Correlated-Data Fusion for Networked Navigation: Theory and Application

Hamid Mokhtarzadeh
PhD Advisor: Professor Demoz Gebre-Egziabher

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**COOPERATIVE NAVIGATION**

**Definition**: Two or more vehicles collaborate to calculate a navigation solution with higher quality than would be attainable otherwise.
**COOPERATIVE NAVIGATION**

**Definition:** Two or more vehicles collaborate to calculate a navigation solution with higher quality than would be attainable otherwise.
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Two implementations: **Centralized** and **Decentralized**
COOPERATIVE NAVIGATION

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Cooperative Navigation

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**COOPERATIVE NAVIGATION**

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COOPERATIVE NAVIGATION

CHALLENGES

Centralized

- Correlations are maintained
- Communication and infrastructure burden

Decentralized

+ Flexible implementation
- No knowledge of correlations

Estimator must handle unknown correlations
**What is Correlation?**

\[ \hat{x}_i = \text{State.} \]

\[ y_{ij} = \text{Measurement.} \]

\[ P_{xy} = \text{Correlation.} \]
Error Loops

Imposing communication restrictions can prevent or delay the formation of error loops.

- Reduced flexibility.
Problem Statement

1. What is a suitable correlated data fusion filter for decentralized cooperative aiding?

2. Case study: GPS/GNSS-denied navigation for small UAVs.
   2.1 What is the concept of operation when dealing with a GPS/GNSS outage?
   2.2 What is the impact of limited communication bandwidth: decentralized performance?
   2.3 Does it make sense to consider centralized implementations? If so, what is the expected performance?
Prior Work

Kalman Filter Order Reduction
Handling lost correlation
Shah 1971

Cooperative Navigation
JTIDs
Fried et al. 1979
Source selection
Rome et al. 1977
Stability issues
Gobbini 1981

Simultaneous Localization And Mapping
Importance of correlations
Castellanos et al. 1997
Distributed centralized implementation
Roumeliotis et al. 2002

Correlated Data Fusion Filters
Covariance Intersection
Julier and Uhlmann 1997
Bounded Covariance Inflation
Hanebeck et al. 2001
Reece and Roberts 2005

Covariance Intersection and SLAM
Julier and Uhlmann 2006
Multi-robot Localization
Carrillo-Arce et al. 2013
## Prior Work

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<td></td>
<td>Distributed centralized implementation</td>
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Correlated Data Fusion
Correlated Data Fusion

1. \( \hat{x}^- \) *a priori* state estimate (covariance \( P^- \))
2. \( y \) measurement (covariance \( R \))
3. \( \hat{x}^+ \) *a posteriori* state estimate (covariance \( P^+ \))

where the correlation between them, \( P_{xy} \), is unknown.
**Correlated Data Fusion**

1. $\hat{x}^-$ *a priori* state estimate (covariance $P^-$)
2. $y$ measurement (covariance $R$)
3. $\hat{x}^+$ *a posteriori* state estimate (covariance $P^+$)

where the correlation between them, $P_{xy}$, is unknown.

**Strategy:** Seek an uncorrelated matrix which bounds the true joint covariance:

$$
\begin{bmatrix}
\bar{P}^- & 0 \\
0 & \bar{R}
\end{bmatrix} \geq
\begin{bmatrix}
P^- & P_{xy} \\
P_{yx} & R
\end{bmatrix}
$$

Result:

1. an unbiased estimate $\hat{x}^+$ and (2) a covariance $\hat{P}^+$ which is an overbound of the true covariance $P^+$, i.e., covariance computed if $P_{xy}$ was known.

