

# GPS, Flying Clocks and Fun with Relativity

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Tom Van Baak (tvb)

[www.LeanSecond.com](http://www.LeanSecond.com)

*“Time Nut”*

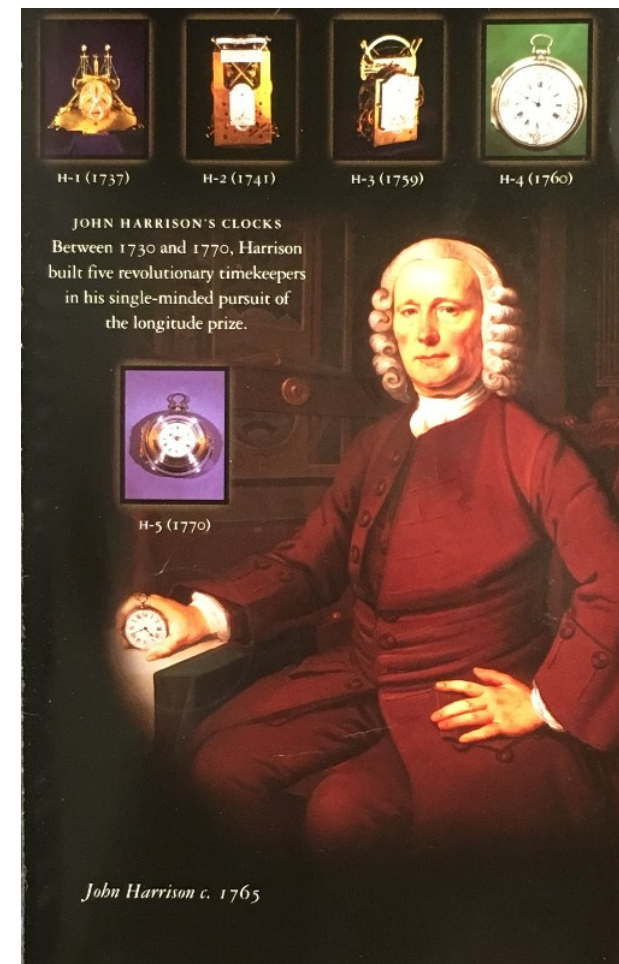
# Outline

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- Part 1 – Navigation and traveling clocks
  - clocks, timing, the ‘T’ in PNT
- Part 2 – GPS and relativity (time dilation)
  - so good it needs clock corrections
- Part 3 – Atomic clock collecting hobby
  - going to extremes at home
- Part 4 – Project GREAT
  - a DIY gravitational time dilation experiment

# Chronometers at sea

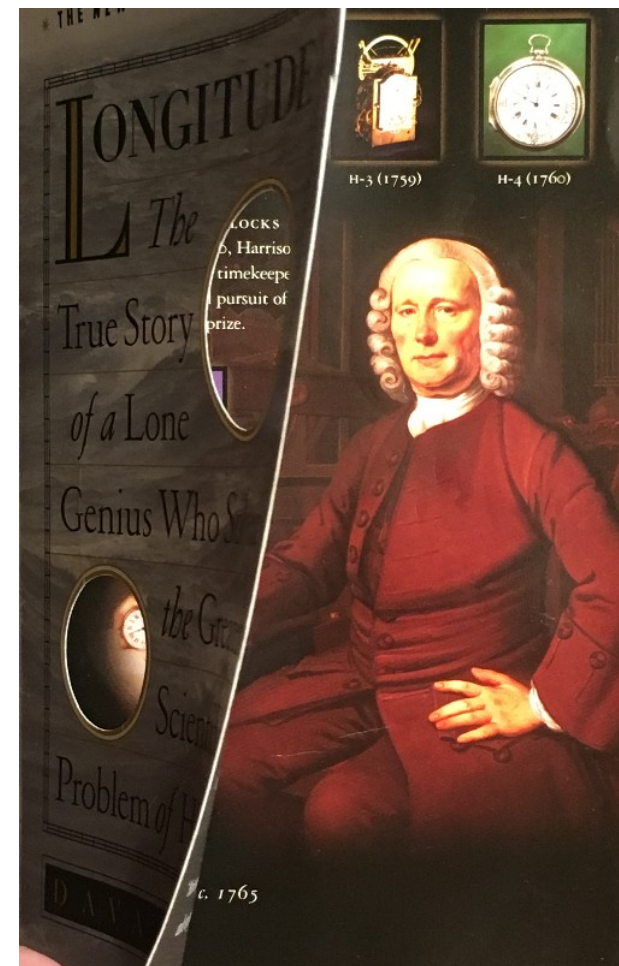
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  - John Harrison
- Why all the clocks?
  - genius clockmaker
  - marine chronometer, accurate
- Longitude prize winner
  - 1759 (250+ years ago)
  - he put the “T” in PNT
- Read.the.book
  - by Dava Sobel



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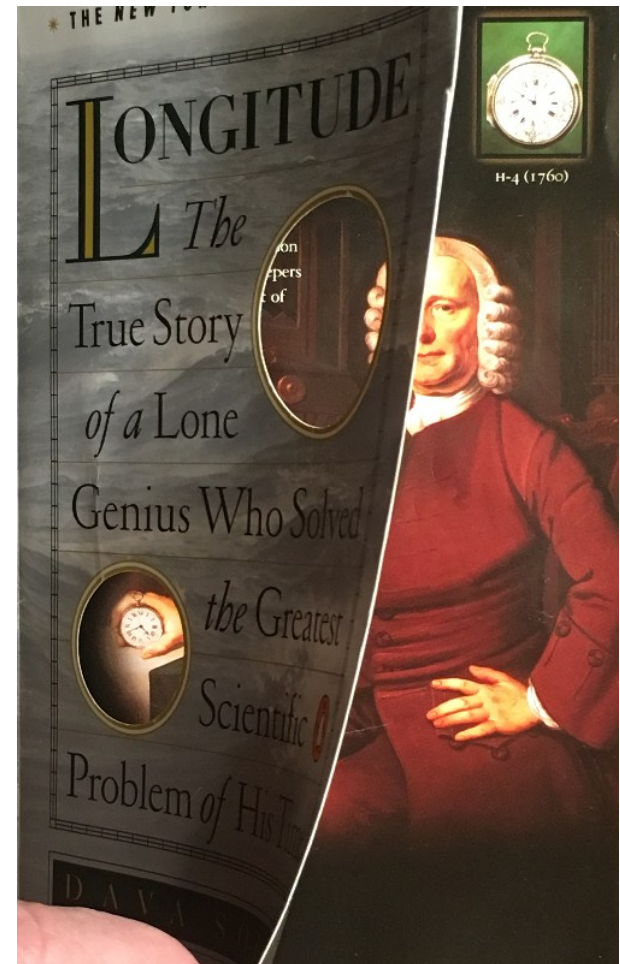




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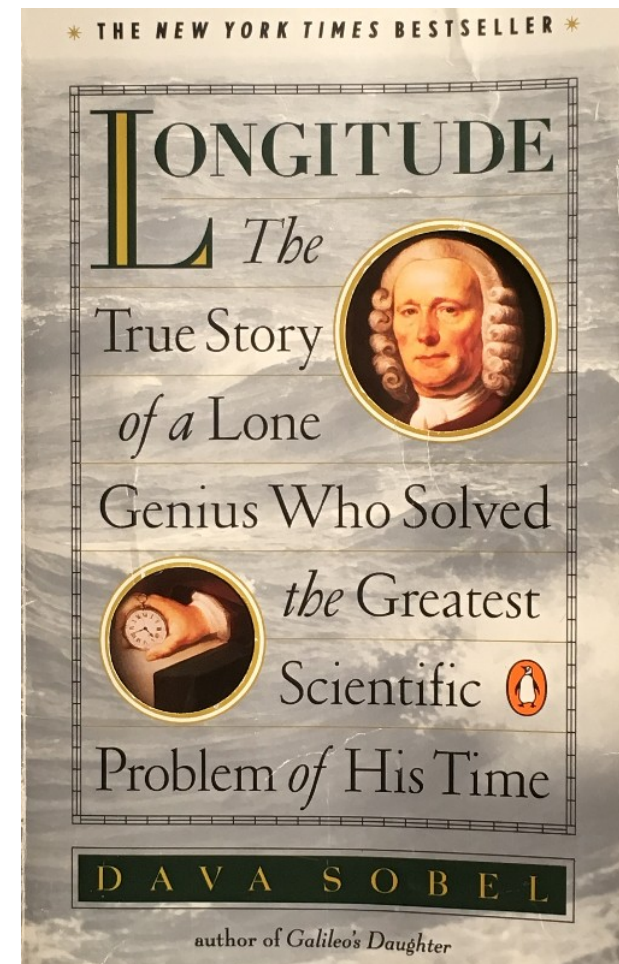
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# Cesium “chronometers” by air

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- Who is this?
  - Len Cutler, *hp* clockmaker
- See the clock?
  - *hp* 5060A cesium atomic
  - batteries & “digital” clock
- Mission?
  - time synchronization
  - cross-country
  - round-the-world
  - 1960’s “flying clock” era



# Cesium “chronometers” by land

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- Who is this?
  - *hp* field engineer
- See the clock?
  - *hp* 5061A cesium
  - atomic clock
- Marketing ad
  - year 1967
  - self-contained
  - portable, rugged
  - accurate  $1\mu\text{s}/\text{month}$



# Flying clocks around-the-world

---

- Who is this?
  - J.Hafele & R.Keating
- See the clocks?
  - 4 @ hp 5061A
  - backup DC power
  - time interval counter
- Relativity experiment
  - year 1971
  - commercial flights
  - RTW, twice! (6 days)





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# Ultimate “flying” clocks: spacecraft

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- GP-A
- NTS-2
- NAVSTAR / GPS



# First space test was GP-A

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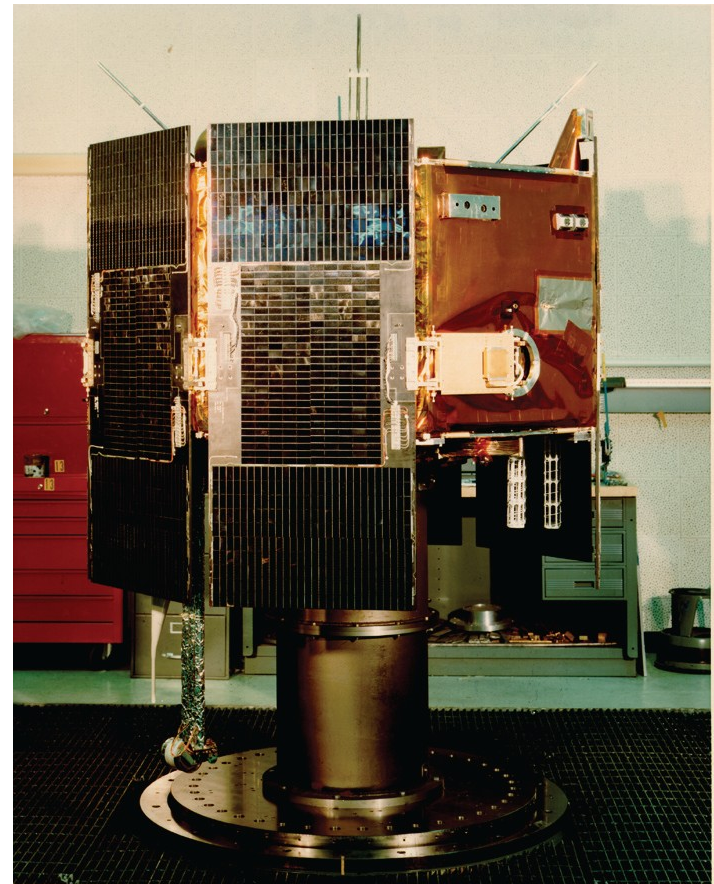
- Gravity Probe “A”
- 1976, first H-maser in space
  - Robert Vessot, clockmaker
- Successful test of relativity
  - science mission
  - launched to 10,000 km
  - 2 hour flight up / down
  - 60 ppm accuracy



# First orbital test was NTS-2

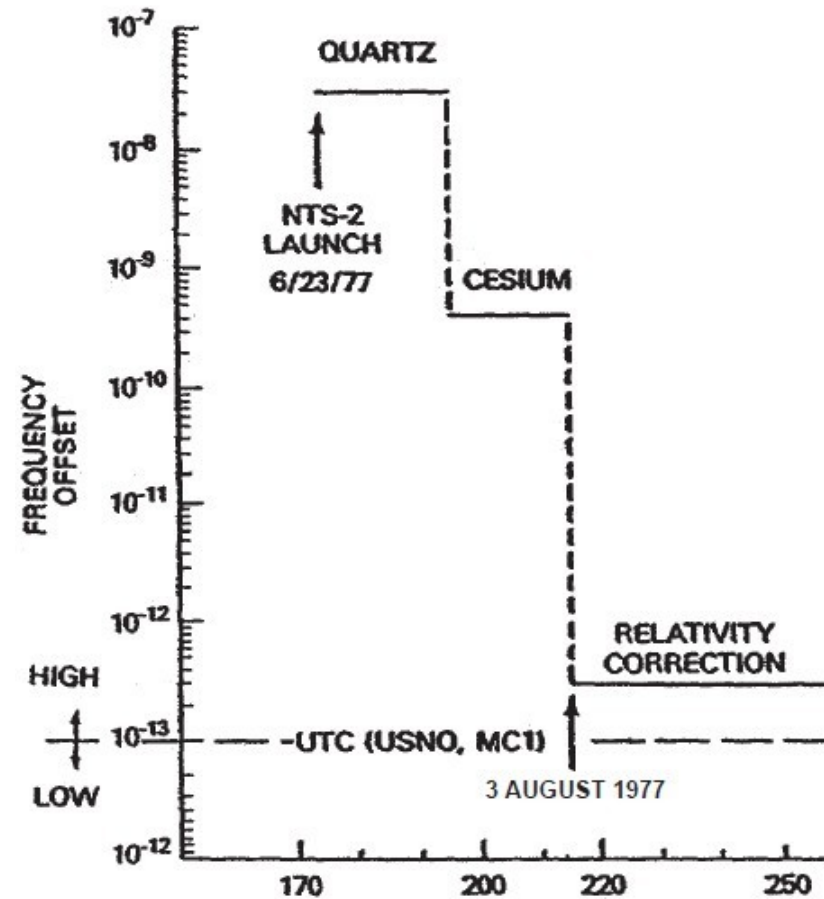
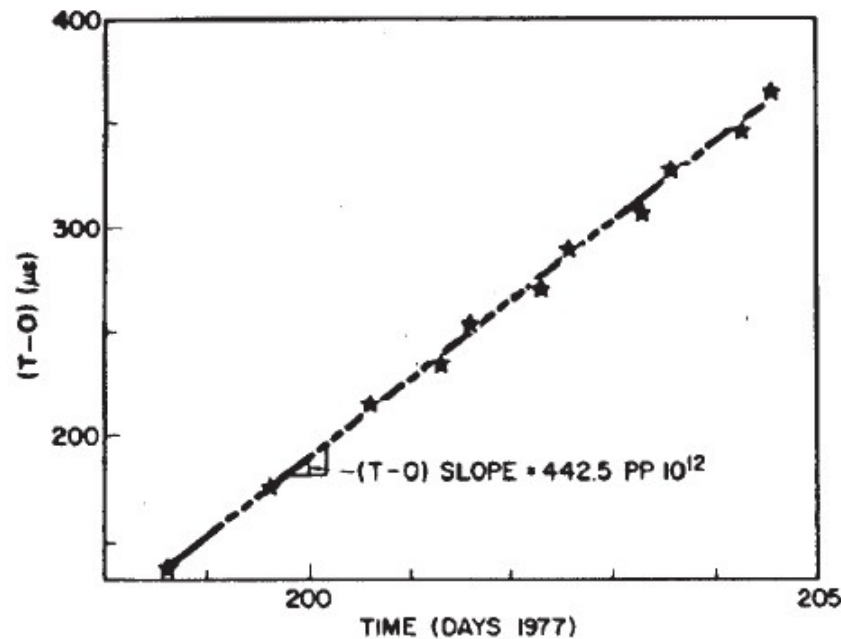
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- “Navigation Technology Satellite”
- 1977, first cesium in orbit
  - Robert Kern, clockmaker
- Successful test of relativity
  - pre-NAVSTAR (GPS)
  - 12 hour orbit
  - similar relativity as GPS
  - cesium clocks, synthesizer



