Correlated-Data Fusion for Networked Navigation: Theory and Application

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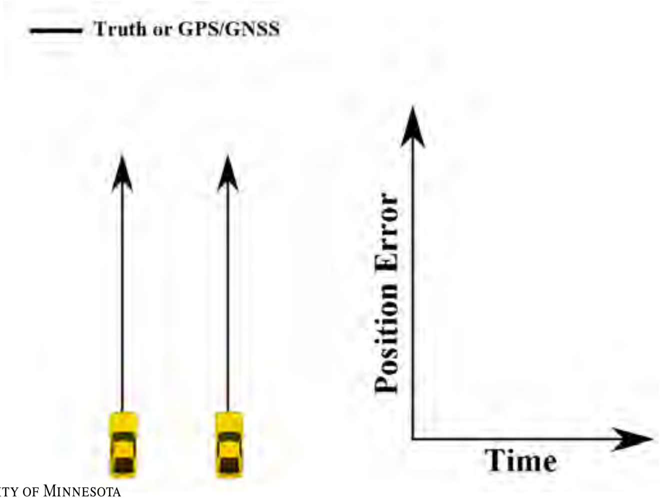
October 28, 2014

Cooperative Navigation

Definition: *Two or more vehicles collaborate to calculate a navigation solution with higher quality than would be attainable otherwise.*



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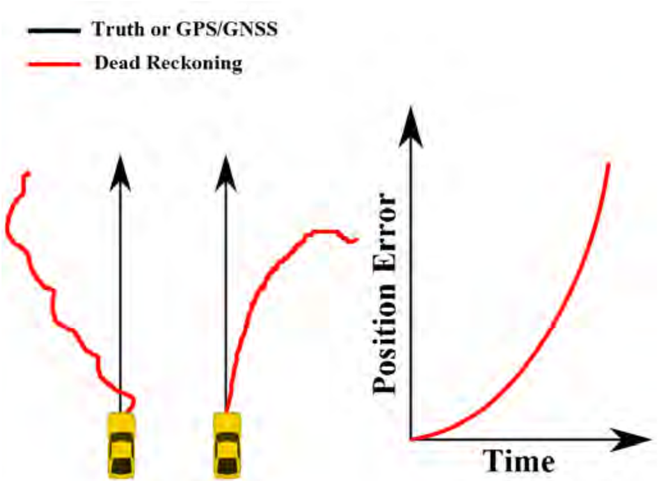

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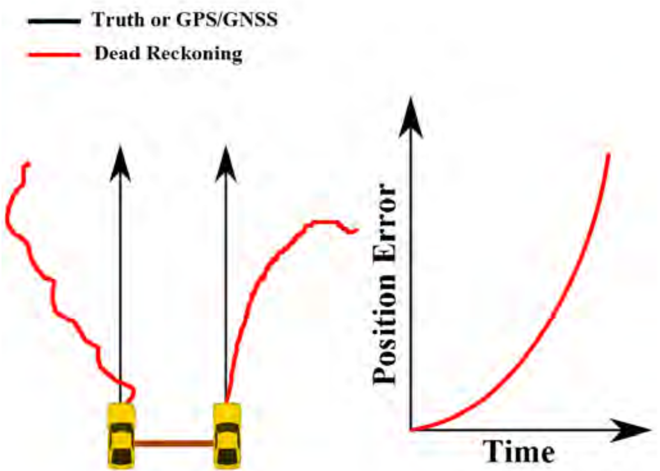

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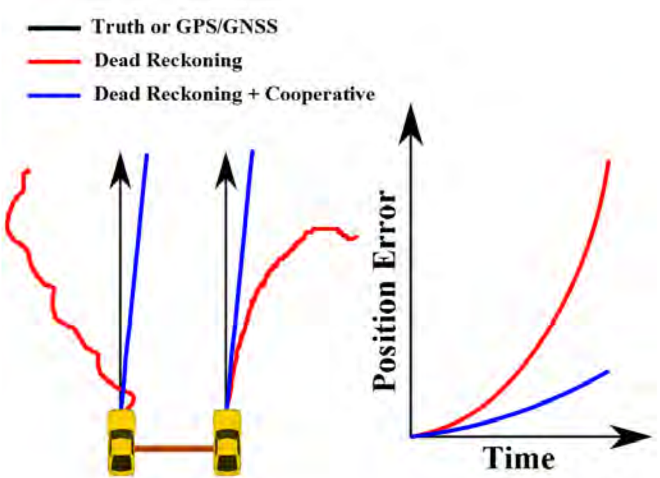

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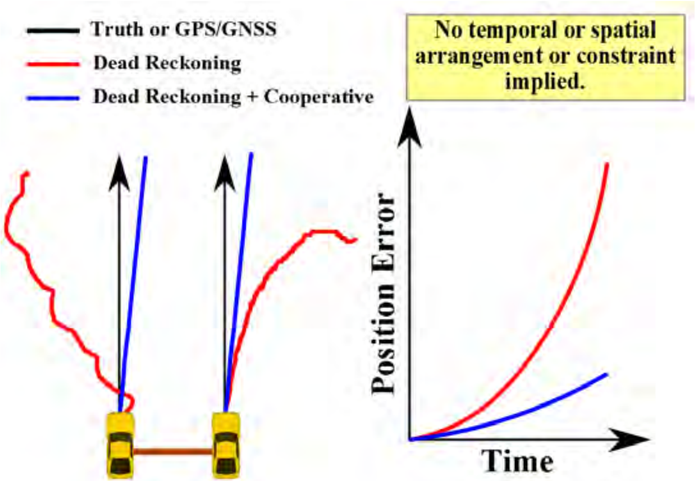
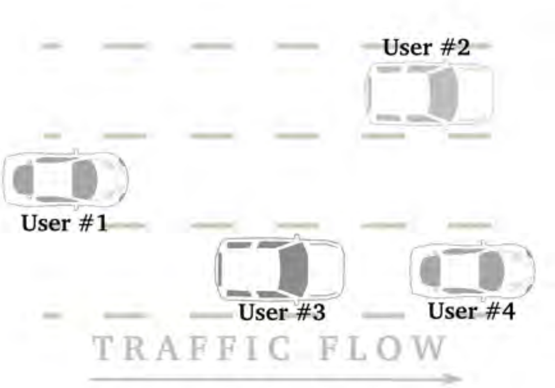


COOPERATIVE NAVIGATION

Two implementations: **Centralized** and **Decentralized**



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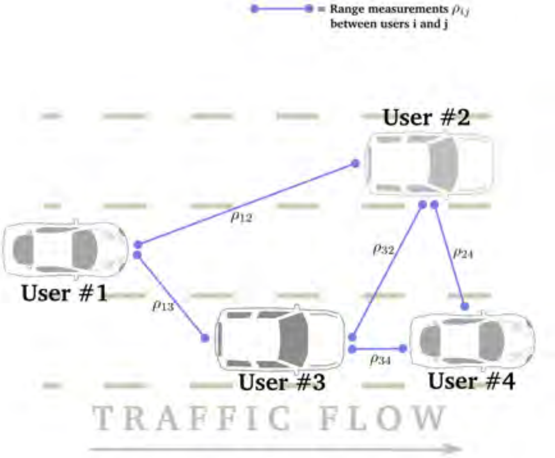

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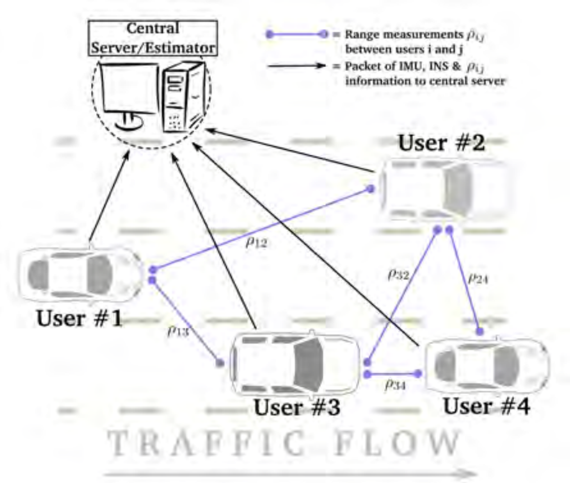

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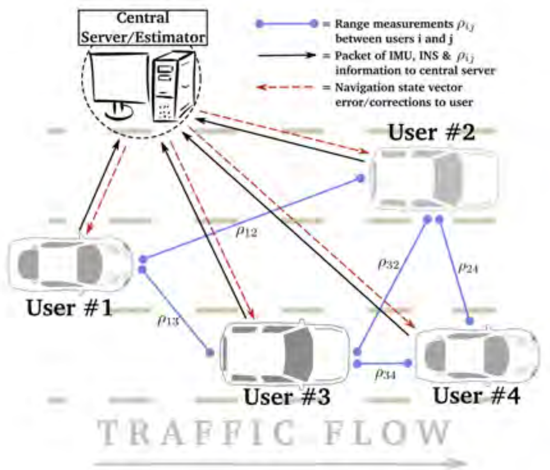

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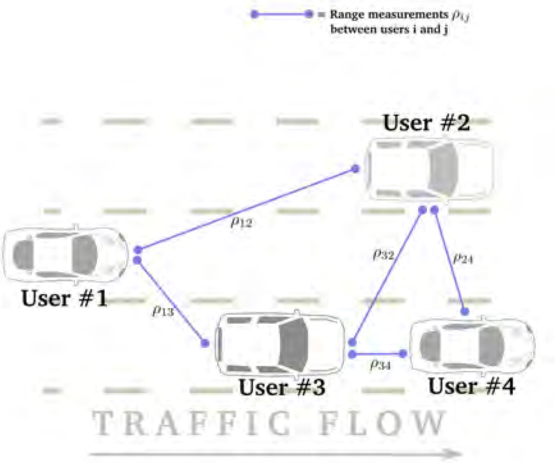

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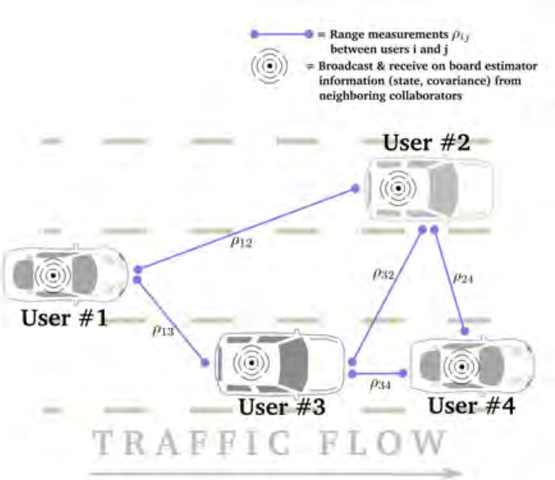
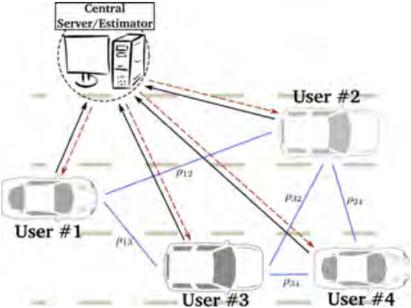


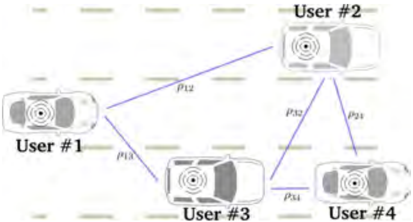
Cooperative Navigation

Challenges

Centralized



Decentralized



\+ Correlations are maintained

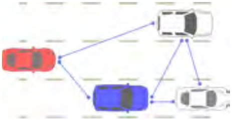
\- Communication and infrastructure burden

\+ Flexible implementation

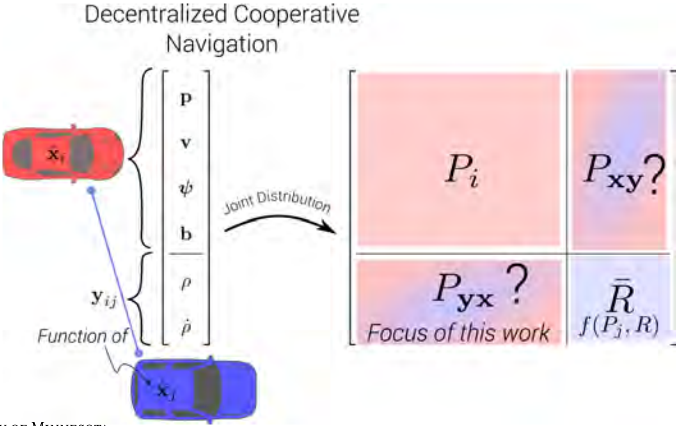
\- No knowledge of correlations

Estimator must handle unknown correlations

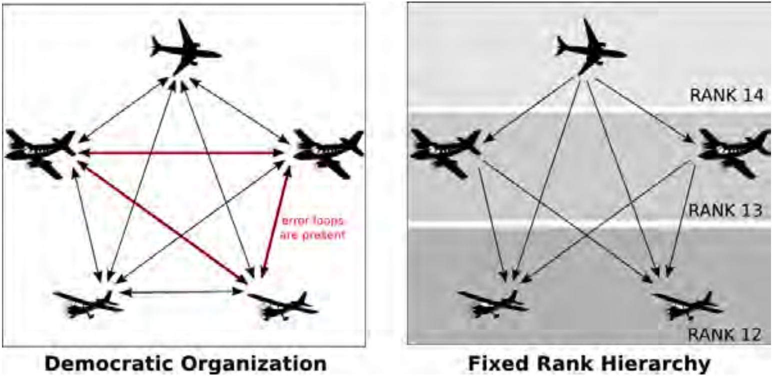
WHAT IS CORRELATION?



$\hat{\mathbf{x}}_i$ = State.
$\mathbf{y}_{ij}$ = Measurement.
$\mathbf{P}_{xy}$ = Correlation.



ERROR LOOPS



Imposing communication restrictions can prevent or delay the formation of error loops.

- Reduced flexibility.

PROBLEM STATEMENT

1. What is a suitable correlated data fusion filter for **decentralized** cooperative aiding?

2. Case study: GPS/GNSS-denied navigation for small UAVs.

    2.1 What is the concept of operation when dealing with a GPS/GNSS outage?

    2.2 What is the impact of limited communication bandwidth: decentralized performance?

    2.3 Does it make sense to consider centralized implementations? If so, what is the expected performance?

Prior Work

Kalman Filter Order Reduction		
Handling lost correlation	Shah	1971
Cooperative Navigation		
JTIDs	Fried et al.	1979
Source selection	Rome et al.	1977
Stability issues	Gobbini	1981
Simultaneous Localization And Mapping		
Importance of correlations	Castellanos et al.	1997
Distributed centralized implementation	Roumeliotis et al.	2002
Correlated Data Fusion Filters		
Covariance Intersection	Julier and Uhlmann	1997
Bounded Covariance Inflation	Hanebeck et al.	2001
	Reece and Roberts	2005
Covariance Intersection and SLAM	Julier and Uhlmann	2006
Multi-robot Localization	Carrillo-Arce et al.	2013

PRIOR WORK

COMMUNICATION	**Kalman Filter Order Reduction**		
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APPLICATION	Covariance Intersection and SLAM	Julier and Uhlmann	2006
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Correlated Data Fusion

CORRELATED DATA FUSION

1. $\hat{\mathbf{x}}^-$ *a priori* state estimate (covariance $\mathbf{P}^-$)
2. $\mathbf{y}$ measurement (covariance $\mathbf{R}$)
3. $\hat{\mathbf{x}}^+$ *a posteriori* state estimate (covariance $\mathbf{P}^+$)

where the correlation between them, P_{xy}, is unknown.

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Strategy: Seek an uncorrelated matrix which bounds the true joint covariance:

$$\begin{bmatrix} \bar{P}^- & \mathbf{0} \\ \mathbf{0} & \bar{R} \end{bmatrix} \geq \begin{bmatrix} P^- & P_{xy} \\ P_{yx} & R \end{bmatrix}$$

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Result: (1) an unbiased estimate $\hat{\mathbf{x}}^+$ and (2) a covariance $\widehat{\mathbf{P}}^+$ which is an *overbound* of the true covariance $\mathbf{P}^+$

- *i.e covariance computed if $\boldsymbol{P_{xy}}$ was known. $\Rightarrow$ Conservative.*

DECENTRALIZED ESTIMATORS

Similar form as the Kalman Filter except:

- (1) Scalar weight, ω and (2) Correlation bound, r_{max}.

$$K = \bar{P}^- H^T (H\bar{P}^- H^T + \bar{R})^{-1} \qquad \text{(Gain calculation)}$$
$$\mathbf{x}^+ = \mathbf{x}^- + K(\mathbf{y} - H\mathbf{x}^-) \qquad \text{(State update)}$$
$$P^+ = (I - KH)\bar{P}^-(I - KH)^T + K\bar{R}K^T \qquad \text{(Covariance update)}$$

where $\bar{P}^-$, $\bar{R}$ are assumed covariance statistics.

