Instantaneous Detection of Spatial Gradient Errors in Differential GNSS

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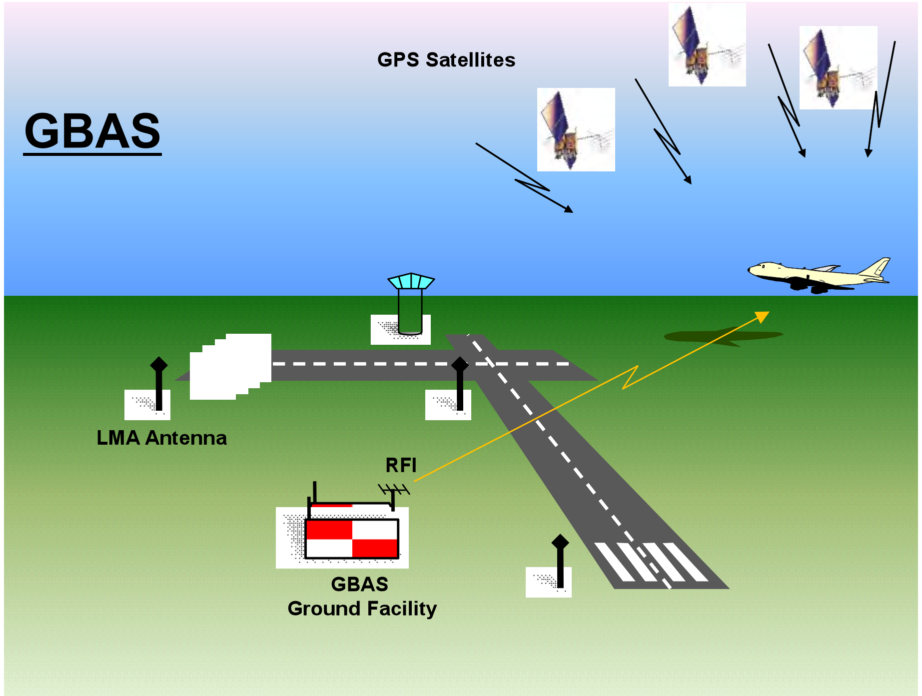
Introduction

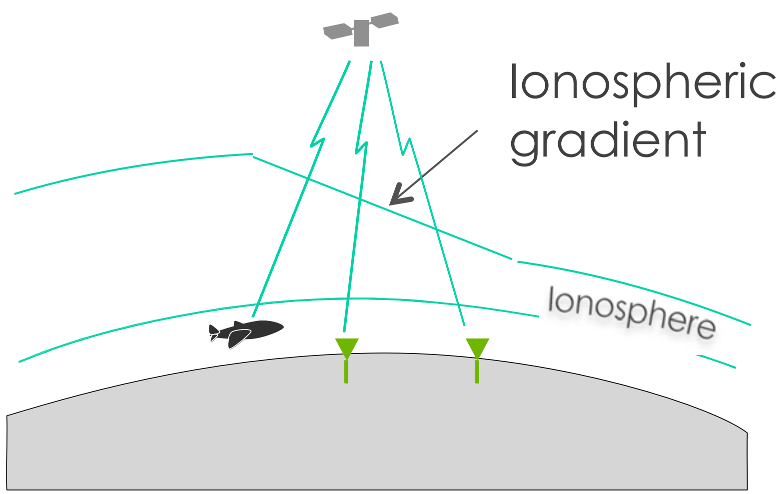
- **Differential GNSS**
  - Commonly used technique for improving GNSS performance
  - Ground Based Augmentation System (GBAS)
  - Includes multiple GPS low multipath antennas for net reduction in ranging error.
  - Antenna separation can be used to detect signal-in-space failures and anomalies:

    Ionospheric fronts
    Ephemeris anomalies



Background

- **Ionospheric fronts**
  - Caused by solar activity
  - Gradients as large as 412 mm/km have been observed (20 m vertical positioning error for GBAS users) [1]
- **Design an instantaneous ground monitor that detects ionospheric gradient errors**
  - Gradient error is relatively small
  - Using carrier phase measurement
- **Extended to detect orbit ephemeris failures.**



[1] Pullen, S., et al, "Impact and mitigation of ionospheric anomalies on ground-based augmentation of GNSS," Radio Science, Vol. 44, RS0A21, doi:10.1029/2008RS004084, 2009.

Single Difference Carrier Phase Measurement

- Start with single difference measurement between satellites for antenna 1

$$\Delta\phi_1 = \tilde{\rho} + \lambda\Delta n_1 + \Delta\upsilon_1$$

- Following the format of antenna 1, the measurement for antenna 2 is

$$\Delta\phi_2 = \tilde{\rho} + \Delta\mathbf{e}^T\mathbf{b}_{12} + \mathbf{b}_{h,12}^T\boldsymbol{\alpha} + \lambda\Delta n_2 + \Delta\upsilon_2$$

$\mathbf{b}$: *3×1 vector between two antennas expressed in local NED frame*

$\mathbf{b}_h$: *2×1 vector composed of horizontal (north and east) components of* $\mathbf{b}$

$\boldsymbol{\alpha}$: *2×1 gradient error vector*

- Need to eliminate the unknown component $\tilde{\rho}$ in the measurements.