# Relativity in NTS-2

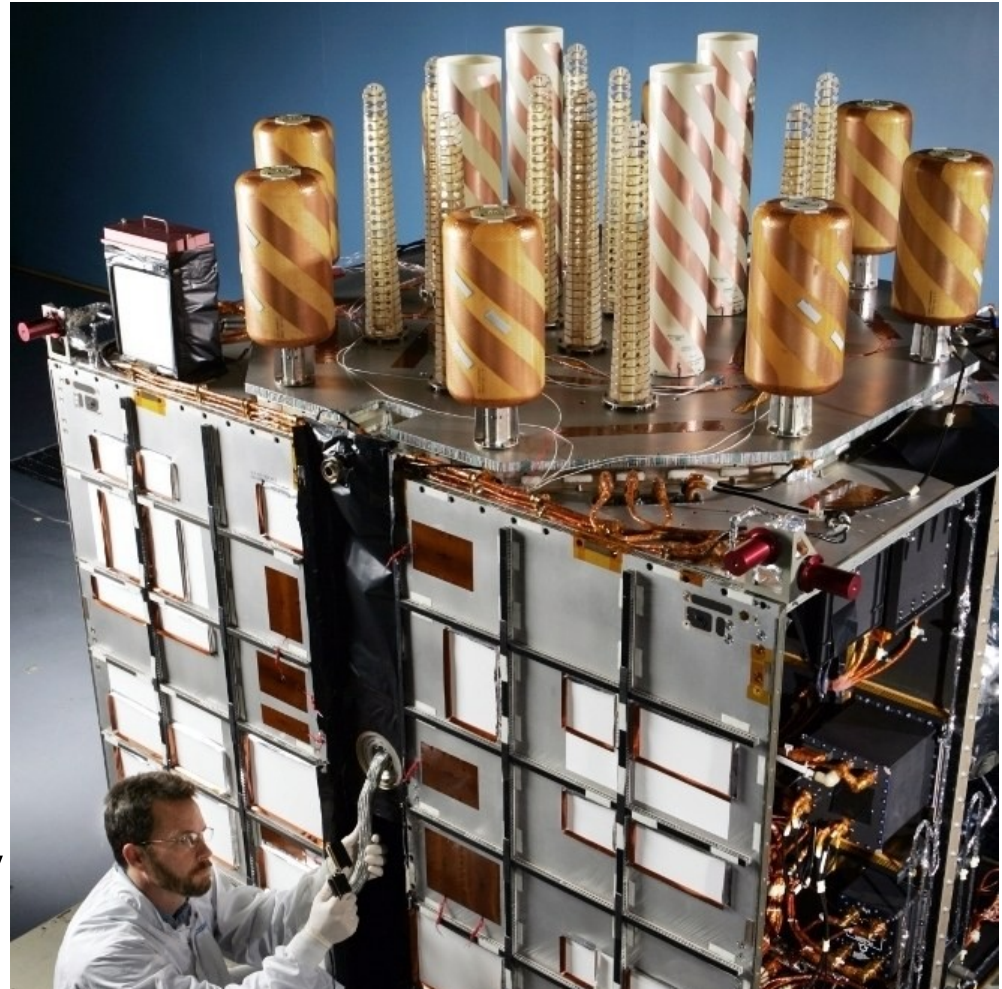
- Plots for NTS-2
- $4.425 \times 10^{-10} = 38 \mu\text{s/day}$



# 32 atomic clocks in space: GPS

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- What's this?
  - GPS IIR-M
  - present era
- 250 year evolution
  - from Harrison to GPS
  - now many GNSS
- Clock accuracy
  - from seconds/day to nanoseconds/day





# Relativity in GPS

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- GPS orbits are extreme:
  - speed 14,000 kph, 8,700 mph (Mach 12)
  - altitude 20,000 km, 12,500 miles ( $\sim 3 \times R_e$ )
  - N.B. GPS speed is only  $0.000013 c$ , “13  $\mu c$ ”
- Large relativistic effects occur:
  - $\Delta f/f$  is  $-8.4 \times 10^{-11}$ , or  $-7.3 \mu s/day$  (kinematic)
  - $\Delta f/f$  is  $+5.3 \times 10^{-10}$ , or  $+45.6 \mu s/day$  (gravitational)
  - $\Delta f/f$  is  $+4.4 \times 10^{-10}$ , or  $+38.3 \mu s/day$  (net effect)
  - 1.023 MHz set to 10.2299999954326 MHz

# How small is $4 \times 10^{-10}$ ?

---

- Relativity correction:

$4.4647 \times 10^{-10}$  (fractional frequency offset)

38.575  $\mu\text{s}$  / day (time interval ratio units)

~1 ms / month

1 s / ~71 years

- In “newspaper” units:

~6 inches / distance to Moon

~1 atom / 1 meter

~1.4  $\text{m}^2$  / area of Rhode Island

~1  $\text{cm}^3$  / volume of Olympic swimming pool

# Relativity at human scale: SR

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- GPS velocity  
14,000 kph, 8,700 mph (Mach 12)  
 $\Delta f/f$  is  $-8.4 \times 10^{-11}$ , or  $-7.3 \mu\text{s/day}$  (kinematic)
- Human-scale velocity  
 $-3 \times 10^{-13}$ ,  $0.02 \mu\text{s/day}$  – flying (500 mph)  
 $-4 \times 10^{-15}$ ,  $0.0003 \mu\text{s/day}$  – driving (60 mph)  
 $-1 \times 10^{-17}$ ,  $0.00000008 \mu\text{s/day}$  – walking (3 mph)



# Relativity at human scale: GR

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- GPS altitude  
20,000 km, 12,500 miles ( $\sim 3 \times R_e$ )  
 $\Delta f/f$  is  $+5.3 \times 10^{-10}$ , or **+45.6**  $\mu\text{s/day}$  (gravitational)
- Human-scale altitude  
 $+9.6 \times 10^{-13}$ , 0.083  $\mu\text{s/day}$  – Mt. Everest (8848 m)  
 $+1.7 \times 10^{-13}$ , **0.015**  $\mu\text{s/day}$  – Denver, CO (1 mile)  
 $+9.5 \times 10^{-15}$ , 0.00082  $\mu\text{s/day}$  – Hoover tower (285 ft)
- “down to earth” relativity  
– Denver 3000 $\times$  less than GPS

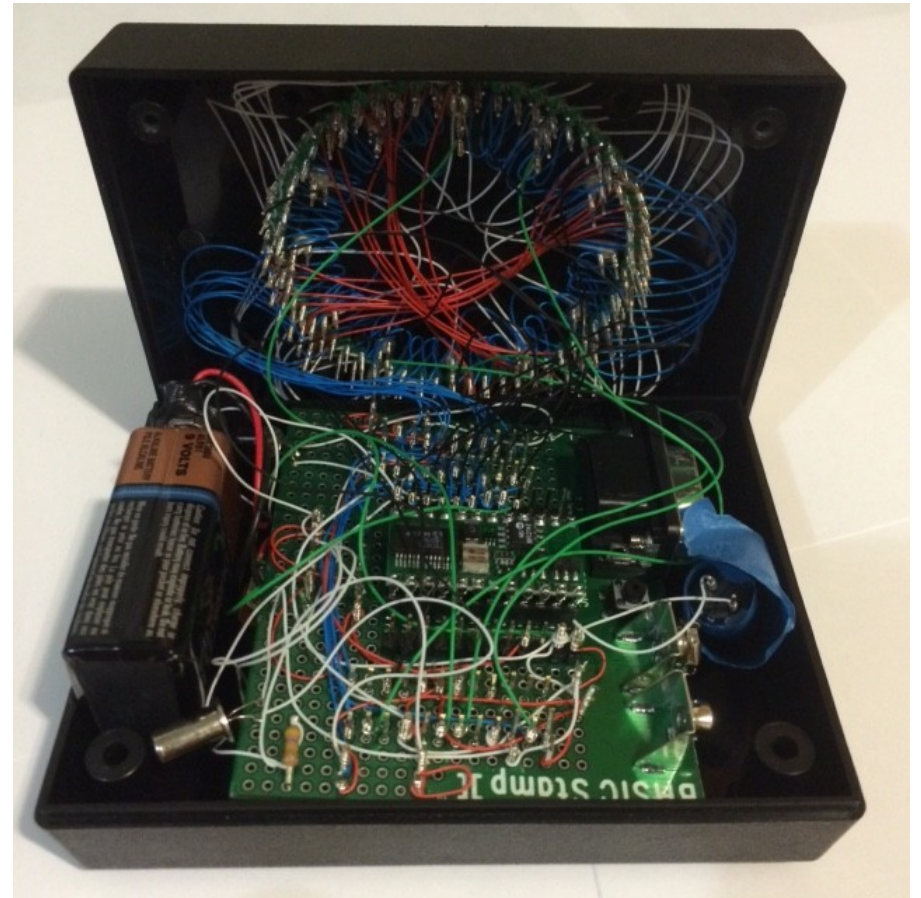
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# Precise time as a hobby (1994)

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# How to keep time?

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- “Timebase” required (quartz oscillator)
  - how accurate is it?
  - how to measure it?
- Use frequency counter
  - how accurate is it?
  - how to measure it?
- Use a reference standard
  - how accurate ...



# Timebase accuracy

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- $0.01 / 10.00 \text{ MHz} = 0.1\%$  (90 sec/day)
- $0.0001 / 10 \text{ MHz} = 10 \text{ ppm}$  (1 sec/day)





# The quest for better oscillators

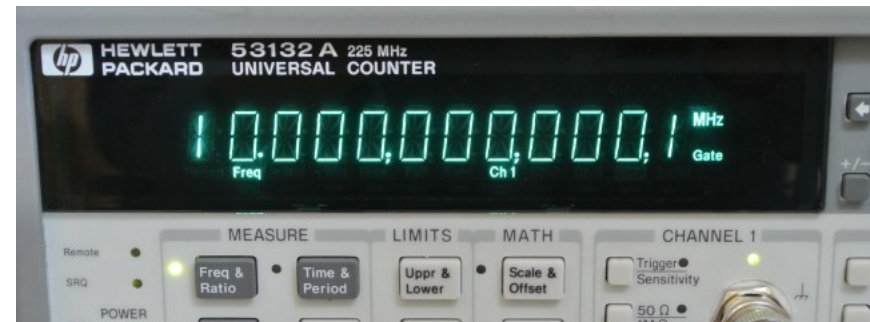
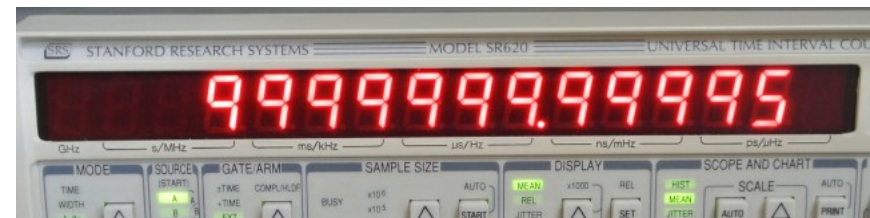
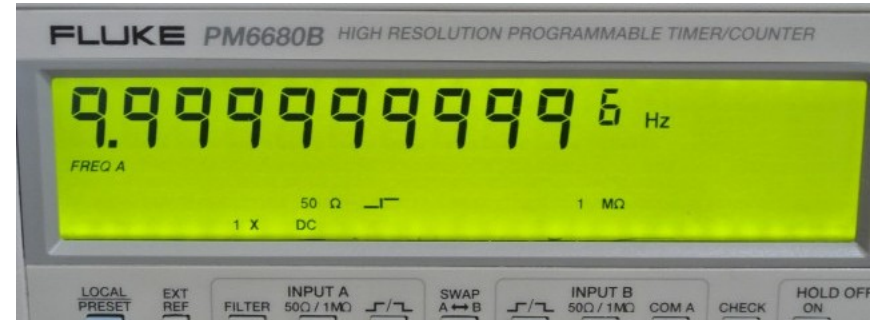


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# The quest for more digits



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# The quest for larger time lab

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# Slippery slope!

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- I started [www.LeanSecond.com](http://www.LeanSecond.com)
  - to share photos, data, software, lab reports, manuals
- Then created the “*time-nuts*” mailing list
  - now 1800 ‘nuts interested in amateur precise time
  - like amateur astronomy, seismology, etc.
- Relatively inexpensive, *time*-consuming hobby
  - measurement concepts work at \$1 as well as \$1000
  - easy to buy, repair, trade, collect interesting gear
  - massive trove of military and telecom surplus ([eBay](http://eBay))

# Vintage *hp* 5061A (eBay)



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# Extremely wide range of precision

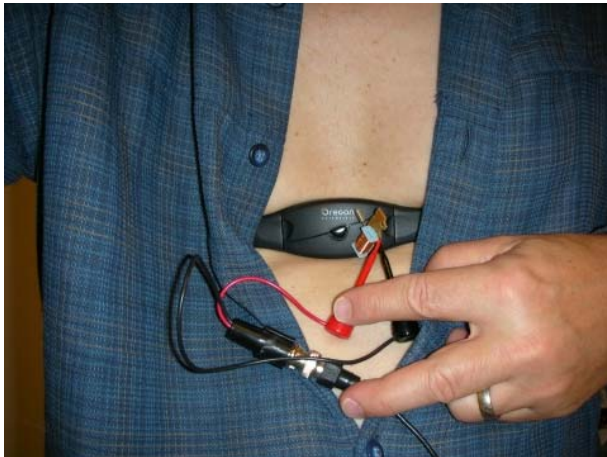
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- $10^{-2} = 1\% \approx 15 \text{ min / day}$
- $10^{-4} = 0.01\% \approx 1 \text{ min / week}$
- $10^{-6} = 1 \text{ ppm} \approx 0.1 \text{ s / day}$
- $10^{-8} \approx 1 \text{ ms / day}$
- $10^{-10} \approx 10 \text{ } \mu\text{s / day}$
- $10^{-12} = 1 \text{ ppt} \approx 100 \text{ ns / day}$
- $10^{-14} \approx \sim 1 \text{ ns / day} \approx 1 \text{ s / 3,000,000 years}$
- $10^{-16} \approx \sim 3 \text{ ns / year} \approx 3 \text{ s / billion years}$

# 10<sup>-2</sup> heart beat

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- The original '1 PPS'
- 10% stability at night, long-term
- 1% stability possible, short-term



```
62.0  
61.0  
61.0  
62.0  
62.0  
62.0  
63.0  
64.0  
65.0  
65.0  
65.0  
65.0  
64.0  
63.0  
62.0  
60.0  
60.0  
59.0  
59.0  
60.0  
60.0  
61.0
```

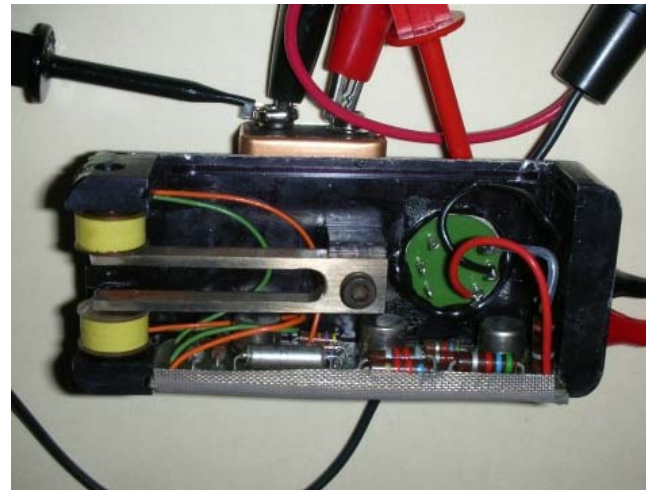


# $10^{-4}$ tuning fork, mains

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- Mechanical oscillator transistorized
- “Four 9’s”

```
999.907,211,67 Hz
999.907,250,33 Hz
999.907,273,16 Hz
999.907,311,01 Hz
999.907,250,27 Hz
999.907,345,09 Hz
N      : 60
STD DEV: 151.812 uHz
MEAN   : 999.907,159,334 Hz
MAX    : 999.907,404,05 Hz
MIN    : 999.906,840,54 Hz
999.907,392,20 Hz
999.907,415,25 Hz
999.907,354,85 Hz
```