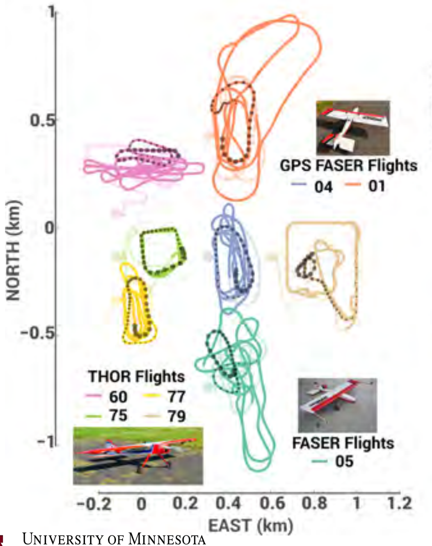
Estimator	Assumption		$\bar{P}^-$	$\bar{R}$
KF/EKF	No correlation	$P_{xy} = \mathbf{0}$	P^-	R
CI	Any allowable correlation	any valid P_{xy}	$\frac{1}{\omega}P^-$	$\frac{1}{1-\omega}R$
BCInf	Bounded correlation	$r_{max}^2 I \geq C_{yx}C_{yx}^T$	$\frac{\omega+(1-\omega)r_{max}}{\omega}P^-$	$\frac{1+\omega(r_{max}-1)}{1-\omega}R$

$\omega \in [0, 1]$: scalar optimization parameter selected to minimize P^+
r_{max} : maximum singular value of the matrix of correlation coefficients (C_{yx})

UAV Case Study Results

Flight Scenario

Data @ http://conservancy.umn.edu/handle/11299/165567



7 UAVs cover 1 km^2 area
Three airframes, flights from 2011 – 2012

Source: www.uav.aem.umn.edu

ON-BOARD NAVIGATION

- Reference: logged 50 Hz *INS/GPS*
- Assumed though simulation *play-back*
  - Equipped with cross-ranging radio modems (1 Hz, $1-\sigma$ ranging accuracy of 5 m)
  - Broadcasts estimated location and covariance at 1 Hz (like ADS-B)

On-Board Sensors
3 axis accelerometer
3 axis gyroscope
3 axis magnetometer
airspeed (*pitot-probe*)
baro-altimeter
GPS *position* and *velocity*

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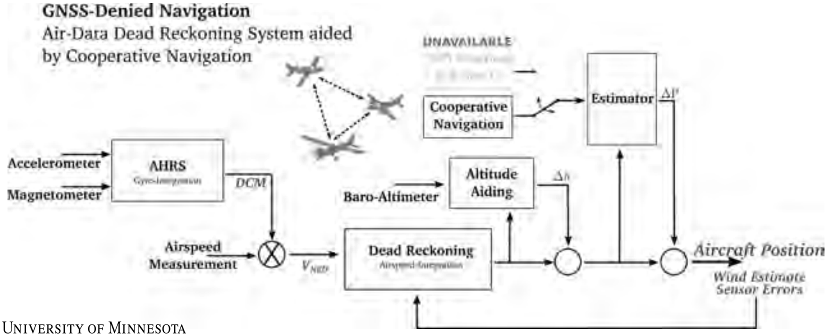
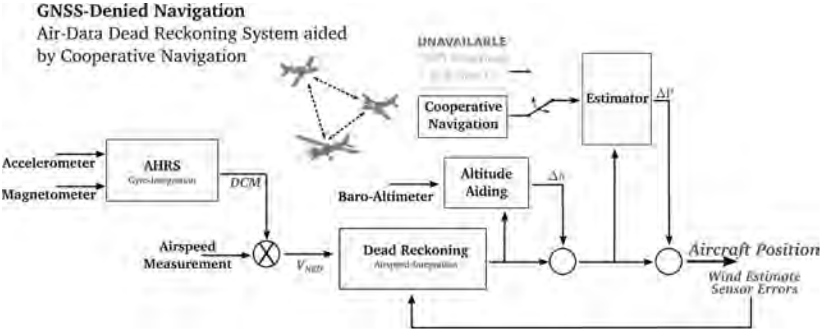


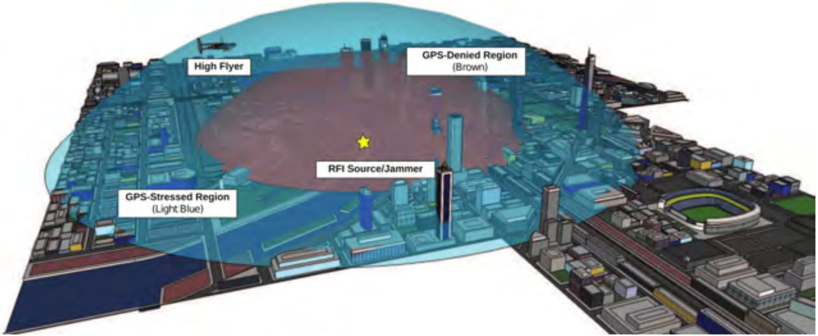
ON-BOARD NAVIGATION

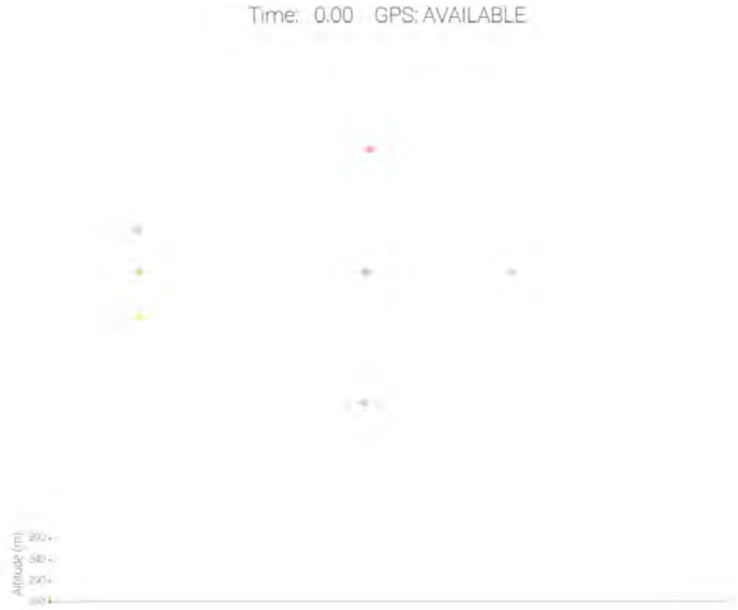
State	Description
$\Delta N, \Delta E, \Delta D$	North-East-Down position error
W_{NS}, W_{EW}	North-South and East-West wind
$\Delta\psi, \Delta\theta$	Yaw-error and pitch-error
$\Delta u, \Delta v, \Delta w$	Body-axes velocity measurement errors
Δh	offset between *geometric-pressure* altitude (e.g GPS vs baro altitude)



Operational Concept

Scenario	GPS Denied	GPS Available
1. **No High Flyer**	7 UAVs	0
2. **One High Flyer:**	6 UAVs	1 UAV
3. **Two High Flyers:**	5 UAVs	2 UAVs




Time: 0.00 GPS: AVAILABLE
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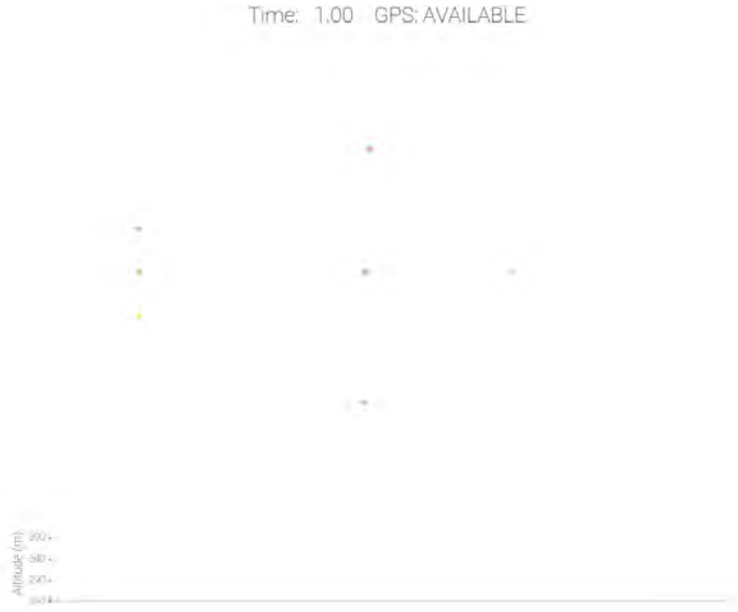
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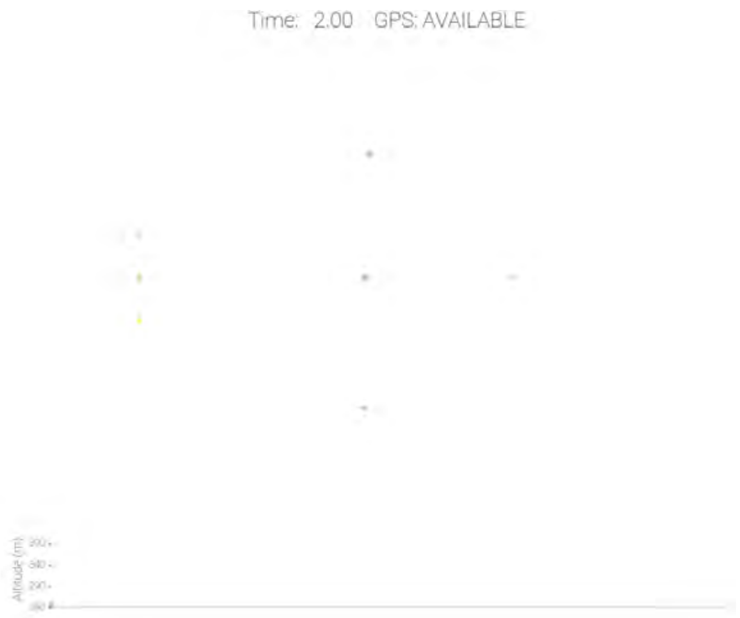
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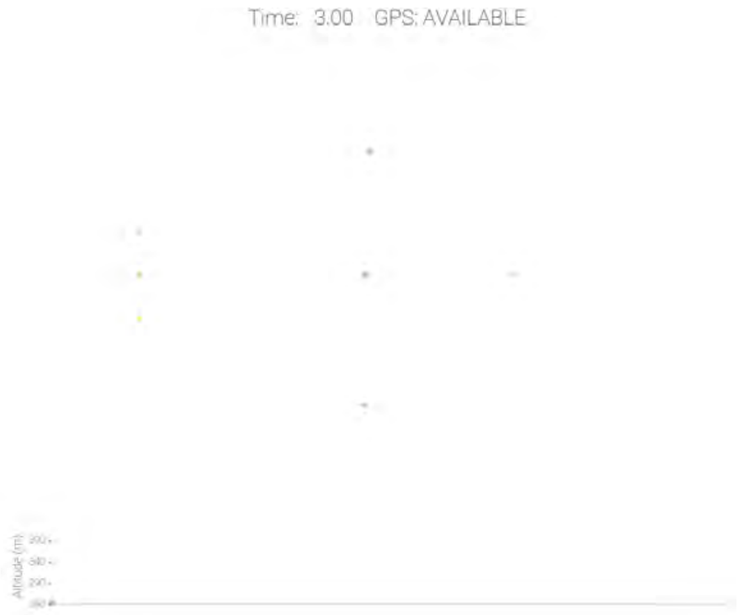
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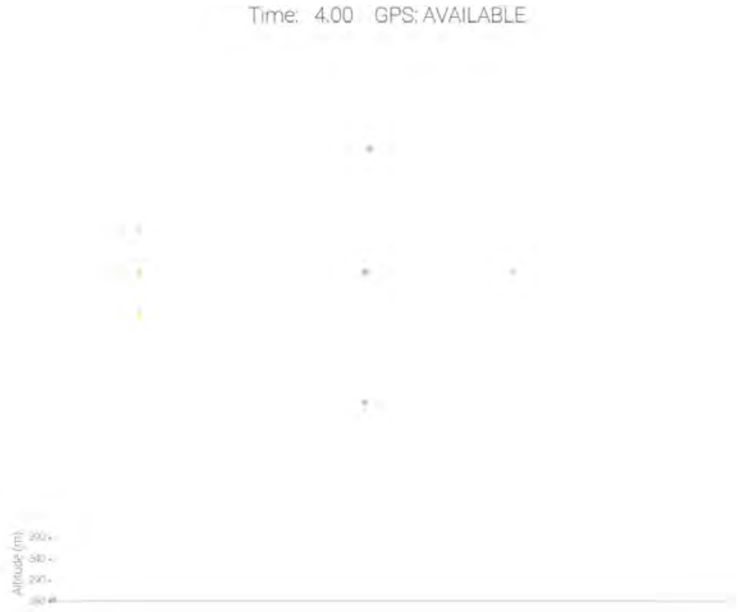
Altitude (m)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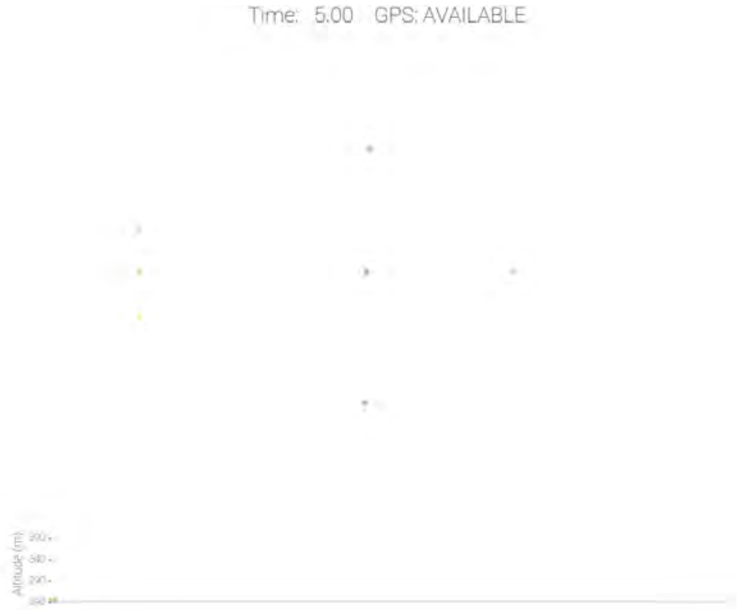
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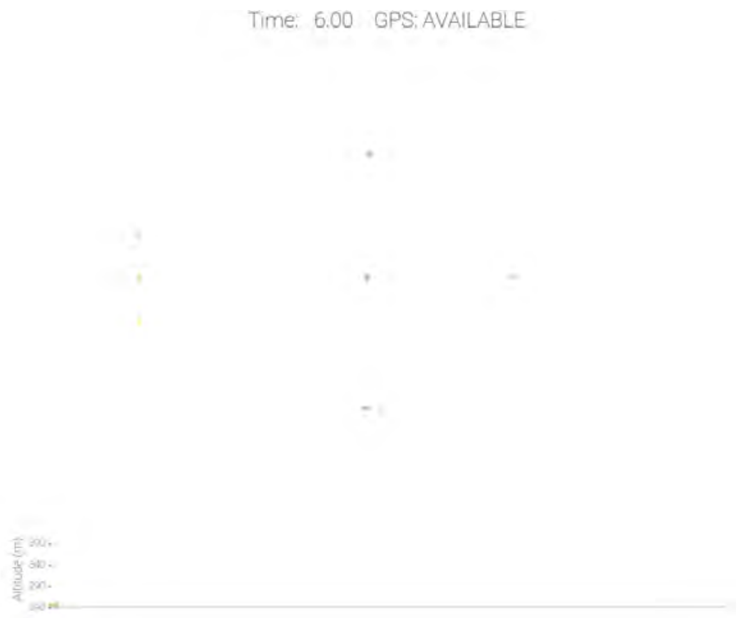
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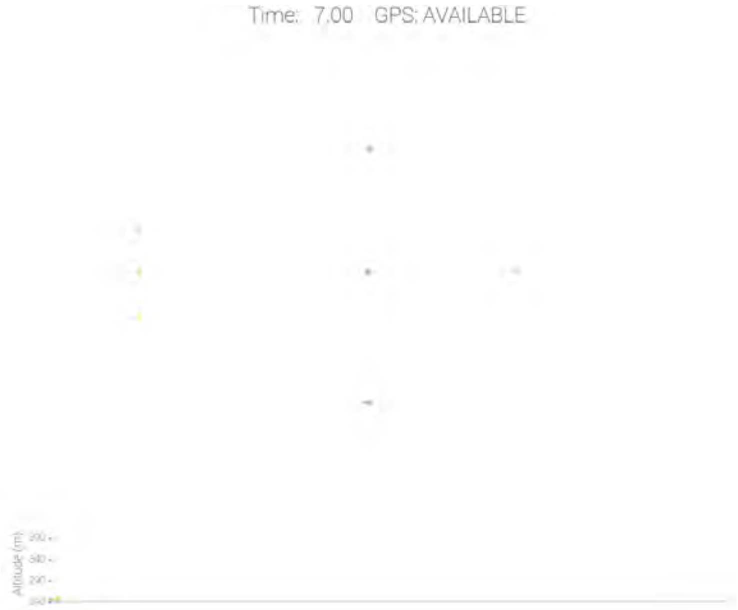
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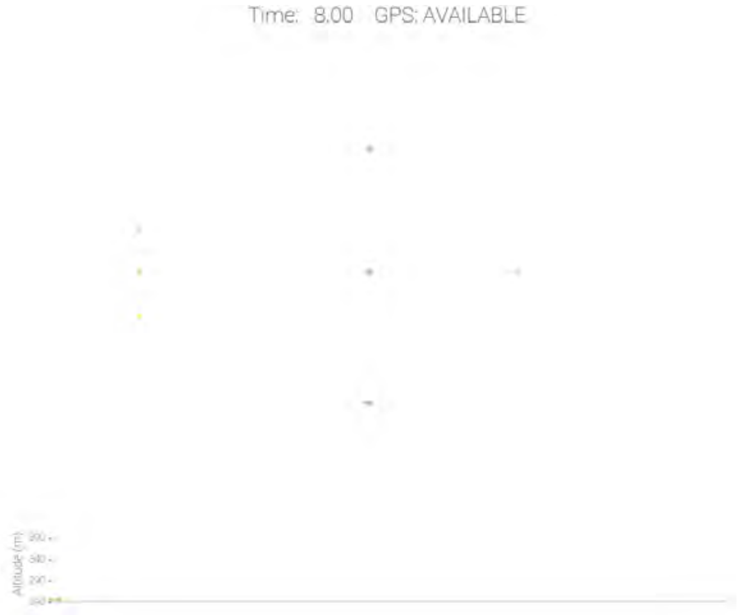
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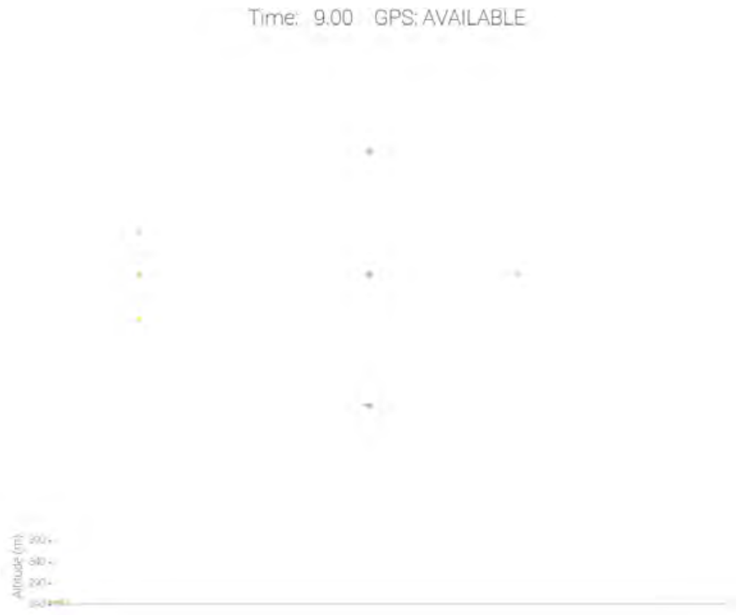
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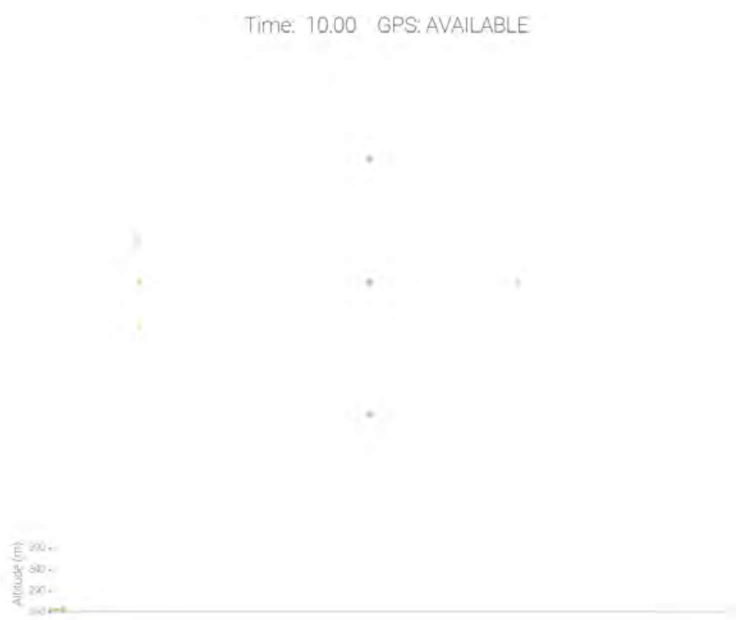
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Time: 11.00 GPS: AVAILABLE
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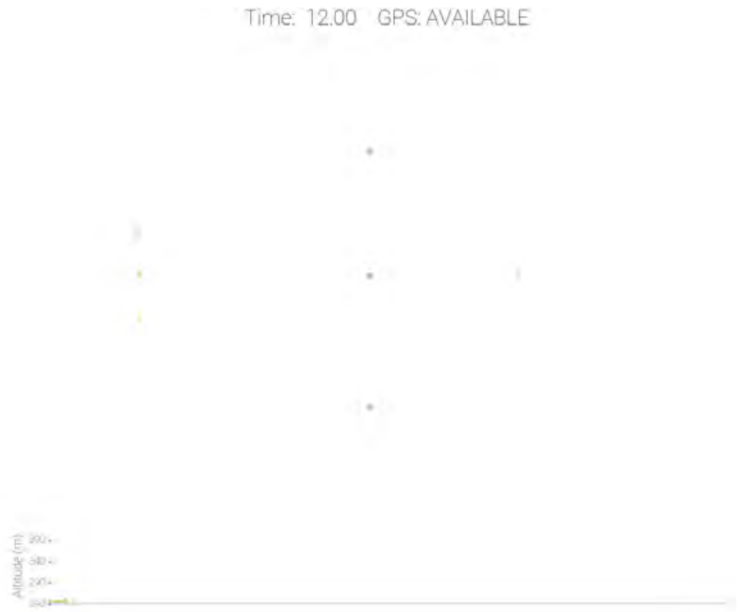
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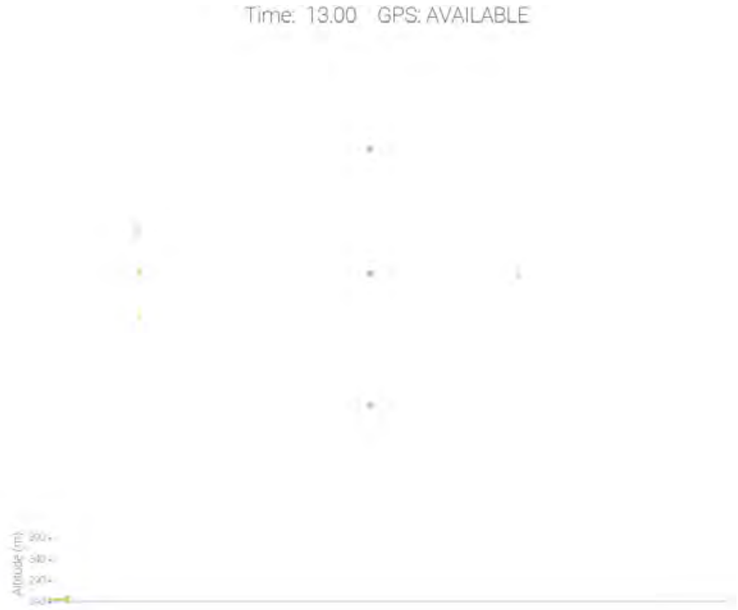
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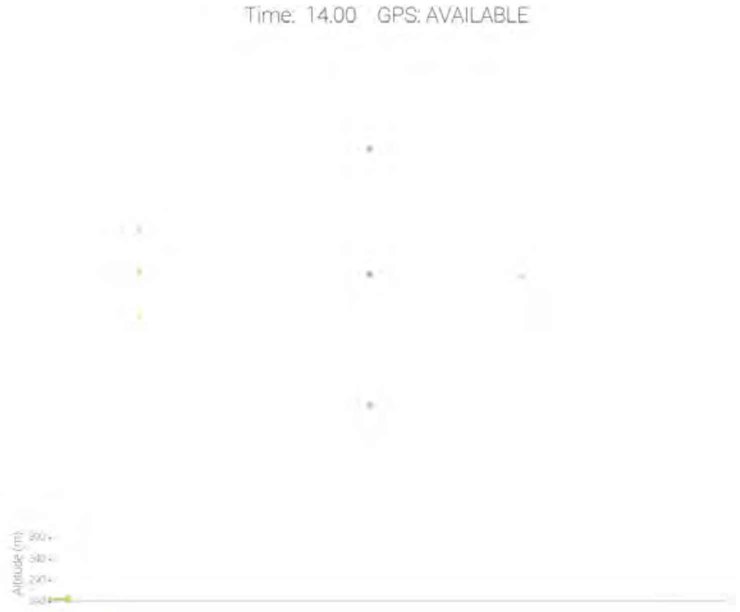
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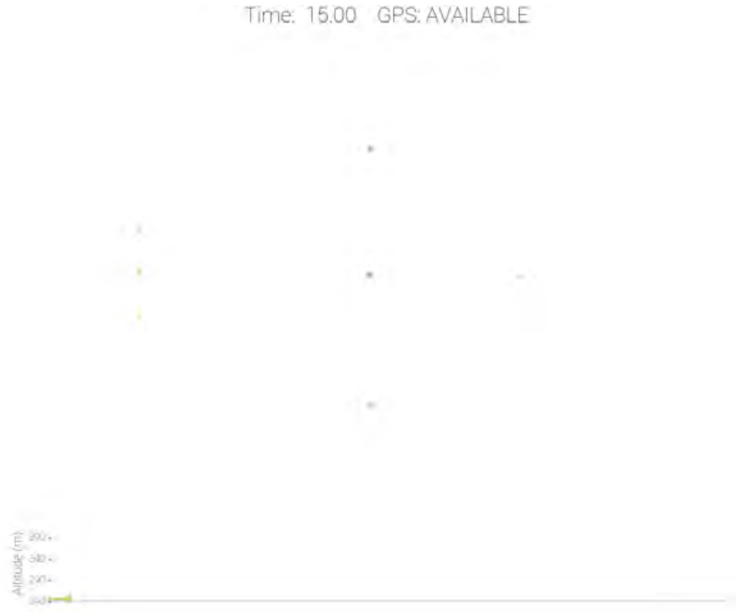
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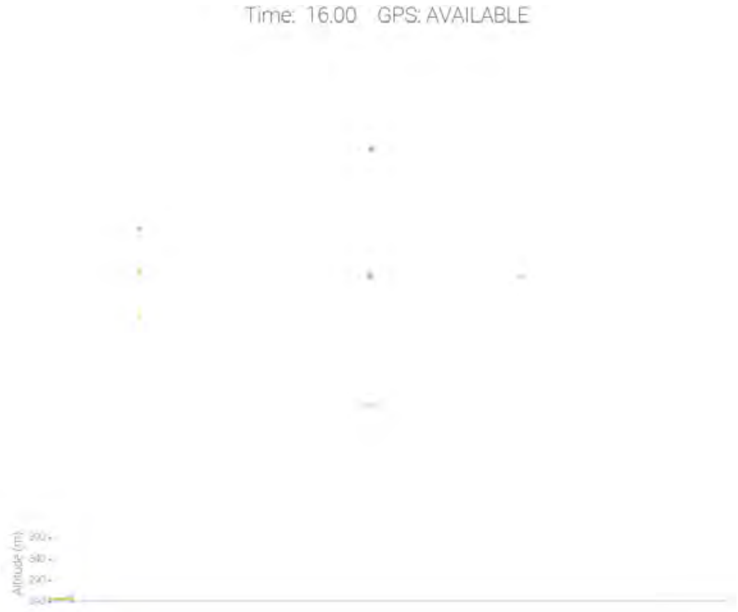
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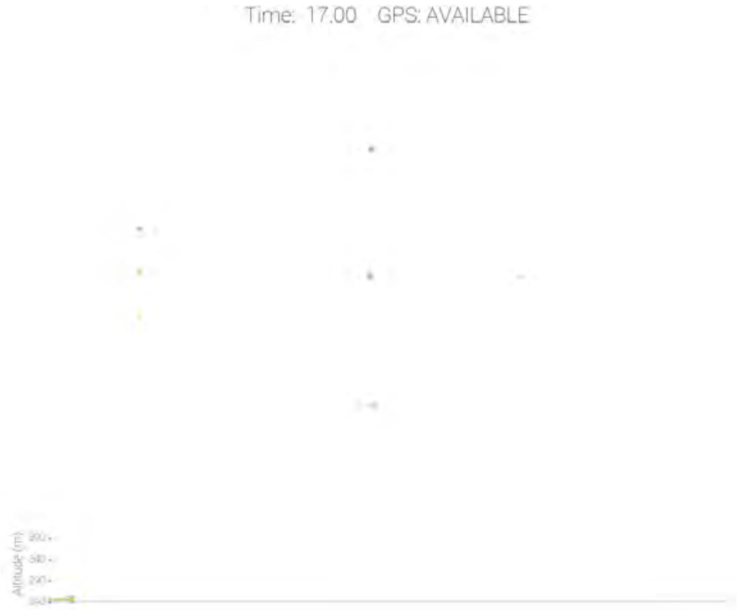
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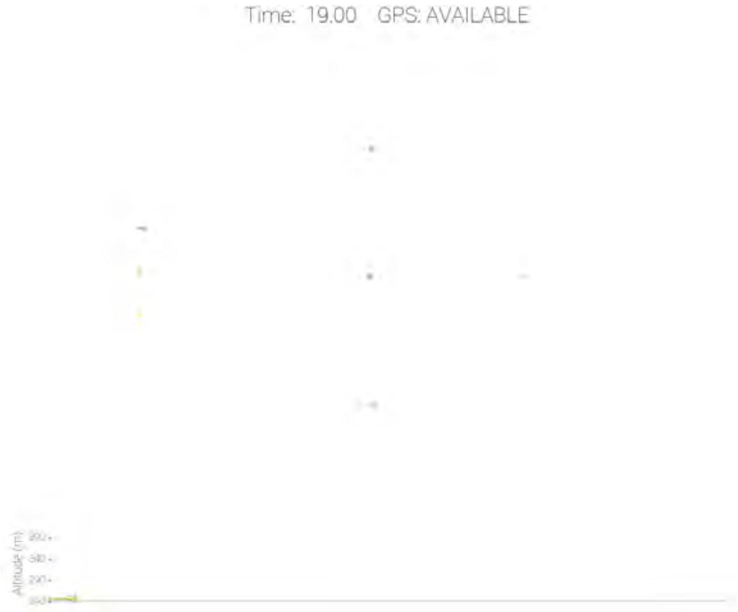
Altitude (m)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Time: 20.00 GPS: AVAILABLE
Altitude (m)