$\Rightarrow$ Conservative.
Correlated Data Fusion

1. $\hat{x}^-$  a priori state estimate (covariance $P^-$)
2. $y$  measurement (covariance $R$)
3. $\hat{x}^+$  a posteriori state estimate (covariance $P^+$)

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P_{yx} & R
\end{bmatrix}
$$

**Result:** (1) an unbiased estimate $\hat{x}^+$ and (2) a covariance $\hat{P}^+$ which is an overbound of the true covariance $P^+$

$\Rightarrow$ *Conservative.*
Decentralized Estimators

Similar form as the Kalman Filter except:

- (1) Scalar weight, $\omega$ and (2) Correlation bound, $r_{\text{max}}$.

$$K = \bar{P}^{-1}H^T(H\bar{P}^{-1}H^T + \bar{R})^{-1}$$  \hspace{1cm} \text{(Gain calculation)}

$$x^+ = x^- + K(y - Hx^-)$$ \hspace{1cm} \text{(State update)}

$$P^+ = (I - KH)\bar{P}^{-1}(I - KH)^T + K\bar{R}K^T$$ \hspace{1cm} \text{(Covariance update)}

where $\bar{P}^{-}$, $\bar{R}$ are assumed covariance statistics.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Assumption</th>
<th>$\bar{P}^{-}$</th>
<th>$\bar{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KF/EKF</td>
<td>No correlation</td>
<td>$P_{xy} = 0$</td>
<td>$P^{-}$</td>
</tr>
<tr>
<td>CI</td>
<td>Any allowable correlation</td>
<td>any valid $P_{xy}$</td>
<td>$\frac{1}{\omega}P^{-}$</td>
</tr>
<tr>
<td>BCInf</td>
<td>Bounded correlation</td>
<td>$r_{\text{max}}^2I \geq C_{yx}C_{yx}^T$ \hspace{1cm} \frac{\omega + (1-\omega)r_{\text{max}}}{\omega} \bar{P}^{-}$</td>
<td>$\frac{1+\omega(r_{\text{max}}-1)}{1-\omega}R$</td>
</tr>
</tbody>
</table>

$\omega \in [0, 1]$ : scalar optimization parameter selected to minimize $P^+$

$r_{\text{max}}$ : maximum singular value of the matrix of correlation coefficients ($C_{yx}$)
UAV Case Study Results
**Flight Scenario**

Data @ http://conservancy.umn.edu/handle/11299/165567

7 UAVs cover 1 km² area
Three airframes, flights from 2011 − 2012
Source: www.uav.aem.umn.edu
On-Board Navigation

- Reference: logged 50 Hz INS/GPS
- Assumed though simulation *play-back*
  - Equipped with cross-ranging radio modems (1 Hz, 1 − σ ranging accuracy of 5 m)
  - Broadcasts estimated location and covariance at 1 Hz (like ADS-B)

<table>
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<th>On-Board Sensors</th>
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</thead>
<tbody>
<tr>
<td>3 axis accelerometer</td>
</tr>
<tr>
<td>3 axis gyroscope</td>
</tr>
<tr>
<td>3 axis magnetometer</td>
</tr>
<tr>
<td>airspeed (<em>pitot-probe</em>)</td>
</tr>
<tr>
<td>baro-altimeter</td>
</tr>
<tr>
<td>GPS position and <em>velocity</em></td>
</tr>
</tbody>
</table>
On-Board Navigation

- Reference: logged 50 Hz INS/GPS
- Assumed though simulation play-back
  - Equipped with cross-ranging radio modems (1 Hz, 1 − σ ranging accuracy of 5 m)
  - Broadcasts estimated location and covariance at 1 Hz (like ADS-B)

On-Board Sensors

- 3 axis accelerometer
- 3 axis gyroscope
- 3 axis magnetometer
- Airspeed (pitot-probe)
- Baro-altimeter
- GPS position and velocity
# On-Board Navigation

