Simulating Real-World Signal Environments

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Presentation Outline

• Motivation
  – What to measure, and why?
  – Existing test methods and drawbacks

• Novel testing approach: fluctuating signals
  – Raw histogram data collection
  – Figures of merit, mathematical tools
  – Differences from conventional methods

• Implementation and results
Motivation

• Object of measurement?
  – Accuracy (2D/3D error: max / mean / 95\textsuperscript{th} percentile) [acq. or nav.]
  – Time to first fix (seconds) [acq. only]

• Goals
  – Predictivity via meaningful metrics (“user experience isn’t everything; it’s the only thing”)
    • Reasonable reproduction of real-world performance in the lab
    • Element of randomness to harness full power of Monte Carlo-type simulations
  – Standardization-friendly alternative to existing approaches
    • Requires no additional equipment
    • Minimal storage requirements (c.f. RF replay: 10Msps @ 16b, 100 reference files @ 20min → reference set \sim 2.4\text{TB})
• 3GPP specs implicitly consider *cumulative distribution functions* (CDFs).
• Requirements formulated as 67th or 95th percentile bounds on 2D error and, for acquisition (as shown here), time-to-first fix (TTFF).
Conventional Method 1: “Apples to Apples” Testing [1]

- Step down power levels on all SVs simultaneously, observe lowest level at which behaviors of interest (navigation, [re-]acquisition, etc.) are possible:

![Graph showing power levels and behaviors]

<table>
<thead>
<tr>
<th>SVID</th>
<th>14</th>
<th>18</th>
<th>21</th>
<th>25</th>
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<th>6</th>
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![Graph showing power levels and SVIDs](image)

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Conventional Method 1: “Apples to Apples” Testing [1]

- Tracking sensitivity

![Graph showing tracking sensitivity over time]

- At each time step (typically 3-4 minutes), all SV signals are at the same power
- Analogous tests also defined for acquisition and reacquisition
- Well-controlled and repeatable, but how useful are these tests practically?

Conventional Method 2: 3GPP A-GPS Testing \[1\] \[2\]

- Primary goals are acquisition (TTFF and accuracy), for push-to-fix-style use cases (coarse/fine time, “lead SV”):

Proposed Alternative Method: $C/N_0$ Histograms (1 of 2)

- Tests just described assume fixed, static signal levels. For example, consider histograms of receiver-reported $C/N_0$ for 3GPP 5.2.1 and 5.2.2:

![Histograms](image)

Key question: How well do these cases represent real-world environments?
Proposed Alternative Method: $C/N_0$ Histograms (2 of 2)

- Experience shows that real-world signal levels are almost *never* uniform or static, especially in difficult environments and/or when the user is moving.

- A histogram of observed $C/N_0$ values may capture the full range and characteristics of signal conditions a device will encounter in a given environment more accurately than a set of static signals.

- Care must be taken in processing the source data to ensure histograms are as accurately representative of the target signal environment as possible.
Measuring Real-world Histograms

- To investigate, gather data from various GPS environments of interest.

Initialize in good visibility to ensure all visible SVs are initially tracked

Trim data manually to ensure homogeneous environment

Assemble C/N0 histograms from Rx output, using P/T data to properly account for blocked SVs

Start

End

Init
$C/N_0$ histogram (open sky*, Stockholm)

- $H[n]$ defined for $n = 0, 1, 2, \ldots, P_{\text{max}}$. Typically $P_{\text{max}} \approx 48$ dBHz for active patch antennas, less for low-efficiency embedded antennas.

- $H[n]$ is normalized: \[ \sum_{n=0}^{P_{\text{max}}} H[n] = 1 \]

$\rightarrow H[n]$ represents a proper probability mass function (PMF) usable for further analysis.

* Antenna used for this data was mounted on a multipath-mitigating choke ring (sharp gain cutoff below ~15° elevation).
$C/N_0$ histogram (suburban, dense foliage)

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$C/N_0$ histogram (urban canyon, San Francisco)

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What Can We Do With $C/N_0$ Histograms?

• Approximate simulation of real-world environments in a compact fashion requiring only existing, widely deployed testing apparatus

• Detailed tuning of acquisition algorithms via knowledge of a priori distributions measured in various environments of interest

• Prediction of certain interesting performance metrics in a way that cannot be done easily (or at all) with conventional methods (e.g. center of mass, probability of blockage, etc.)

• Easily quantifiable measurement of both raw data and test results via simple mathematical tools
Example: Acquisition and \textit{a priori} Distributions

- Acquisition is typically considered over 3D search space:
  \[
  \left( PRN, \hat{\tau}, f_d \right)
  \]
  ... but search space is really 4D:
  \[
  \left( PRN, \hat{\tau}, f_d, T_{\text{int}} \right)
  \]
- Together with $C/N_0$, dwell time $T_{\text{int}}$ determines detection threshold and affects acquisition performance
  - Search too shallow and risk missing a signal ($P_{md}$ too high).
  - Search too deep and risk getting false lock ($P_{fa}$ too high), wasting power and CPU resources.