Time: 21.00 GPS: AVAILABLE
Altitude (m)


Time: 22.00 GPS: AVAILABLE


Time: 23.00 GPS: AVAILABLE
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Time: 24.00 GPS: AVAILABLE
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Time: 25.00 GPS: AVAILABLE
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Time: 26.00 GPS: AVAILABLE
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Time: 27.00 GPS: AVAILABLE
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Time: 28.00 GPS: AVAILABLE
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Time: 29.00 GPS: AVAILABLE
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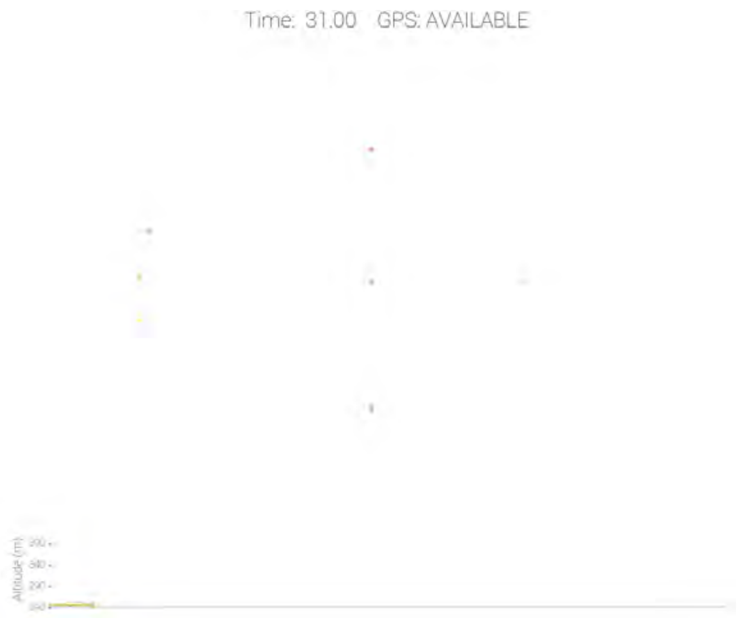
Time: 30.00 GPS: AVAILABLE
Altitude (m)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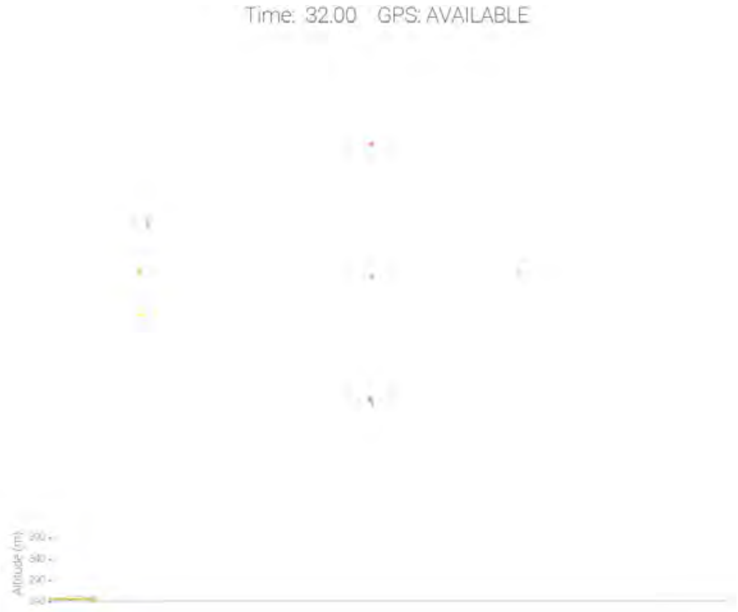
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Time: 33.00 GPS: AVAILABLE
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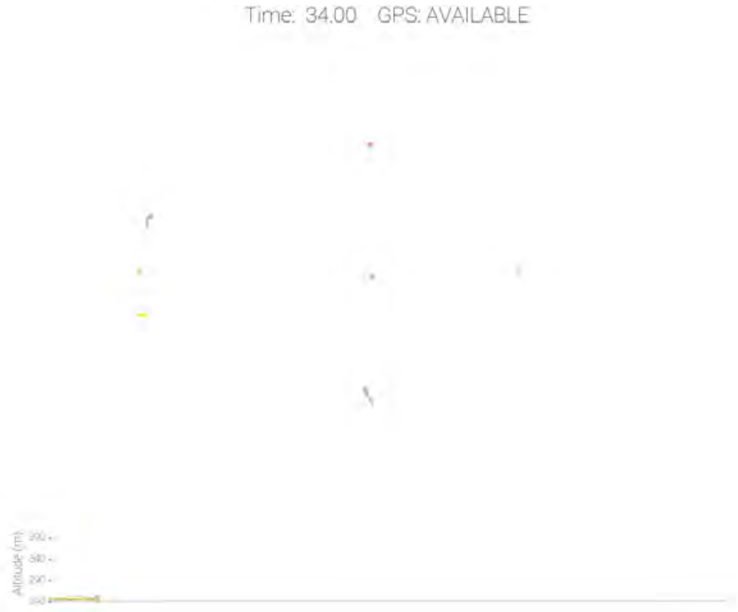
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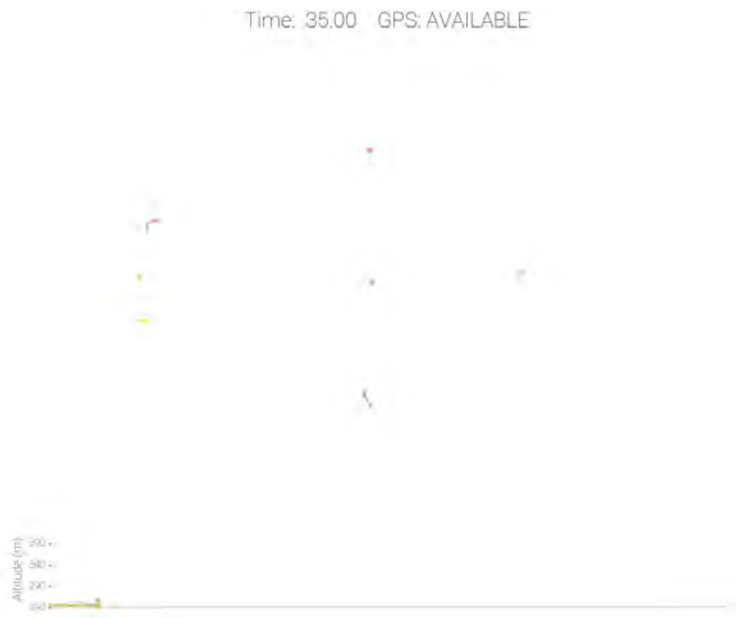
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Time: 36.00 GPS: AVAILABLE
Altitude (m)


Time: 37.00 GPS: AVAILABLE
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Time: 38.00 GPS: AVAILABLE
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Time: 48.00 GPS: AVAILABLE
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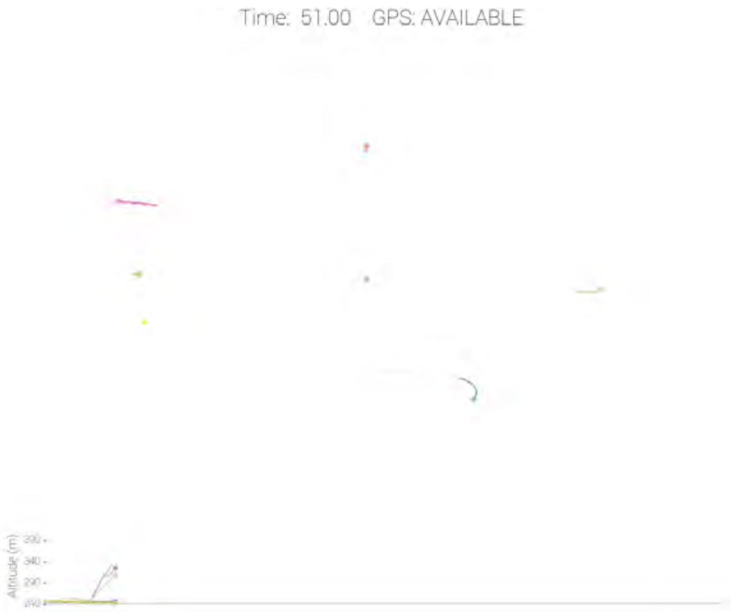
Time: 49.00 GPS: AVAILABLE
Altitude (m)