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta N$, $\Delta E$, $\Delta D$</td>
<td>North-East-Down position error</td>
</tr>
<tr>
<td>$W_{NS}$, $W_{EW}$</td>
<td>North-South and East-West wind</td>
</tr>
<tr>
<td>$\Delta \psi$, $\Delta \theta$</td>
<td>Yaw-error and pitch-error</td>
</tr>
<tr>
<td>$\Delta u$, $\Delta v$, $\Delta w$</td>
<td>Body-axes velocity measurement errors</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>Offset between geometric-pressure altitude (e.g. GPS vs baro altitude)</td>
</tr>
</tbody>
</table>

![Diagram of GNSS-Denied Navigation](image.png)
## Operational Concept

<table>
<thead>
<tr>
<th>Scenario</th>
<th>GPS Denied</th>
<th>GPS Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No High Flyer</td>
<td>7 UAVs</td>
<td>0</td>
</tr>
<tr>
<td>2. One High Flyer:</td>
<td>6 UAVs</td>
<td>1 UAV</td>
</tr>
<tr>
<td>3. Two High Flyers:</td>
<td>5 UAVs</td>
<td>2 UAVs</td>
</tr>
</tbody>
</table>
Time: 13:00   GPS: AVAILABLE
Time: 14.00  GPS: AVAILABLE
Time: 20.00   GPS: AVAILABLE
INTRODUCTION

CORRELATED DATA FUSION

RESULTS

CONCLUSION
Time: 28.00  GPS: AVAILABLE
Time: 37.00  GPS: AVAILABLE
Time: 44.00  GPS: AVAILABLE
Time: 58.00  GPS: AVAILABLE
Time: 62.00  GPS: AVAILABLE
Time: 65.00  GPS: AVAILABLE
Time: 68.00  GPS: AVAILABLE
Time: 69.00  GPS: AVAILABLE
Time: 71.00  GPS: AVAILABLE
Time: 74.00  GPS: AVAILABLE
Time: 75.00   GPS: AVAILABLE
Time: 76.00  GPS: AVAILABLE
Time: 77.00   GPS: AVAILABLE
Time: 78.00  GPS: AVAILABLE
Time: 79.00  GPS: AVAILABLE
Time: 81.00  GPS: AVAILABLE
Time: 84.00  GPS: AVAILABLE

Altitude (m): 240 - 350
Time: 86.00   GPS: AVAILABLE
Time: 91.00  GPS: AVAILABLE

University of Minnesota
Driven to Discover
Time: 92.00   GPS: AVAILABLE
Time: 93.00  GPS: AVAILABLE
Time: 98.00  GPS: AVAILABLE
Time: 105.00  GPS: AVAILABLE
Time: 111.00  GPS: DENIED
Time: 112.00   GPS: DENIED
Time: 116.00  GPS: DENIED

Altitude (m)
Time: 117.00   GPS: DENIED
Time: 118.00  GPS: DENIED
Time: 119.00  GPS: DENIED
Time: 121.00   GPS: DENIED
Time: 125.00    GPS: DENIED
Introduction

Correlated Data Fusion

Results

Conclusion
Time: 130.00  GPS: DENIED
Time: 131.00  GPS: DENIED
Time: 135.00  GPS: DENIED

Altitude (m)
Time: 136.00  GPS: DENIED
Time: 143.00  GPS: DENIED
Time: 144.00  GPS: DENIED
Time: 146.00  GPS: DENIED
Time: 147.00   GPS: DENIED

[Diagram showing various circular data points with lines connecting them]

Altitude (m)
240
260
280
300
320
340
360
Time: 152.00  GPS: DENIED
Time: 153.00   GPS: DENIED
Time: 154.00   GPS: DENIED
Time: 155.00   GPS: DENIED
Time: 157.00   GPS: DENIED
Time: 158.00   GPS: DENIED
Time: 159.00  GPS: DENIED
Time: 160.00  GPS: DENIED
Time: 161.00  GPS: DENIED
Time: 163.00  GPS: DENIED
Time: 165.00   GPS: DENIED
Time: 166.00   GPS: DENIED

