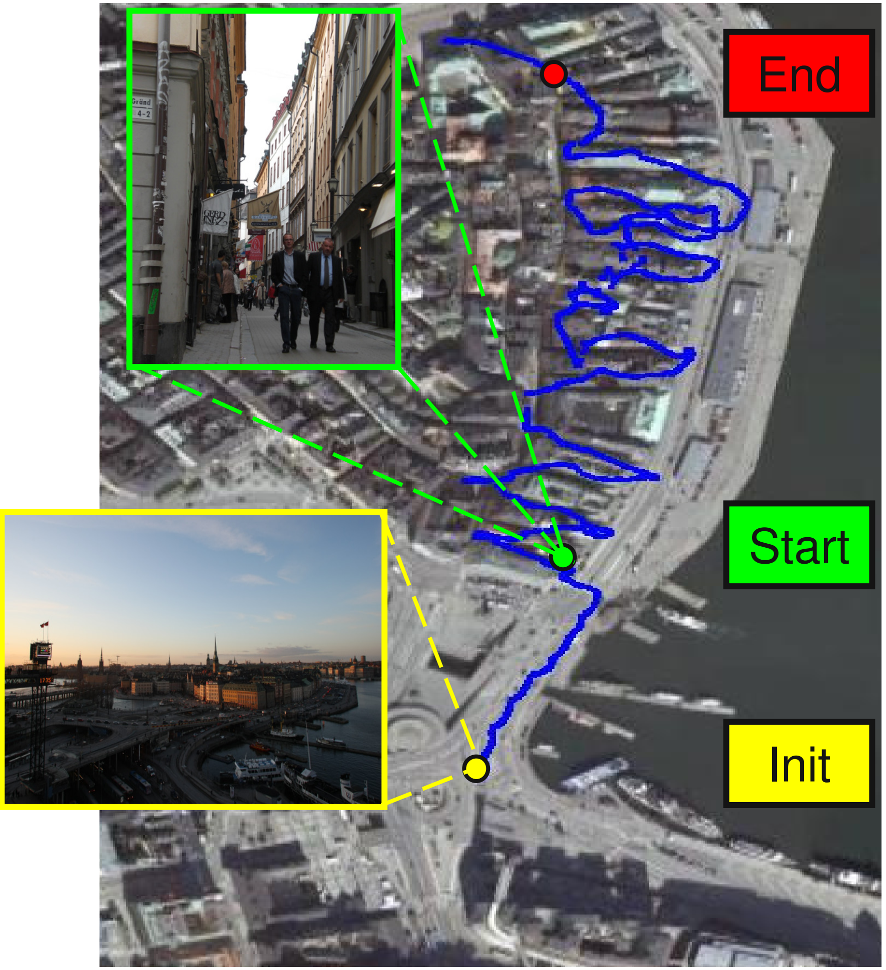
Proposed Alternative Method: C/N_0 Histograms (2 of 2)

- Experience shows that real-world signal levels are almost *never* uniform or static, especially in difficult environments and/or when the user is moving.

- A histogram of observed C/N_0 values may capture the full range and characteristics of signal conditions a device will encounter in a given environment more accurately than a set of static signals.

- Care must be taken in processing the source data to ensure histograms are as accurately representative of the target signal environment as possible.

- To investigate, gather data from various GPS environments of interest.



Initialize in good visibility to ensure all visible SVs are initially tracked

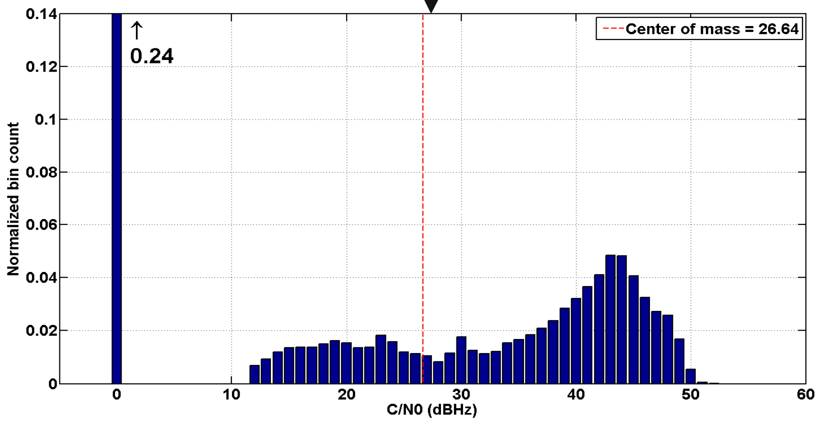
↓

Trim data manually to ensure homogeneous environment

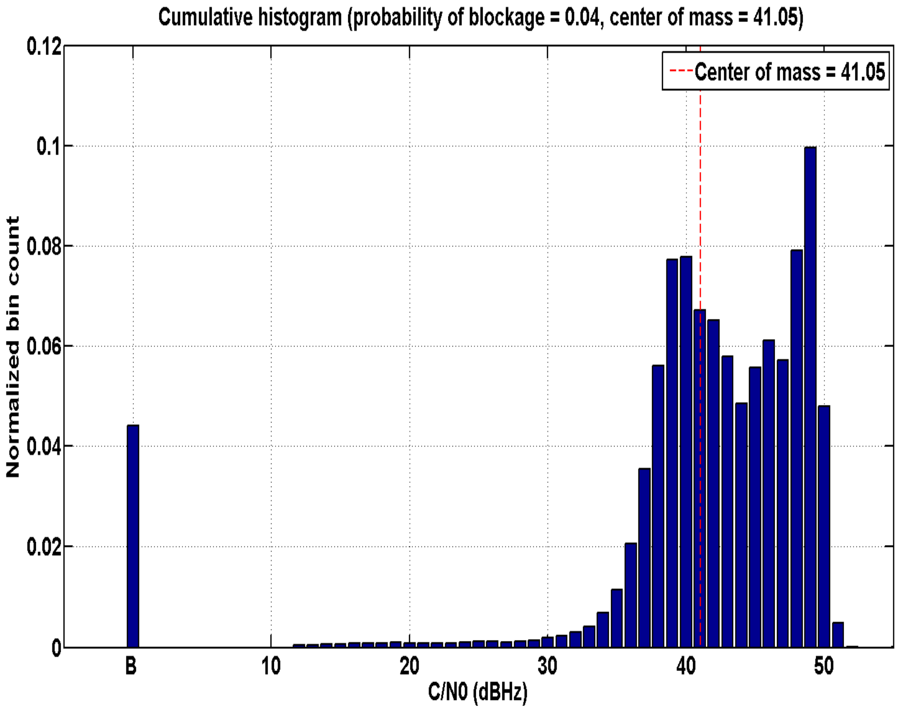
↓

Assemble C/N0 histograms from Rx output, using P/T data to properly account for blocked SVs

↓



C/N_0 histogram (open sky*, Stockholm)

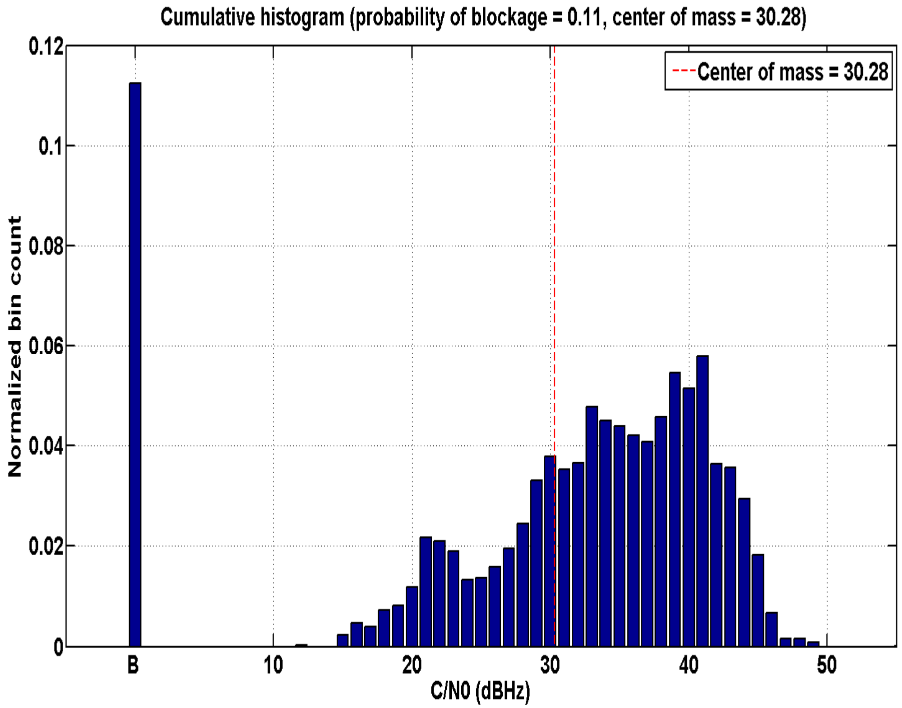


- $H[n]$ defined for n = 0, 1, 2, …, $P_{\max}$. Typically $P_{\max}$ ≈ 48 dBHz for active patch antennas, less for low-efficiency embedded antennas.
- $H[n]$ is normalized: $\sum_{n=0}^{P_{\max}} H[n] = 1$

→ $H[n]$ represents a proper probability mass function (PMF) usable for further analysis.