Double Difference Measurement

- **To eliminate the unknown component $\tilde{\rho}$ in the single difference measurement**
  - Double difference measurement is utilized

$$\Delta^2\phi_{12} = \Delta\mathbf{e}^T\mathbf{b}_{12} + \tilde{\mathbf{b}}_{h,12}^T\boldsymbol{\alpha} + \lambda\Delta^2 n_{12} + \Delta^2 v_{12}$$

- ***Goal: detect the presence of gradient error, α***
  - Two traditional methods

Traditional algorithm I : detection theory

Traditional algorithm II: estimation theory

Traditional Algorithm I

- ***Define test statistic and compare to threshold [1]***
  - Multiple antennas are needed to detect multi-dimensional gradients.
  - Test statistics are correlated among antennas, which makes it impossible to assign a unique threshold to each baseline test.

$$\bar{P}_{fa} = P\left(s_{D,1} > T_1 \cup \cdots \cup s_{D,m} > T_m | \bar{F}\right)$$

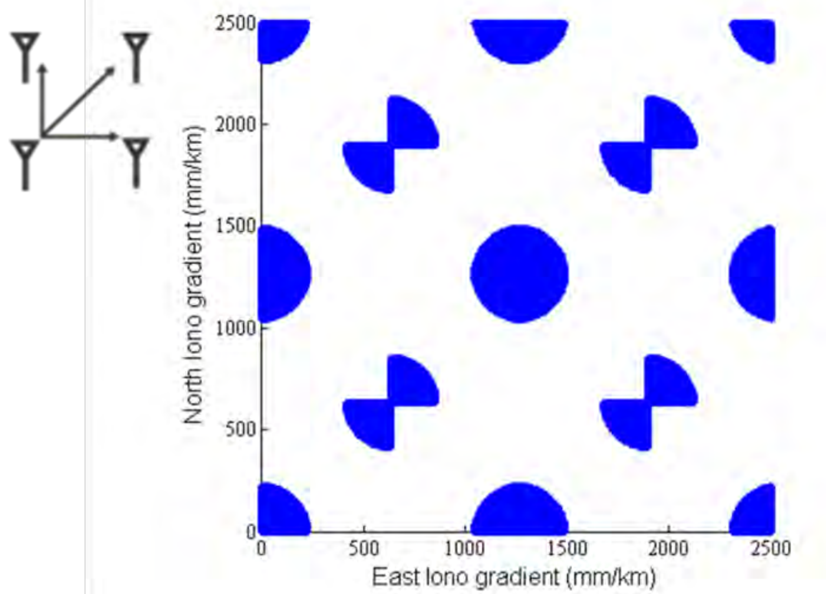
$$\bar{P}_{md} = P\left(s_{D,1} < T_1 \cap \cdots \cap s_{D,m} < T_m | F\right)$$

  - Conservative allocation makes performance dependent on the antenna pair selection.

[1] Jing, J., et al., "Carrier Phase Null Space Monitor for Ionospheric Gradient Detection," IEEE Transactions on Aerospace and Electronic Systems, in press.

Traditional Algorithm II

- ***Directly estimate gradient [1]***
  - Determine the cycle ambiguity through a rounding process. (non-linear)
  - Add unexpected undetectable region.
  - The performance still depends on the antenna pair selection.



[1] Jing, J., et al., "Carrier Phase Null Space Monitor for Ionospheric Gradient Detection," IEEE Transactions on Aerospace and Electronic Systems, in press.

Null Space Monitor Algorithm (I)

- **Stacking measurements from all antennas (m+1 antennas)**

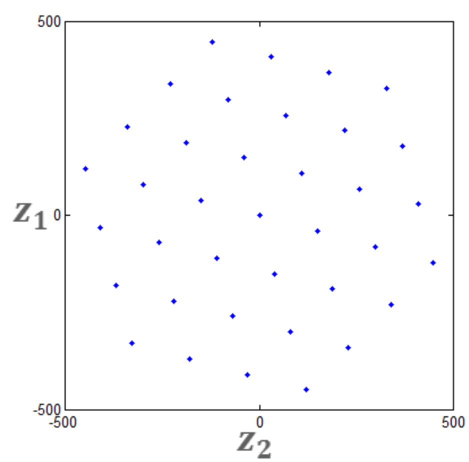
$$\bar{\mathbf{z}} = \begin{bmatrix} \Delta\phi_1 \\ \Delta\phi_2 - \Delta\mathbf{e}^T\mathbf{b}_{12} \\ \vdots \\ \Delta\phi_{m+1} - \Delta\mathbf{e}^T\mathbf{b}_{1(m+1)} \end{bmatrix} = \begin{bmatrix} \tilde{\rho} \\ \tilde{\rho} \\ \vdots \\ \tilde{\rho} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{b}_{h,12}^T \\ \vdots \\ \mathbf{b}_{h,1(m+1)}^T \end{bmatrix} \boldsymbol{\alpha} + \lambda \begin{bmatrix} \Delta n_1 \\ \Delta n_2 \\ \vdots \\ \Delta n_{m+1} \end{bmatrix} + \begin{bmatrix} \Delta\upsilon_1 \\ \Delta\upsilon_2 \\ \vdots \\ \Delta\upsilon_{m+1} \end{bmatrix}$$

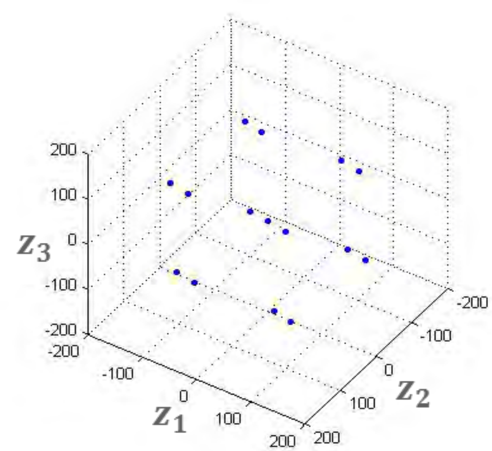
$$= \tilde{\rho}\mathbf{1} + \mathbf{B}\boldsymbol{\alpha} + \lambda\mathbf{n} + \bar{\boldsymbol{\upsilon}}$$

- **Using orthonormal null space matrix L of vector 1, $\mathbf{L1} = \mathbf{0}$**

$$\mathbf{z} = \mathbf{L}\bar{\mathbf{z}} = \mathbf{LB}\boldsymbol{\alpha} + \lambda\mathbf{Ln} + \boldsymbol{\upsilon}$$

$\mathbf{L}$ is the lattice generator.