Time: 50.00 GPS: AVAILABLE
Altitude (m)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Altitude (m)
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Time: 52.00 GPS: AVAILABLE
Altitude (m)
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Time: 53.00 GPS: AVAILABLE
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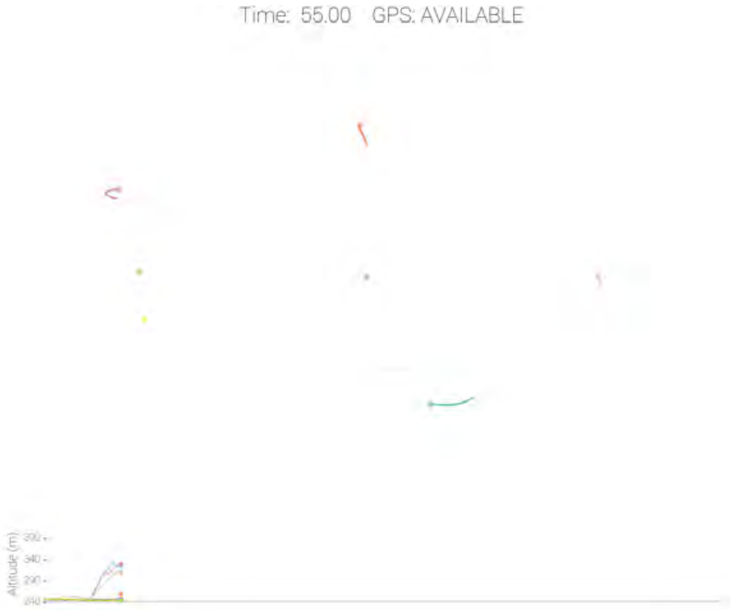
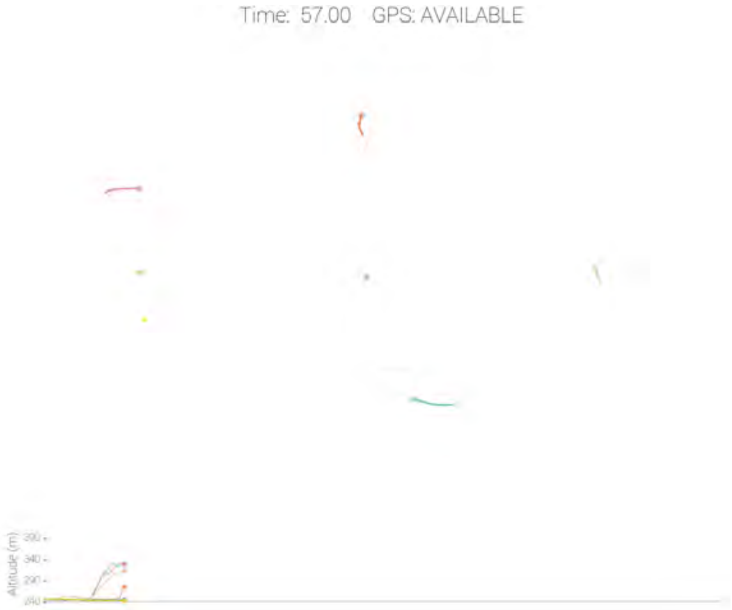
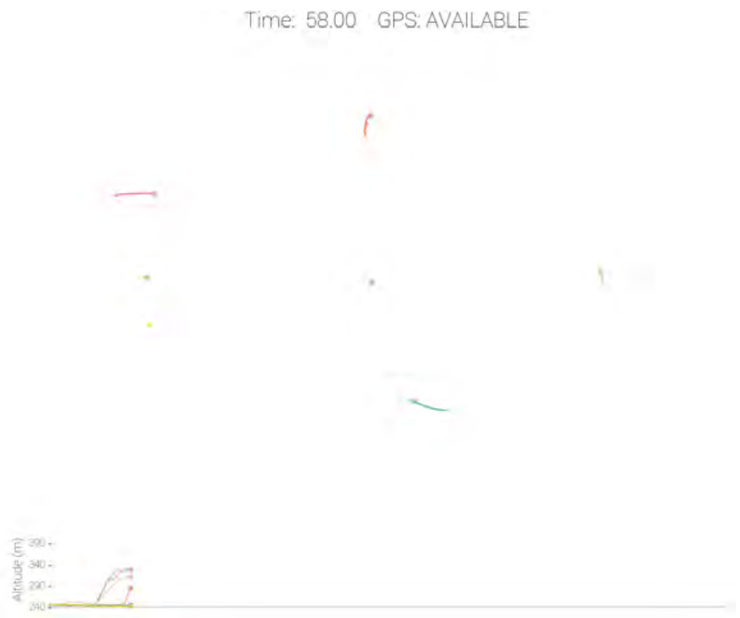
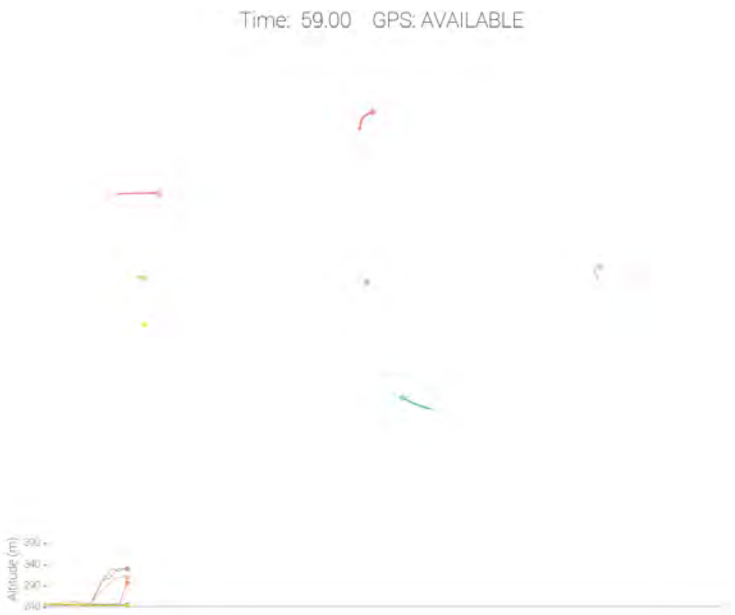

Time: 55.00 GPS: AVAILABLE
Altitude (m)


Time: 56.00 GPS: AVAILABLE
Altitude (m)


Time: 57.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 58.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 59.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

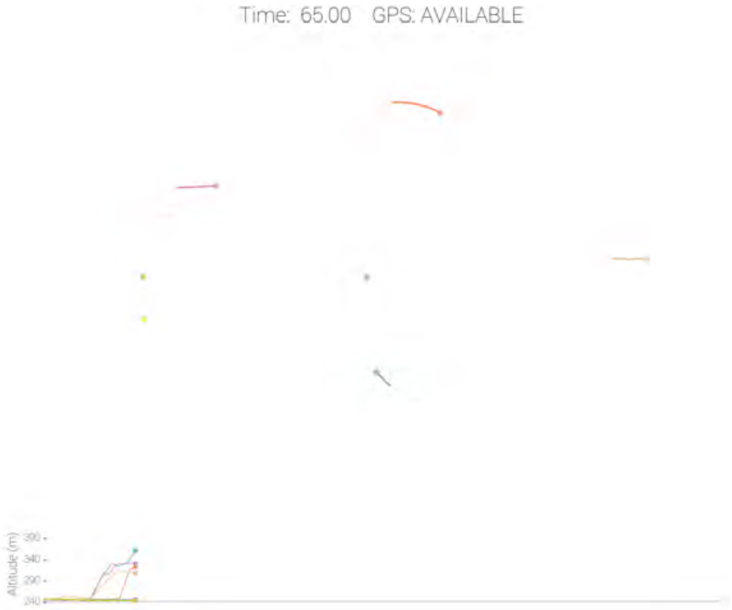

Time: 60.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

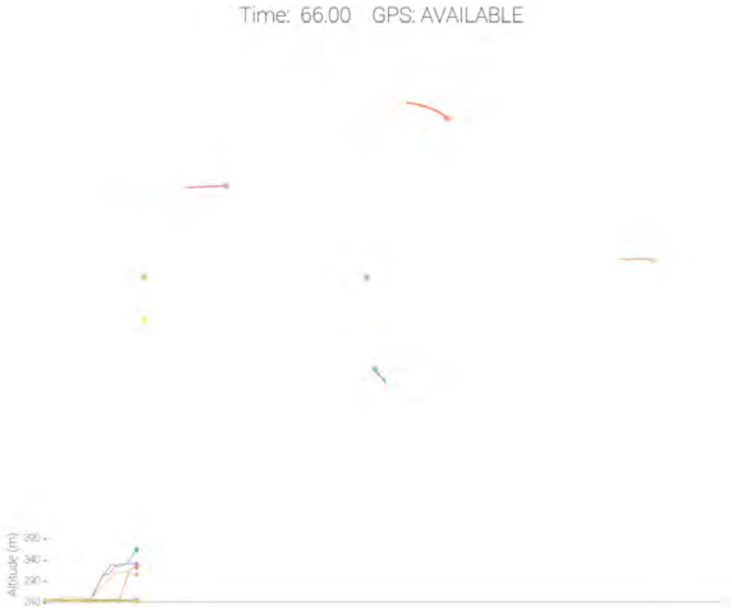

Time: 61.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 62.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 63.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 64.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 65.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 66.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 67.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 68.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 69.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 70.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 71.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 72.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 73.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

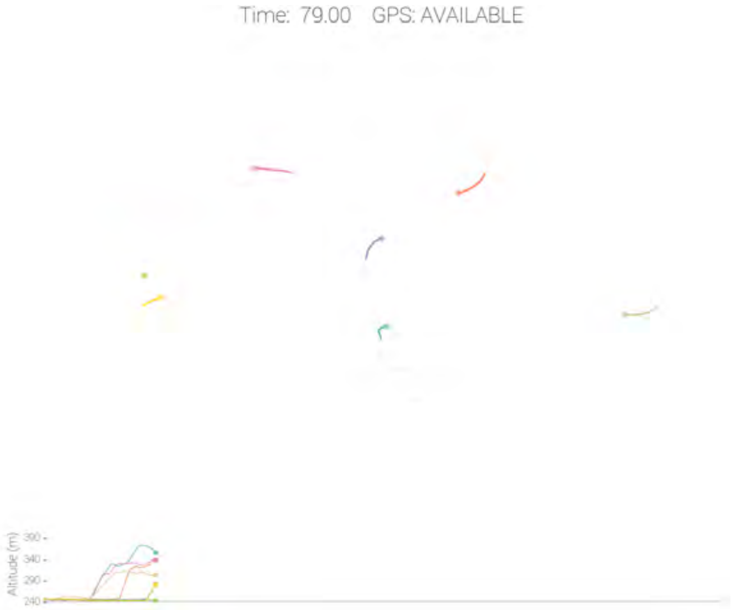

Time: 74.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 75.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 76.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 77.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 78.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 79.00 GPS: AVAILABLE
Altitude (m)
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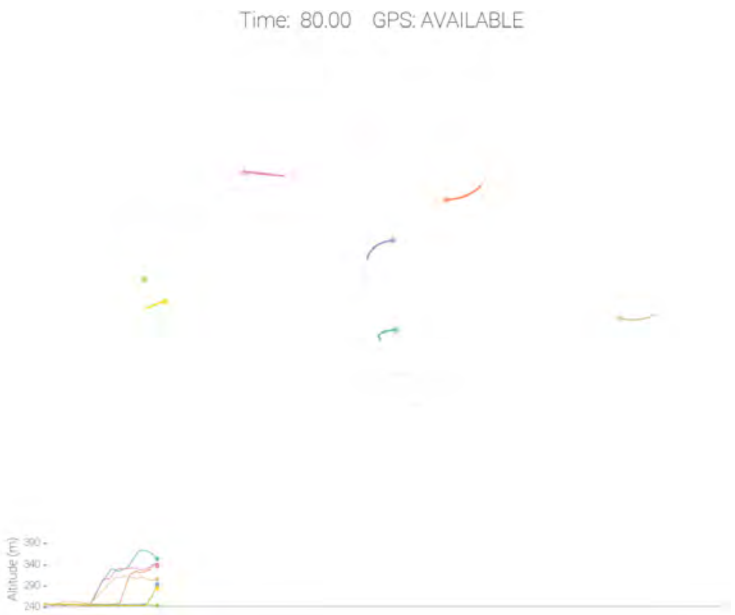
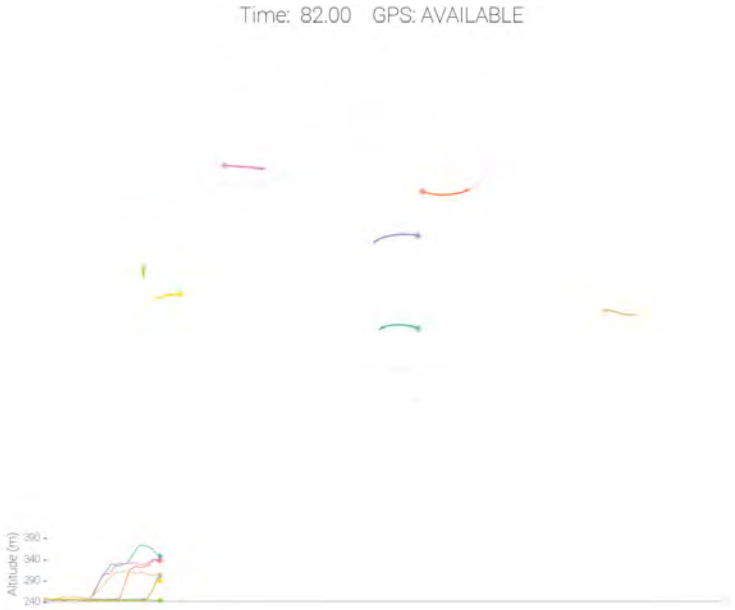
340
290
240


Time: 80.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 81.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 82.00 GPS: AVAILABLE
Altitude (m)
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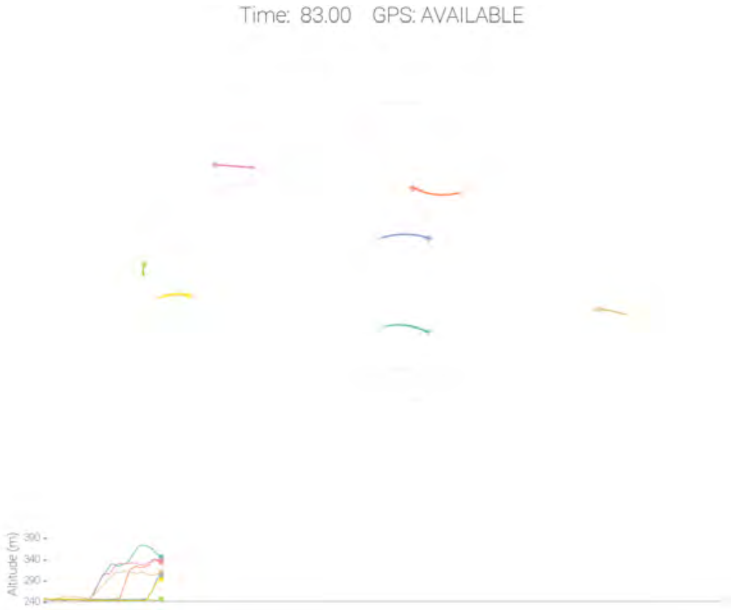
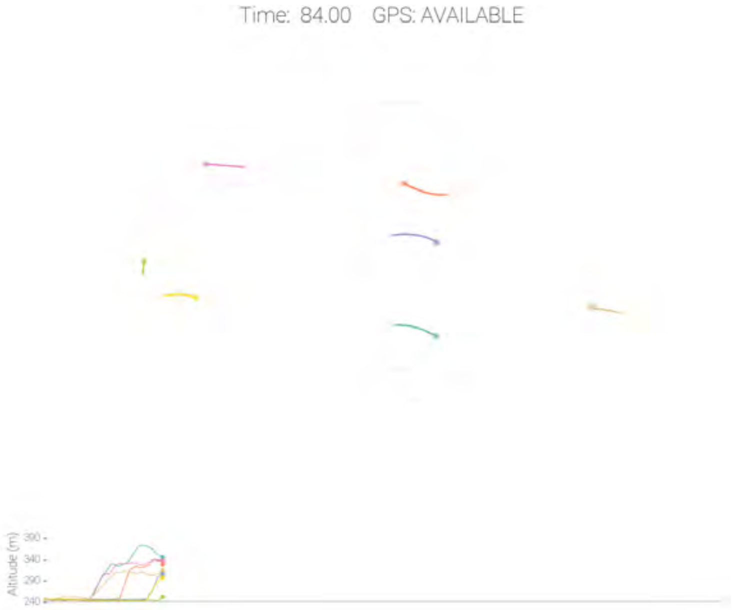
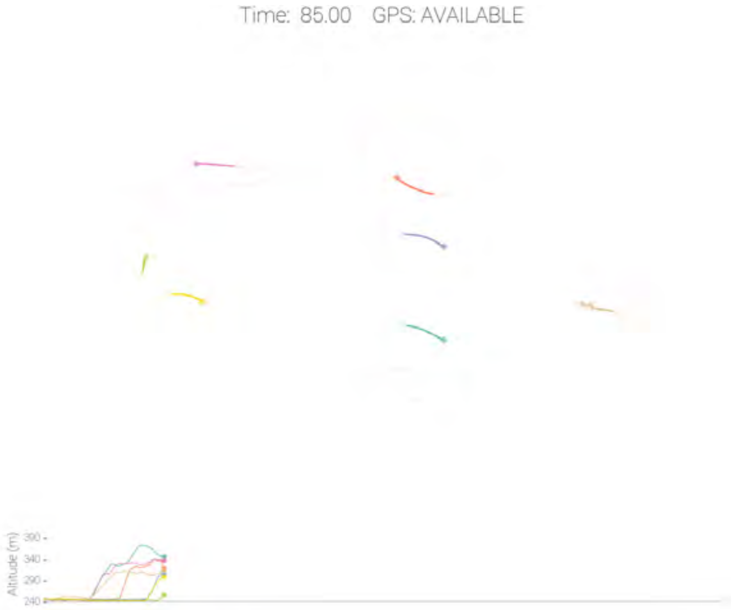
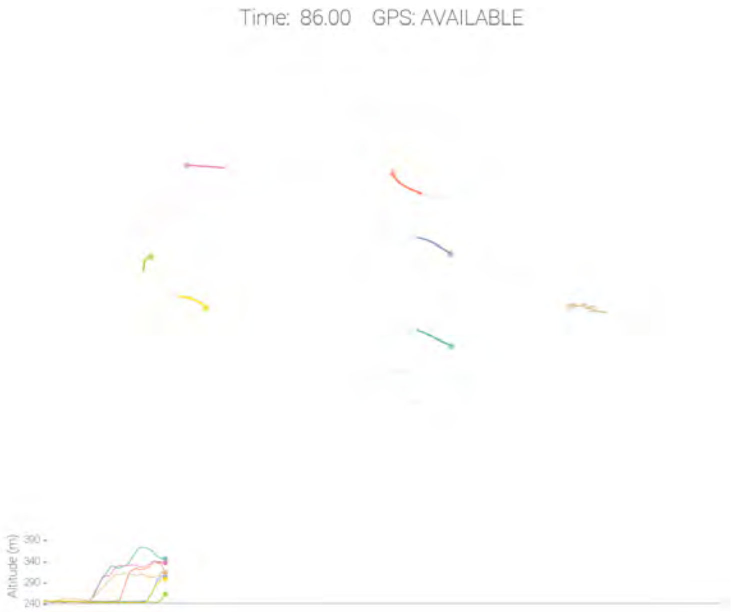
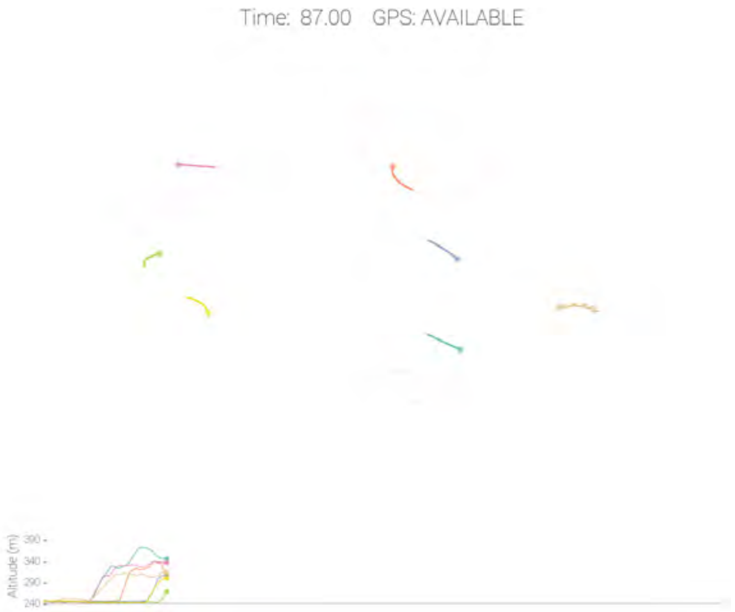
340
290
240