* Antenna used for this data was mounted on a multipath-mitigating choke ring (sharp gain cutoff below ~15° elevation).

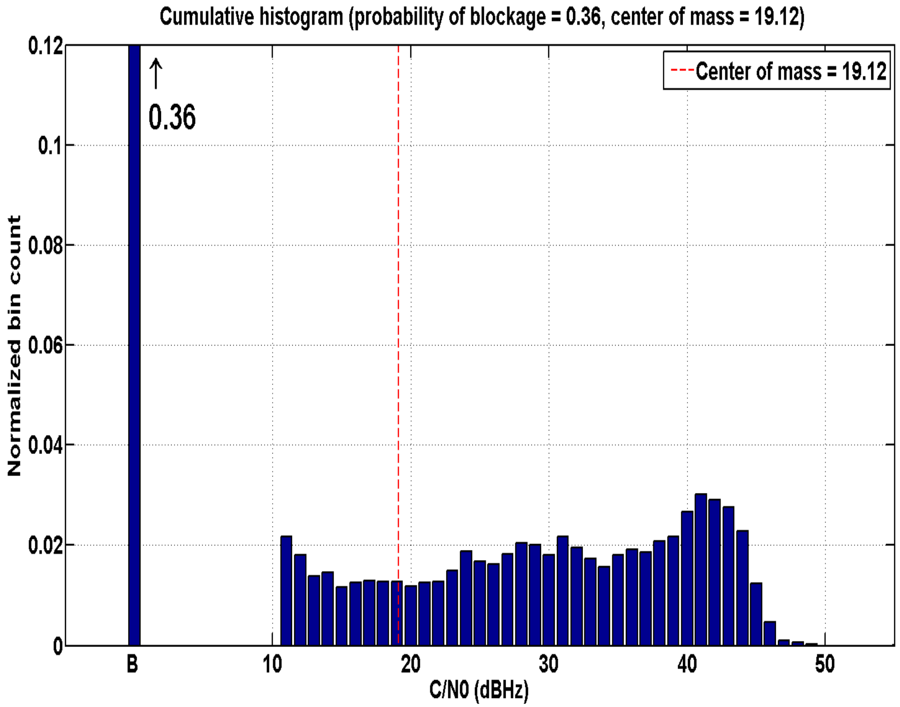
C/N_0 histogram (suburban, dense foliage)



- $H[n]$ defined for $n = 0, 1, 2, \ldots, P_{\max}$. Typically $P_{\max} \approx 48$ dBHz for active patch antennas, less for low-efficiency embedded antennas.
- $H[n]$ is normalized: $$\sum_{n=0}^{P_{\max}} H[n] = 1$$

→ $H[n]$ represents a proper probability mass function (PMF) usable for further analysis.

C/N_0 histogram (urban canyon, San Francisco)



- $H[n]$ defined for n = 0, 1, 2, …, $P_{\max}$. Typically $P_{\max}$ ≈ 48 dBHz for active patch antennas, less for low-efficiency embedded antennas.
- $H[n]$ is normalized: $\displaystyle\sum_{n=0}^{P_{\max}} H[n] = 1$

→ $H[n]$ represents a proper probability mass function (PMF) usable for further analysis.

What Can We Do With C/N_0 Histograms?

- Approximate simulation of real-world environments in a compact fashion requiring only existing, widely deployed testing apparatus

- Detailed tuning of acquisition algorithms via knowledge of *a priori* distributions measured in various environments of interest

- Prediction of certain interesting performance metrics in a way that cannot be done easily (or at all) with conventional methods (*e.g.* center of mass, probability of blockage, etc.)

- Easily quantifiable measurement of both raw data and test results via simple mathematical tools

Example: Acquisition and *a priori* Distributions

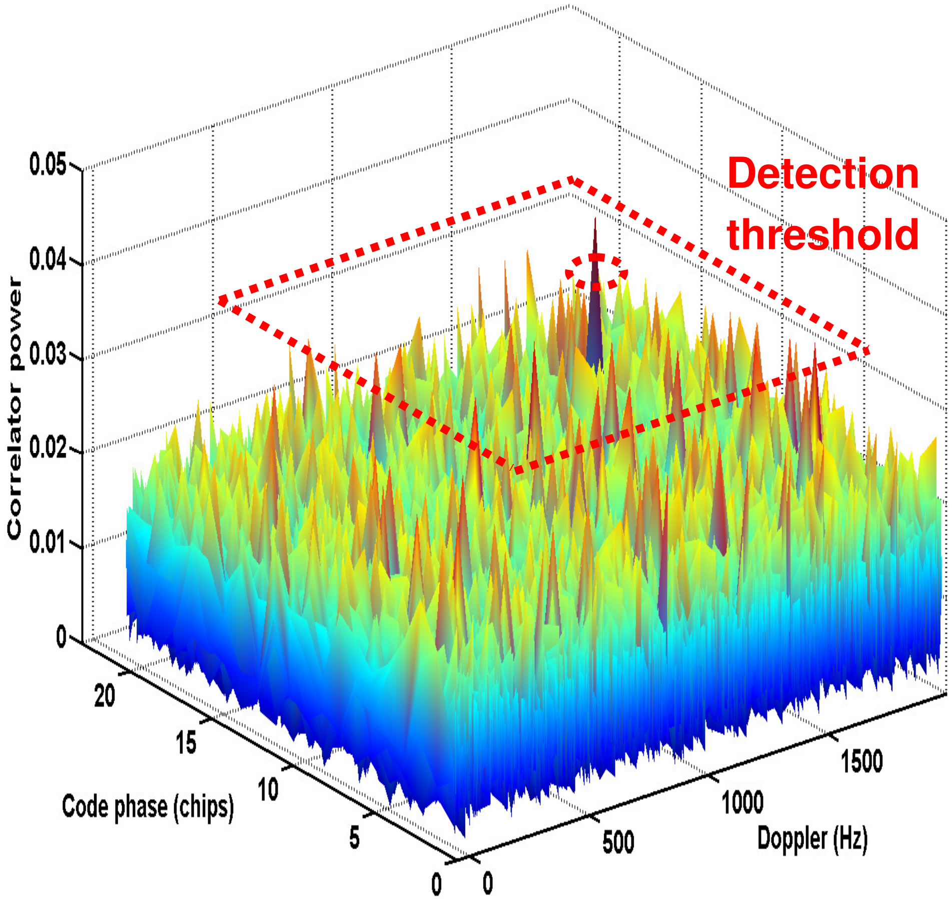
- Acquisition is typically considered over 3D search space:

$$\left(PRN, \hat{\tau}, f_d\right)$$

… but search space is really 4D:

$$\left(PRN, \hat{\tau}, f_d, T_{\mathrm{int}}\right)$$

- Together with C/N_0, dwell time T_{int} determines detection threshold and affects acquisition performance
  - Search too shallow and risk missing a signal (P_{md} too high).
  - Search too deep and risk getting false lock (P_{fa} too high), wasting power and CPU resources.



Full search space for a single PRN

Histograms provide information on prior probabilities of real-world C/N_0 values.

Example: Probability of Successful Fix (1 of 4)

- **Question**: What is the probability that a receiver will be able to get a fix in a given environment?

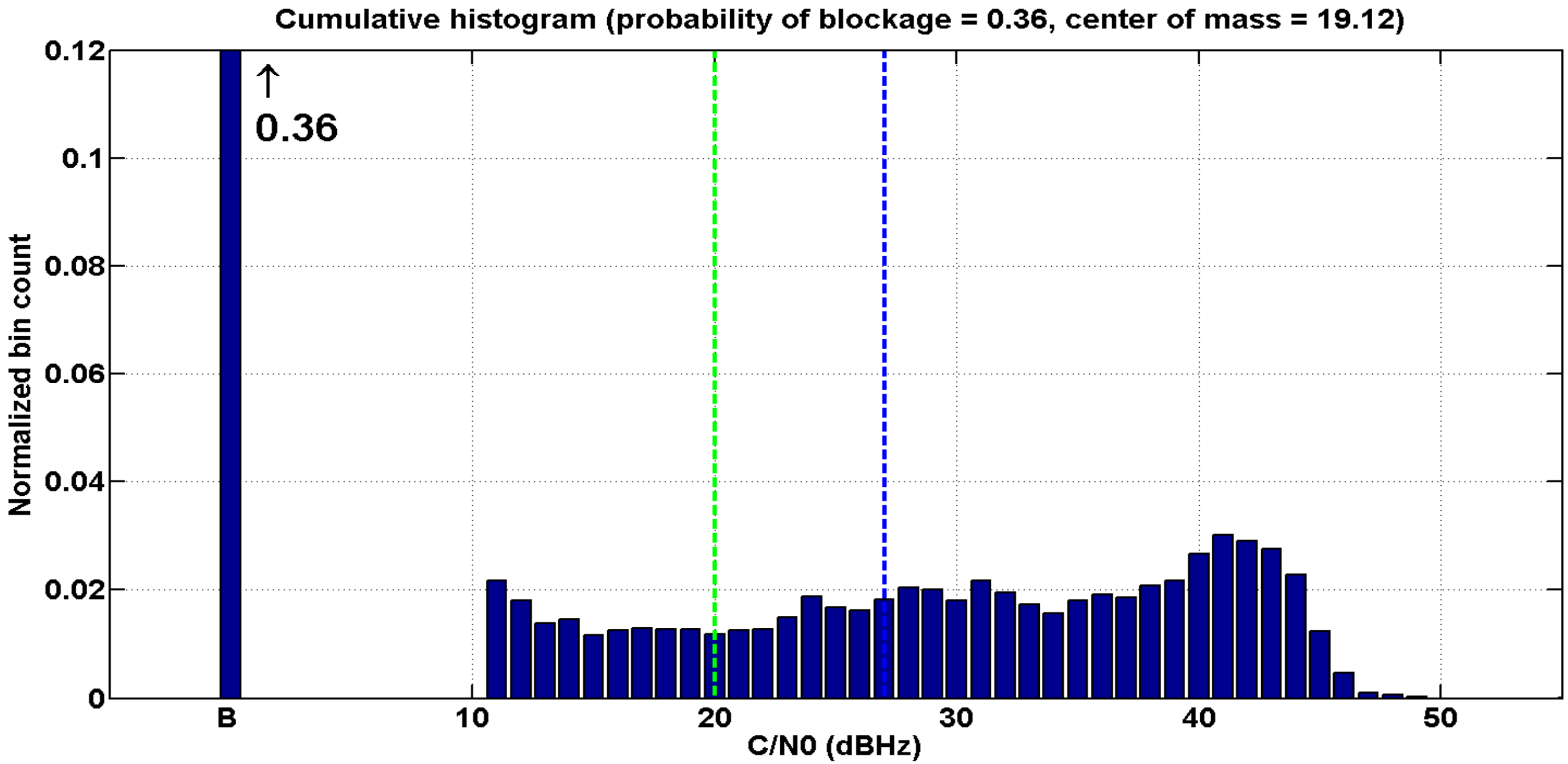
- **Analysis**: A receiver's tracking loops are typically designed to work down to a particular C/N_0 (below which the loop could work, but would yield unacceptable position accuracy). Consider two thresholds:

  – P_i = min. C/N_0 where receiver can acquire a signal independently

  – P_d = min. C/N_0 where receiver can acquire a signal assuming another signal is already tracked ($P_d < P_i$). This is the so-called "lead SV" case.

Using a histogram collected from a particular environment, we can predict the probability of a successful fix as a function of the signal levels we expect to see in that environment. (Not so easy to do this with conventional methods…)

Example: Probability of Successful Fix (2 of 4)

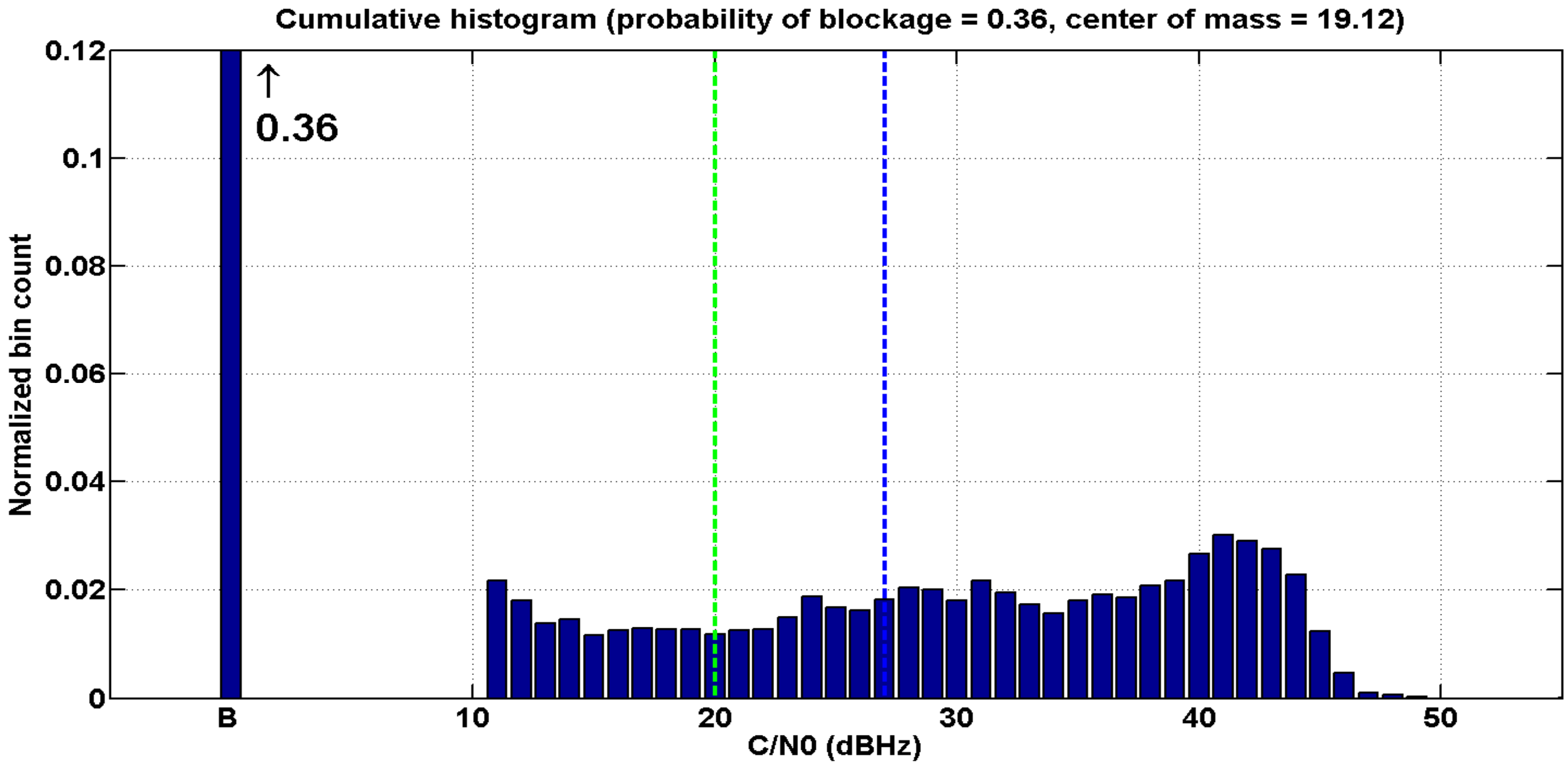


- P_i = 27 dBHz, P_d = 20 dBHz, N = floor(avg. # of SVs in view) = 8. Then:

$$\Pr(\text{have a lead SV}) = \Pr(\text{at least 1 SV} \geq P_i) = 1 - \Pr(\text{no SVs} \geq P_i)$$

$$= 1 - \left(\sum_{n=0}^{P_i - 1} H[n] \right)^N = 1 - (0.58)^{\lfloor 8.1 \rfloor} \approx 0.987$$

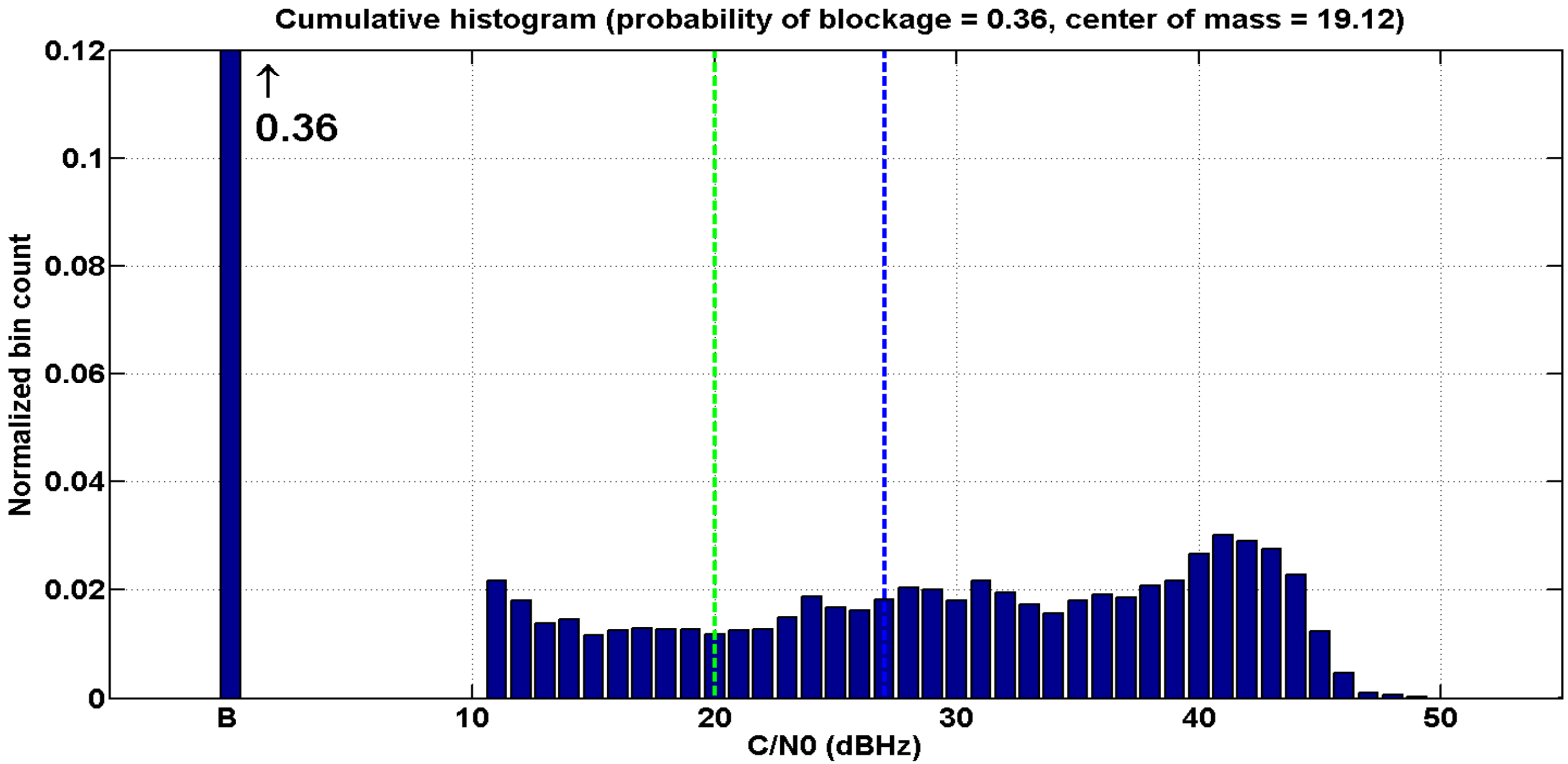
Example: Probability of Successful Fix (3 of 4)