Null Space Monitor Algorithm (II)

- **Measurements**

$$\mathbf{z} = \mathbf{L}\bar{\mathbf{z}} = \mathbf{LB}\boldsymbol{\alpha} + \lambda\mathbf{Ln} + \boldsymbol{v}$$

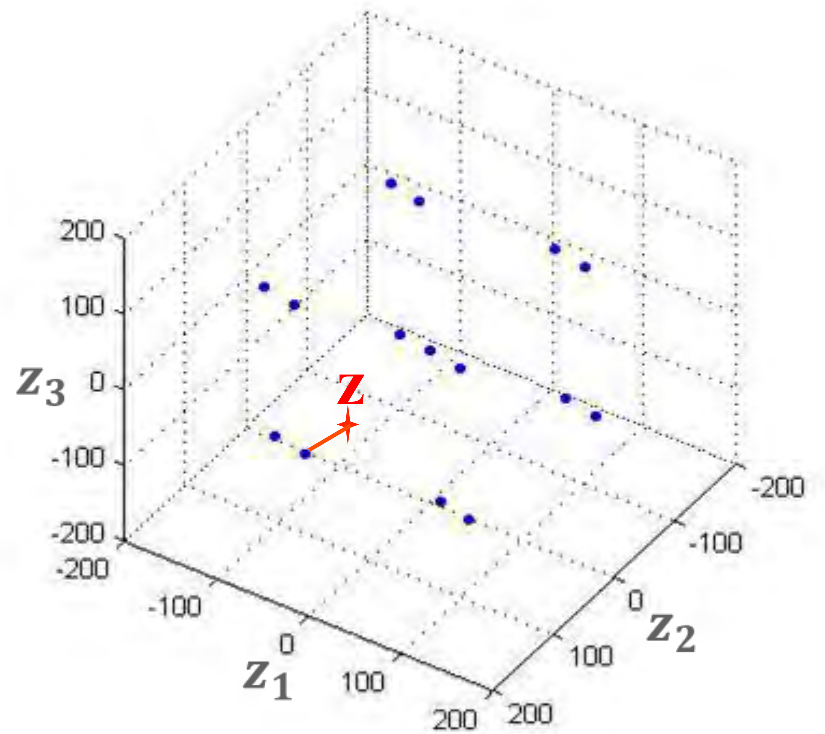
$\mathbf{Ln}$ is no longer an integer vector

- **Test statistic**

$$s = \|\mathbf{z} - \lambda\mathbf{L}\hat{\mathbf{n}}\|^2$$

$$\hat{\mathbf{n}} = \arg\min\|\mathbf{z} - \lambda\mathbf{Ln}\|^2$$

- **Detection threshold**
  - Under fault-free conditions: $s/\sigma^2_{\Delta\phi} \sim \chi^2(m) \rightarrow$ Threshold T

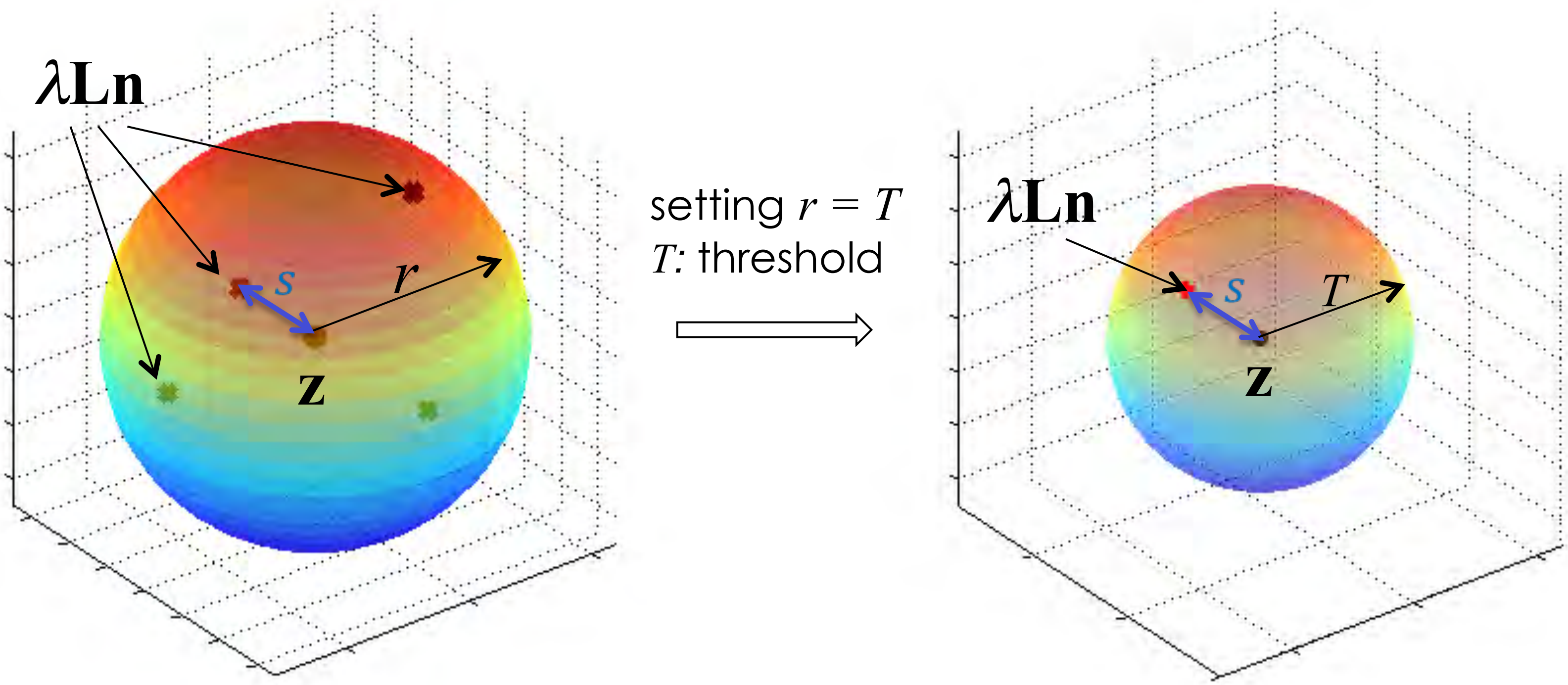


- **Searching for Closest Lattice Point (CLP problem)**

Null Space Monitor Algorithm (III)