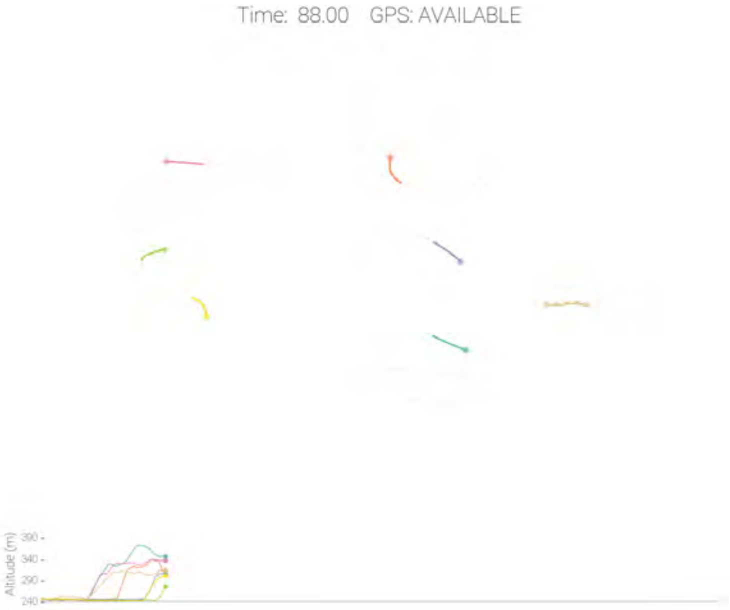

Time: 83.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

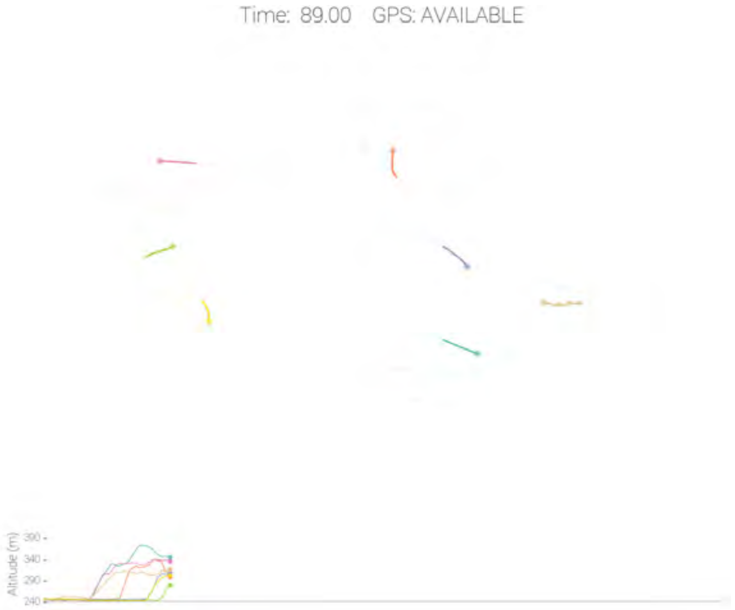

Time: 84.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

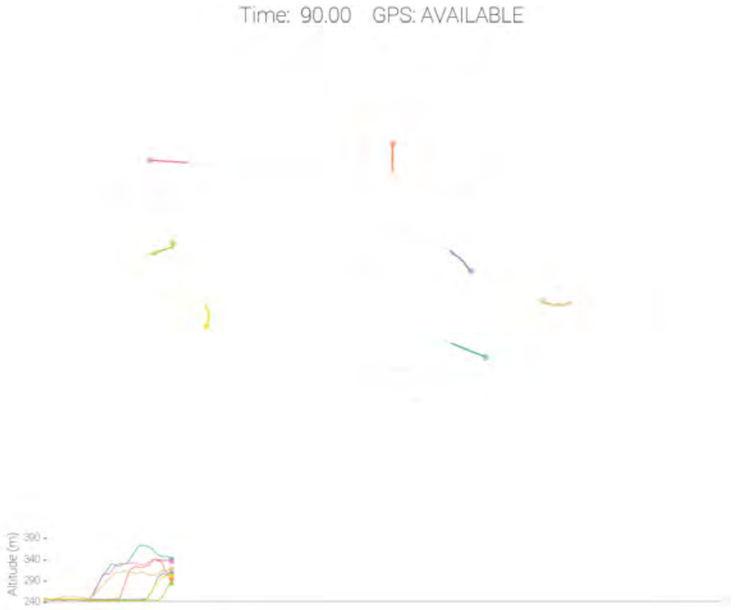

Time: 85.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

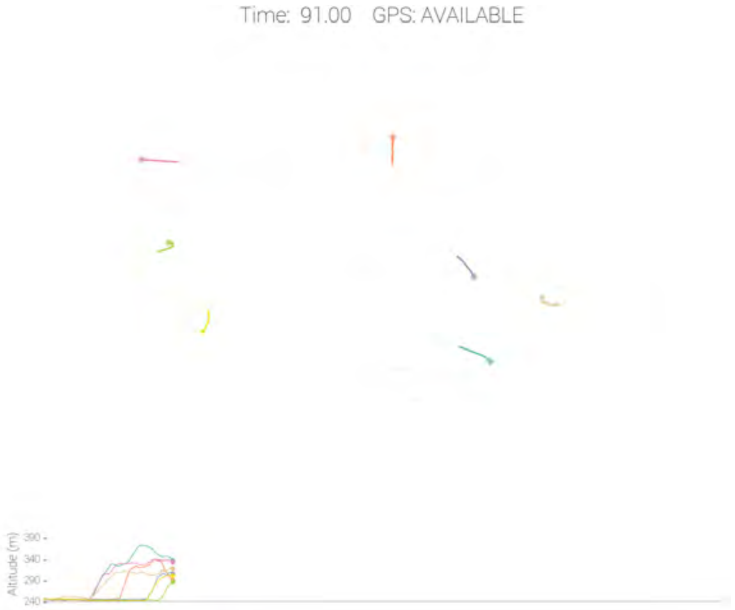

Time: 86.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

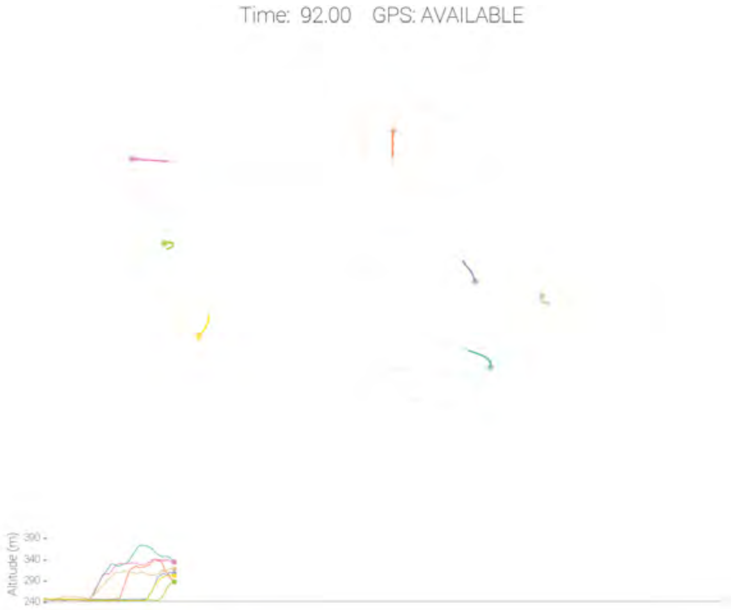

Time: 87.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

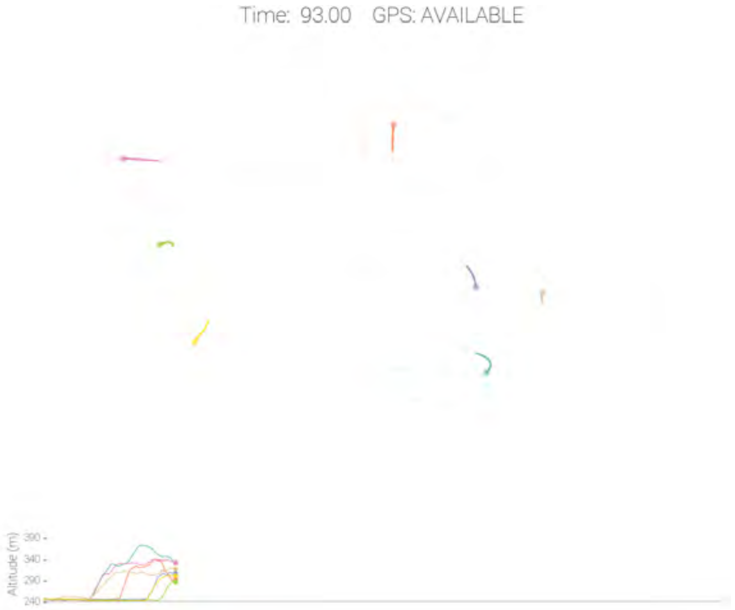

Time: 88.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

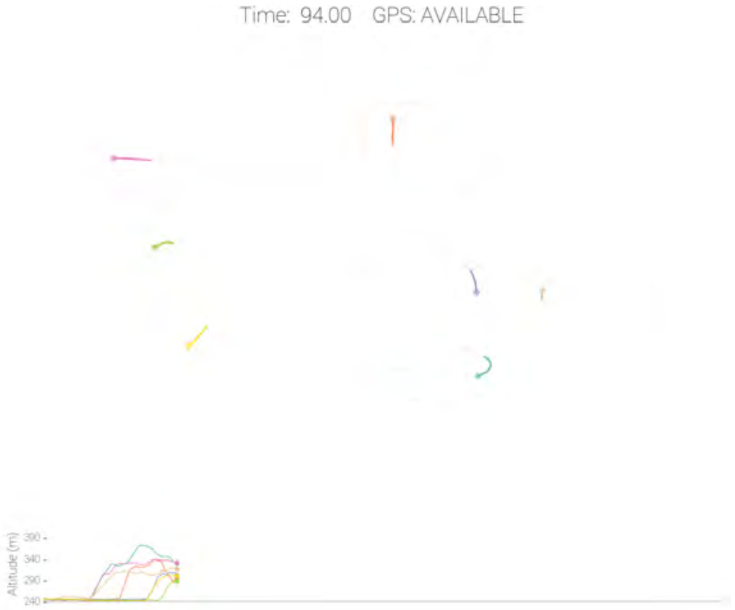

Time: 89.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

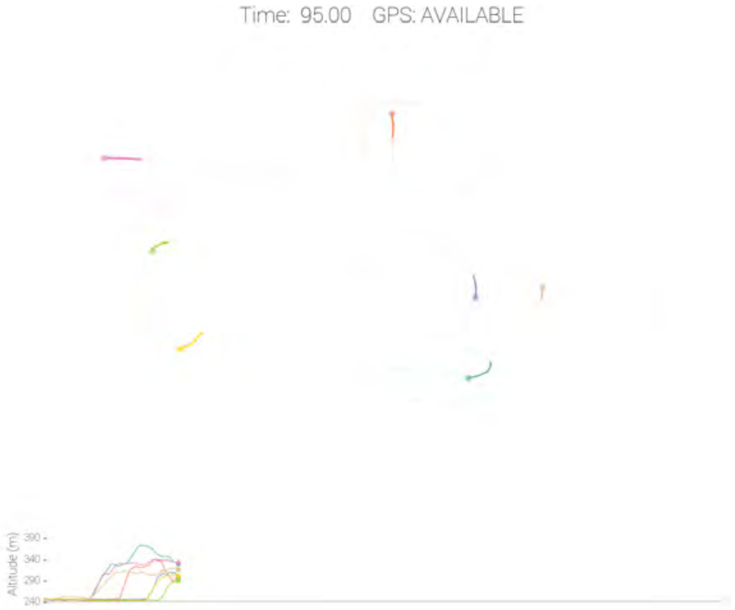

Time: 90.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 91.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

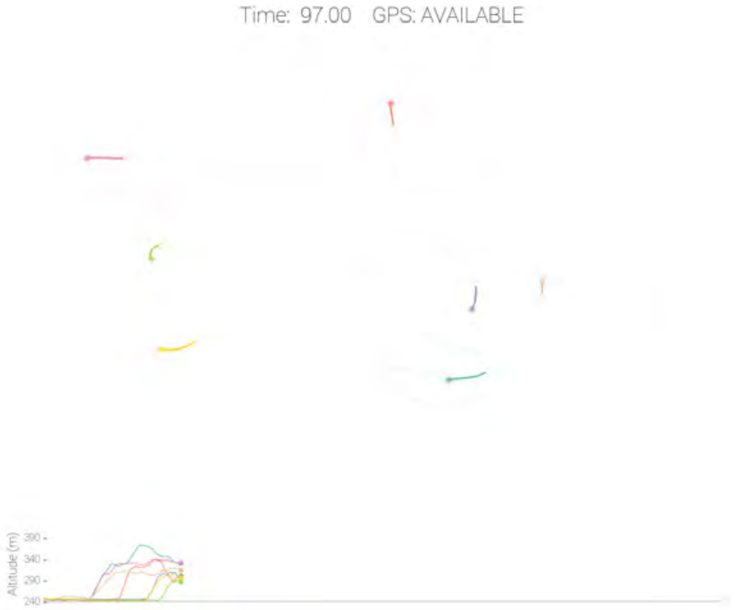

Time: 92.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 93.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 94.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 95.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

Time: 96.00 GPS: AVAILABLE


Time: 97.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 98.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 99.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 100.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 101.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 102.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 103.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 104.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

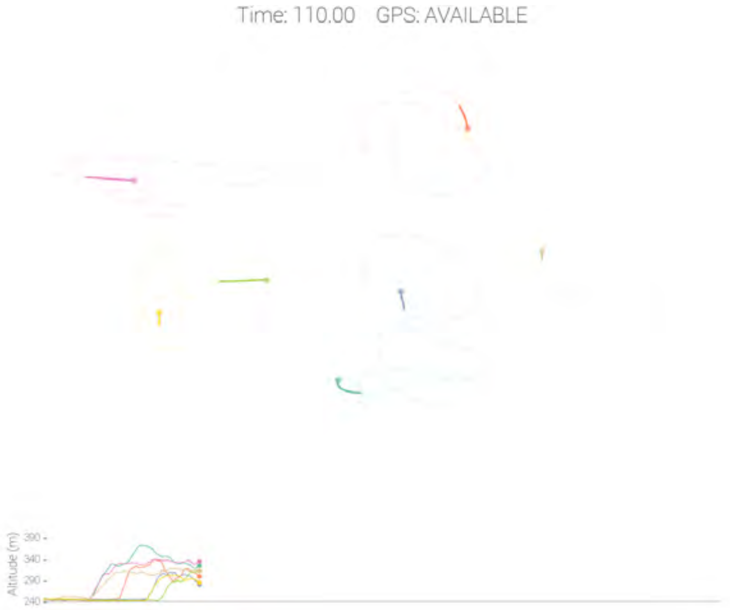

Time: 105.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

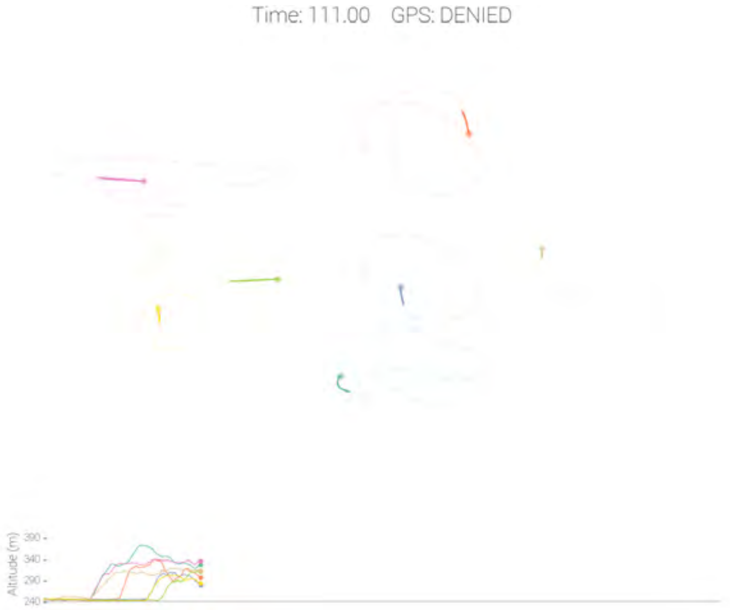

Time: 106.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 107.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