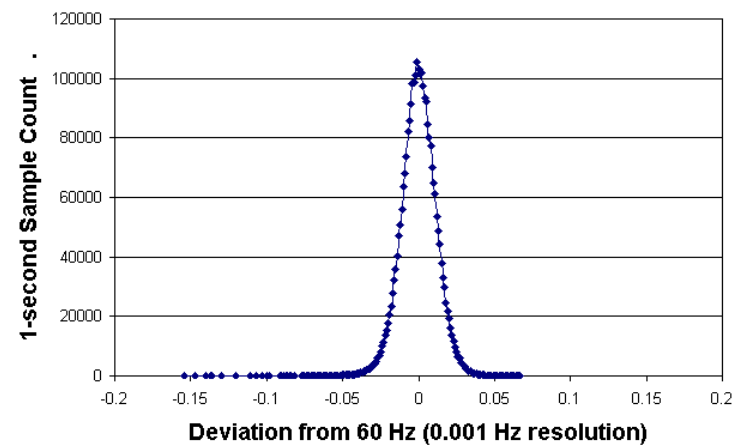
# 10<sup>-4</sup> tuning fork, mains

- Power line frequency: 60± Hz

```
60.003,640,720,5 Hz
60.009,491,393,8 Hz
60.000,431,181,6 Hz
59.992,198,219,9 Hz
59.987,371,509,5 Hz
59.993,148,200,6 Hz
59.999,032,462,5 Hz
59.985,892,634,1 Hz
59.995,727,396,2 Hz
N      : 36
STD DEV: 0.006,765,596,40 Hz
MEAN   : 59.999,554,563,23 Hz
MAX    : 60.010,390,980,5 Hz
MIN    : 59.985,892,634,1 Hz
59.996,011,518,6 Hz
59.999,536,139,7 Hz
```



60 Hz Mains Frequency Deviation Histogram  
2.7 million one second samples (~1 month)

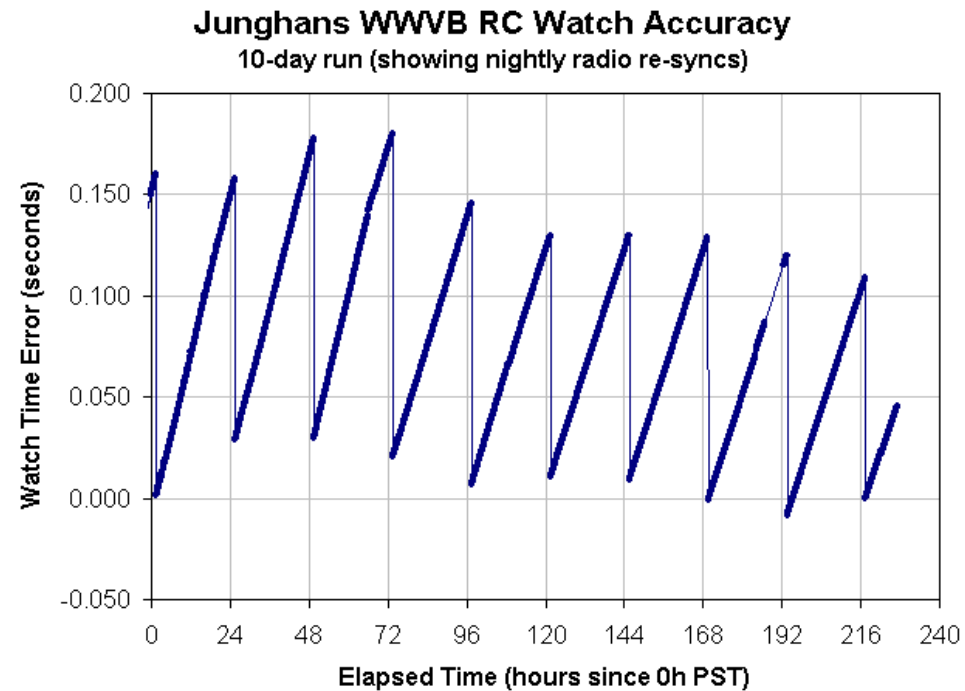




# $10^{-6}$ quartz watch, chronometer

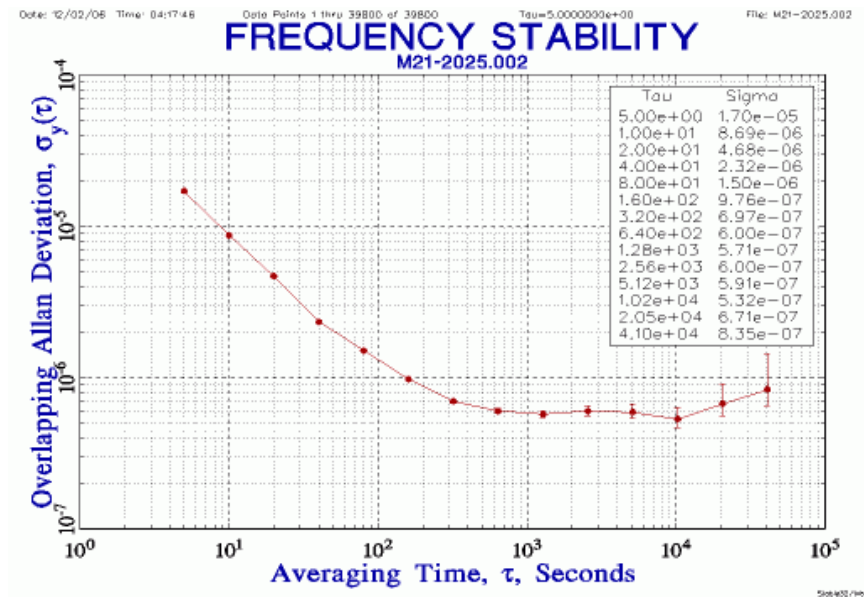
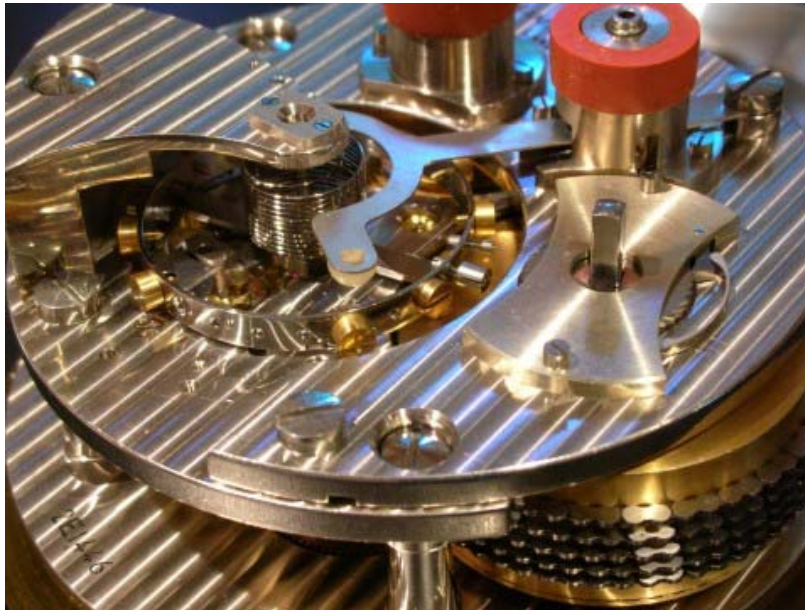
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- +160 ms/d = +1.85 ppm



# $10^{-6}$ quartz watch, **chronometer**

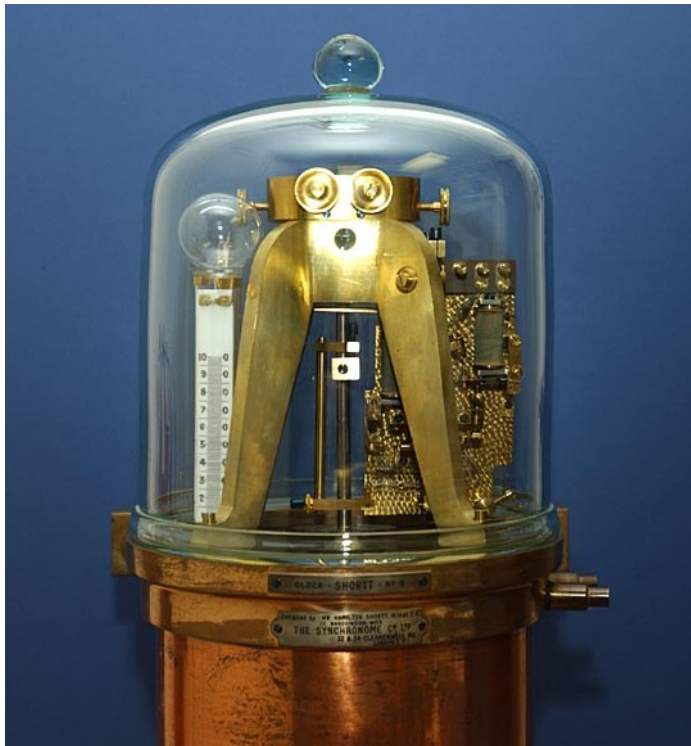
- Conservatively rated  $\frac{1}{4}$  sec/day deviation



# $10^{-8}$ **pendulum clock**, earth

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- Shortt-Synchronome
- 1 second / year



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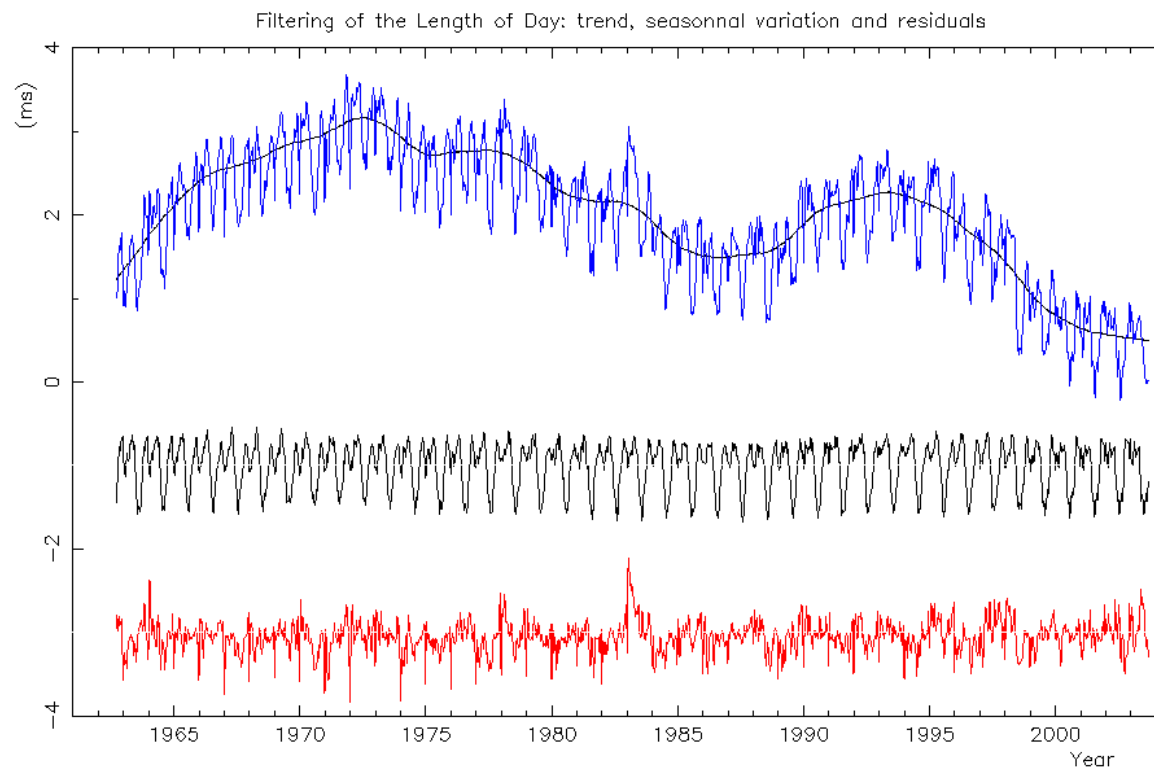
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# $10^{-8}$ pendulum clock, earth