- **To solve CLP problem, a sphere decoder [2] is applied**
  - finding all lattice points within a given sphere centered at $\mathbf{z}$

$$\|\mathbf{z} - \lambda \mathbf{L}\hat{\mathbf{n}}\|^2 \leq r^2$$



[2] Mohamed Oussama Damen. "On Maximum-Likelihood Detection and the Search for the Closest Lattice Point." IEEE Transactions on Information Theory, VOL. 49, NO. 10, Oct. 2003.

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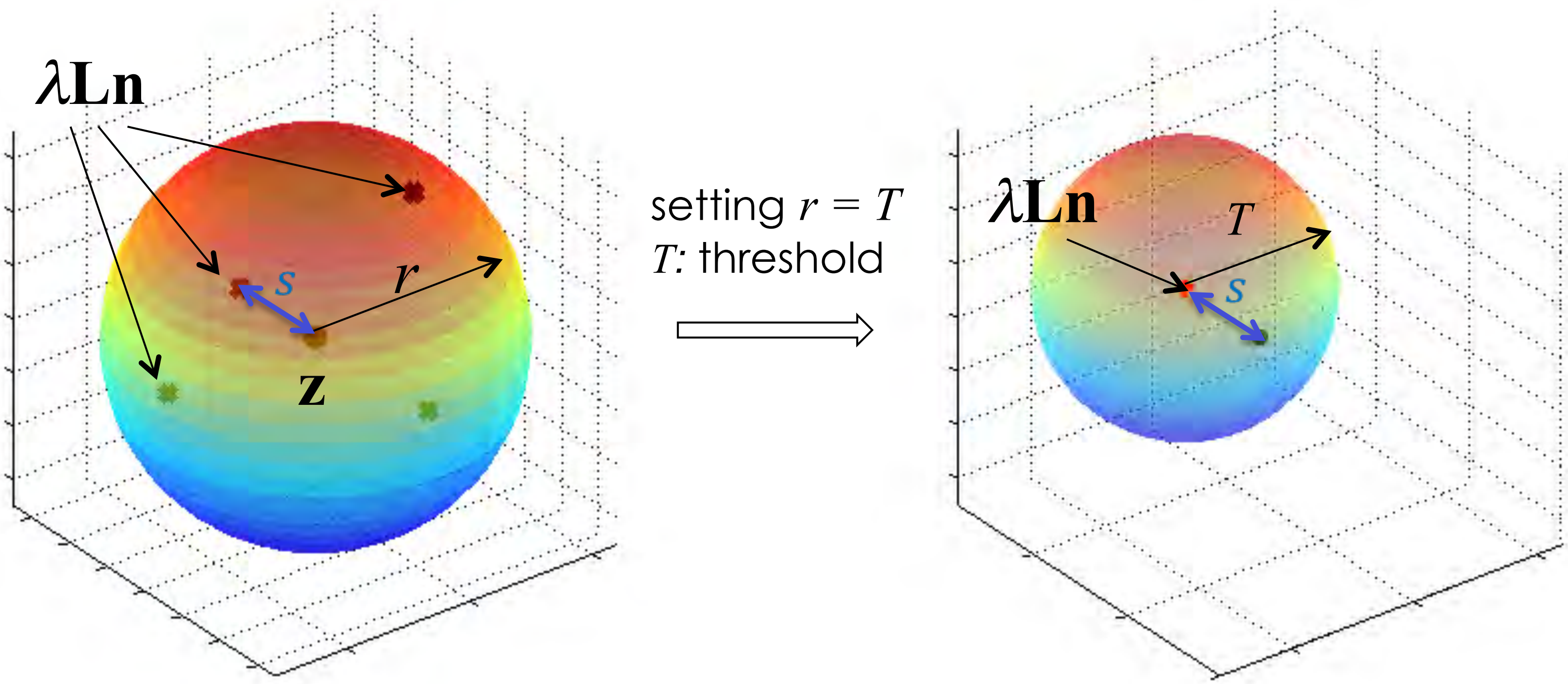

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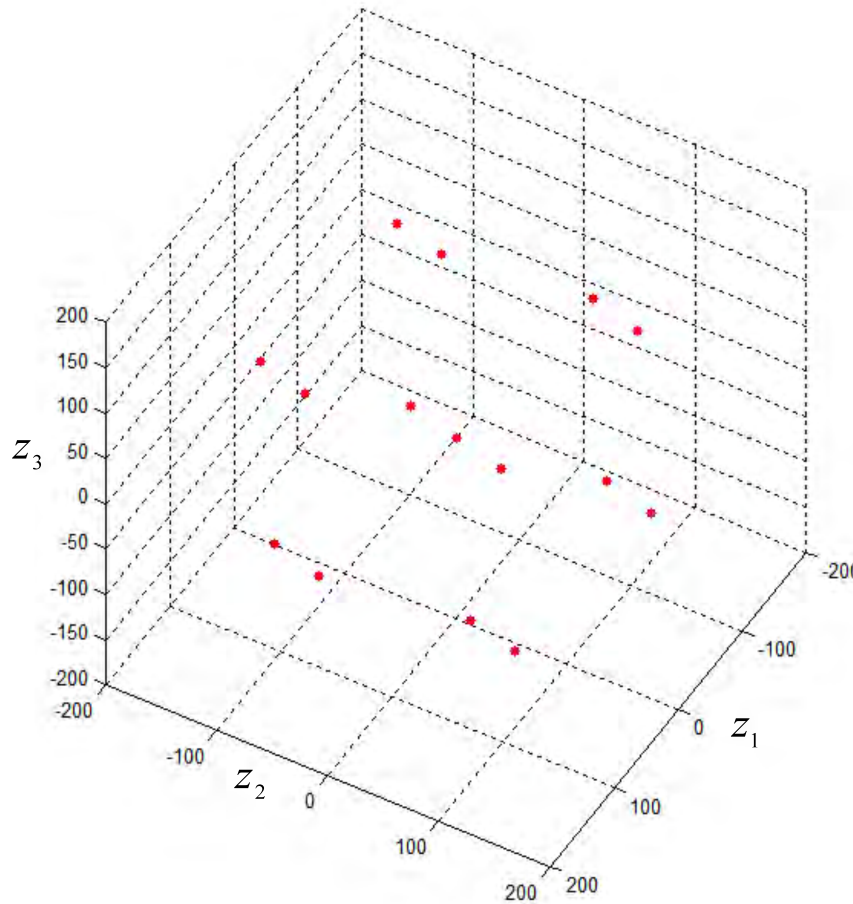
Performance Analysis