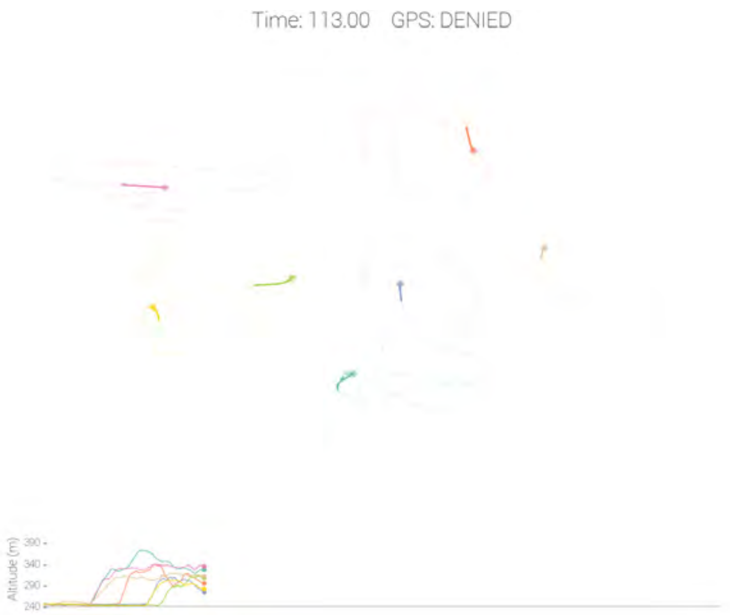

Time: 108.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

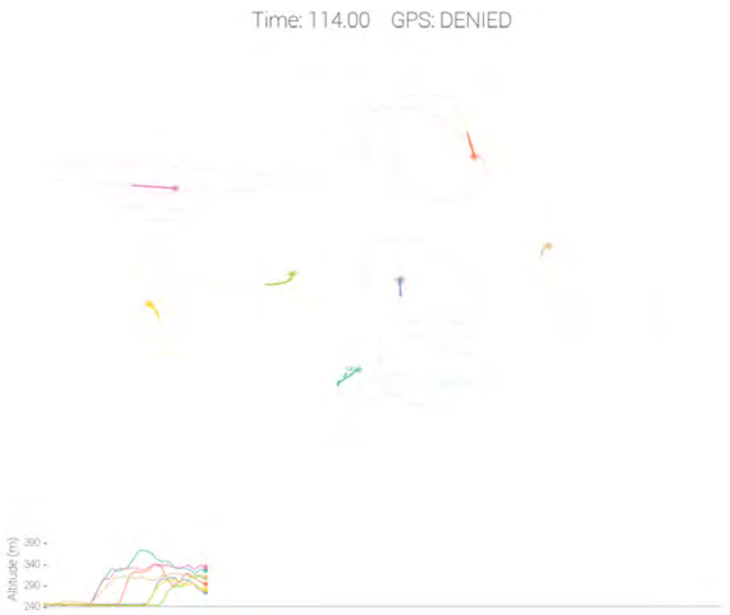

Time: 109.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240

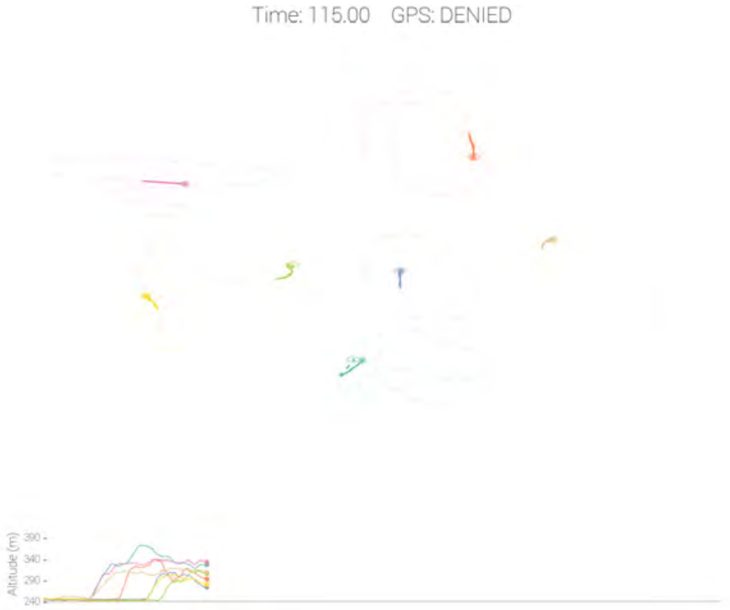

Time: 110.00 GPS: AVAILABLE
Altitude (m)
390
340
290
240


Time: 111.00 GPS: DENIED
Altitude (m)
390
340
290
240

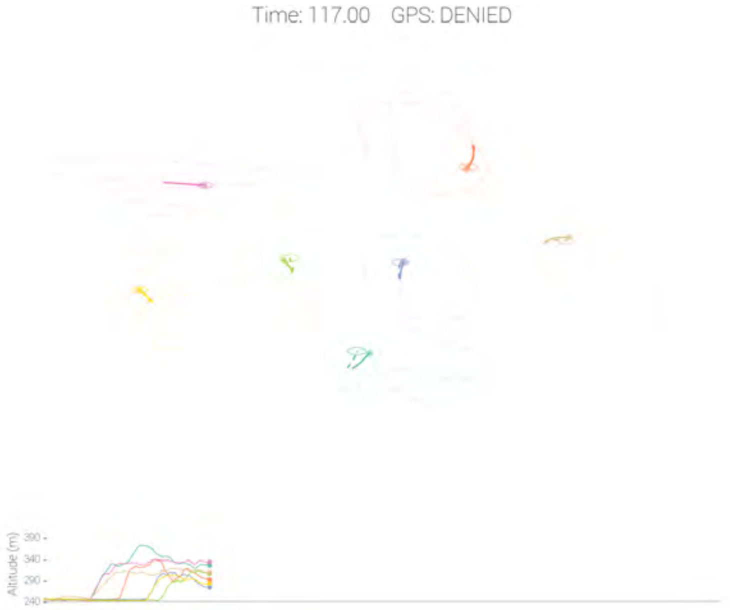

Time: 112.00 GPS: DENIED
Altitude (m)
390
340
290
240

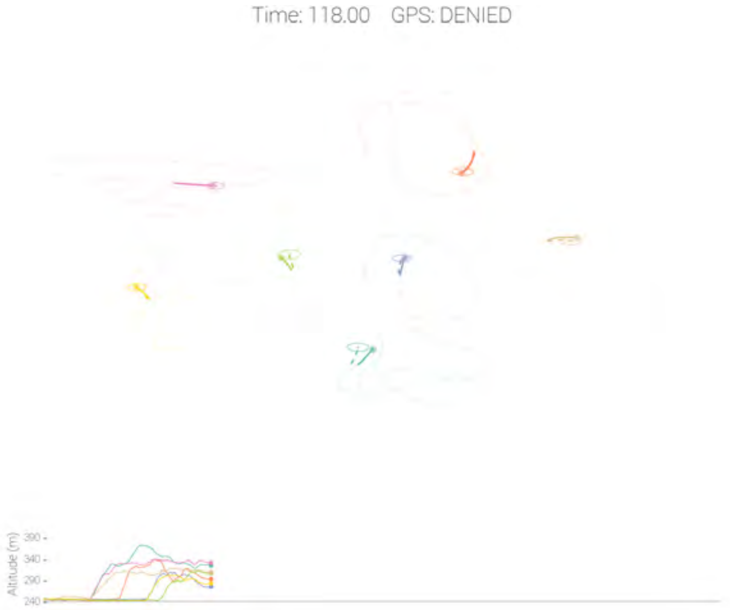

Time: 113.00 GPS: DENIED
Altitude (m)
390
340
290
240

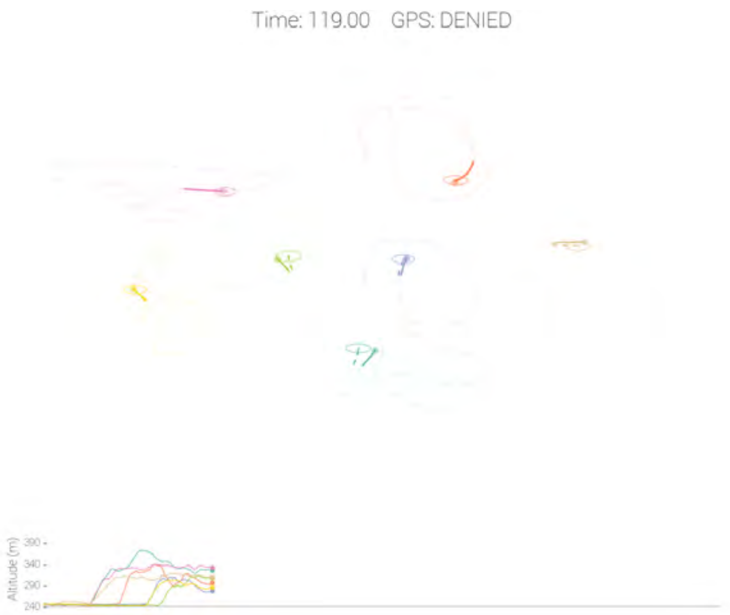

Time: 114.00 GPS: DENIED
Altitude (m)
390
340
290
240

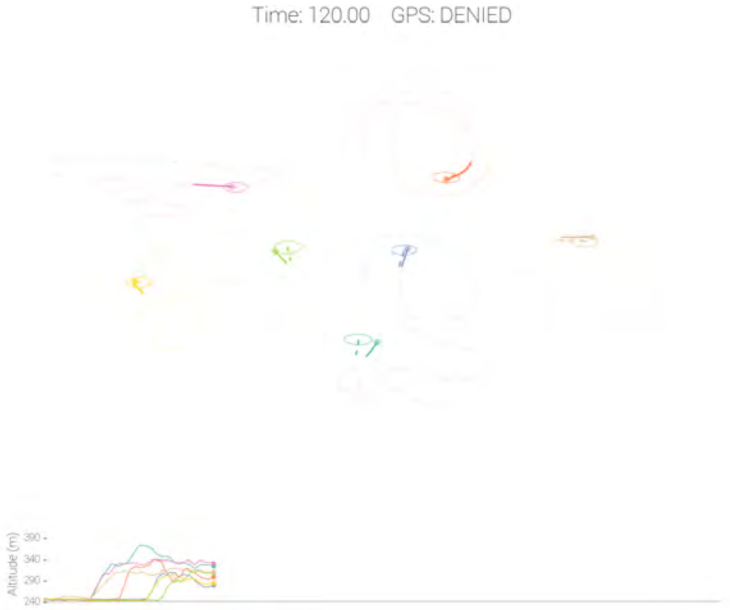

Time: 115.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 116.00 GPS: DENIED
Altitude (m)
390
340
290
240

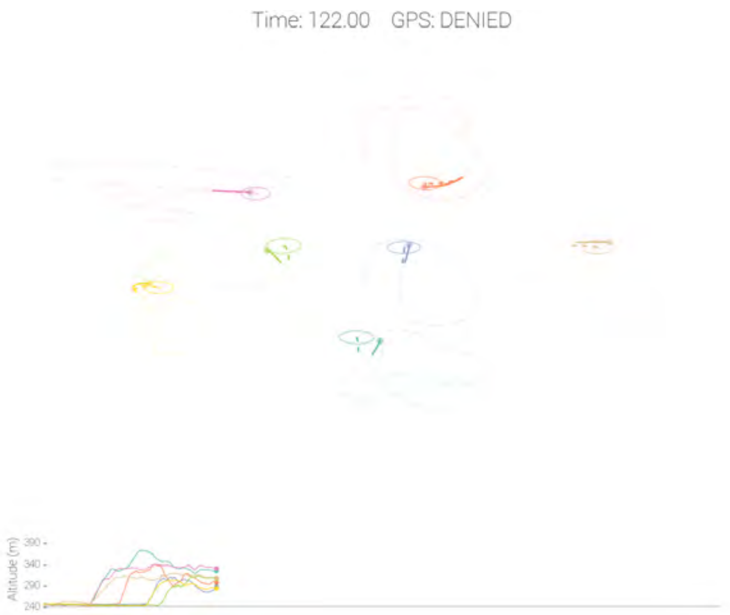

Time: 117.00 GPS: DENIED
Altitude (m)
390
340
290
240

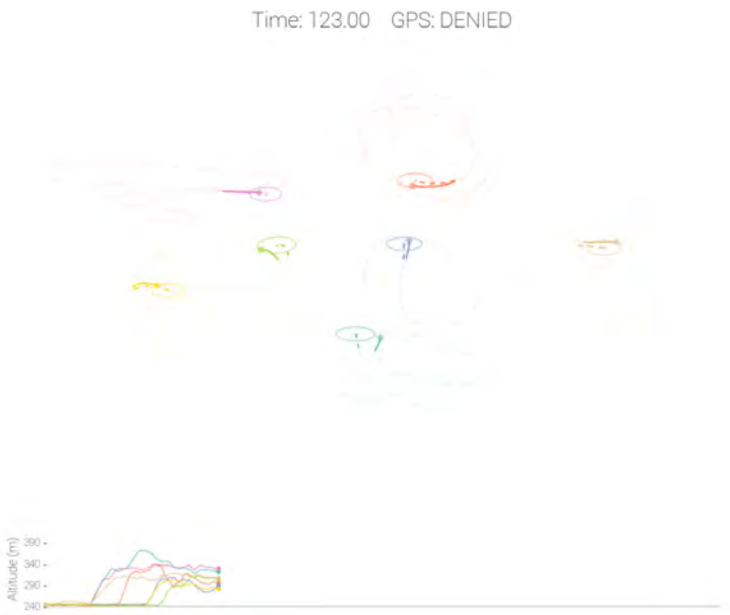

Time: 118.00 GPS: DENIED
Altitude (m)
390
340
290
240

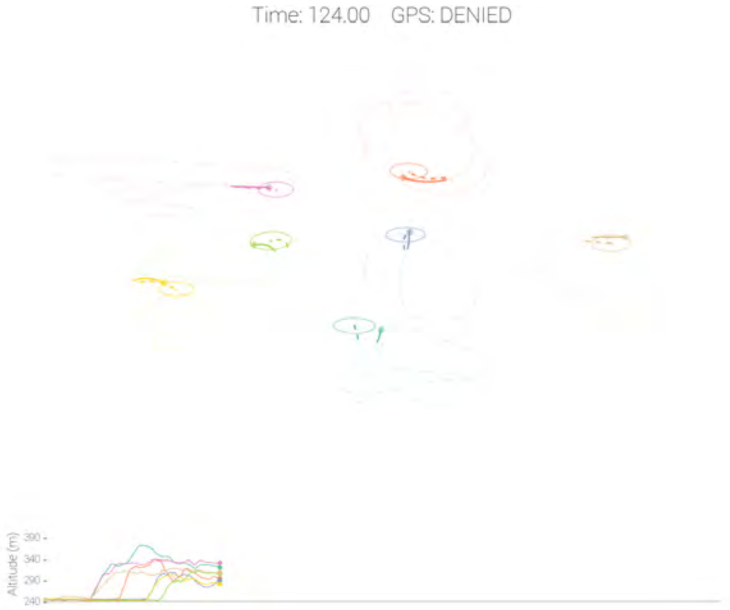

Time: 119.00 GPS: DENIED
Altitude (m)
390
340
290
240

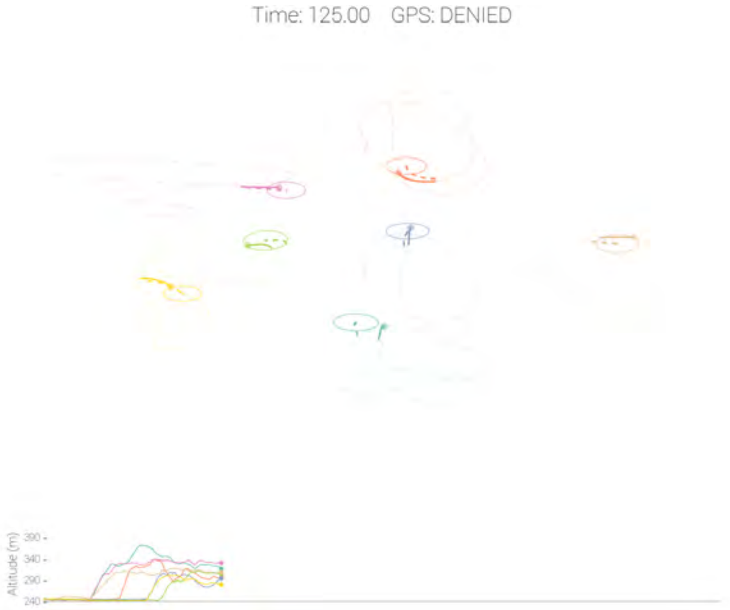

Time: 120.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 121.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 122.00 GPS: DENIED
Altitude (m)
390
340
290
240

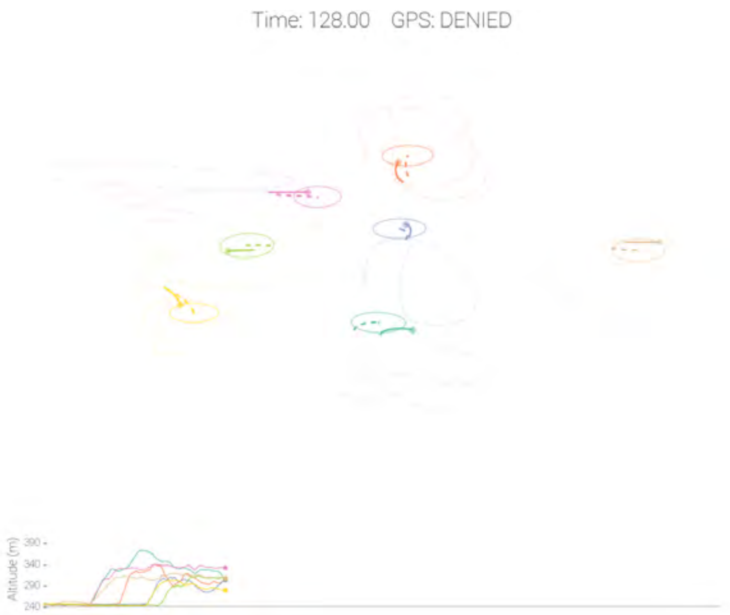

Time: 123.00 GPS: DENIED
Altitude (m)
390
340
290
240

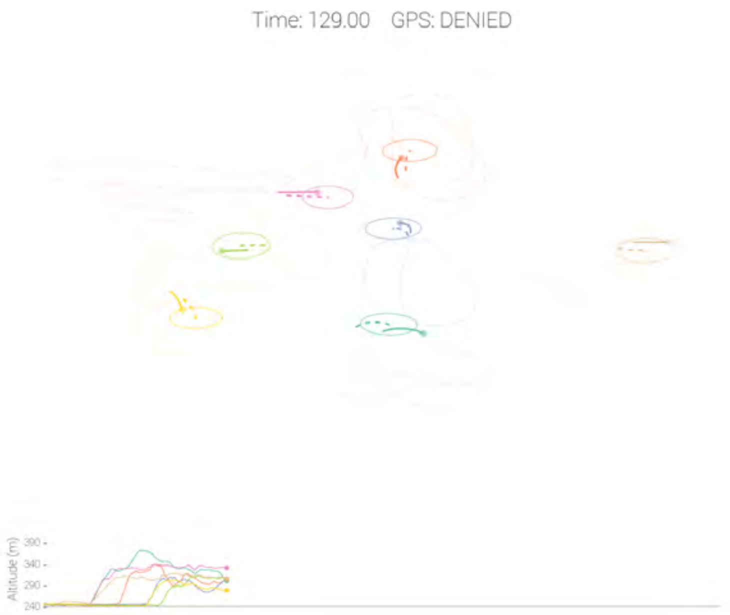

Time: 124.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 125.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 126.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 127.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 128.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 129.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 130.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 131.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 132.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 133.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 134.00 GPS: DENIED
Altitude (m)
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340
290
240


Time: 135.00 GPS: DENIED
Altitude (m)
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340
290
240


Time: 136.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 137.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 138.00 GPS: DENIED
Altitude (m)
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340
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240


Time: 139.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 140.00 GPS: DENIED
Altitude (m)
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340
290
240


Time: 141.00 GPS: DENIED
Altitude (m)
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340
290
240


Time: 142.00 GPS: DENIED
Altitude (m)
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340
290
240


Time: 143.00 GPS: DENIED
Altitude (m)
390
340
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240


Time: 144.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 145.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 146.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 147.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 148.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 149.00   GPS: DENIED
Altitude (m)
390
340
290
240


Time: 150.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 151.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 152.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 153.00 GPS: DENIED
Altitude (m)
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340
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240


Time: 154.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 155.00 GPS: DENIED
Altitude (m)
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Time: 156.00 GPS: DENIED
Altitude (m)
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Time: 157.00 GPS: DENIED
Altitude (m)
390
340
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240


Time: 158.00 GPS: DENIED
Altitude (m)
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340
290
240


Time: 159.00 GPS: DENIED
Altitude (m)
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Time: 160.00 GPS: DENIED
Altitude (m)
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340
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Time: 161.00 GPS: DENIED
Altitude (m)
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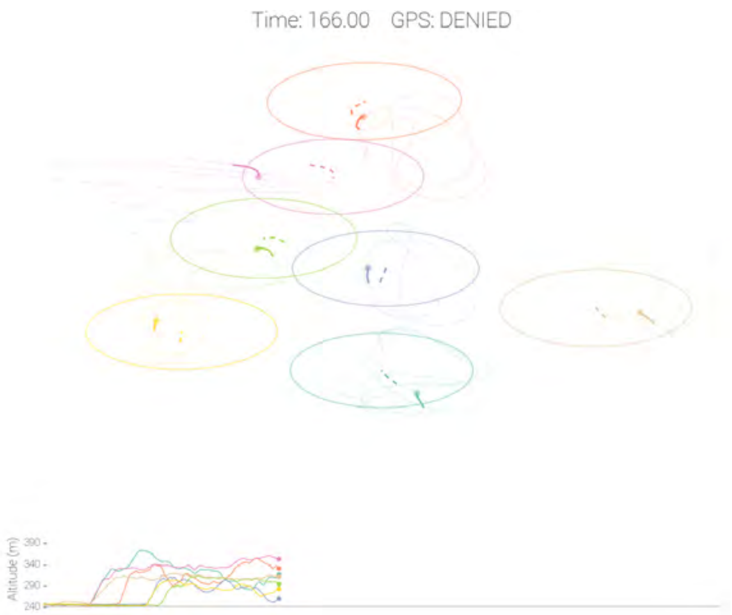
Time: 162.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 163.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 164.00 GPS: DENIED
Altitude (m)
390
340
290
240

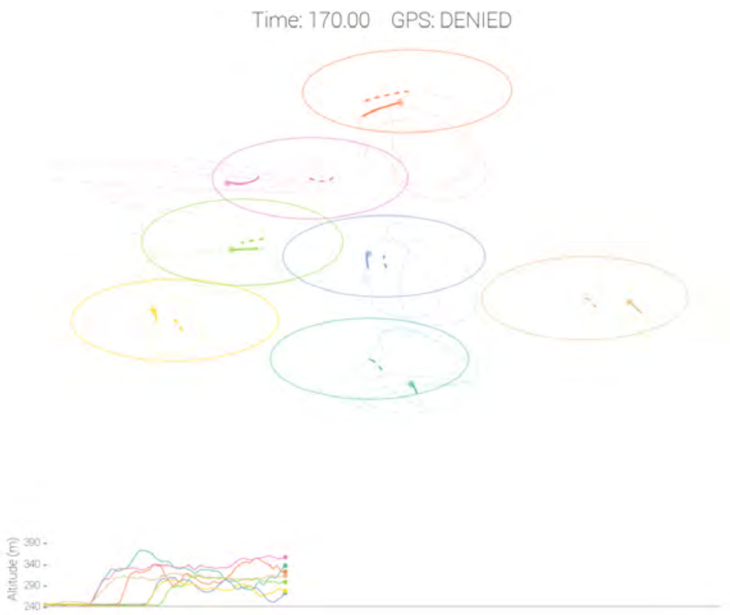

Time: 165.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 166.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 167.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 168.00 GPS: DENIED
Altitude (m)
390
340
290
240