- ~1 second / year, irregular



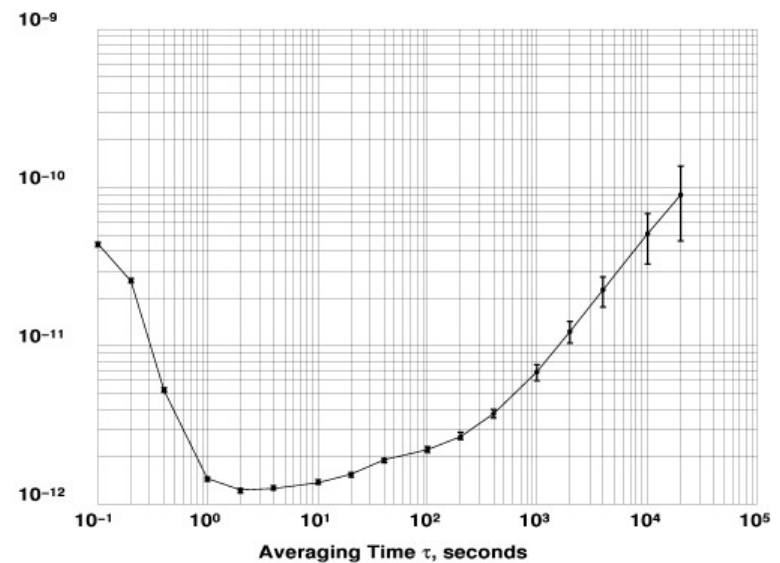
# $10^{-10}$ ovenized quartz

- $10^{-10} \dots 10^{-13}$  short-term
- $5 \times 10^{-10}/\text{d}$  drift



26 Oct 2003 19:58:45

Allan Deviation  $\sigma_y(\tau)$



Ch A: 5.0 MHz 2.7 V<sub>pp</sub>  
Averaged Phase

Ch B: 10.2 MHz 1.4 V<sub>pp</sub>  
B/A=2.04600007663435

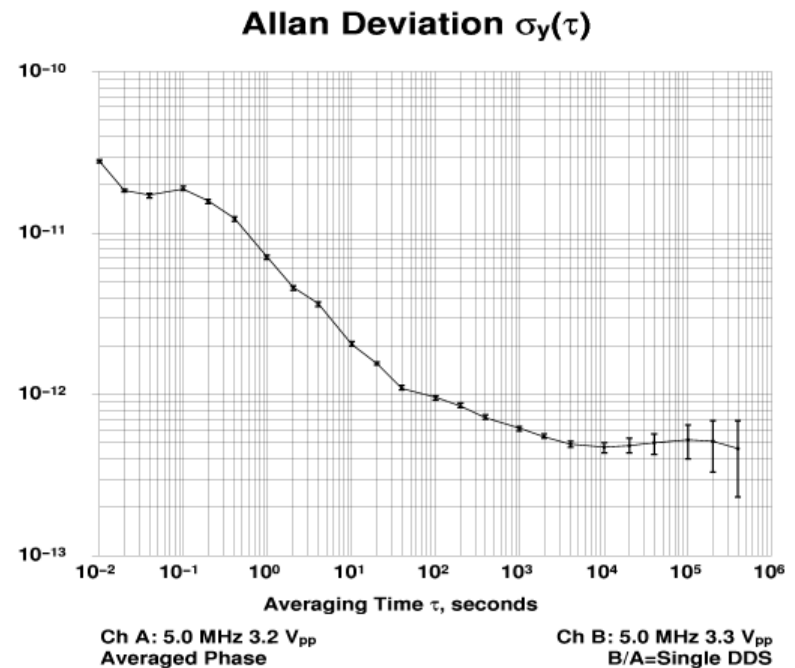
C:\twb\SCPNT\log4165.gif

# $10^{-12}$ rubidium (atomic)

- $\sim 10^{-13}$  mid-term
- $\sim 10^{-11}$ /month drift



01 Dec 2006 23:05:04

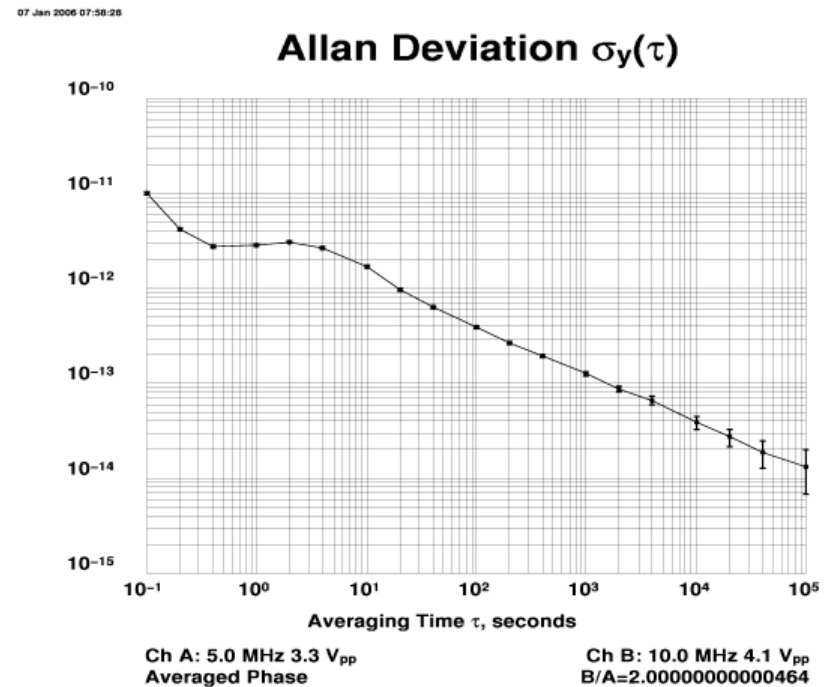


C:\twb\TSCP101\Log1578.gif



# $10^{-14}$ cesium (atomic)

- $\sim 10^{-13}$  mid-term
- $1 \times 10^{-14}$  at 1 day



C:\twb\Tscplot\Log23362.gif

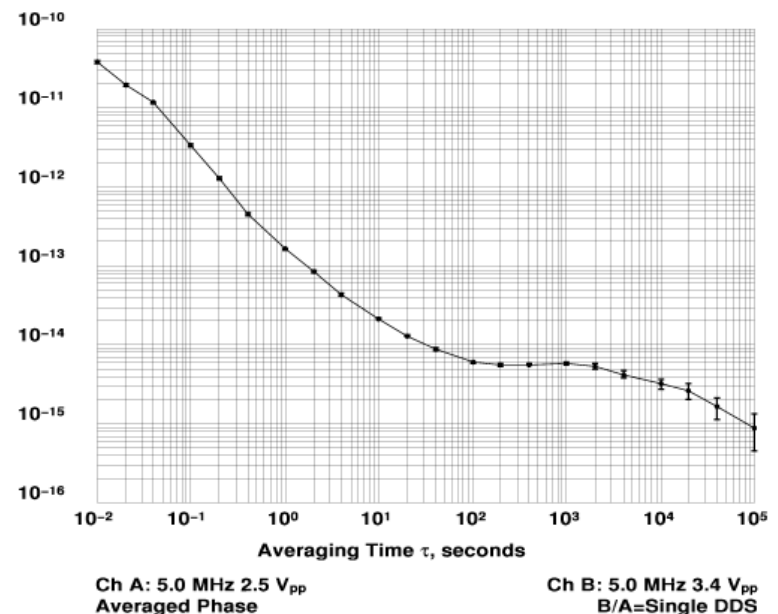
# $10^{-16}$ hydrogen maser

- Most stable (but non-portable)
- $5 \times 10^{-16}$  possible



05 Apr 2003 09:38:02

Allan Deviation  $\sigma_y(\tau)$



C:\twb\TSCP\ot\Log20148.gif

# FYI: cesium (caesium)

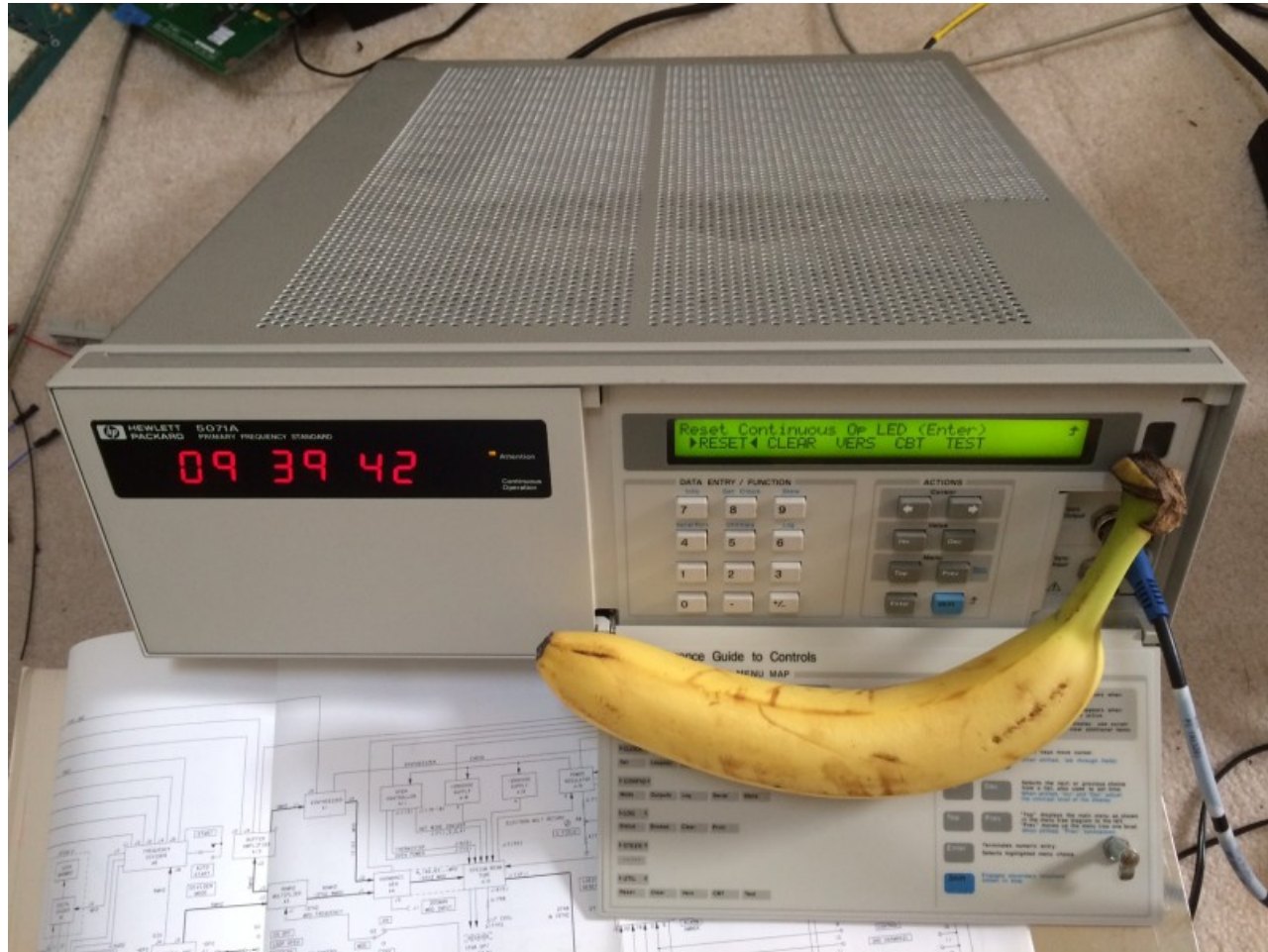
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- Cesium atomic clocks are ***not*** radioactive
- They use a natural, stable  $\text{Cs}^{133}$  atom, not the dangerous man-made *radioisotope*  $\text{Cs}^{137}$
- Analogy:  $\text{C}^{12}$  vs.  $\text{C}^{14}$
- $\text{K}^{39}$  vs.  $\text{K}^{40}$  (banana)
- “hyperfine transition”  
9,192,631,770 Hz
- Solid / liquid [28 °C]



# First radioactive cesium clock!

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# Outline

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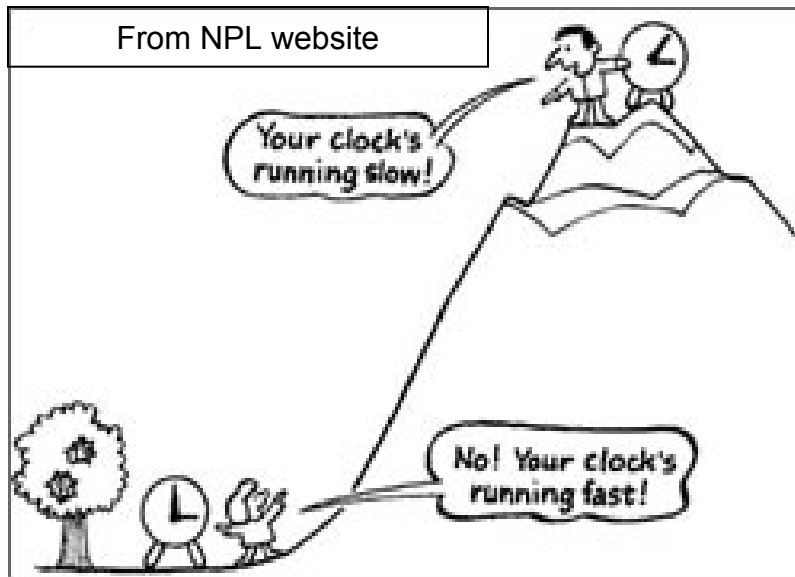
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# Clocks, mountains, relativity

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- Is relativity detectable at human scale?
  - have I accumulated enough toys by now?
  - can time dilation be measured by an amateur?
  - can I travel fast enough, or high enough?



# A great idea

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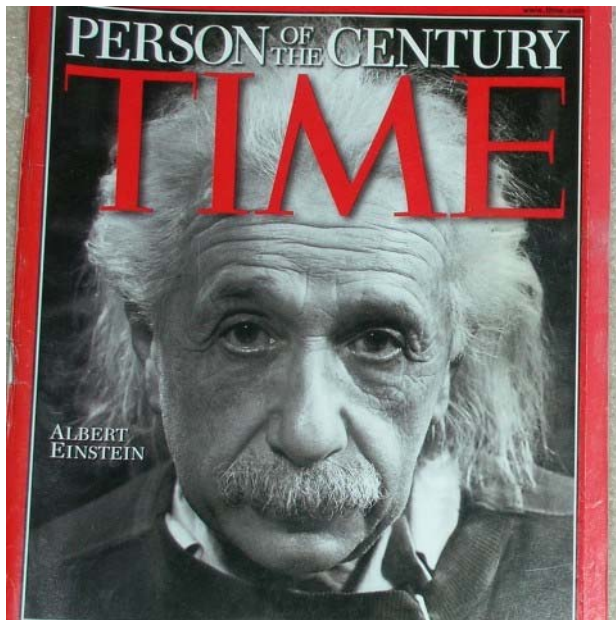
- Take our 3 kids with *portable cesium* clocks high up Mt Rainier
- See if Einstein was right about *gravity* and *time*
- See if clocks really run faster up there
- In 2005



# Einstein and 2005

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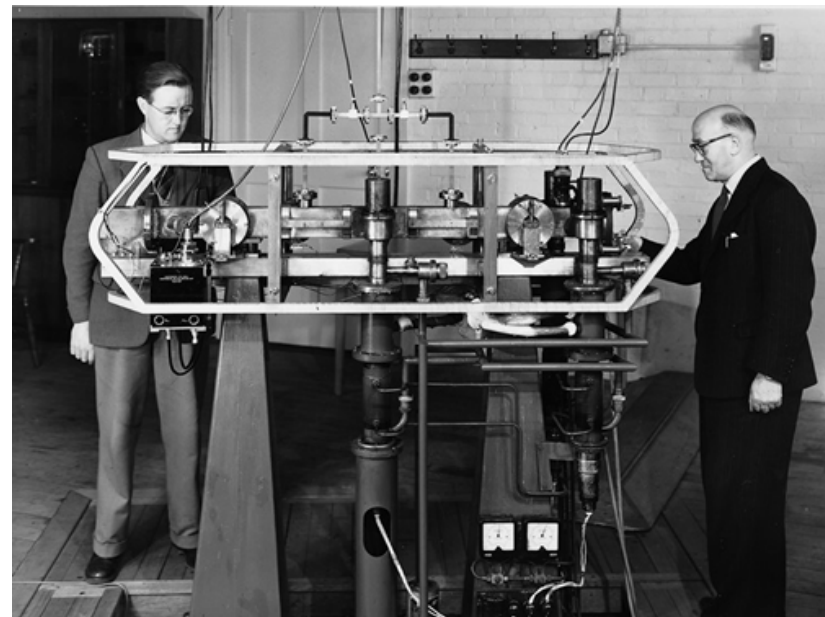
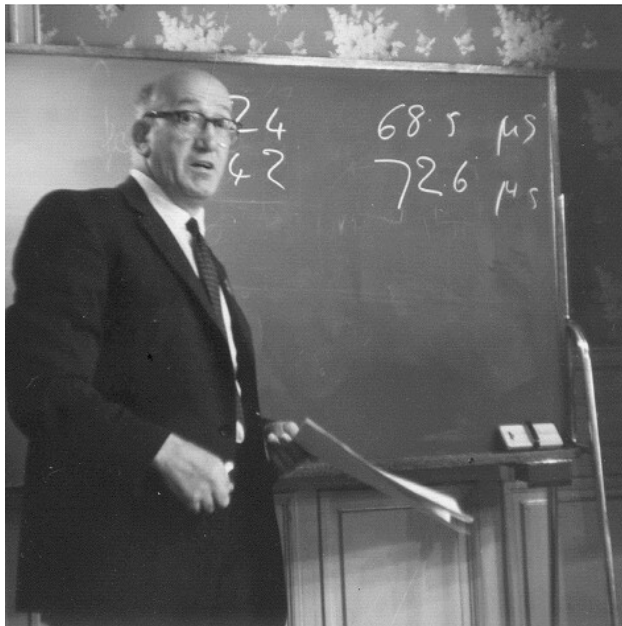
- 100<sup>th</sup> anniversary of relativity: books, magazines, radio, TV, web sites, “Physics Year”, lectures...