- **Measurements**

$$\mathbf{z} = \mathbf{LB\alpha} + \lambda\mathbf{Ln} + \boldsymbol{\upsilon}$$

- **If $\mathbf{z}$ is close to any $\lambda\mathbf{Ln}$, this monitor cannot detect it**
  - $\lambda\mathbf{Ln}$ *will create a point lattice*

Four antenna example



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- **With the inclusion of nominal meas. noise**
  - *spheres centered at each lattice point whose radius is computed to satisfy probability of false alarm and missed detection*

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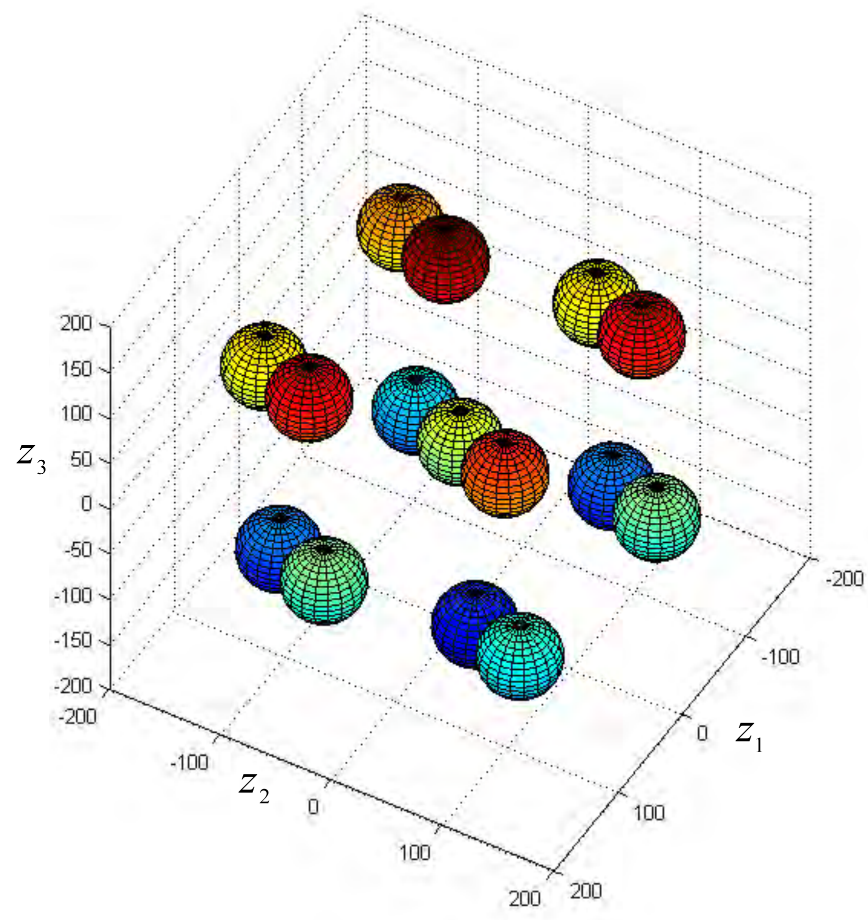

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- **In the presence of a gradient fault**
  - *The term* $\mathbf{LB\alpha}$ *defines a 2-D plane, because* $\boldsymbol{\alpha}$ *is a* 2×1 *gradient vector*

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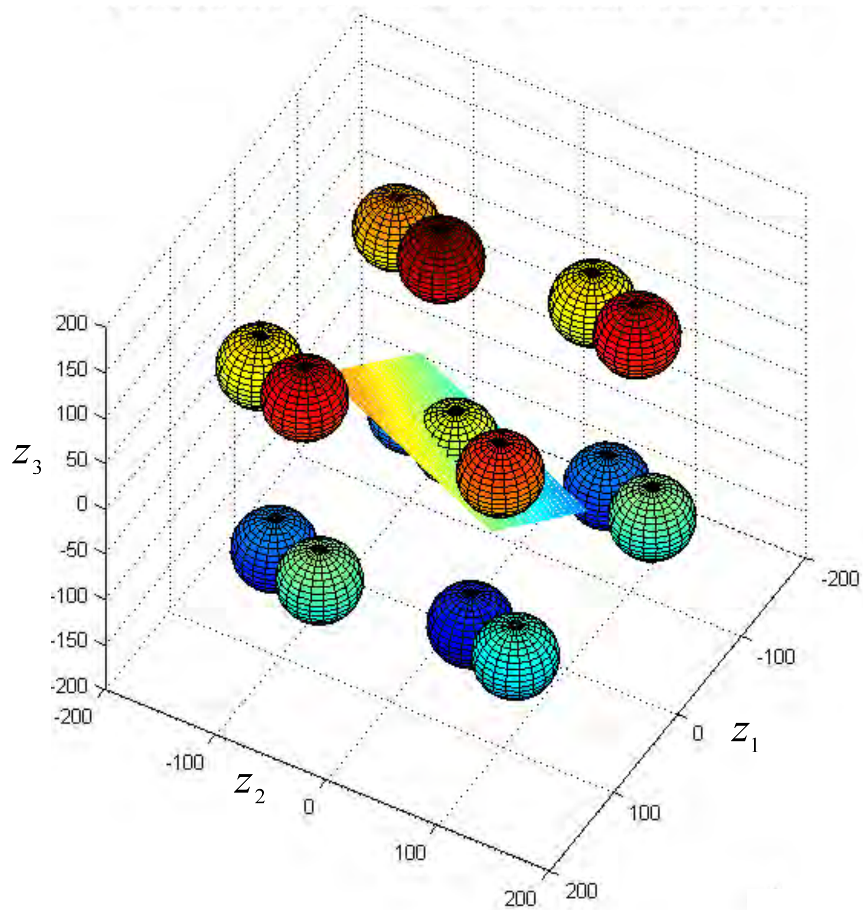