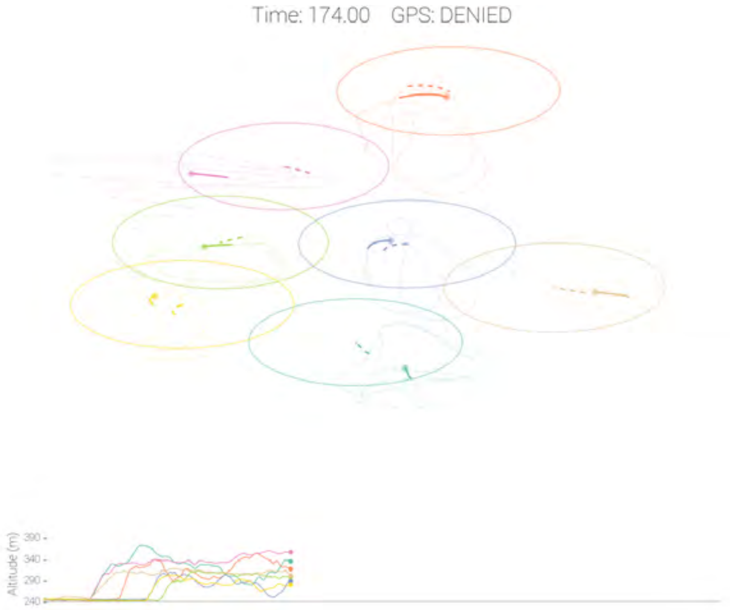

Time: 169.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 170.00 GPS: DENIED
Altitude (m)
390
340
290
240

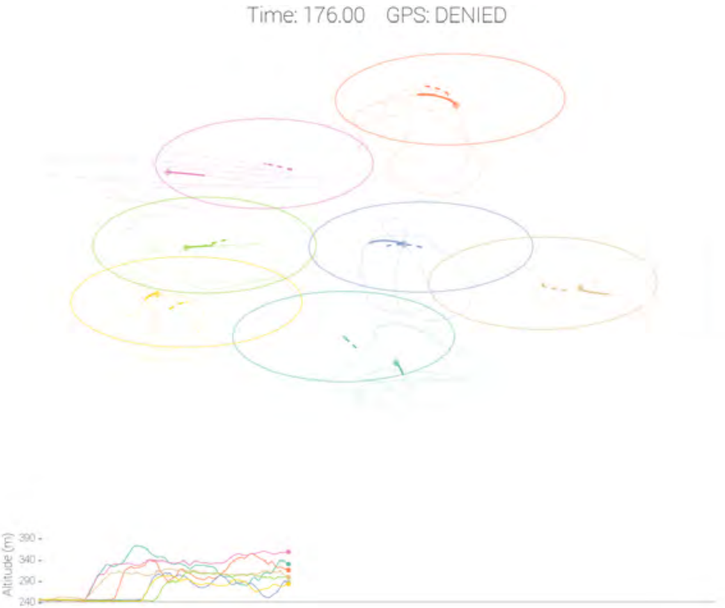

Time: 171.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 172.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 173.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 174.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 175.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 176.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 177.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 178.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 179.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 180.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 181.00 GPS: DENIED
Altitude (m)
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Time: 182.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 183.00 GPS: DENIED
Altitude (m)
390
340
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240


Time: 184.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 185.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 186.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 187.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 188.00 GPS: DENIED
Altitude (m)
390
340
290
240

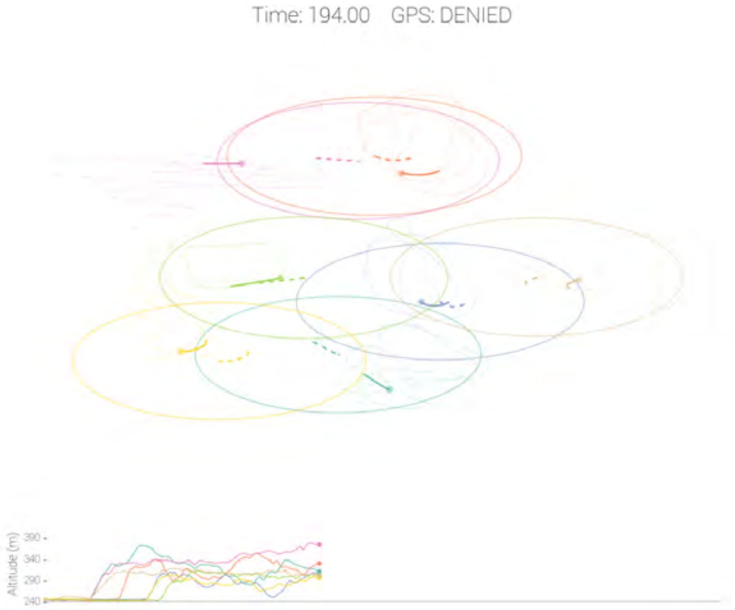

Time: 189.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 190.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 191.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 192.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 193.00 GPS: DENIED
Altitude (m)
390
340
290
240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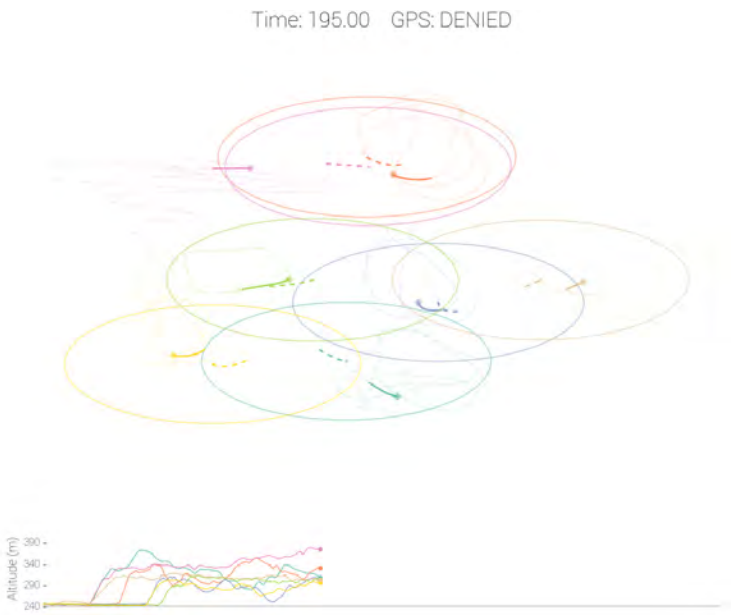
340
290
240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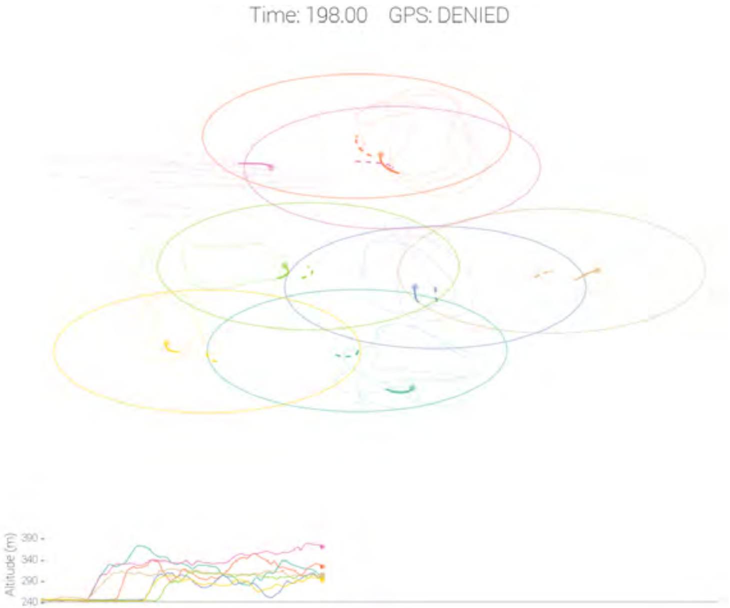
Altitude (m)
390
340
290
240


Time: 196.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 197.00 GPS: DENIED
Altitude (m)
390
340
290
240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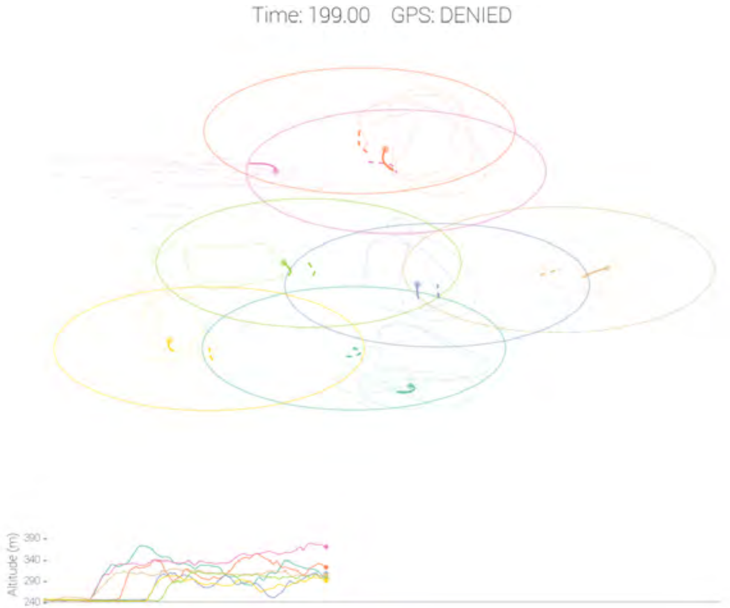
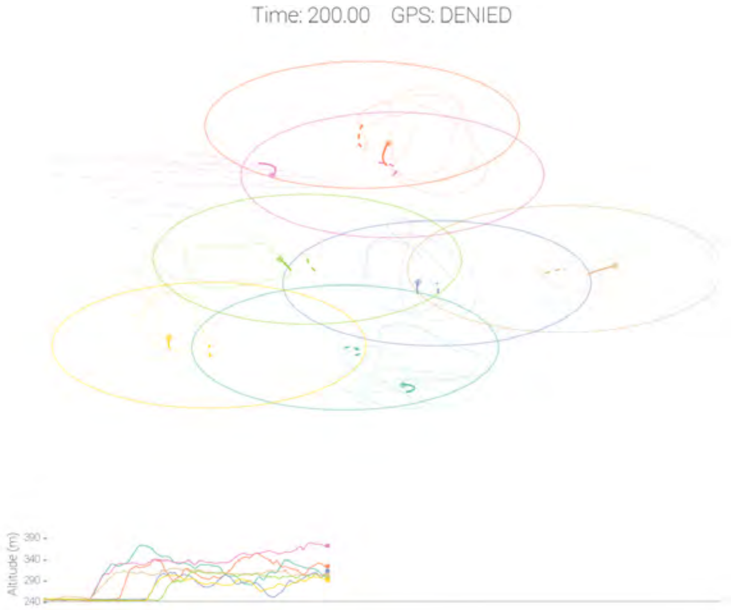
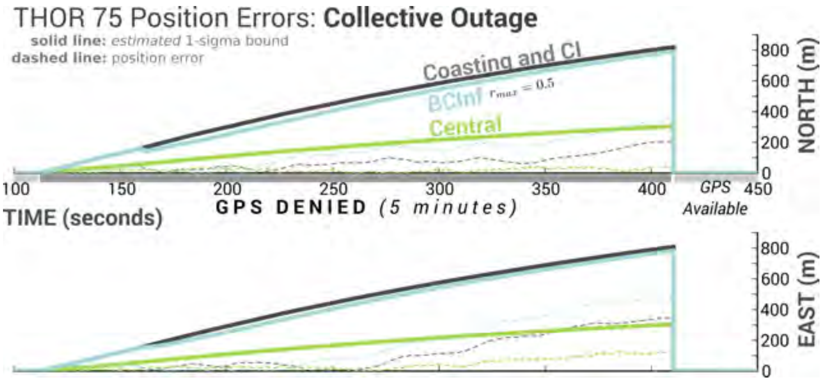
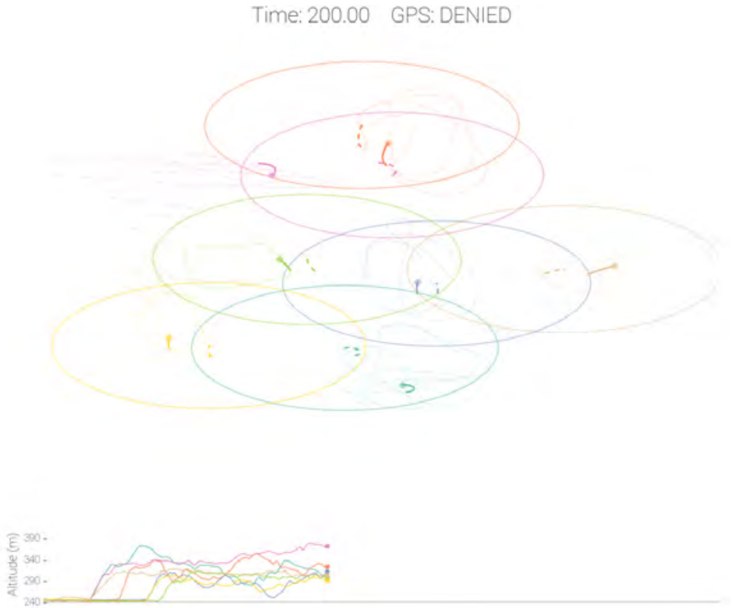
Altitude (m)
390
340
290
240


Time: 199.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 200.00 GPS: DENIED
Altitude (m)
390
340
290
240

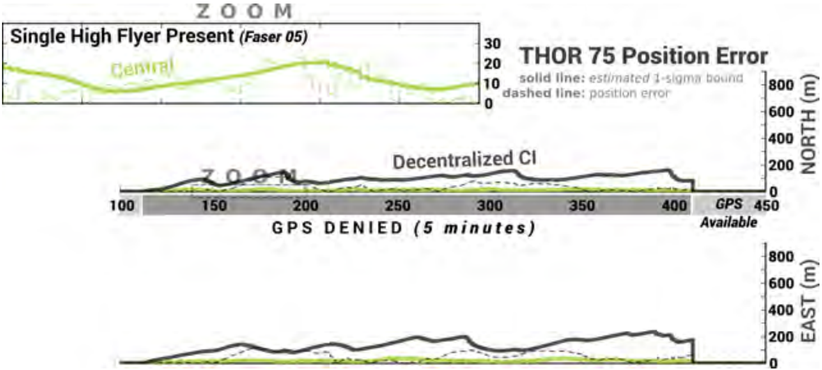
Scenario 1: No High Flyer



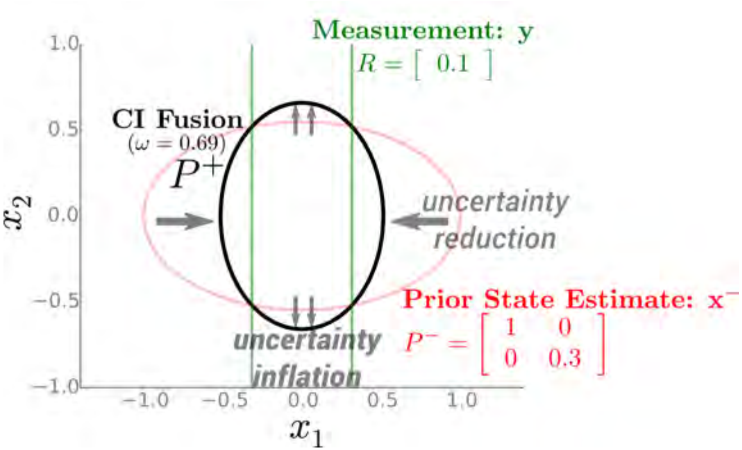

Time: 200.00 GPS: DENIED
Altitude (m)
390
340
290
240


Time: 200.00 GPS: DENIED
Altitude (m)
390
340
290
240

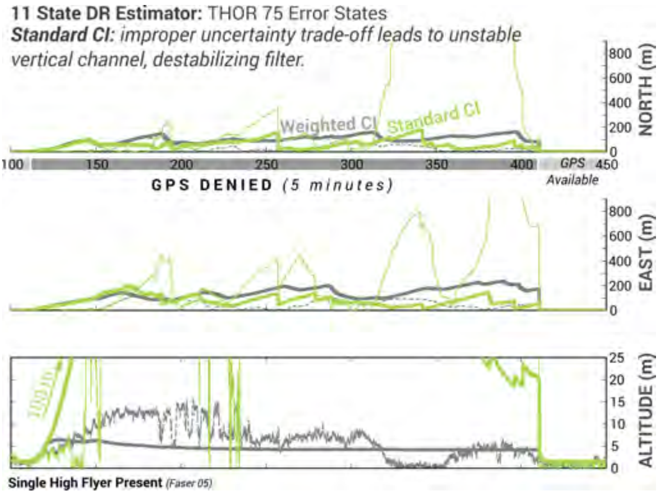
Scenario 2: Single High Flyer



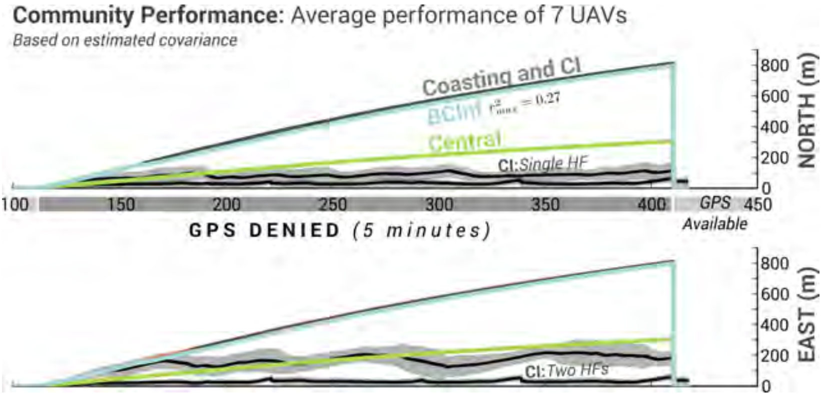
UNCERTAINTY TRADE-OFF



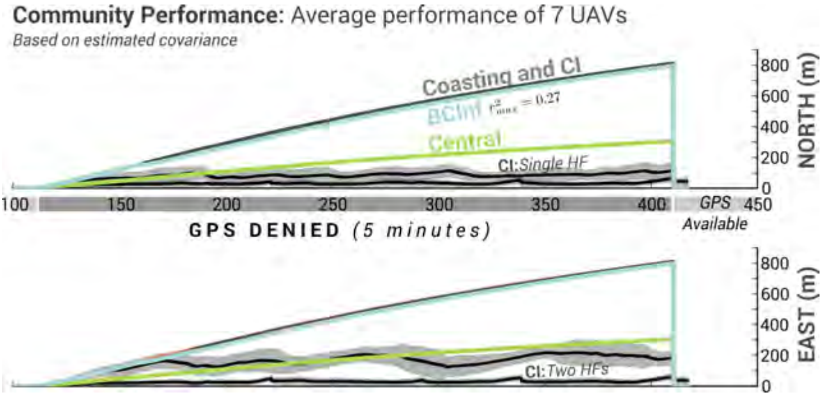
SCENARIO 2: COVARIANCE NORMALIZATION



SCENARIO 3: TWO HIGH FLYERS



SCENARIO 3: TWO HIGH FLYERS



Investing in maintained presence of High Flyers better than centralized implementation!

Conclusion

- Cooperative Navigation can mitigate GNSS-outage
- **Centralized Implementation:** advantageous for predefined community
- **Decentralized Implementation:** suited for large and dynamic community
  - covariance intersection + covariance normalization
  - requires presence of high-quality users

FUTURE WORK

- Real-time implementation - handling delays
- Fault detection and integrity monitoring for decentralized implementations
- Investigate other bounding techniques
  - More *degrees of freedom* may help overcome uncertainty trade-off challenge

$$\bar{P}^{-} = \frac{1}{\omega}P^{-} \quad \bar{R} = \frac{1}{1-\omega}R$$

Correlated data fusion estimators have expanded applications!