- Now a fix is possible if we have a lead <u>and</u> at least three other SVs are above the lower limit. The second part is a simple binomial expansion:

$$p = \sum_{n=P_d}^{P_{\max}} H[n] \approx 0.521 \quad \Rightarrow \quad P_{3\,\text{more}} \approx 1 - \sum_{k=0}^{2} \binom{N-1}{k} p^k (1-p)^{N-1-k} \approx 0.805$$

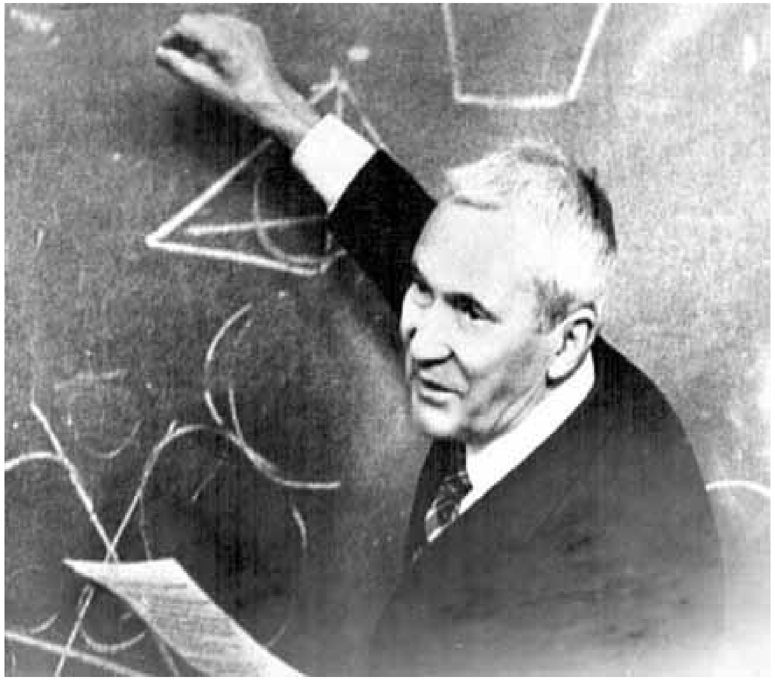
Example: Probability of Successful Fix (4 of 4)



- Finally, the probability of a fix is just *0.988 * 0.805 = 0.795 =* ***79.5%*** in this signal environment.

- Note that this calculation is specific to each receiver in each specific set of signal conditions.

Comparing histograms: Kolmogorov–Smirnov Test

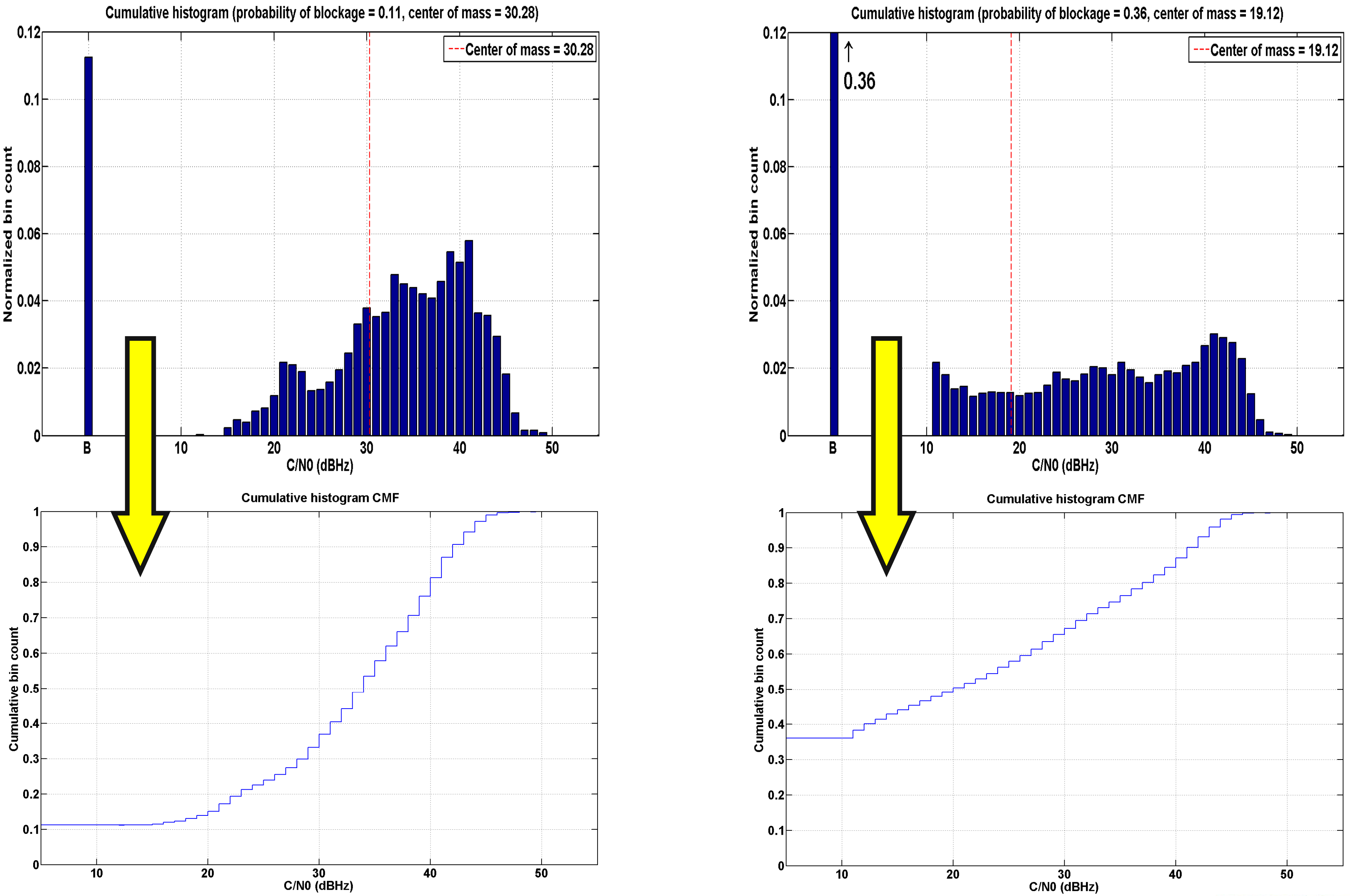


Andrey Nikolaevich Kolmogorov
(1903-1987)