# Louis Essen (UK) and 2005

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- 50<sup>th</sup> anniversary of cesium clock (NPL)
- “famous for a second” 9 192 631 770 Hz



# Project GRE<sup>2</sup>AT

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- **G**eneral **R**elativity **E**instein/**E**ssen **A**nniversary **T**est (2005)
  - 100<sup>th</sup> anniversary (Einstein) theory of relativity
  - 50<sup>th</sup> anniversary (Essen) first cesium clock
- Combine atomic clock hobby, physics, history, technology, math, computers, children, car trip, vacation, and family fun
- First “home made” general relativity test



# Back-of-envelope calculation

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- Turn *infinitesimal* into *measurable*
- Frequency change  $\Delta f/f \approx gh/c^2$   
 $\Delta f/f \approx 1.09 \times 10^{-16}$  s/s/meter
- But if you go up 1 km instead of 1 m, then  
 $\Delta f/f = 1.1 \times 10^{-13} = 0.11$  ps/s  
note: 4000× less than GPS
- And if you stay up there 24 hours, then  
 $\Delta T = \Delta f/f \times 86400 \text{ s} = 9.5 \times 10^{-9} \text{ s} = 9.5 \text{ ns}$
- Rule-of-thumb: 1 km elevation  $\approx 10$  ns/day

# Magnify 0.00000000000000000001

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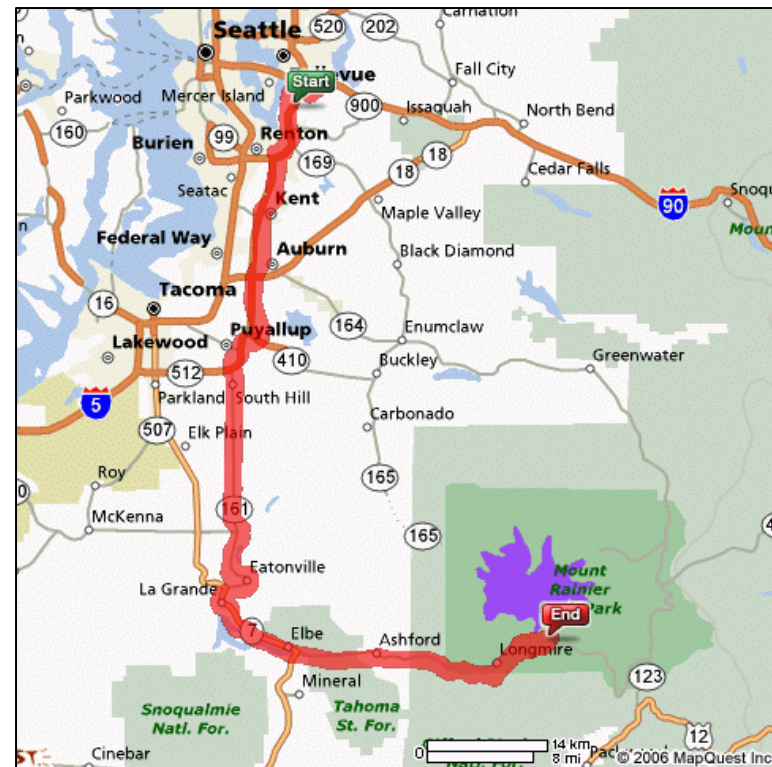
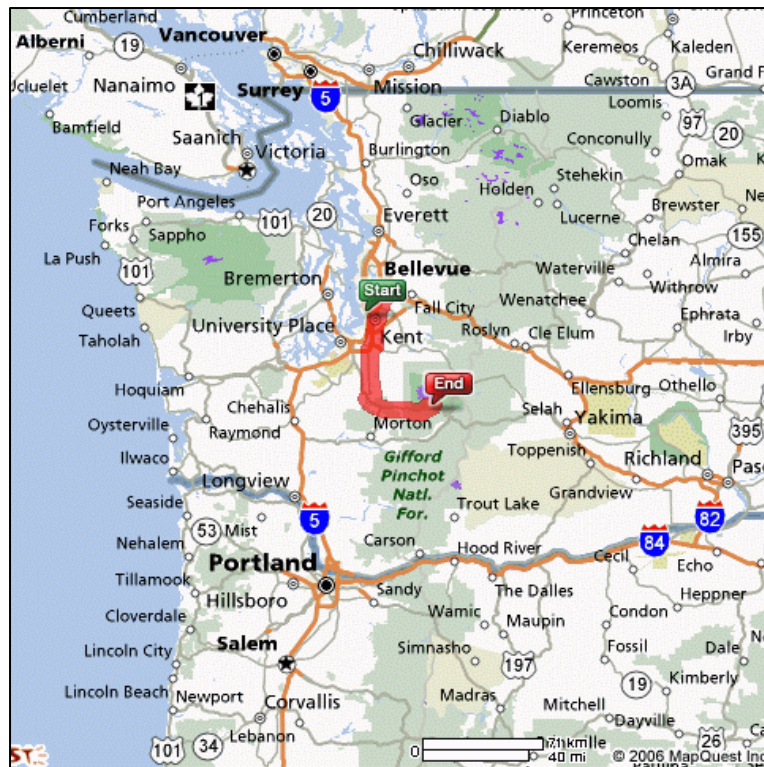
- Go as **high** as possible
- Stay as **long** as possible
- Measure as **precisely** as possible
- Use the **best** clock(s) possible



Cartoon by Dusan Petricic  
Scientific American column Wonders by Philip and Phyllis Morrison  
<http://www.sciam.com/1998/0298issue/0298wonders.html>

# Bellevue to Mt Rainier

- Just 100 miles away (~2½ hours)



# The GREAT trip, day 1

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- Carrying synchronized, running cesium clock downstairs. Repeat. Repeat.

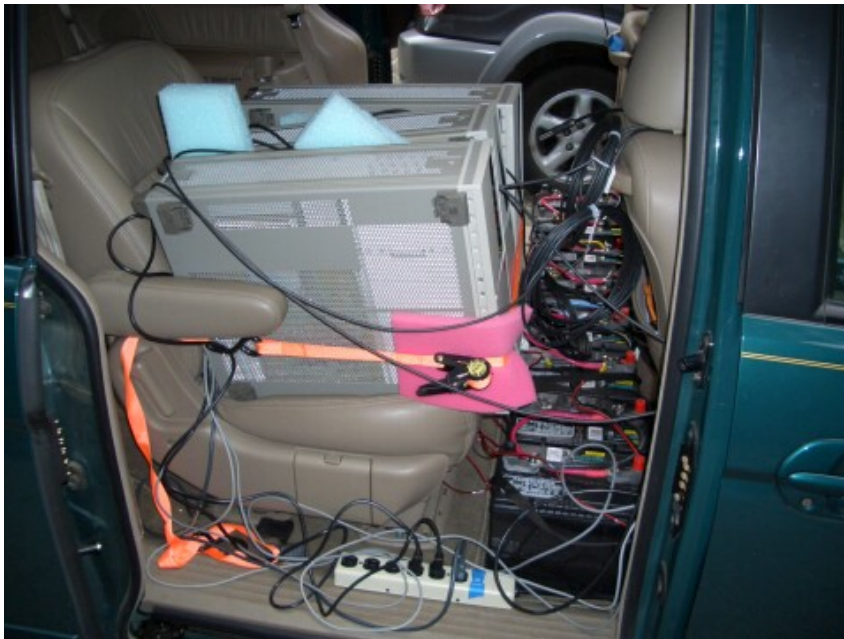




# The GREAT trip, day 1

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- 3 clocks in the middle. Batteries on the floor. Monitoring instruments in front.





# The GREAT trip, day 1

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- 3 kids in the back. Dad making final clock BNC connections. Mom says goodbye.



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# The GREAT trip, day 1

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- Detail of TIC's and laptop in front seat and clocks in middle seat. 23:33:48 UTC





# The GREAT trip, day 2

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- Paradise Inn is at 5400' elevation. Large parking lot for car & precious clocks.



# The GREAT trip, day 2

---

- Classic old Northwest inn. Wonderful place to visit during the summer.





# The GREAT trip, day 2

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- Wonderful hiking trails and climbing. Lucky to have clear weather.





# The GREAT trip, day 2

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- Avoid a ticket and move the car again. Worried about running out of fuel.



# The GREAT trip, day 3

---

- More hiking, exploring, playing. It's a fun place for a weekend.





# The GREAT trip, day 3

---

- 42 hours is up; time to leave. We're all tired. Can this really work? Go home.



# Time dilation: *prediction*

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- Home clock and mountain clock elevations
- 5400 ft – 1000 ft = 4400 ft (1340 m)
- $\Delta f/f = 1.46 \times 10^{-13}$  times 42 hours = +22 ns



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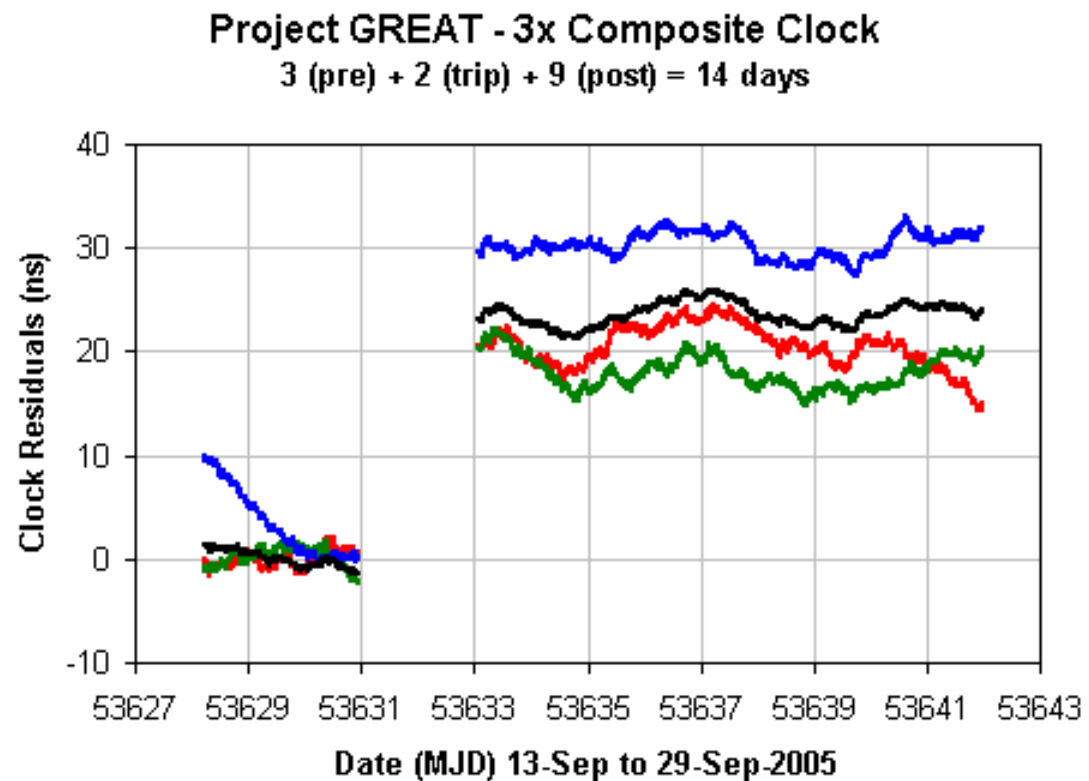
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# Time dilation: *measured*

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- 3 clock mean  
**23.2 ns**  
 $\pm 4$  ns
- Prediction  
**22.4 ns**
- Wow!





# Project GRE<sup>2</sup>AT – summary

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- Einstein was right; time dilation is real!
  - clocks (and we) came back 22 ns **older**
  - gravitational effect (elevation, *not* velocity)
  - unexpected press: WIRED, Physics Today, Reddit, Scientific American, even a physics textbook, etc.
- Now *"relativity is child's play"*
- And then a decade later...

# GREAT 2016a – Lemmon

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- Stephen Hawking “GENIUS” series (PBS / BBC)
  - episode on space-time, clocks, time dilation
  - they asked me to repeat the GREAT experiment
- Mt Lemmon, Tucson, AZ (January 2016)
  - 9160 ft (2790 m) summit, dormitory
  - 2600 ft ( 790 m) base camp, hotel
  - UK film crew, tight schedule, no re-takes
- $2000 \text{ m} \times 24 \text{ hours} = \sim 20 \text{ ns}$  time dilation
  - used 3+3 cesium clocks



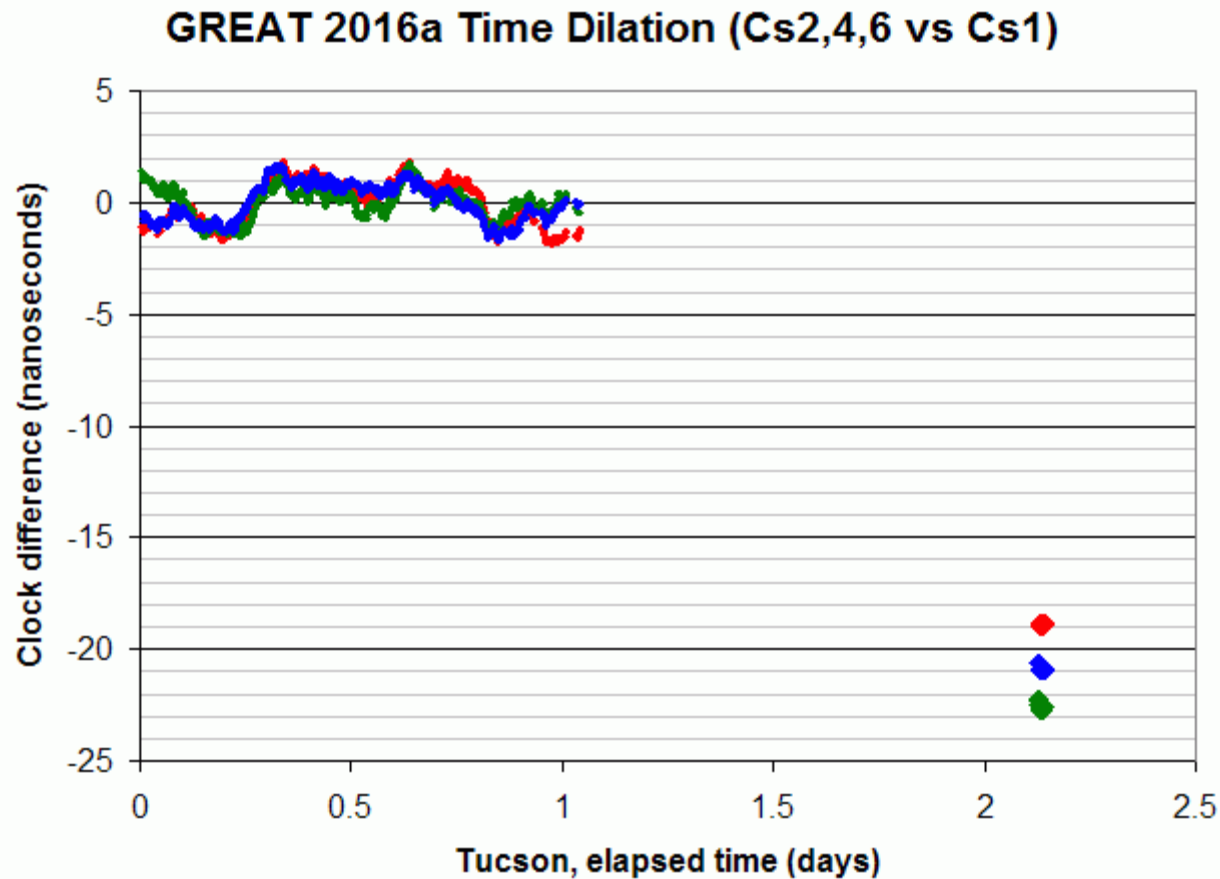
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# 2016a – time dilation results

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# 2016a – summary

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- It worked! (much relieved)
  - up-down-up vs. down-up-down
  - different elevation and latitude (earth rotation)
- RIP Stephen Hawking (1942–2018)
- Show available on iTunes or *pbs.org*



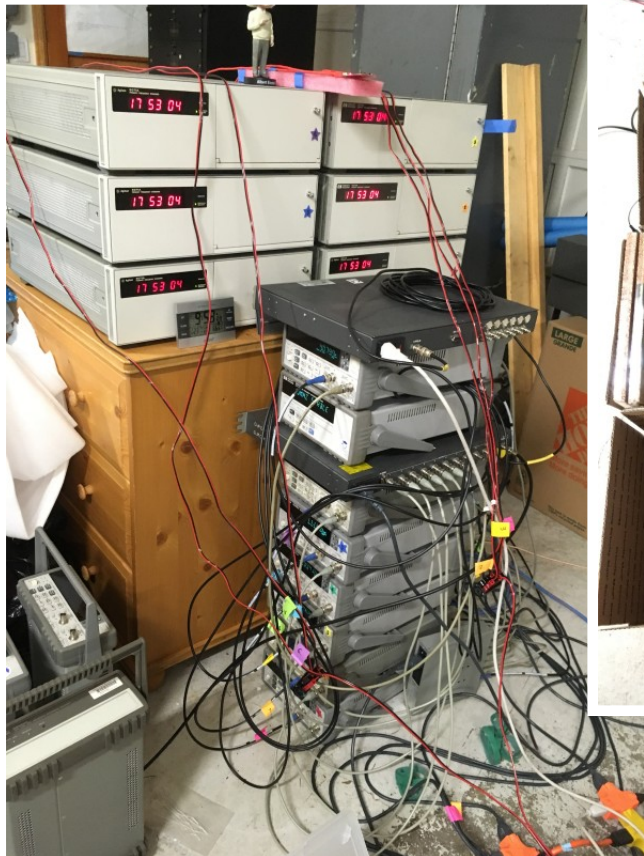
# GREAT 2018a – Palomar

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- Earlier this year (2018), another request
  - History channel, “In Search Of”, Zachary Quinto
- Demonstrate time dilation
  - sort of related to their “time travel” theme
- Palomar Mountain, CA
  - low clocks: Oceanside (sea level)
  - high clocks: Palomar Mountain (~5500 ft)
- Predicted time dilation: ~15 ns
  - used 2+2 cesium clocks