[Graph depicting multiple data points over time and altitude]
Time: 167.00   GPS: DENIED
Time: 169.00  GPS: DENIED
Time: 171.00  GPS: DENIED
Time: 173.00  GPS: DENIED
Time: 175.00  GPS: DENIED
Time: 177.00   GPS: DENIED
Time: 181.00  GPS: DENIED
Time: 183.00   GPS: DENIED
Time: 186.00  GPS: DENIED
Time: 187.00   GPS: DENIED
**RESULTS**

Time: 188.00  GPS: DENIED

[Graph showing data fusion results with overlapping circles and altitude graphs]
Time: 189.00  GPS: DENIED
Time: 191.00  GPS: DENIED

[Graph showing data fusion results]
Time: 192.00  GPS: DENIED
Time: 196.00   GPS: DENIED
Time: 197.00   GPS: DENIED
Scenario 1: No High Flyer
Time: 200.00   GPS: DENIED
Scenario 2: Single High Flyer
Uncertainty Trade-Off

\[
R = \begin{bmatrix} 0.1 \end{bmatrix}
\]

\[
P^+ = \begin{bmatrix} 1 & 0 \\ 0 & 0.3 \end{bmatrix}
\]

\[
prior\ State\ Estimate: \ x^-\]

\[
certainty\ inflation\]

\[
certainty\ reduction\]

\[
CI\ Fusion\ (\omega = 0.69)\]

\[
Measurement: \ y
\]}
**Scenario 2: Covariance Normalization**

11 State DR Estimator: THOR 75 Error States
*Standard CI:* improper uncertainty trade-off leads to unstable vertical channel, destabilizing filter.
SCENARIO 3: TWO HIGH FLYERS
SCENARIO 3: TWO HIGH FLYERS

Investing in maintained presence of High Flyers better than centralized implementation!
CONCLUSION

- Cooperative Navigation can mitigate GNSS-outage
- **Centralized Implementation:** advantageous for predefined community
- **Decentralized Implementation:** suited for large and dynamic community
  - covariance intersection + covariance normalization
  - requires presence of high-quality users
Future Work

- Real-time implementation - handling delays
- Fault detection and integrity monitoring for decentralized implementations
- Investigate other bounding techniques
  - More degrees of freedom may help overcome uncertainty trade-off challenge

\[
\bar{P} = \frac{1}{\omega} P \quad \bar{R} = \frac{1}{1-\omega} R
\]

Correlated data fusion estimators have expanded applications!

Thank you!
**Contribution:** Enable engineer, familiar with standard Kalman Filter, to determine whether CI/BCInf might work in their application.

- Certain properties are surprising or counter-intuitive

**Topics Covered**

1. Inherent *uncertainty trade-off*
   - effect of number of states on trade-off
2. Necessity of units normalization
3. BCInf and choosing correlation bound $r_{max}$
SCENARIO 2: COVARIANCE NORMALIZATION

11 State DR Estimator: THOR 75 Error States
Standard CI: improper uncertainty trade-off leads to unstable vertical channel, destabilizing filter.

GPS DENIED (5 minutes)

Single High Flyer Present (Faser 05)
SCENARIO 2: COVARIANCE NORMALIZATION

8 State (Horizontal Channel) DR Estimator: THOR 75 Error States

Standard CI: stable, though aggressive position aiding inflates wind and airspeed uncertainty estimates and causes poor time-update.

GPS DENIED (5 minutes)

Zoom

Single High Flyer Present (Faser 05)
Scenario 2: Covariance Normalization
Scenario 2: Covariance Normalization

4 State (Horizontal Channel) DR Estimator: THOR 75 Error States
Standard CI: further degradation in time-update. Fewer states available to 'slow' aggressive position aiding.