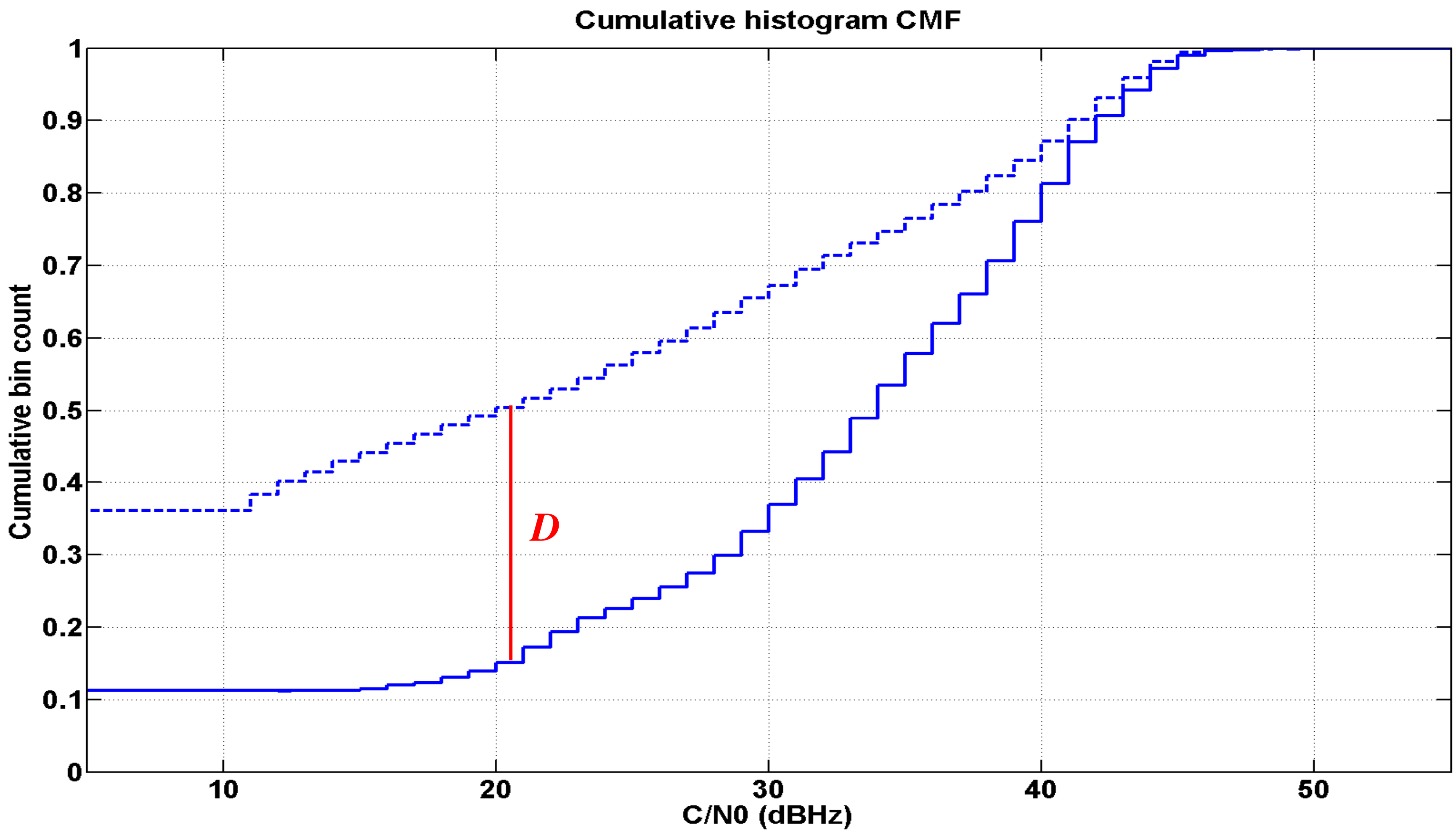
Nikolay Vasilievich Smirnov
(1900-1966)

- A simple, generic way to measure the similarity of two sets of random samples.
- Usable for any well-behaved normalized distributions.
- Can be adapted to compare both histograms and TTFF / 2D error CDFs.

Kolmogorov–Smirnov Test: Example (1 of 2)

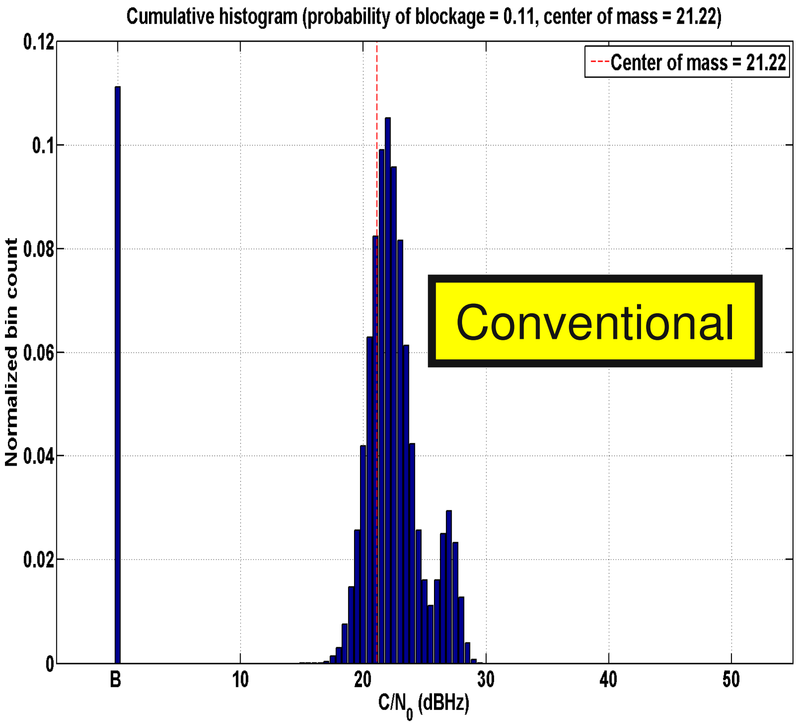


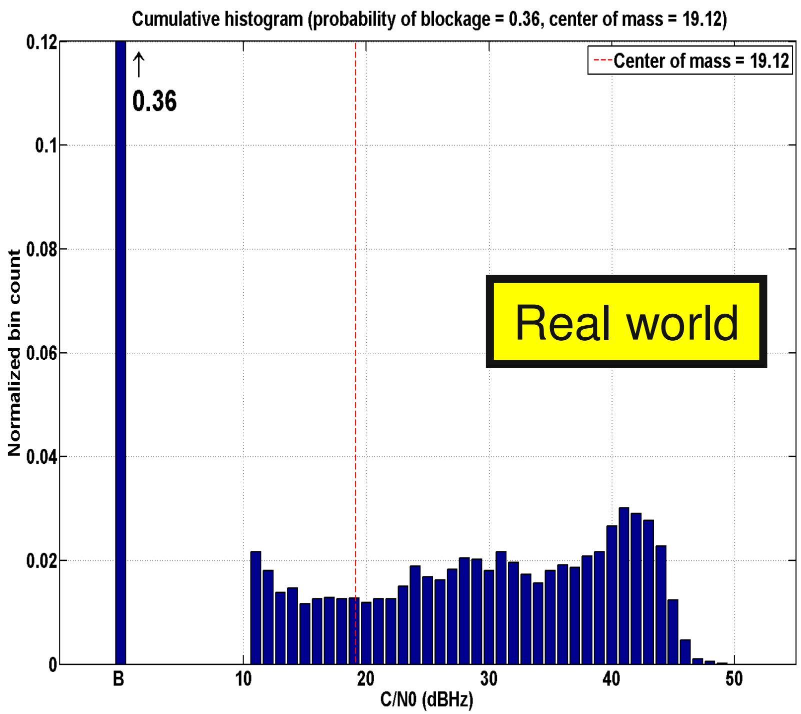
Kolmogorov–Smirnov Test: Example (2 of 2)



- Normalized histogram → CMF → range is [0 .. 1] by construction.
- The parameter D is the maximum vertical difference, and indicates degree of similarity (or lack thereof). For identical distributions we expect $D = 0$.

Conventional vs. real-world histograms





- 3GPP 34.171 §5.2.1 "coarse time aiding" test (left) vs. downtown San Francisco urban canyon (right).
- Similar center of mass, but signal environments are substantially different here: (same scale on both axes). Difference is quantified by K-S test: ***D = 0.36.***

<u>Conclusion</u>: Histograms suggest that signal levels in conventional tests bear little resemblance to those in real-world environments.

A Histogram-Based Test Methodology For The Lab

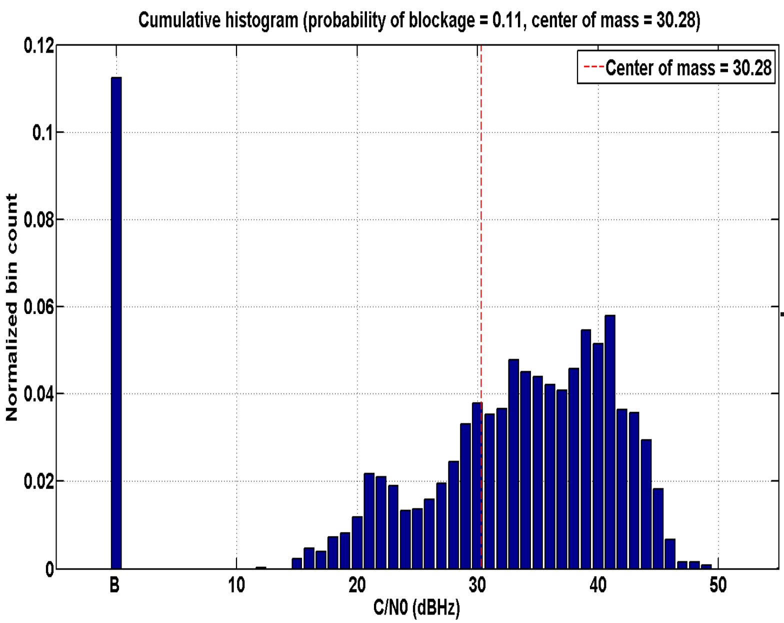
- All previous results (as well as all 3GPP tests for A-GPS) have two undesirable characteristics in common:
  - Distribution of SV power levels is unrealistically contrived
  - Power levels remain constant throughout each acquisition attempt
- Alternative approach: control essential scenario features independently
  - **How much power?**
    - Create randomized power profiles based on empirically observed distributions (*i.e.* histograms collected from real-world data).
    - New draw each time to target distribution
    - Initial draw to target distribution, followed by random walk
  - **When does it change (what is the timing model)?**
    - Constant interval (simple; unlikely to be physically realistic)
    - Poisson arrival process (exponential distribution) ~ raindrops incident on unit area or customer arrivals
    - Parallel threads or single thread
- Validate by comparing empirically measured CDF curves (for TTFF and position accuracy) versus same results from simulated data

Generating Random Levels with Desired Distribution (1 of 2)

- **<u>Goal</u>**: design a randomizer that generates power levels matching an arbitrary (observed) distribution. Assume a generator of uniformly distributed random samples is available.