# 2018a – prep

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# 2018a – battery backup

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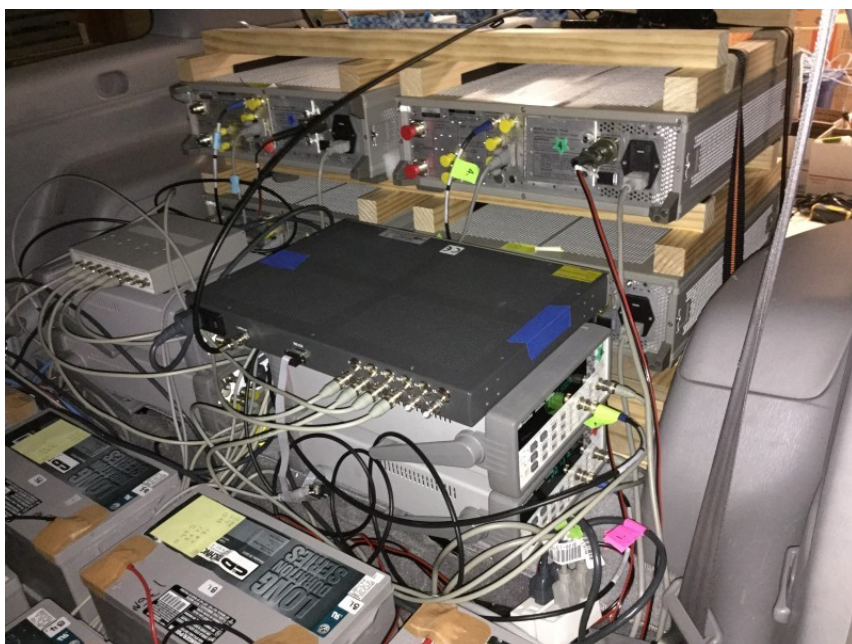


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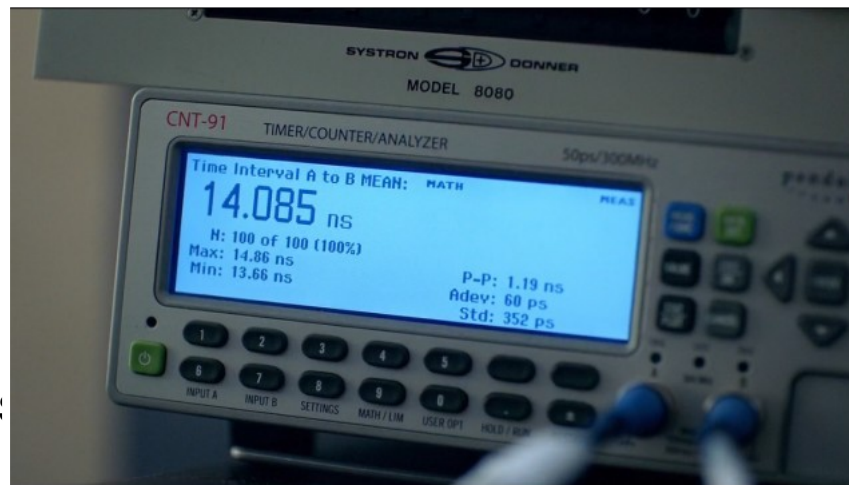
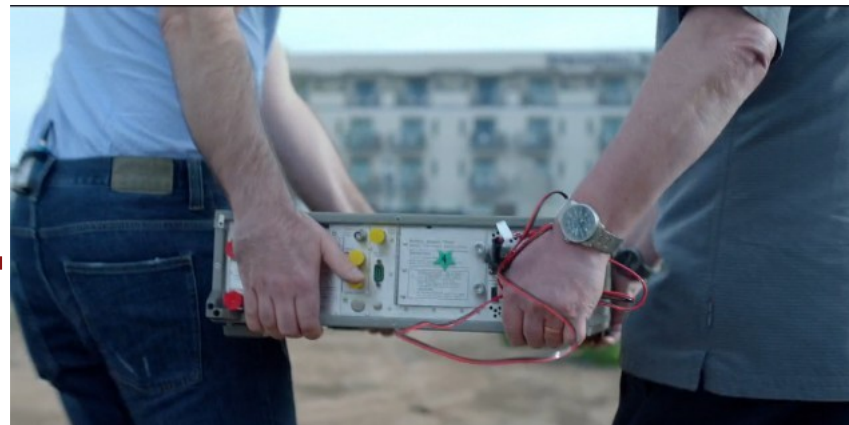


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# 2018a – summary

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- Predicted time dilation: 15.5 ns
  - based on recorded elevation and dwell time
- Measured time dilation: 14.1 ns
  - another success!
  - Cs1 to Cs4: 14.3 ns
  - Cs3 to Cs5: 13.8 ns
- Show available on iTunes or *history.org*

# Conclusion

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- Welcome to the world of amateur precise time
  - fascination with time, vintage and atomic clocks, counters, experiments, GPS, relativity, etc.
- GR time dilation experiments
  - just for fun, with the kids, and
  - larger audience via talks, TV episode or two
- Looking for interesting new venues
  - below sea level, or deep underground?
  - 50<sup>th</sup> anniversary of Hafele-Keating in 2021

# Thanks

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- Thanks to Tom Langenstein, Leo Hollberg, and Ralph Devoe (via *time-nuts*).
- Thanks for your time...
- Contact: [tvb@LeapSecond.com](mailto:tvb@LeapSecond.com)
- Website: [www.LeapSecond.com](http://www.LeapSecond.com)
- Questions?





# Backup slides

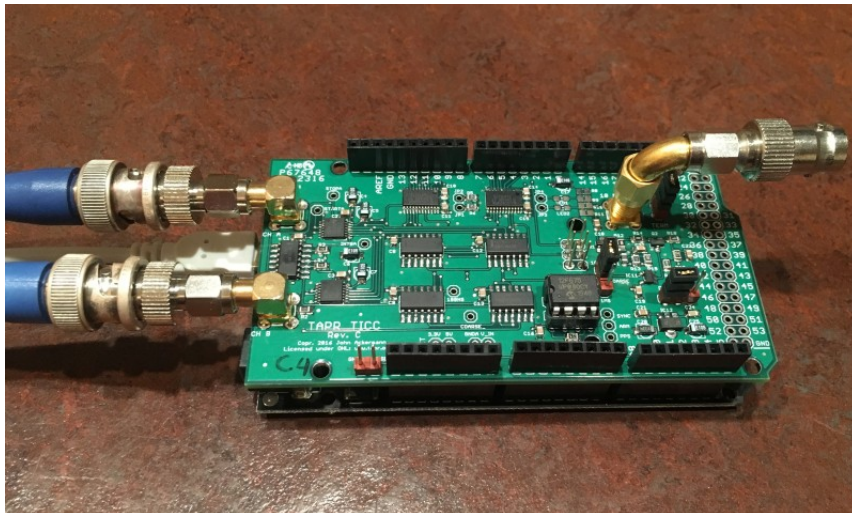
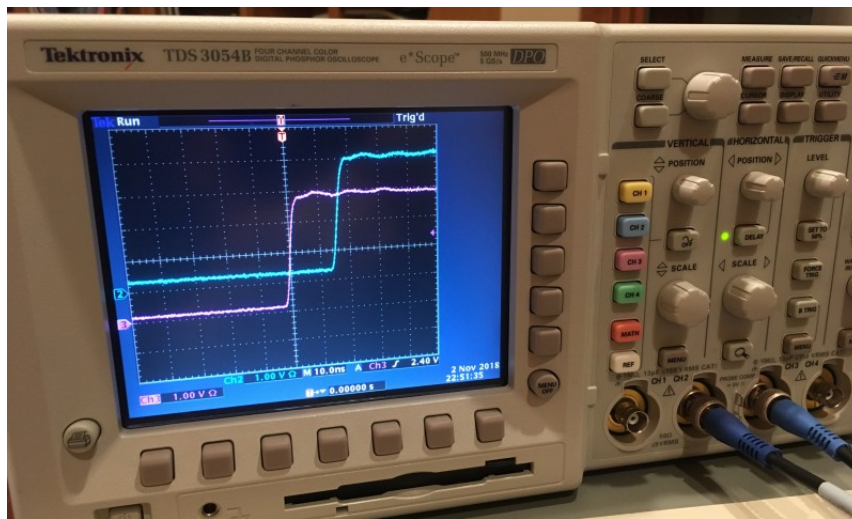
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# Time interval counter (TIC)

- Time-To-Digital
  - start input
  - stop input
- Trigger
  - pos/neg slope
  - AC/DC input
  - trigger level
- Resolution



# Comparing clocks / nanosecond



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# Terrestrial relativity math, easy

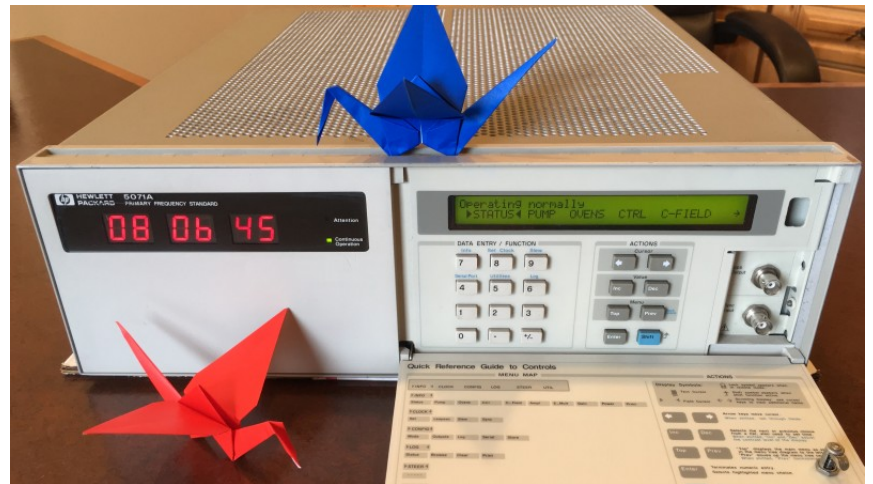
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- At modest speeds and modest elevation
  - time dilation equations become very simple
- Kinematic effect (moving clocks run slower)  
 $\Delta f/f \approx -\frac{1}{2}v^2/c^2$
- Gravitational effect (lower clocks run slower)  
 $\Delta f/f \approx +gh/c^2$
- Sign? SI second defined at sea level
  - so clocks speed up with higher altitude
  - “speed up” is actually “slow down less”



# Power of $c^2$

- Speed of light
  - $c \approx 300\,000\,000$
  - $c^2 \approx 90\,000\,000\,000\,000\,000$
  - $1/c^2 \approx 0.000\,000\,000\,000\,000\,01$
- $E = mc^2$ 
  - $c^2$  in numerator
  - very, very large
- $\Delta f/f = gh/c^2$ 
  - $c^2$  in denominator
  - very, very small
- As they say:  
“make time dilation, not war”





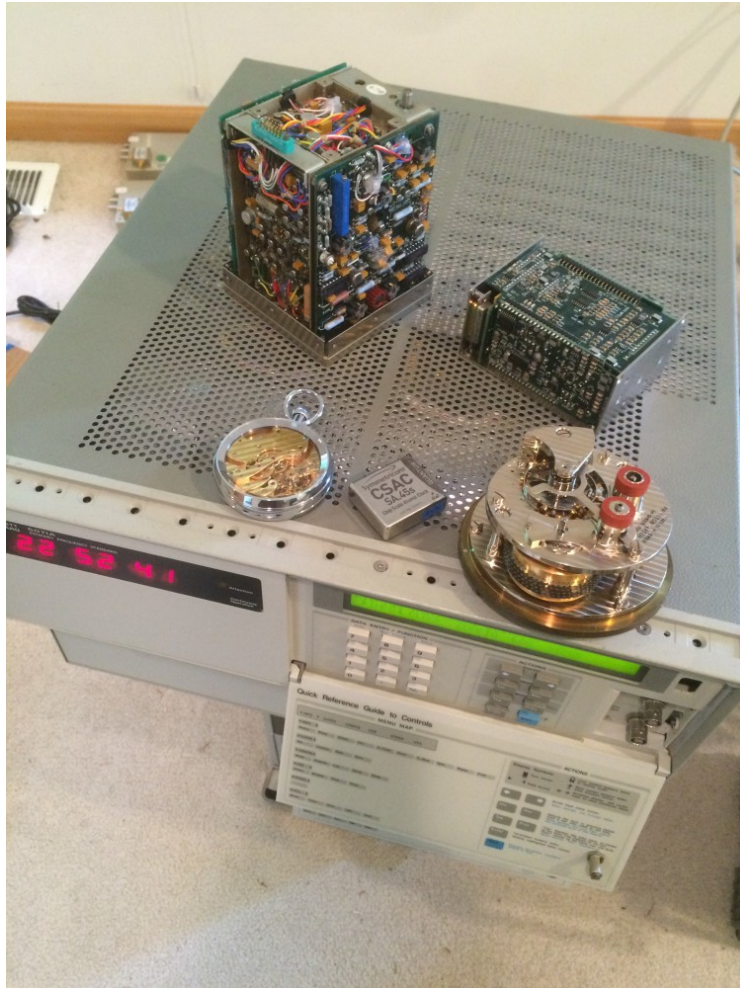
# First “atomic” wristwatch

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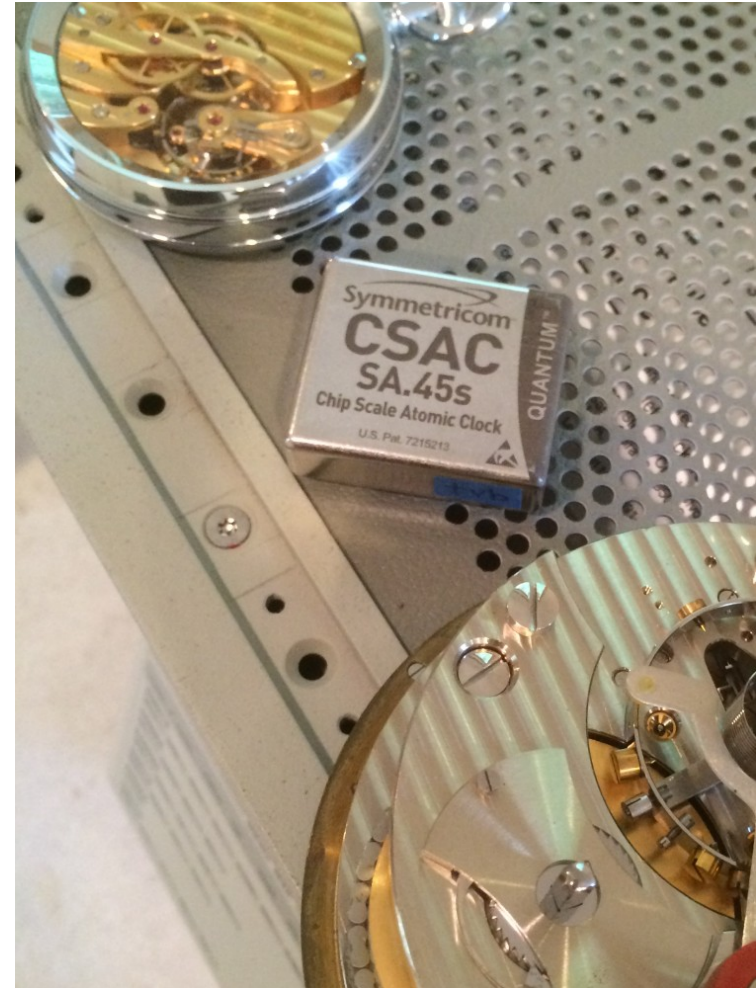