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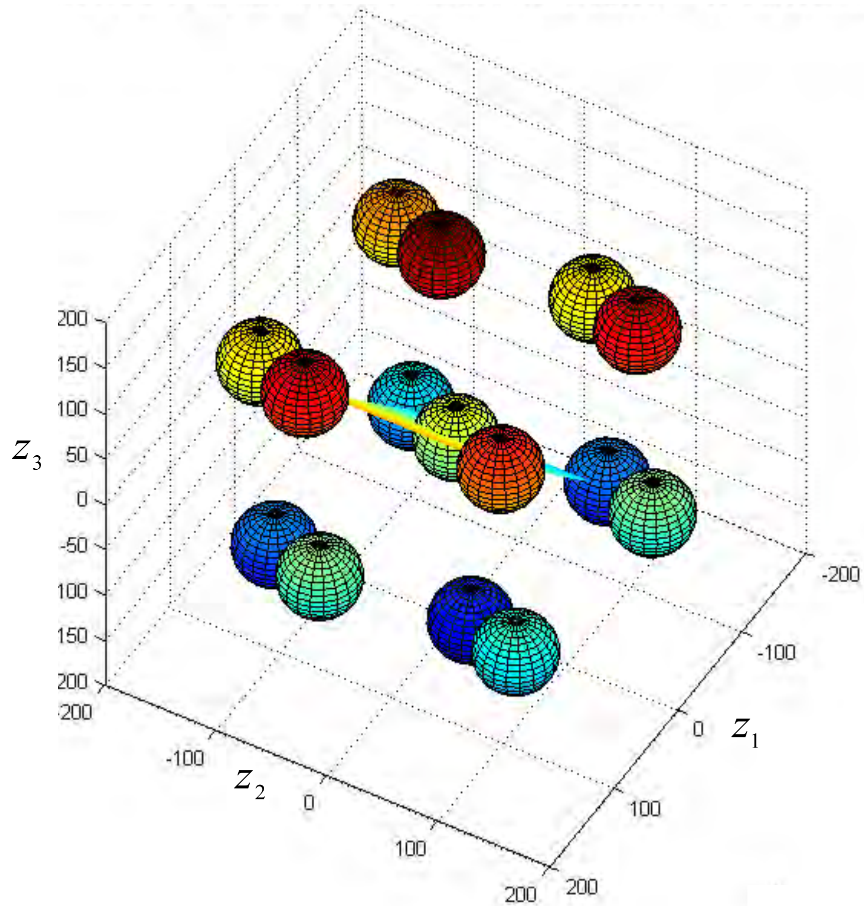
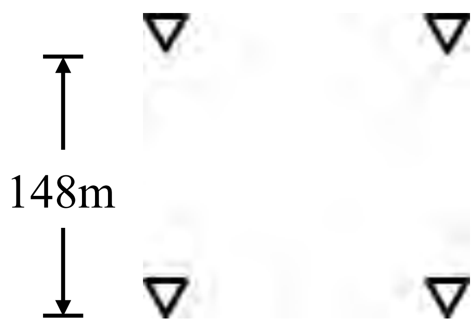
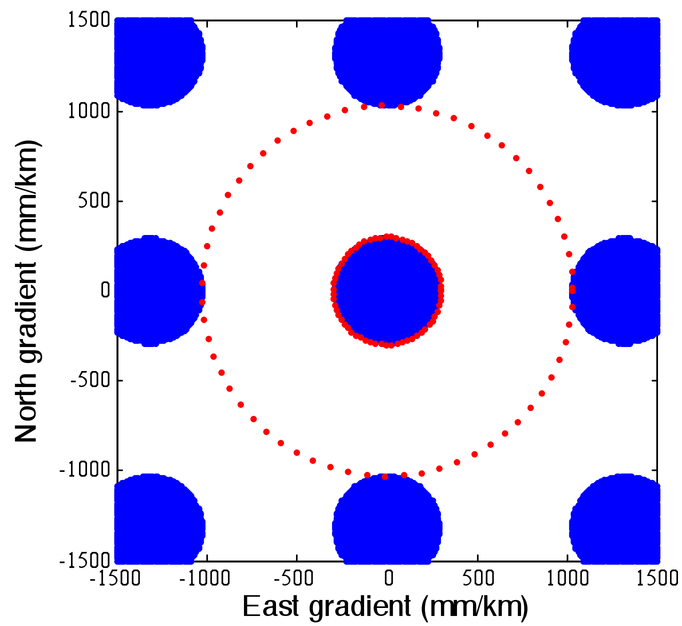


Null Space Monitor Performance (2-D)