Histograms provide information on prior probabilities of real-world $C/N_0$ values.
Example: Probability of Successful Fix (1 of 4)

- **Question**: What is the probability that a receiver will be able to get a fix in a given environment?

- **Analysis**: A receiver’s tracking loops are typically designed to work down to a particular $C/N_0$ (below which the loop could work, but would yield unacceptable position accuracy). Consider two thresholds:

  - $P_i = \min. \ C/N_0$ where receiver can acquire a signal independently
  - $P_d = \min. \ C/N_0$ where receiver can acquire a signal assuming another signal is already tracked ($P_d < P_i$). This is the so-called “lead SV” case.

Using a histogram collected from a particular environment, we can predict the probability of a successful fix as a function of the signal levels we expect to see in that environment. (Not so easy to do this with conventional methods…)
• \( P_i = 27 \text{ dBHz}, \quad P_d = 20 \text{ dBHz}, \quad N = \text{floor( avg. # of SVs in view)} = 8. \) Then:

\[
\Pr(\text{have a lead SV}) = \Pr(\text{at least 1 SV} \geq P_i) = 1 - \Pr(\text{no SVs} \geq P_i)
\]

\[
= 1 - \left( \sum_{n=0}^{P_i-1} H[n] \right)^N = 1 - (0.58)^{8.1} \approx 0.987
\]
• Now a fix is possible if we have a lead and at least three other SVs are above the lower limit. The second part is a simple binomial expansion:

\[
p = \sum_{n=P_d}^{P_{\text{max}}} H[n] \approx 0.521 \quad \Rightarrow \quad P_{3\text{more}} \approx 1 - \sum_{k=0}^{2} \binom{N-1}{k} p^k (1-p)^{N-1-k} \approx 0.805
\]
Finally, the probability of a fix is just $0.988 \times 0.805 = 0.795 = 79.5\%$ in this signal environment.

Note that this calculation is specific to each receiver in each specific set of signal conditions.
Comparing histograms: Kolmogorov–Smirnov Test

Andrey Nikolaevich Kolmogorov
(1903-1987)

Nikolay Vasilievich Smirnov
(1900-1966)

• A simple, generic way to measure the similarity of two sets of random samples.
• Usable for any well-behaved normalized distributions.
• Can be adapted to compare both histograms and TTFF / 2D error CDFs.
Kolmogorov–Smirnov Test: Example (1 of 2)
Normalized histogram $\rightarrow$ CMF $\rightarrow$ range is $[0..1]$ by construction.

The parameter $D$ is the maximum vertical difference, and indicates degree of similarity (or lack thereof). For identical distributions we expect $D = 0$. 

Kolmogorov–Smirnov Test: Example (2 of 2)
Conventional vs. real-world histograms

- 3GPP 34.171 §5.2.1 “coarse time aiding” test (left) vs. downtown San Francisco urban canyon (right).
- Similar center of mass, but signal environments are substantially different here: (same scale on both axes). Difference is quantified by K-S test: $D = 0.36$.

**Conclusion:** Histograms suggest that signal levels in conventional tests bear little resemblance to those in real-world environments.
A Histogram-Based Test Methodology For The Lab

• All previous results (as well as all 3GPP tests for A-GPS) have two undesirable characteristics in common:
  – Distribution of SV power levels is unrealistically contrived
  – Power levels remain constant throughout each acquisition attempt

• Alternative approach: control essential scenario features independently
  – **How much power?**
    • Create randomized power profiles based on empirically observed distributions (*i.e.* histograms collected from real-world data).
    • New draw each time to target distribution
    • Initial draw to target distribution, followed by random walk
  – **When does it change (what is the timing model)?**
    • Constant interval (simple; unlikely to be physically realistic)
    • Poisson arrival process (exponential distribution) ~ raindrops incident on unit area or customer arrivals
    • Parallel threads or single thread

• Validate by comparing empirically measured CDF curves (for TTFF and position accuracy) versus same results from simulated data
- **Goal**: design a randomizer that generates power levels matching an arbitrary (observed) distribution. Assume a generator of uniformly distributed random samples is available.

- **Approach**: use *probability integral transform* (*a.k.a.* rejection sampling).

\[ [u_1, u_2, u_3, \ldots], \text{ where } u_n \sim \text{U}[0,1] \]

\[ \text{PIT function: } P_k = H^{-1}[u] \]

\[ [p_1, p_2, p_3, \ldots] \]

where \(p_n\) has the same ensemble distribution as the empirically observed signal data.
Generating Random Levels with Desired Distribution (2 of 2)

GNSS Simulator Channel Status (12 SV scenario)

Normalized cumulative histogram (1000/1000)

\[ D = 0.0087 \]
**Fluctuation Timing (1 of 4)**

- Ensemble distribution of signal levels and timing of individual transitions (fluctuations) can be treated independently.