# Smaller and smaller ... CSAC

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# Museum of *hp* clocks

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# HP quartz

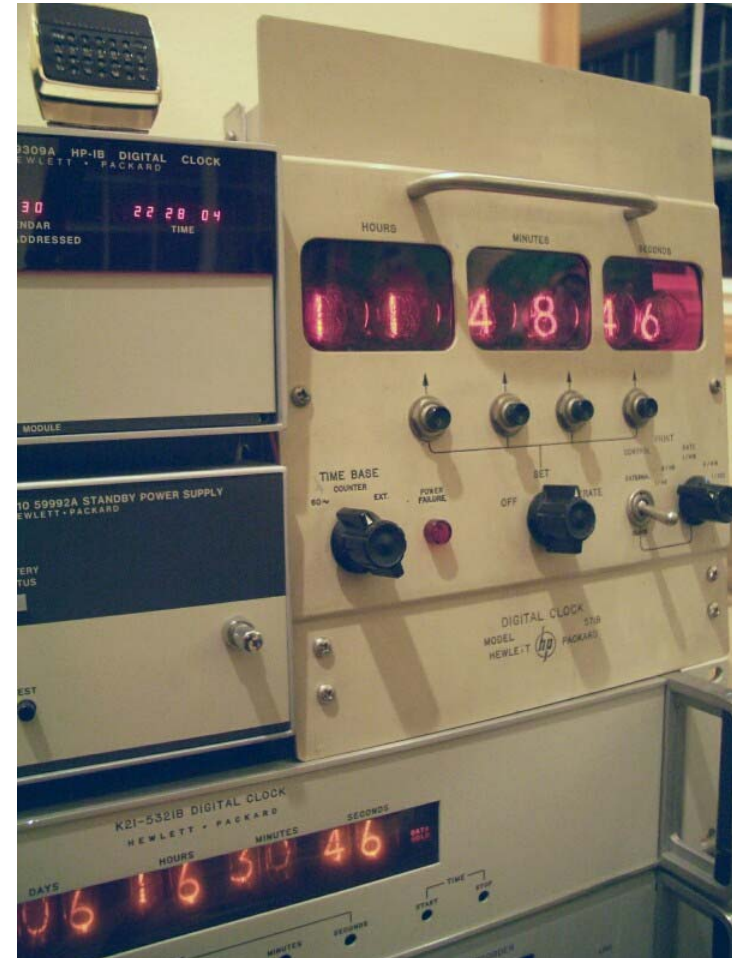
- 105B
- 107BR
- 106B
- 104AR
- 103AR
- 101A
- 100ER





# HP clocks

- HP01
- 571B
- 5321
- 117A
- 114BR
- 115BR
- 113AR

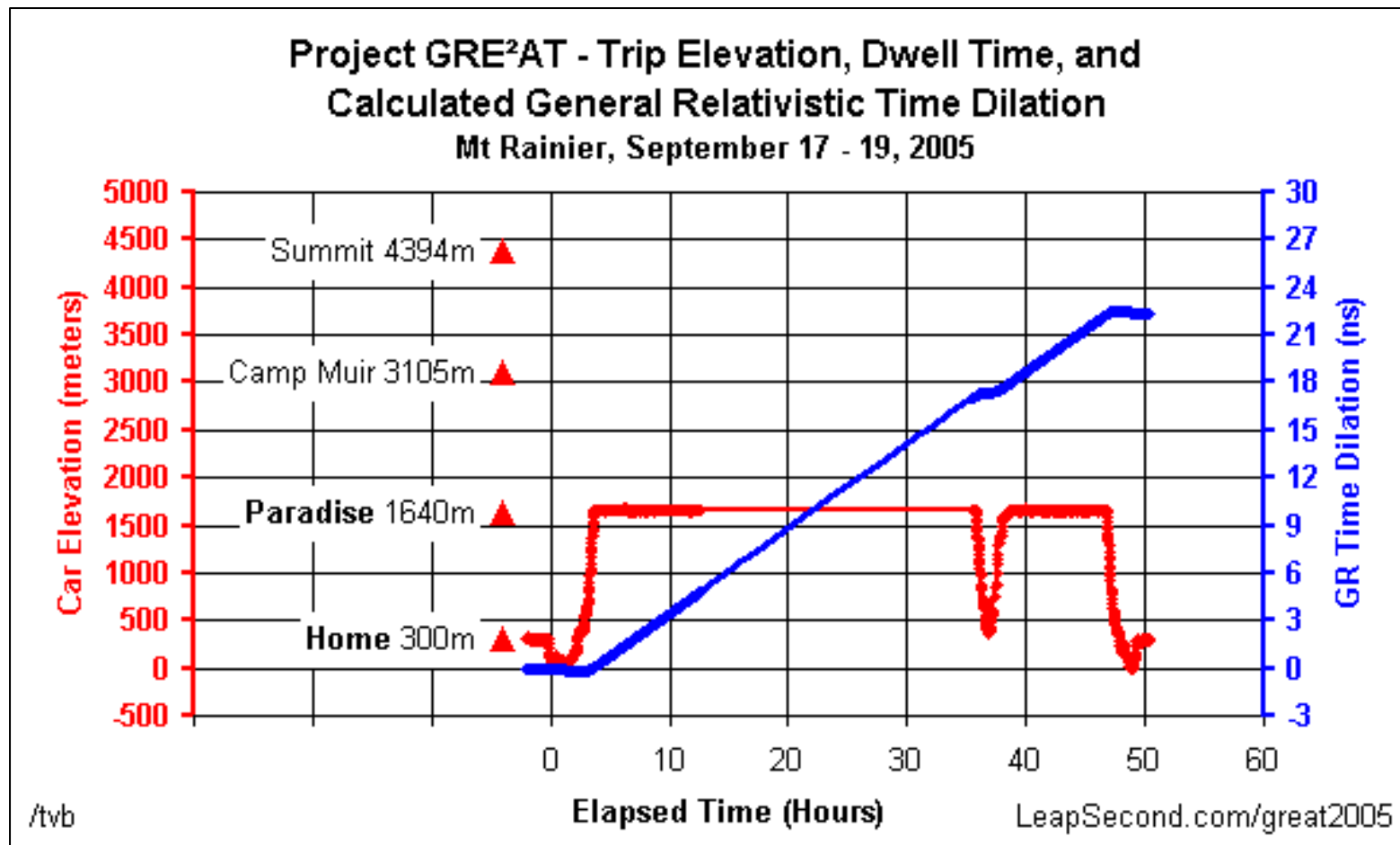


# HP cesium & rubidium

- 5071A
- 5065A
- 5062c
- 5061B
- 5061A
- 5060A



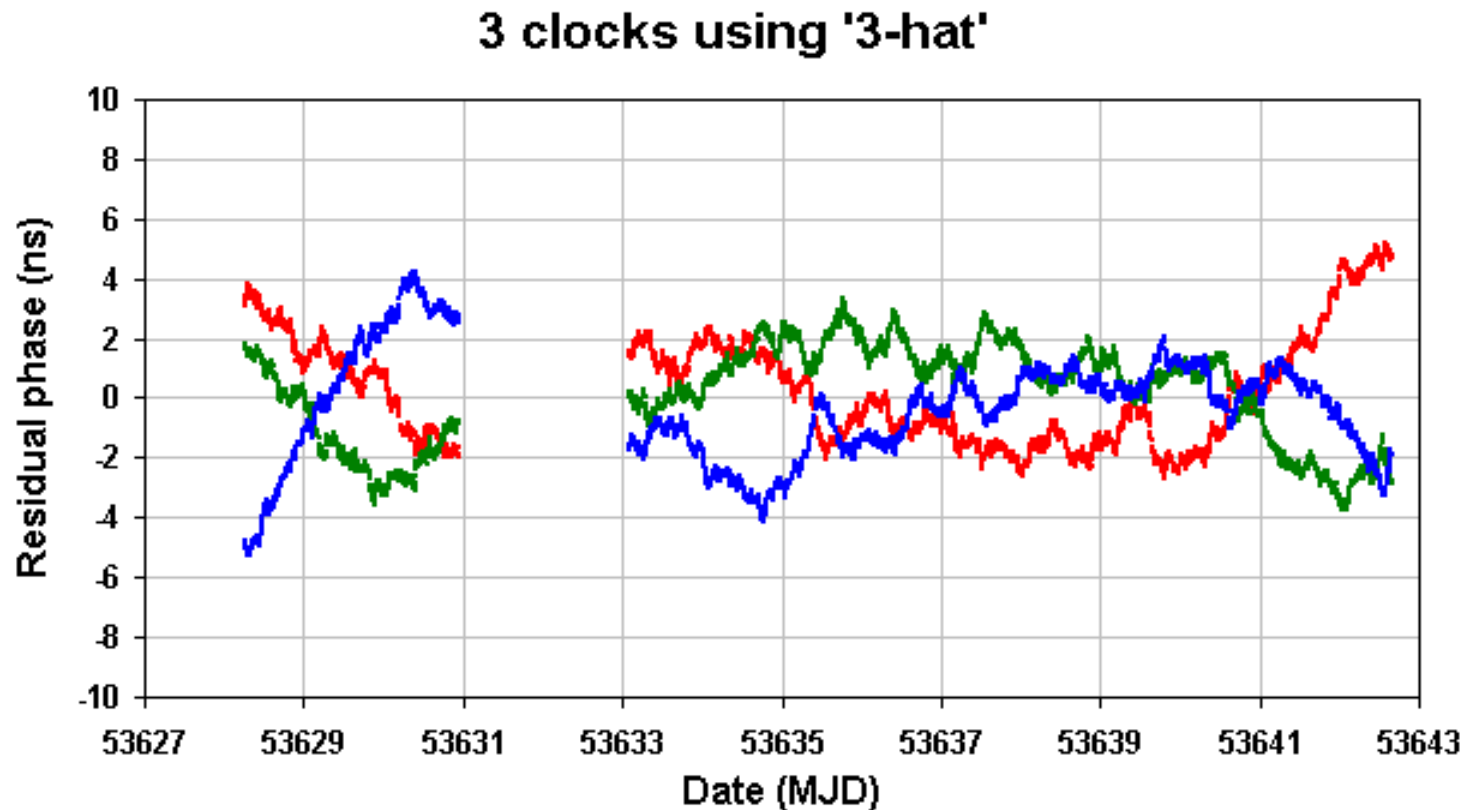
# Elevation and *predicted* dilation



# 3-hat, residuals (home)

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- $Cs_i - Cs_j$  via lab reference

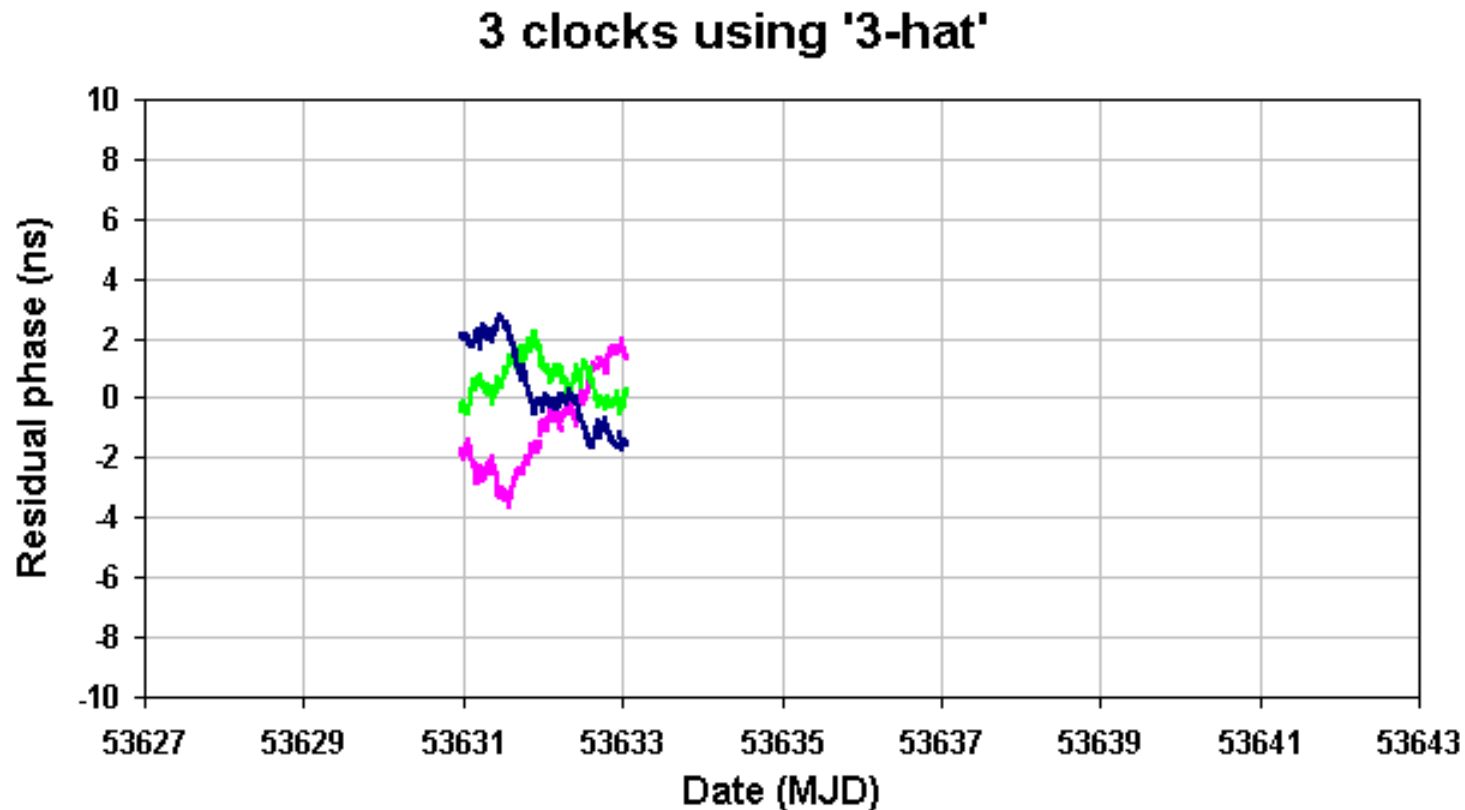




# 3-hat, residuals (away)

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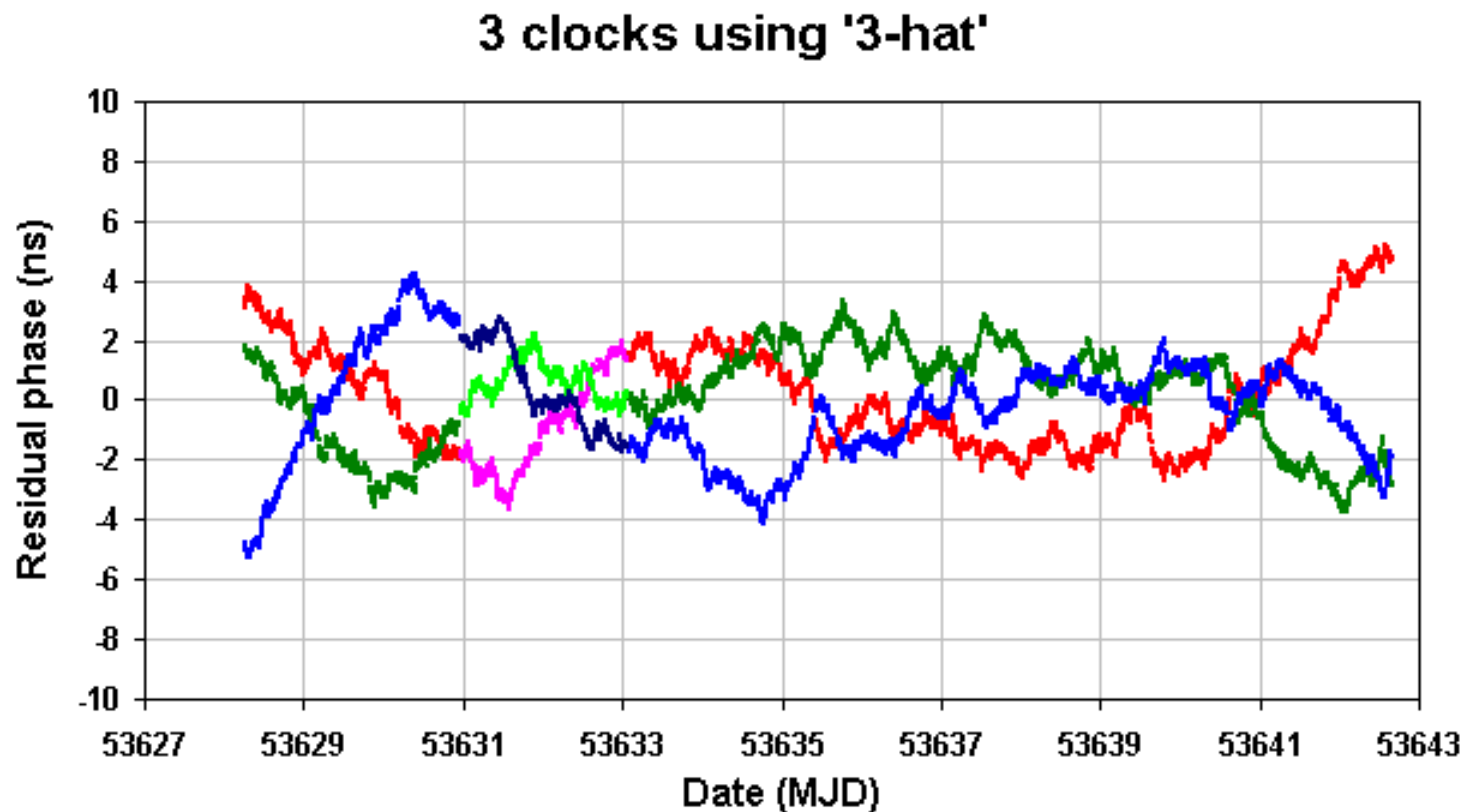
- $Cs_i - Cs_j$  via mutual-comparisons