- **Monitor performance**



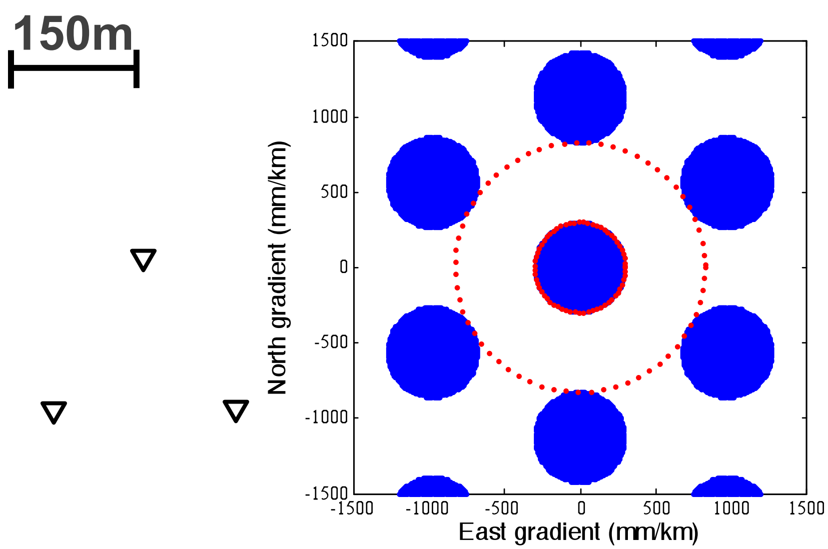


: undetectable gradient

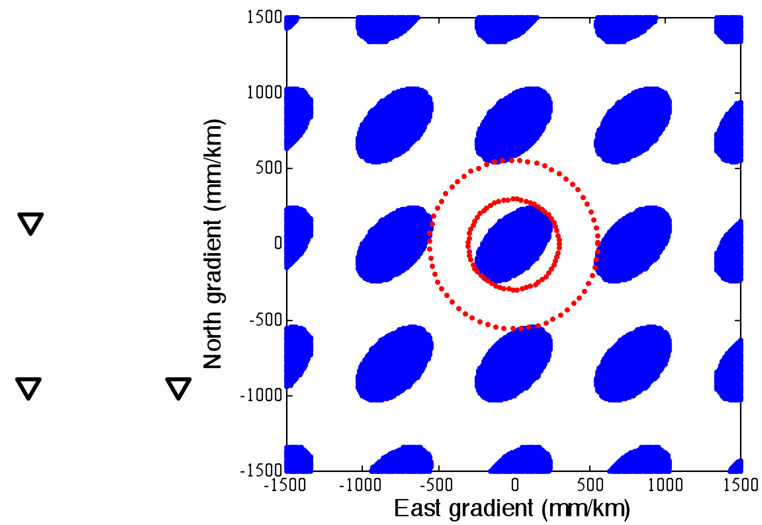
Detectable range:300 and 1030 mm/km

Antenna Topology Selection (I)

- **Null space monitor is designed**
  - Detection performance is highly dependent on the antenna topology



Detectable range:300 - 828 mm/km



Detectable range:300 and 559 mm/km

- **Find the antenna topology that maximizes the detectable gradient**

Antenna Topology Selection (II)

- **To maximize the detectable gradient in any direction, the cost function is developed [3]**

$$\max_{\mathbf{B}} \min_{\mathbf{n}} \{ \| [(\mathbf{LB})^T \mathbf{LB}]^{-1} (\mathbf{LB})^T \lambda \mathbf{Ln} \|_2 - \alpha_i \}$$

$$\text{s.t.} \quad (\lambda \mathbf{Ln})^T \mathbf{p} < \lambda_{\text{MDE}}$$

$$\sqrt{\text{eig}([(\mathbf{LB})^T \mathbf{LB}]^{-1})}\, \lambda_{\text{MDE}} = \alpha_{req}$$

$$[(\mathbf{LB})^T \mathbf{LB}]^{-1} \text{ has repeated eigenvalues}$$

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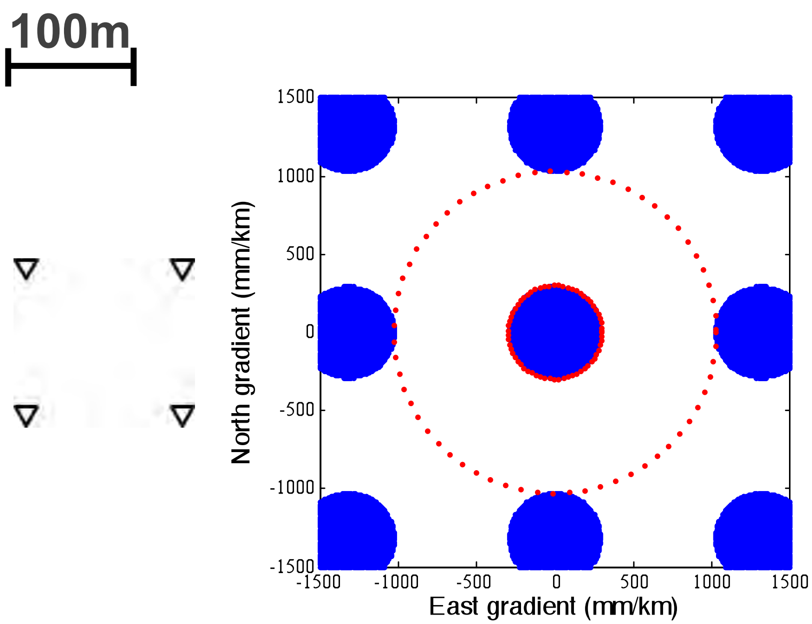
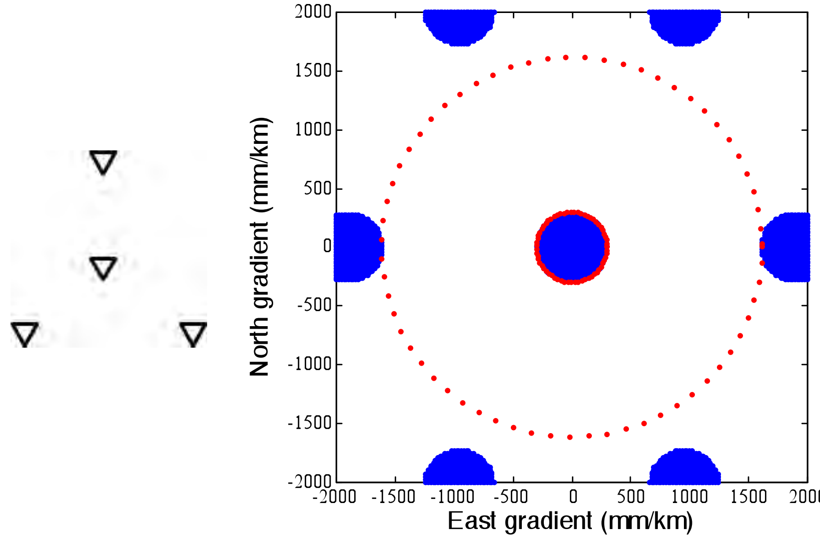
[3]Jing, J., et al., "Multi-dimensional Ionospheric Gradient Detection for GBAS," Proceedings of the 2013 International Technical Meeting of The Institute of Navigation, San Diego, California, January 2013, pp. 121-128.