- Different ways to simulate fluctuation timing
  - All SVs fluctuated simultaneously (synchronously)
  - Individual SVs or groups of SVs fluctuated independently (asynchronously)

- Timing models for intervals
  - Uniform / constant
  - Experimentally determined from observed Allan Variance
  - Exponential distribution (derived from Poisson arrival process, which arises in queuing theory, time-sequence analysis of random events, etc.).

Probability distribution function:

\[
p(t, \lambda) = \begin{cases} 
\lambda e^{-\lambda t}, & t \geq 0 \\
0, & t < 0 
\end{cases},
\]

where \( \lambda \) is a characteristic time scale (in seconds\(^{-1}\)).
• **Single thread, multiple SVs**: timing intervals are modelled as
  \[ t_n = \text{time between events} \sim \text{exponential} \left( \lambda^{-1} \right) \]
• **Single thread, all SVs**: timing intervals are modelled as
  \[ t_n = \text{time between events} \sim \text{exponential} (\lambda^{-1}) \]
**Multiple threads, one per visible SV (N total):** timing intervals are modelled as

\[ t_n^i = \text{time between events on } i\text{-th SV} \sim \text{exponential } \{(N\lambda)^{-1}\}\]

and \( t_n = \text{time between events } \sim \text{exponential } (\lambda^{-1})\), as before.
Overall Fluctuating Test Design Considerations

• PIT method produces a series of random numbers whose aggregate histogram matches a target histogram.

• Timing model(s) determine when to adjust power levels on simulator channels. Amplitudes are determined by the output of the PIT method.

• Scenario characteristics are treated separately:
  – Location, time, date: determine number of SVs in view and their geometry.
  – For a given number of SVs, geometry (DOP) affects the 2D error CDF, but not the TTFF CDF – receiving antennas are generally not directional!
  – The number of visible SVs does affect the TTFF CDF\(^1\).
  – These features are controlled independently of both power level distribution \((C/N_0)\) and timing model.

Scenario sanity check: Histogram level

- Compare target histogram [24 hours rooftop data] (left) with continuous data from fluctuating simulator scenario with $\lambda^{-1} = 5.0$ (right)
- Shapes look very similar once ~4.0 dB offset due to NF+IL of RUT is measured, then incorporated into generation of fluctuating scenario.
- K-S test indicates these histograms are very similar ($D = 0.042$).
### Quasi-realistic sensitivity test matrix

#### Simulated Antenna Position

<table>
<thead>
<tr>
<th>SV Power Levels</th>
<th>Static Antenna</th>
<th>Moving Antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Randomised once @ beginning of each start attempt</td>
<td><em>Quasi-stationary start in difficult environment (mobile phone case)</em></td>
<td>(not physically realistic)</td>
</tr>
<tr>
<td>Fluctuating randomly throughout all start attempts</td>
<td>(not physically realistic)</td>
<td><em>Fix-and-go start in difficult environment (PND case)</em></td>
</tr>
</tbody>
</table>
Fluctuating test (“Multiple SV” variant)

This result represents a first-cut attempt to bring a realistic urban environment into the lab in a compact, easily realizable way.
Next steps toward “closing the loop”

• Gather data from a variety of environments (fully characterized to date: one urban canyon with patch antenna, one open sky with choke ring)
  – Additional urban canyons (differences in construction materials, characteristic distance to nearby reflectors, etc. are expected to yield substantially different profiles)
  – Additional “dense foliage” environments
  – Additional antenna types (including “slave” antennas from mobile phones to realistically model reception patterns)

• Generate simulator scenarios from continuous data, assuming, e.g., exponential timing model for fluctuations. Investigate a range of $\lambda^{-t}$ values.

• Subject pool of receivers, ideally from a wide range of manufacturers, to simulated scenario and test acquisition performance.

• Quantify similarity between live and fluctuation-based CDF curves for TTFF and 2D error using K-S at, e.g., 95% confidence bound.
• Existing test methods (3GPP and related variants) do not always capture the essential features of real-world signal environments.

• The fluctuating histogram approach models signal distributions more realistically and provides *a priori* information about levels likely to be encountered in a given environment.

• This approach is easily implementable with existing equipment, requiring neither significant additional RF hardware nor storage.

• What’s next?
  – Multipath (should primarily affect 2D error; TTFF to a lesser extent)
  – Other timing models; full covariance analysis
  – Much more empirical data
  – Proposal to standards committees (3GPP RAN, GERAN, etc.)
Questions? Comments?

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