- **<u>Approach</u>**: use *probability integral transform* (*a.k.a.* rejection sampling).

$[u_1, u_2, u_3, \ldots]$, where $u_n \sim U[0,1]$



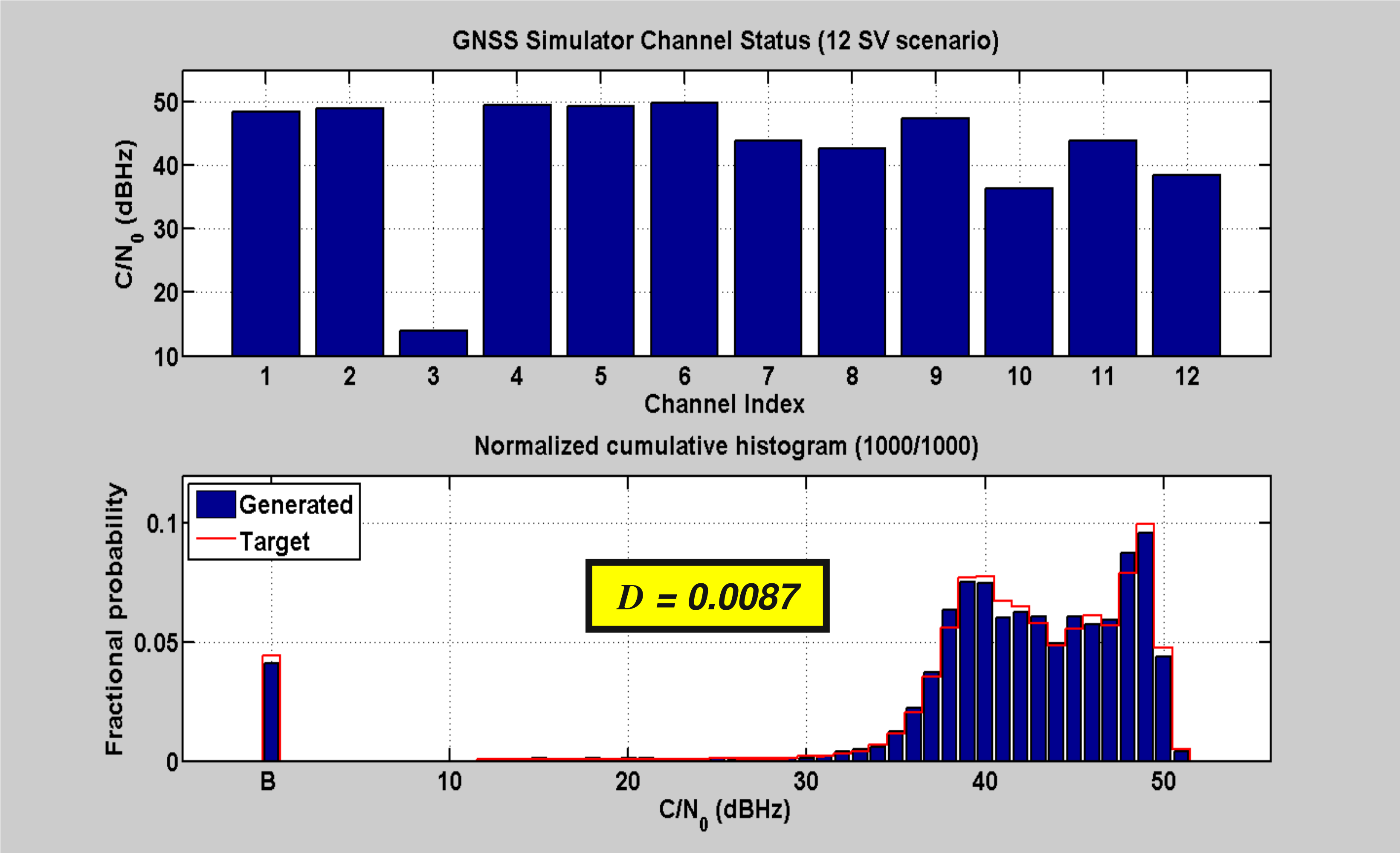
PIT function:

$P_k = H^{-1}[u]$

$[p_1, p_2, p_3, \ldots]$

where p_n has the same ensemble distribution as the empirically observed signal data

Generating Random Levels with Desired Distribution (2 of 2)



Fluctuation Timing (1 of 4)

- Ensemble distribution of signal levels and timing of individual transitions (fluctuations) can be treated independently.

- Different ways to simulate fluctuation timing
  - All SVs fluctuated simultaneously (synchronously)
  - Individual SVs or groups of SVs fluctuated independently (asynchronously)

- Timing models for intervals
  - Uniform / constant
  - Experimentally determined from observed Allan Variance
  - Exponential distribution (derived from Poisson arrival process, which arises in queuing theory, time-sequence analysis of random events, etc.). Probability distribution function:

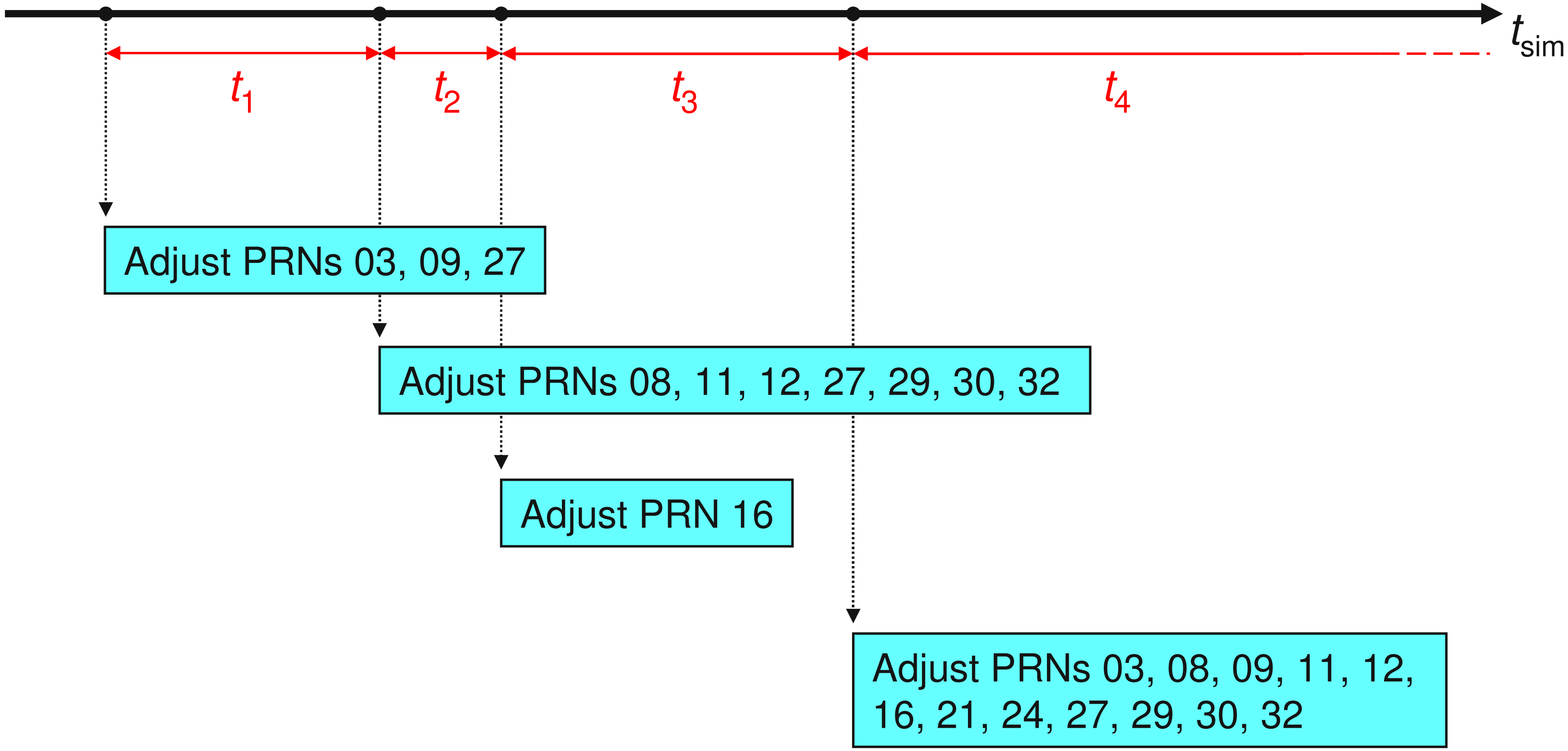
$$p(t,\lambda) = \begin{cases} \lambda e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases},$$

where λ is a characteristic time scale (in seconds^{-1}).