Antenna Topology Selection (III)

- **Monitor performance with four antennas**





Blue region: undetectable gradient

Detectable range:300 - 1030mm/km

Detectable range:300 and 1613 mm/km

Code and Dual Frequency Carrier Phase monitors (I)

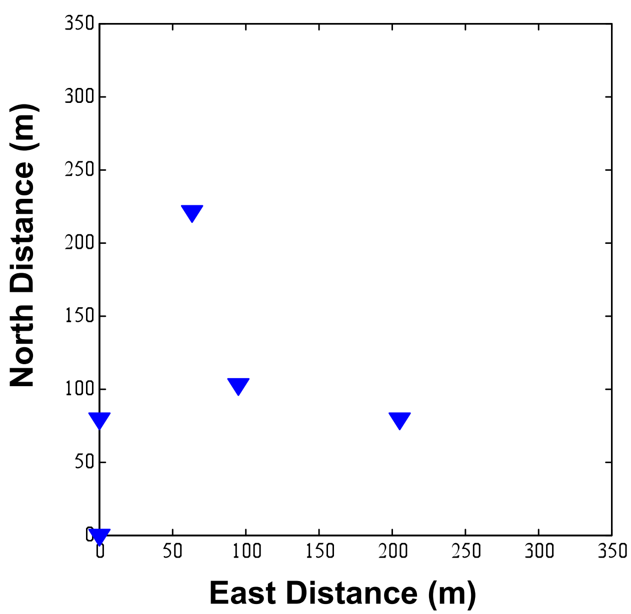
- **The null space monitor is designed to detect ionospheric fronts, but can potentially also be used to detect orbit ephemeris failures.**
  - No upper limit, can be infinitely large
- **Code phase monitor [4]**
  - Without cycle resolution, the carrier phase monitor cannot detect all gradients
  - 7 antennas are needed to detect all gradient errors
- **Dual Frequency carrier phase monitor [5]**
  - Reduce the antennas needed

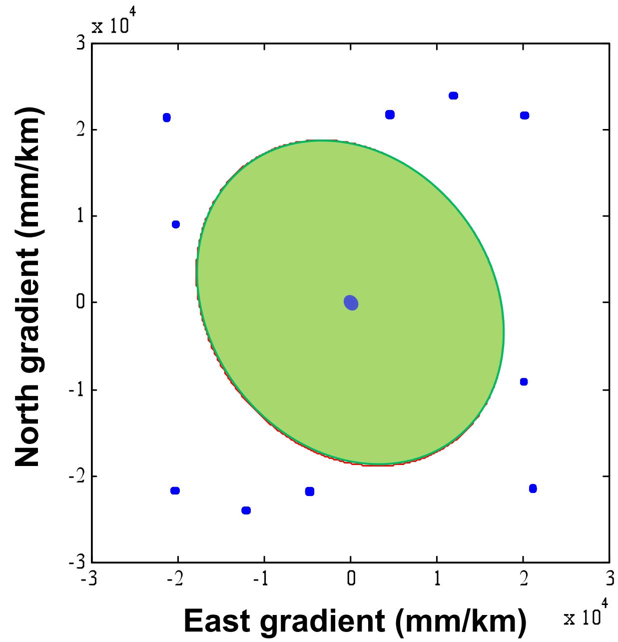
[4]Jing, J., et al., "Null Space Ephemeris Monitor for GBAS," Proceedings of the ION 2013 Pacific PNT Meeting, Honolulu, Hawaii, April 2013, pp. 978-985.

[5]Jing, J., et al., "Dual Frequency Ephemeris Ground Monitor for GBAS," Proceedings of the 2014 International Technical Meeting of The Institute of Navigation, San Diego, California, January 2014, pp. 265-271.

Code and Dual Frequency Carrier Phase monitors (II)

- **Code + Dual frequency carrier measurements**
  - **Example:** Five antenna topology





: Undetectable region of code phase monitor

: Undetectable region of carrier phase monitor

Conclusion

- **Ionospheric gradient monitor**
  - A null space monitor is developed to instantaneously detect ionospheric gradient using carrier phase measurement
  - The performance is consistent with the antenna topology and provides superior detection performance
  - Optimized antenna topology is obtained that maximizes the detectable gradient while satisfying requirements.
- **Extend to orbit ephemeris failure detection**
  - Code phase monitor is developed to detect larger gradients
  - Dual frequency carrier phase monitor is designed to reduce the number of needed antennas
  - Using 5 antennas can detect all ephemeris failures with code and dual frequency null space monitor.

- Thank you!