GPSDenied (5 minutes)
Correlated Data Fusion
Replace original statistics (correlation unknown) with inflated uncorrelated statistics.
Family of Inflated Statistics - Bounding all Possible Correlations

- $\omega = 0.1$
- $\omega = 0.3$
- $\omega = 0.5$
- $\omega = 0.7$
- $\omega = 0.9$
**Introduction**

**Correlated Data Fusion**

**Results**

**Conclusion**

---

**Family of Inflated Statistics - Bounding all Possible Correlations**

$\omega = 0.1$

$\omega = 0.3$

$\omega = 0.5$

$\omega = 0.7$

$\omega = 0.9$

**Apply Kalman Filter Measurement Update Equations**

(on inflated statistics)

$tr\{P^+\} = 3.88$

$tr\{P^+\} = 3.06$

$tr\{P^+\} = 2.92$

$tr\{P^+\} = 3.19$

$tr\{P^+\} = 4.06$
2. Family of Inflated Statistics - Bounding all Possible Correlations

\[ \omega = 0.1, \quad \omega = 0.3, \quad \omega = 0.5, \quad \omega = 0.7, \quad \omega = 0.9 \]

3. Apply Kalman Filter Measurement Update Equations (on inflated statistics)

\[ \text{trace} \{P^+\} = 3.88, \quad \text{trace} \{P^+\} = 3.06, \quad \text{trace} \{P^+\} = 2.92, \quad \text{trace} \{P^+\} = 3.19, \quad \text{trace} \{P^+\} = 4.06 \]

4. Pick trace minimizing \( \omega \).
   Proceed with estimate mean update.
Cost function for $N$-state \textit{a priori} $x^-$ and measurement $y$:

$$J(\omega) = tr\{P^+\} = \frac{rp_{11}}{\omega r + (1 - \omega)p_{11}} + \frac{1}{\omega} \sum_{i=2}^{N} p_{ii}$$

### Different Units Implementations

<table>
<thead>
<tr>
<th>State</th>
<th>Covariance Element</th>
<th>Units Implemented</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>$p_{11}$</td>
<td>$m$ $nmi$</td>
</tr>
<tr>
<td>East</td>
<td>$p_{22}$</td>
<td>$m$ $nmi$</td>
</tr>
<tr>
<td>Heading</td>
<td>$p_{33}$</td>
<td>$rad$ $deg$</td>
</tr>
<tr>
<td>Clock Bias</td>
<td>$p_{44}$</td>
<td>$s$ $ms$</td>
</tr>
<tr>
<td>...</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>State $N$</td>
<td>$p_{NN}$</td>
<td>units A units B</td>
</tr>
</tbody>
</table>

For the same measurement \textbf{the optimal $\omega$ will vary}
Results in different fusions between $x^-$ and $y$. For example:

**Proposed Solution:** covariance normalization to conduct a meaningful uncertainty trade-off
UNITS AND SCALING
PROPOSED SOLUTION

Prior to forming cost function $J(\omega)$ normalize *a priori* covariance

1. Define linear transformation matrix $T$:

   \[
   \tilde{x}^- = Tx^-
   \]
   \[
   \tilde{P}^- = TP^-T^T
   \]
   \[
   \tilde{H} = HT^{-1}
   \]

2. Form cost function using $\tilde{P}^-, R, \tilde{H}$

   \[
   J(\omega) = \text{tr} \left\{ P^+(\tilde{P}^-, R, \tilde{H}, \omega) \right\}
   \]

Proceed with the selection of $\omega$ using $\tilde{P}^-, R, \tilde{H}$. The $\omega$ found together with original parameters (*not normalized*) used to complete state and covariance update.
Use Desired Uncertainty Sizes

$T = \begin{bmatrix}
\frac{1}{\sigma_{1\text{goal}}} \\
\frac{1}{\sigma_{2\text{goal}}} \\
\vdots \\
\frac{1}{\sigma_{N\text{goal}}}
\end{bmatrix}$

Give slack if uncertainty better than required. Extract information in the directions where the performance is currently below the desired standards.
**Units and Scaling**