Fluctuation Timing (2 of 4)

- **Single thread, multiple SVs**: timing intervals are modelled as

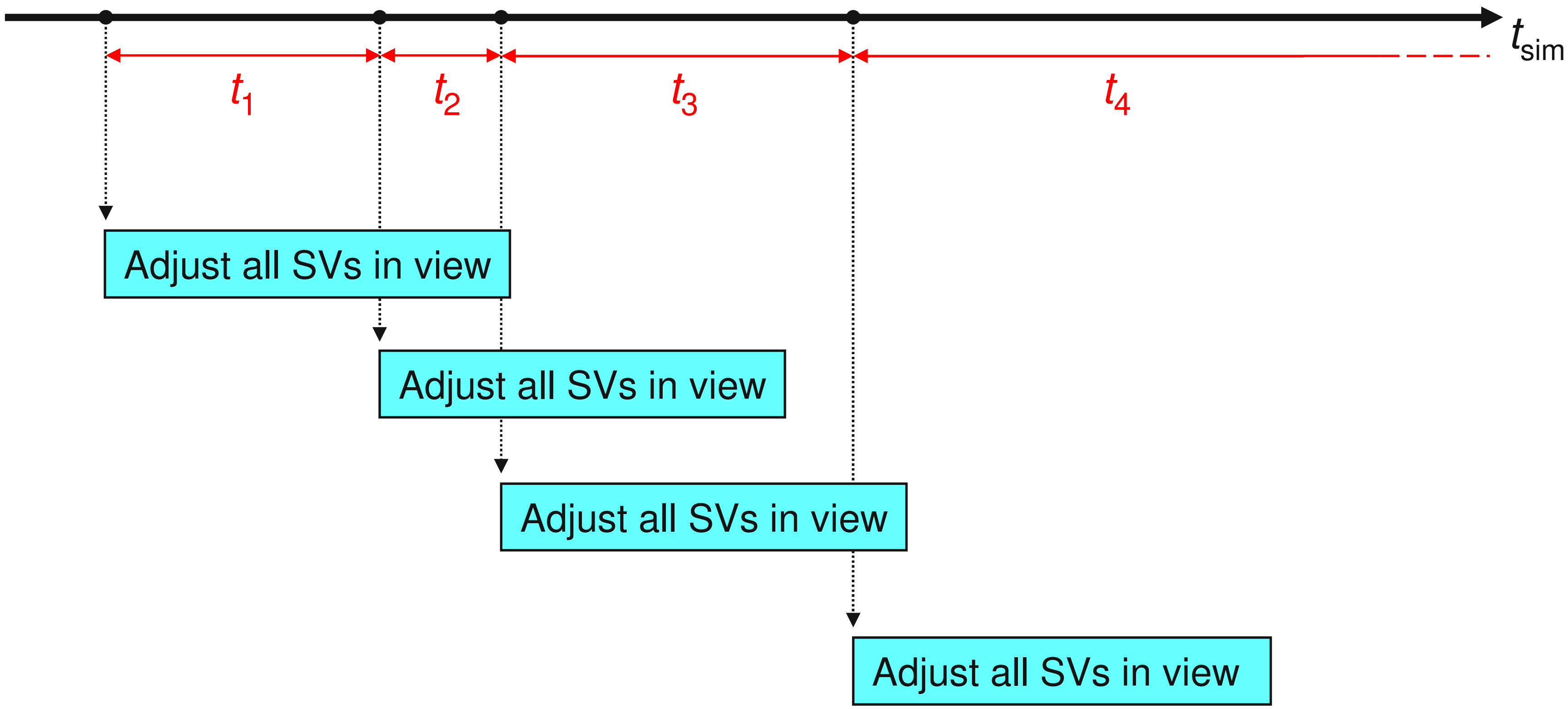
$$t_n = \text{time between events} \sim \text{exponential}\ (\lambda^{-1})$$



Fluctuation Timing (3 of 4)

- **Single thread, all SVs**: timing intervals are modelled as

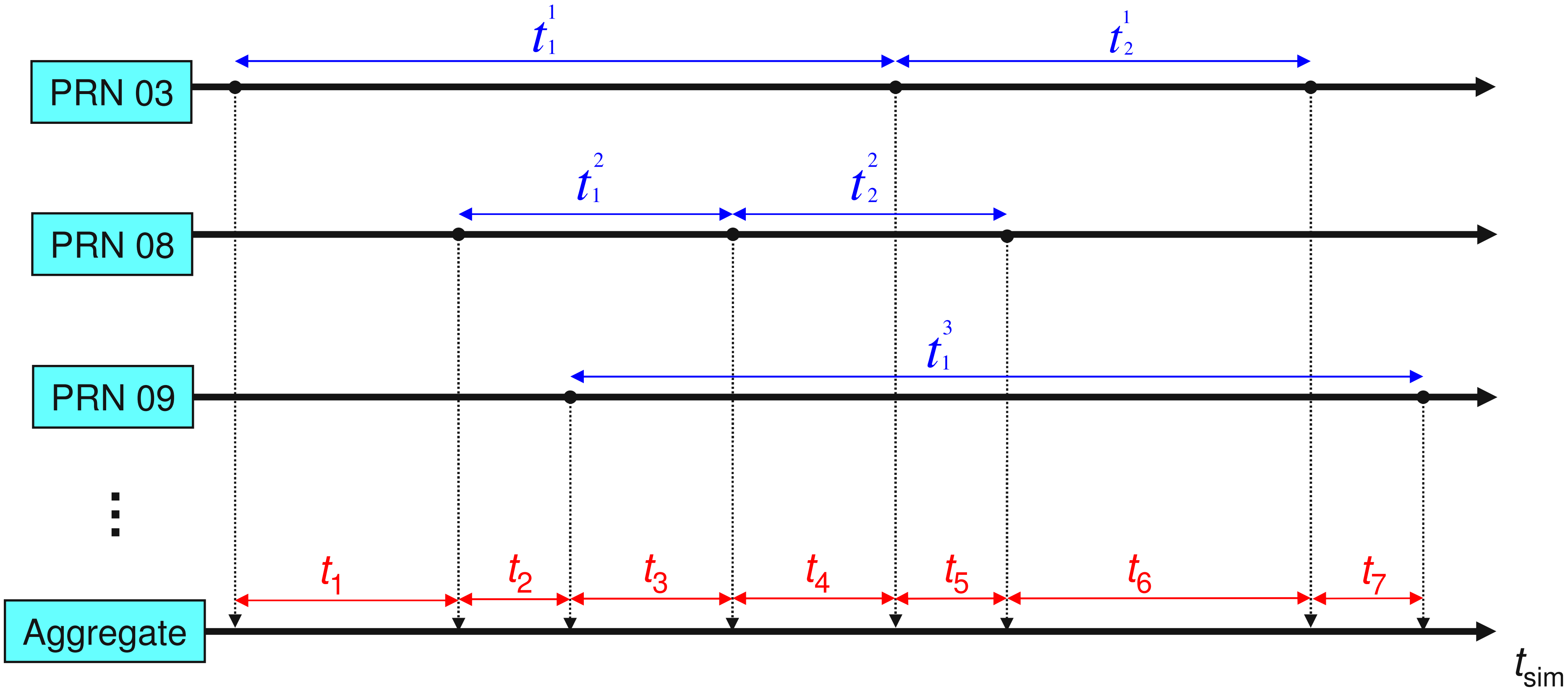
$$t_n = \text{time between events} \sim \text{exponential}\ (\lambda^{-1})$$



Fluctuation Timing (4 of 4)

- **Multiple threads, one per visible SV (*N* total):** timing intervals are modelled as

$$t_n^i = \text{time between events on } i\text{-th SV} \sim \text{exponential } \{(N\lambda)^{-1}\}$$



and t_n = time between events ~ exponential (λ^{-1}), as before.

Overall Fluctuating Test Design Considerations

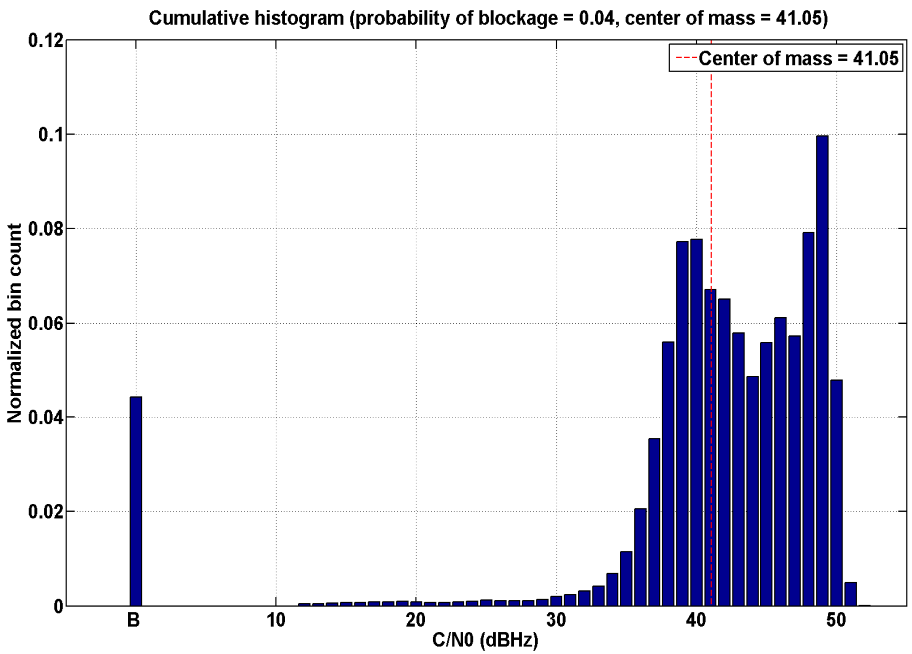
- PIT method produces a series of random numbers whose aggregate histogram matches a target histogram.