Thank you!

Backup Slides

Correlated Data Fusion

Considerations

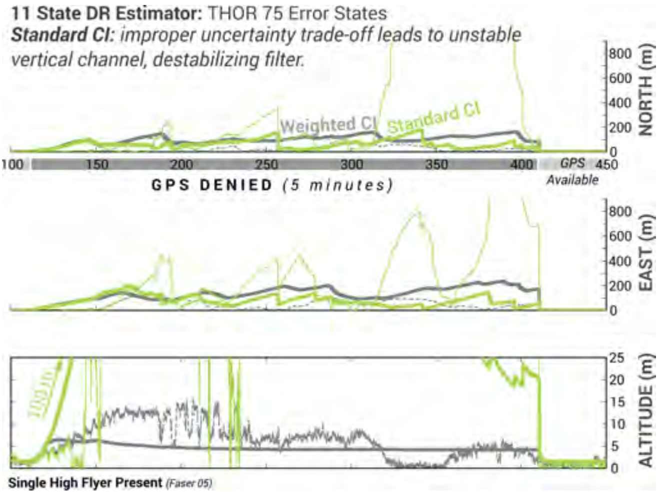
Contribution: Enable engineer, familiar with standard Kalman Filter, to determine whether CI/BCInf might work in their application.

- Certain properties are surprising or counter-intuitive

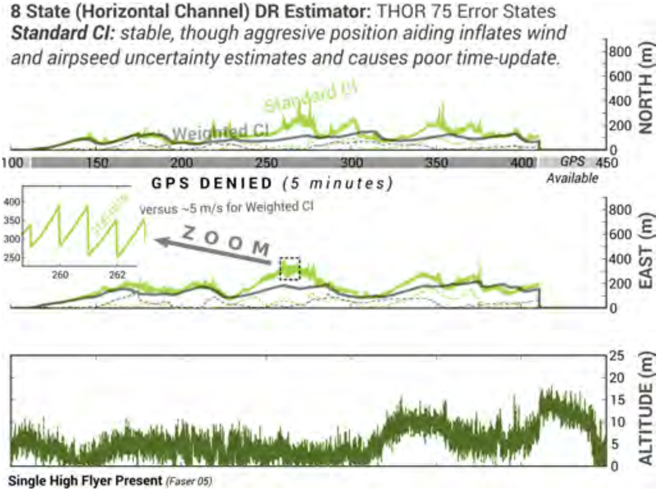
Topics Covered

1. Inherent *uncertainty trade-off*
   - effect of number of states on trade-off
2. Necessity of units normalization
3. BCInf and choosing correlation bound r_{max}

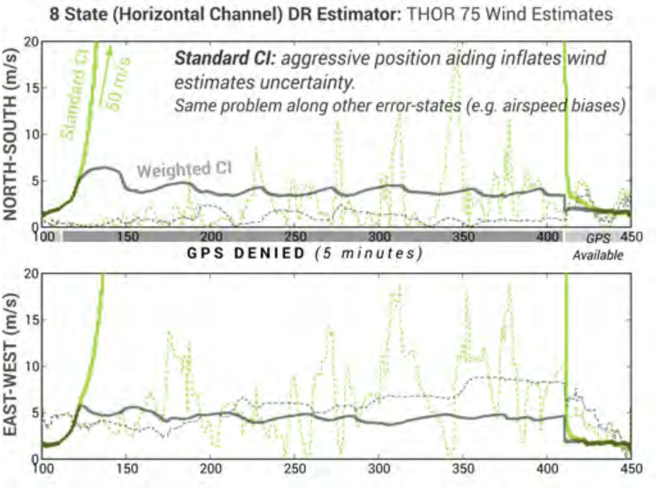
SCENARIO 2: COVARIANCE NORMALIZATION



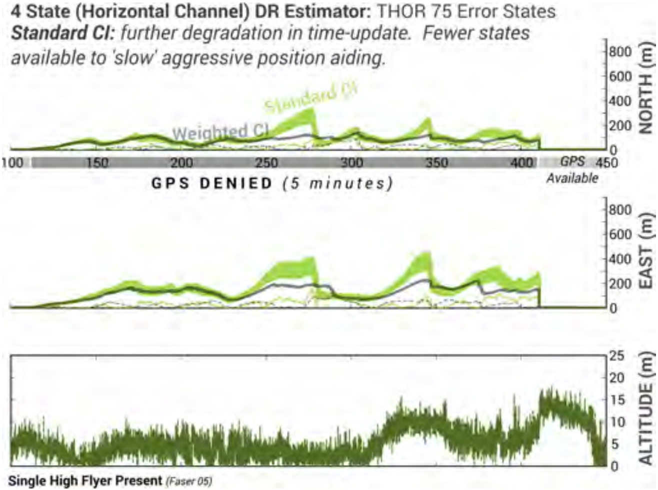
SCENARIO 2: COVARIANCE NORMALIZATION



Scenario 2: Covariance Normalization



SCENARIO 2: COVARIANCE NORMALIZATION

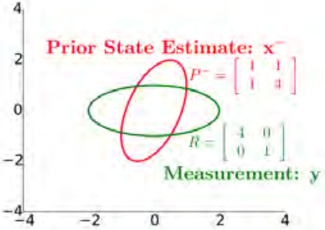


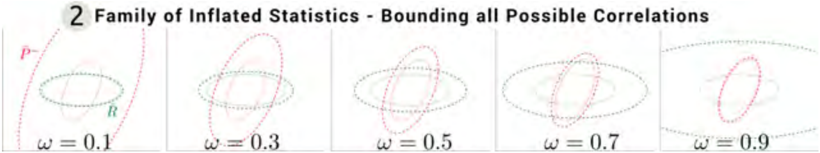
Correlated Data Fusion

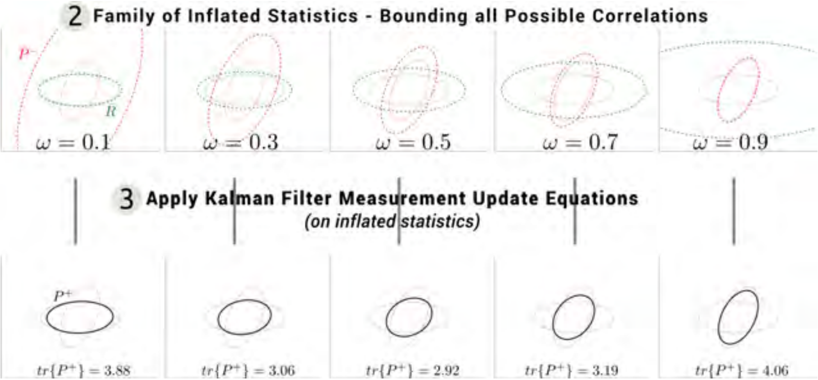
Replace original statistics *(correlation unknown)* with inflated uncorrelated statistics

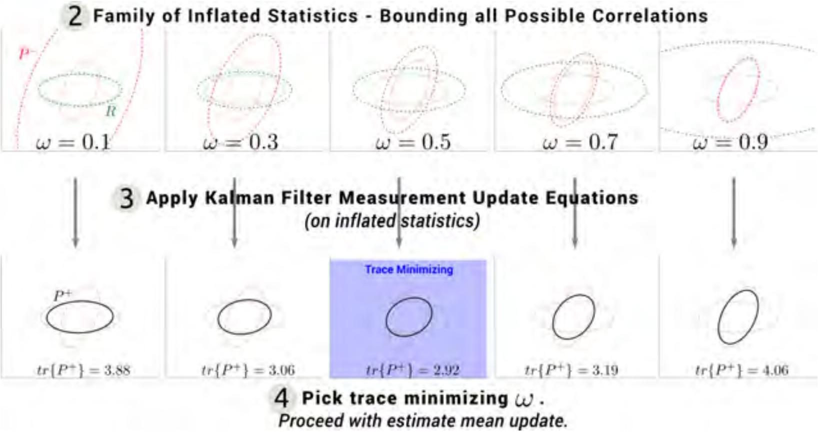
1

$$\begin{bmatrix} P^- & ? \\ ? & R \end{bmatrix} \leq \begin{bmatrix} \bar{P}^- & \mathbf{0} \\ \mathbf{0} & \bar{R} \end{bmatrix}$$




2 Family of Inflated Statistics - Bounding all Possible Correlations
$\omega = 0.1$
$\omega = 0.3$
$\omega = 0.5$
$\omega = 0.7$
$\omega = 0.9$


2 Family of Inflated Statistics - Bounding all Possible Correlations
P^-
R
$\omega = 0.1$
$\omega = 0.3$
$\omega = 0.5$
$\omega = 0.7$
$\omega = 0.9$
3 Apply Kalman Filter Measurement Update Equations
(on inflated statistics)
P^+
$tr\{P^+\} = 3.88$
$tr\{P^+\} = 3.06$
$tr\{P^+\} = 2.92$
$tr\{P^+\} = 3.19$
$tr\{P^+\} = 4.06$


2 Family of Inflated Statistics - Bounding all Possible Correlations
P^-
R
$\omega = 0.1$
$\omega = 0.3$
$\omega = 0.5$
$\omega = 0.7$
$\omega = 0.9$
3 Apply Kalman Filter Measurement Update Equations
(on inflated statistics)
Trace Minimizing
P^+
$tr\{P^+\} = 3.88$
$tr\{P^+\} = 3.06$
$tr\{P^+\} = 2.92$
$tr\{P^+\} = 3.19$
$tr\{P^+\} = 4.06$
4 Pick trace minimizing ω.
Proceed with estimate mean update.

Cost function for N-state *a priori* $\mathbf{x}^-$ and measurement y:

$$J(\omega) = tr\{\mathbf{P}^+\} = \frac{rp_{11}}{\omega r + (1-\omega)p_{11}} + \frac{1}{\omega}\sum_{i=2}^{N} p_{ii}$$

Different Units Implementations

State	Covariance Element	Units Implemented		
North	p_{11}	m	nmi	...
East	p_{22}	m	nmi	...
Heading	p_{33}	rad	deg	...
Clock Bias	p_{44}	s	ms	...
⋮	⋮			
State N	p_{NN}	units A	units B	...

For the same measurement the optimal ω will vary

Results in different fusions between $\mathbf{x}^{-}$ and y. For example:

Proposed Solution: covariance normalization to conduct a meaningful uncertainty trade-off

UNITS AND SCALING

PROPOSED SOLUTION

Prior to forming cost function $J(\omega)$ normalize *a priori* covariance

1. Define linear transformation matrix T:

$$\begin{aligned}\tilde{\mathbf{x}}^- &= T\mathbf{x}^- \\ \tilde{P}^- &= TP^-T^T \\ \tilde{H} &= HT^{-1}\end{aligned}$$

2. Form cost function using $\tilde{P}^-$, R, and $\tilde{H}$

$$J(\omega) = \mathrm{tr}\left\{P^+(\tilde{P}^-, R, \tilde{H}, \omega)\right\}$$

Proceed with the selection of ω using $\tilde{P}^-$, R, and $\tilde{H}$.
The ω found together with original parameters *(not normalized)* used to complete state and covariance update.

UNITS AND SCALING

PROPOSED METHOD FOR PICKING T

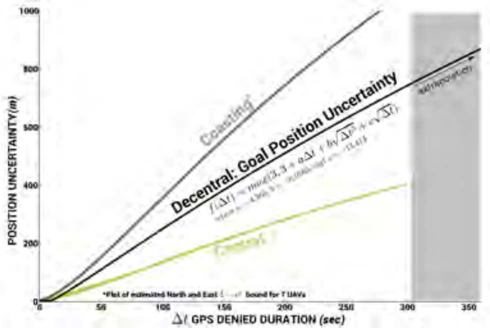
Use Desired Uncertainty Sizes

$$T = \begin{bmatrix} \frac{1}{\sigma_{1_{goal}}} & & & \\ & \frac{1}{\sigma_{2_{goal}}} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_{N_{goal}}} \end{bmatrix}$$

Give slack if uncertainty better than required. Extract information in the directions where the performance is currently below the desired standards.

UNITS AND SCALING

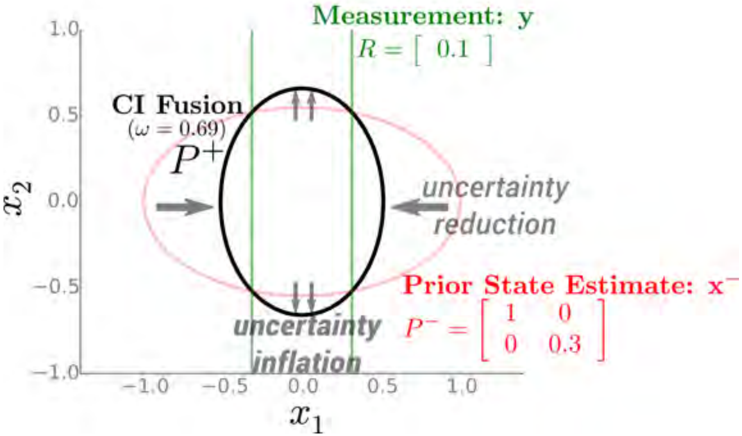
Time-Varying



Fixed

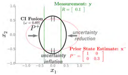
Error-State	Goal Uncertainty	Markov σ_{ss}
Altitude	5 m	-
Wind	8 m/s	5 m/s
Yaw Bias	10°	7.6°
Pitch Bias	5°	2°
Airspeed Bias	5 m/s	2 m/s
Altitude Offset	5 m	4 m

UNCERTAINTY TRADE-OFF



UNCERTAINTY TRADE-OFF

EXPLANATION



Explanation:

Write covariance update equation in *information form*:

$$
\begin{aligned}
P^+ &= \left(\omega \left(P^-\right)^{-1} + (1-\omega)H^T R^{-1} H\right)^{-1} \\
\mathcal{I}^+ &= \omega \mathcal{I}^- + (1-\omega)H^T \mathcal{I}^y H
\end{aligned}
$$

- ω decreased from 1 towards 0 to take advantage of the measurement high-information states
- This will down-weight existing *a priori* high information states.

Uncertainty Trade-off: inflating the uncertainty along one state in order to decrease the uncertainty in another state.

UNCERTAINTY TRADE-OFF

NUMBER OF STATES

N-state estimator
a priori $\mathbf{x}^-$

$$\mathbf{P}^- = \begin{bmatrix} p_{11} & & & \\ & p_{22} & & \\ & & \ddots & \\ & & & p_{NN} \end{bmatrix}$$

Uncertainty Trade-Off

Number of States

N-state estimator
a priori $\mathbf{x}^-$

$$\mathbf{P}^- = \begin{bmatrix} p_{11} & & & \\ & p_{22} & & \\ & & \ddots & \\ & & & p_{NN} \end{bmatrix}$$

Fuse scalar measurement y
$H = [1, 0, \ldots, 0]$ with variance r

$$\mathbf{P}^+ = \left[\omega(\mathbf{P}^-)^{-1} + (1-\omega)r\mathbf{H}^T\mathbf{H}\right]^{-1}$$

UNCERTAINTY TRADE-OFF

NUMBER OF STATES

N-state estimator
a priori $\mathbf{x}^-$

$$\mathbf{P}^- = \begin{bmatrix} p_{11} & & & \\ & p_{22} & & \\ & & \ddots & \\ & & & p_{NN} \end{bmatrix}$$

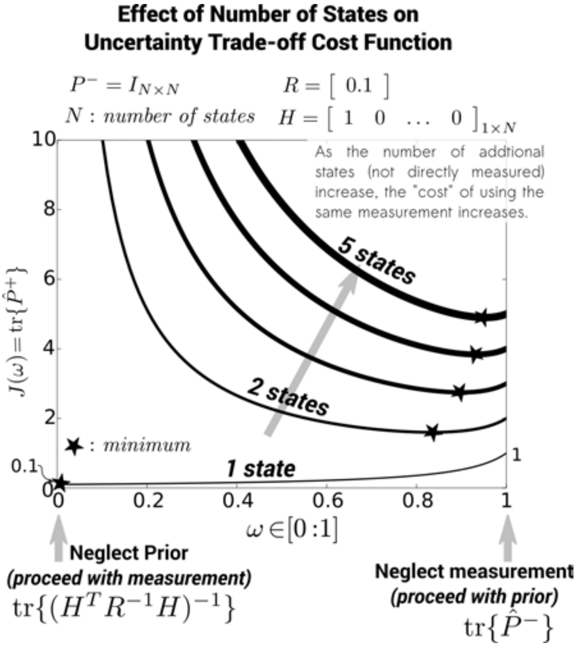
Fuse scalar measurement y
$H = [1, 0, \ldots, 0]$ with variance r

$$\mathbf{P}^+ = \left[\omega(\mathbf{P}^-)^{-1} + (1-\omega)r\mathbf{H}^T\mathbf{H}\right]^{-1}$$

Associated cost function

$$J(\omega) = tr\{\mathbf{P}^+\} = \frac{rp_{11}}{\omega r + (1-\omega)p_{11}} + \frac{1}{\omega}\sum_{i=2}^{N} p_{ii}$$

As N increases, cost function minimum shifts towards $\omega = 1$



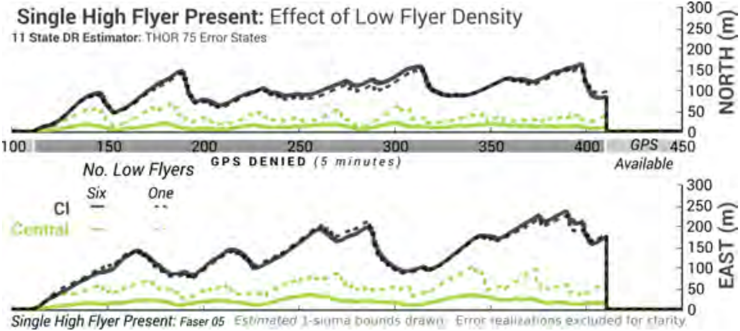
Effect of Number of States on
Uncertainty Trade-off Cost Function
$P^- = I_{N\times N}$
N : number of states
$R = [\ 0.1\ ]$
$H = [\ 1\ \ 0\ \ \dots\ \ 0\ ]_{1\times N}$
As the number of addtional states (not directly measured) increase, the "cost" of using the same measurement increases.
5 states
2 states
1 state
★ : minimum
0.1
1
$J(\omega) = \mathrm{tr}\{\hat{P}^+\}$
$\omega \in [0:1]$
Neglect Prior
(proceed with measurement)
$\mathrm{tr}\{(H^T R^{-1} H)^{-1}\}$
Neglect measurement
(proceed with prior)
$\mathrm{tr}\{\hat{P}^-\}$

UAV Results

Effect of High Flyer density

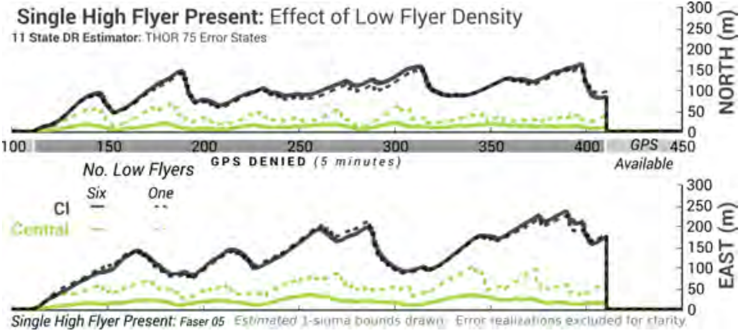
Single High Flyer

Effect of Low Flyer Density



SINGLE HIGH FLYER

EFFECT OF LOW FLYER DENSITY



- **Centralized:** Presence of *average users* are advantageous
- **Decentralized:** more *average users* contributes little or none
  - If small cooperation range, may serve to transmit high flyer information to other low flyers