**Time-Varying**

- Altitude: 5 m
- Wind: 8 m/s
- Yaw Bias: 10°
- Pitch Bias: 5°
- Airspeed Bias: 5 m/s
- Altitude Offset: 5 m

**Fixed**

- Goal Uncertainty
  - Altitude: 5 m
  - Wind: 8 m/s
  - Yaw Bias: 10°
  - Pitch Bias: 5°
  - Airspeed Bias: 5 m/s
  - Altitude Offset: 5 m

**Markov**
- $\sigma_{ss}$
- -
- 5 m/s
- 7.6°
- 2°
- 2 m/s
- 4 m
Uncertainty Trade-Off
Explanation:
Write covariance update equation in information form:

\[
\begin{align*}
    p^+ &= \left( \omega \left( p^- \right)^{-1} + (1 - \omega)H^TR^{-1}H \right)^{-1} \\
    \mathcal{I}^+ &= \omega\mathcal{I}^- + (1 - \omega)H^T\mathcal{I}^yH
\end{align*}
\]

- \( \omega \) decreased from 1 towards 0 to take advantage of the measurement high-information states

- This will down-weight existing a priori high information states.

Uncertainty Trade-off: inflating the uncertainty along one state in order to decrease the uncertainty in another state.
UNCERTAINTY TRADE-OFF

NUMBER OF STATES

N-state estimator

\( a \text{ priori } x^- \)

\[
P^- = \begin{bmatrix} p_{11} & p_{22} & \cdots & p_{1N} \\ & p_{22} & \cdots & \vdots \\ & & \cdots & \vdots \\ & & & p_{NN} \end{bmatrix}
\]
UNCERTAINTY TRADE-OFF

NUMBER OF STATES

N-state estimator
\( a \text{ priori } x^- \)

Fuse scalar measurement \( y \)
\( H = [1, 0, \ldots, 0] \) with variance \( r \)

\[
\begin{align*}
\mathbf{P}^- &= \begin{bmatrix}
  p_{11} & & \\
  & p_{22} & \\
  & & \ddots & \ddots \\
  & & & p_{NN}
\end{bmatrix} \\
\mathbf{P}^+ &= \left[ \omega (\mathbf{P}^-)^{-1} + (1 - \omega) r \mathbf{H}^T \mathbf{H} \right]^{-1}
\end{align*}
\]
Uncertainty Trade-Off
Number of States

N-state estimator
a priori $x^-$

Fuse scalar measurement $y$
$H = [1, 0, \ldots, 0]$ with variance $r$

Associated cost function

$$J(\omega) = tr\{P^+\} = \frac{rp_{11}}{\omega r + (1 - \omega)p_{11}} + \frac{1}{\omega} \sum_{i=2}^{N} p_{ii}$$

As $N$ increases, cost function minimum shifts towards $\omega = 1$
Effect of Number of States on Uncertainty Trade-off Cost Function

\[ \begin{align*}
    P^- &= I_{N \times N} \\
    R &= \begin{bmatrix} 0.1 \end{bmatrix} \\
    H &= \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}_{1 \times N}
\end{align*} \]

As the number of additional states (not directly measured) increase, the "cost" of using the same measurement increases.

- 1 state
- 2 states
- 5 states

\[ J(\omega) = \text{tr}\{\hat{P}^+\} \]

Neglect Prior
(proceed with measurement)
\[ \text{tr}\{(H^T R^{-1} H)^{-1}\} \]

Neglect measurement
(proceed with prior)
\[ \text{tr}\{\hat{P}^-\} \]}
UAV Results

Effect of High Flyer density
**Single High Flyer**

**Effect of Low Flyer Density**
**Single High Flyer**

**Effect of Low Flyer Density**

- **Centralized:** Presence of *average users* are advantageous
- **Decentralized:** more *average users* contributes little or none
  - If small cooperation range, may serve to transmit high flyer information to other low flyers