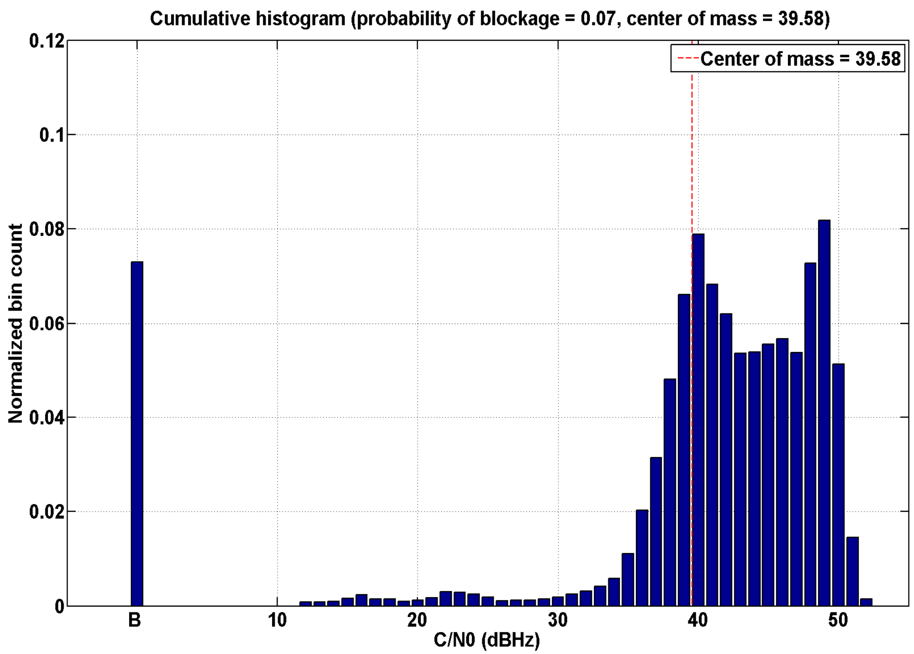
- Timing model(s) determine when to adjust power levels on simulator channels. Amplitudes are determined by the output of the PIT method.

- Scenario characteristics are treated separately:
  - Location, time, date: determine number of SVs in view and their geometry.
  - For a given number of SVs, geometry (DOP) affects the 2D error CDF, but *not* the TTFF CDF – receiving antennas are generally not directional!
  - The number of visible SVs *does* affect the TTFF CDF[1].
  - These features are controlled independently of both power level distribution (C/N_0) and timing model.

[1] "Apples to Apples: Standardized Testing For High Sensitivity Receivers," GPS World, Vol. 19, No. 1, p. 33, January 2008.

Scenario sanity check: Histogram level





- Compare target histogram [24 hours rooftop data] (left) with continuous data from fluctuating simulator scenario with $\lambda^{-1} = 5.0$ (right)

- Shapes look very similar once ~4.0 dB offset due to NF+IL of RUT is measured, then incorporated into generation of fluctuating scenario.

- K-S test indicates these histograms are very similar ($\boldsymbol{D} = 0.042$).

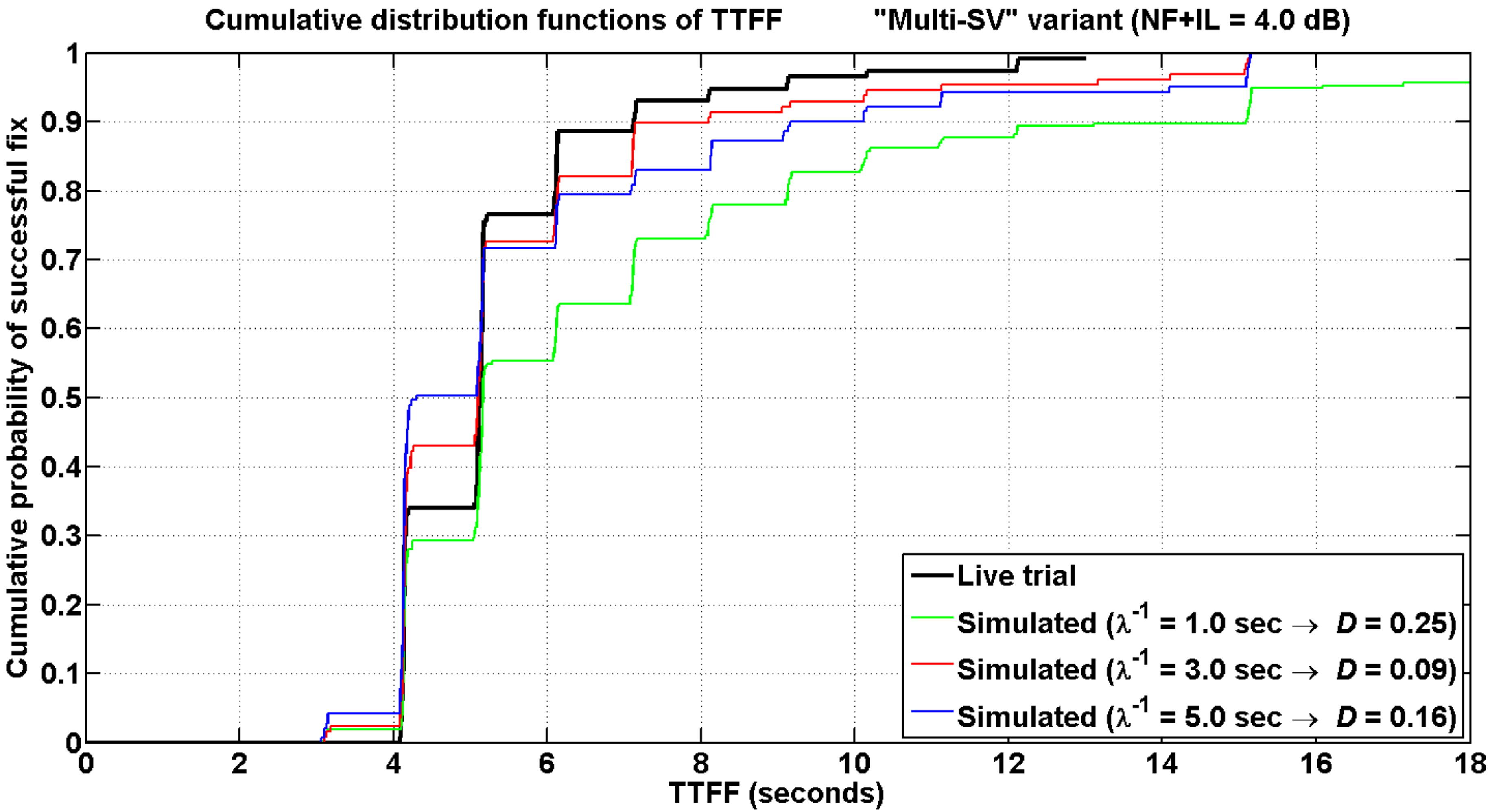
Quasi-realistic sensitivity test matrix

Simulated Antenna Position

SV Power Levels

	Static Antenna	Moving Antenna
Randomised once @ beginning of each start attempt	*Quasi-stationary start in difficult environment (mobile phone case)*	(not physically realistic)
Fluctuating randomly throughout all start attempts	(not physically realistic)	*Fix-and-go start in difficult environment (PND case)*

Fluctuating test ("Multiple SV" variant)



This result represents a first-cut attempt to bring a realistic urban environment into the lab in a compact, easily realizable way.

Next steps toward "closing the loop"

- Gather data from a variety of environments (fully characterized to date: *one* urban canyon with patch antenna, *one* open sky with choke ring)
  - Additional urban canyons (differences in construction materials, characteristic distance to nearby reflectors, etc. are expected to yield substantially different profiles)
  - Additional "dense foliage" environments
  - Additional antenna types (including "slave" antennas from mobile phones to realistically model reception patterns)

- Generate simulator scenarios from continuous data, assuming, *e.g.*, exponential timing model for fluctuations. Investigate a range of λ^{-1} values.

- Subject pool of receivers, ideally from a wide range of manufacturers, to simulated scenario and test acquisition performance.

- Quantify similarity between live and fluctuation-based CDF curves for TTFF and 2D error using K-S at, *e.g.,* 95% confidence bound.

Histograms: Summary

- Existing test methods (3GPP and related variants) do not always capture the essential features of real-world signal environments.
- The fluctuating histogram approach models signal distributions more realistically and provides *a priori* information about levels likely to be encountered in a given environment.
- This approach is easily implementable with existing equipment, requiring neither significant additional RF hardware nor storage.
- What's next?
  - Multipath (should primarily affect 2D error; TTFF to a lesser extent)
  - Other timing models; full covariance analysis
  - Much more empirical data
  - Proposal to standards committees (3GPP RAN, GERAN, etc.)

Questions? Comments?

alexander (dot) mitelman (at-sign) csr (dot) com

sasha (dot) mitelman (at-sign) stanfordalumni (dot) org