# 3-hat, residuals (combined)

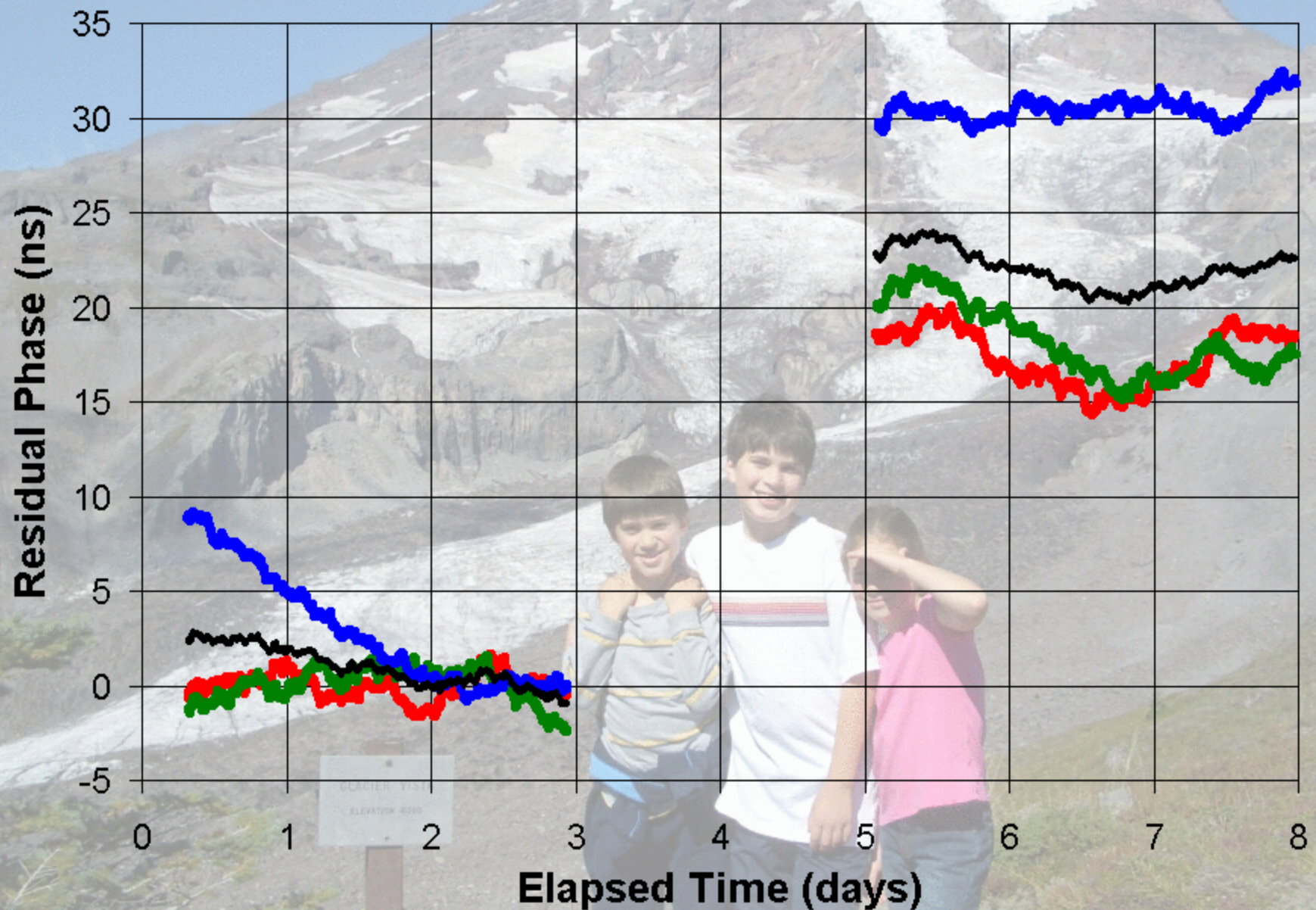
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- $Cs_i - Cs_j$



# Kids, Clocks, and Relativity on Mt Rainier

Three Cesium Clocks: Red Green Blue & Mean





# Chronometer adjust (time, rate)

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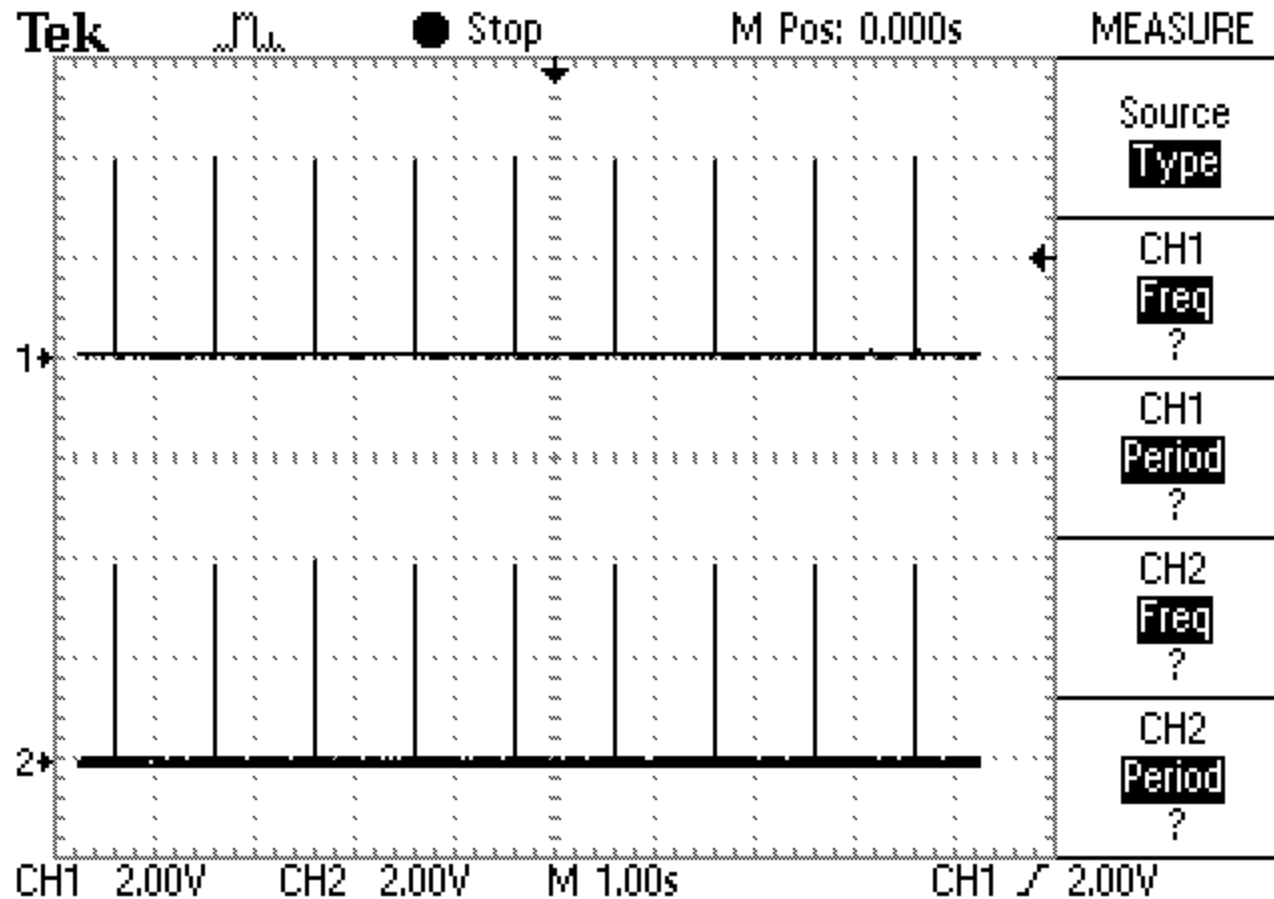


# Frequency adjust

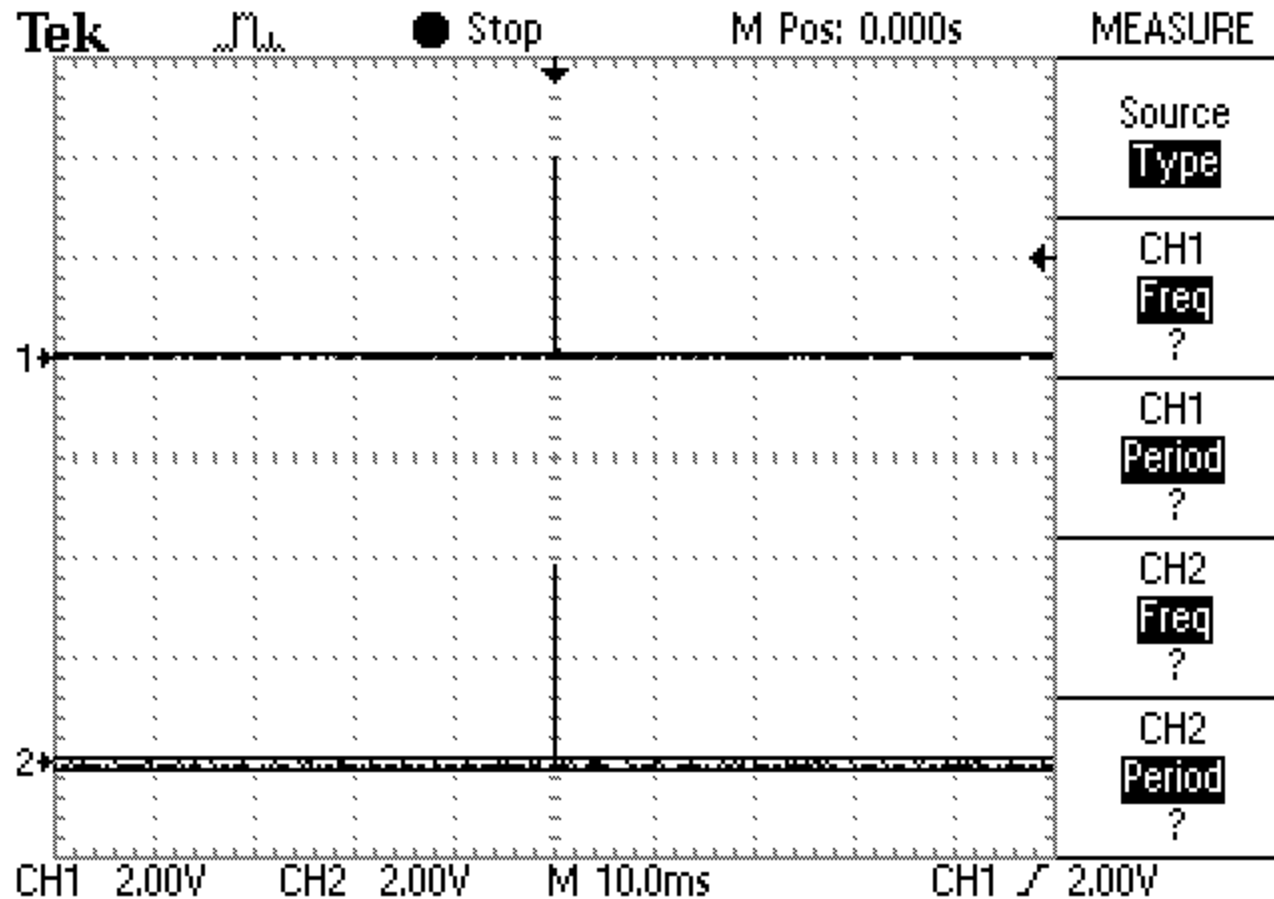
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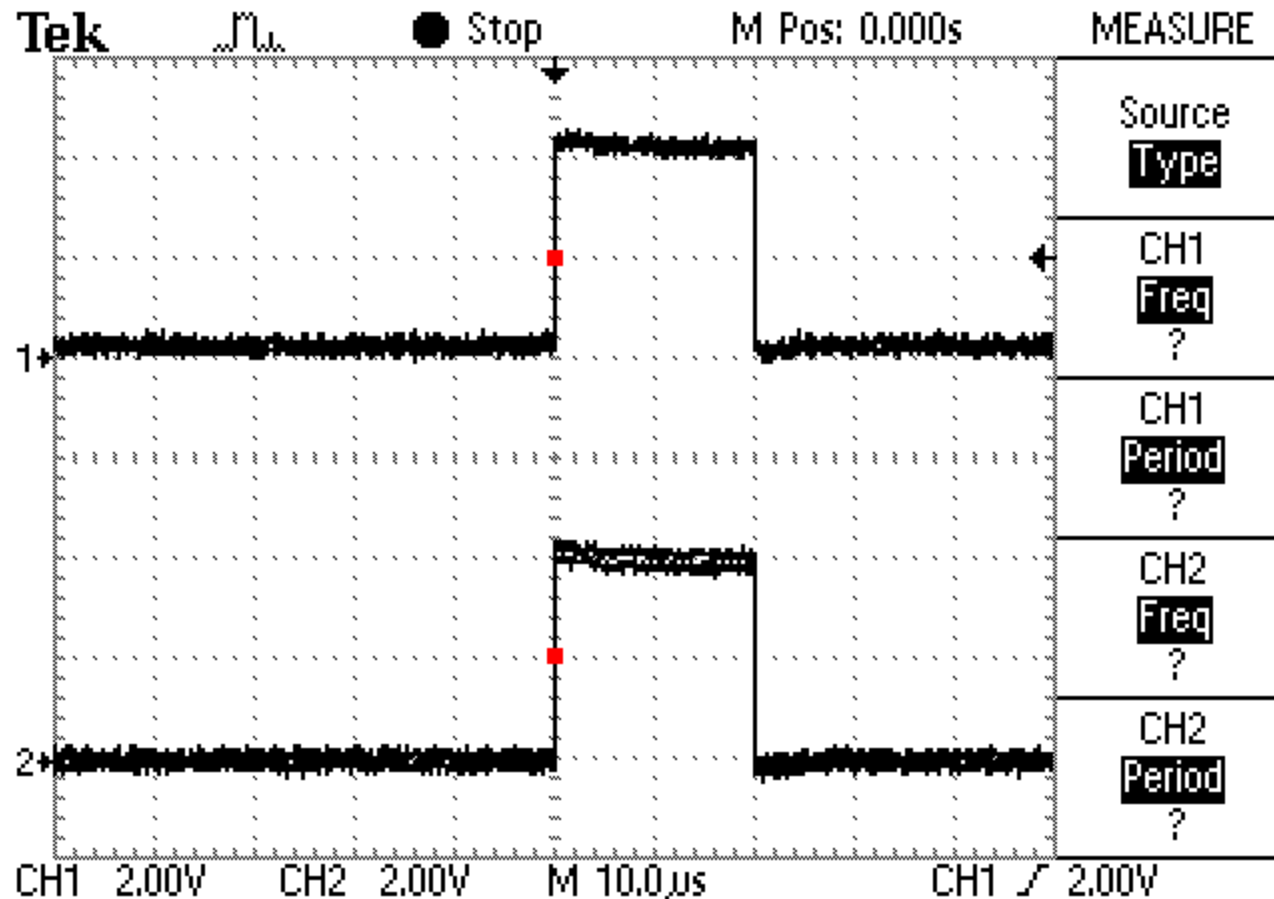
# Atomic clock 1PPS – 10 s



# Atomic clock 1PPS – 10 ms/div

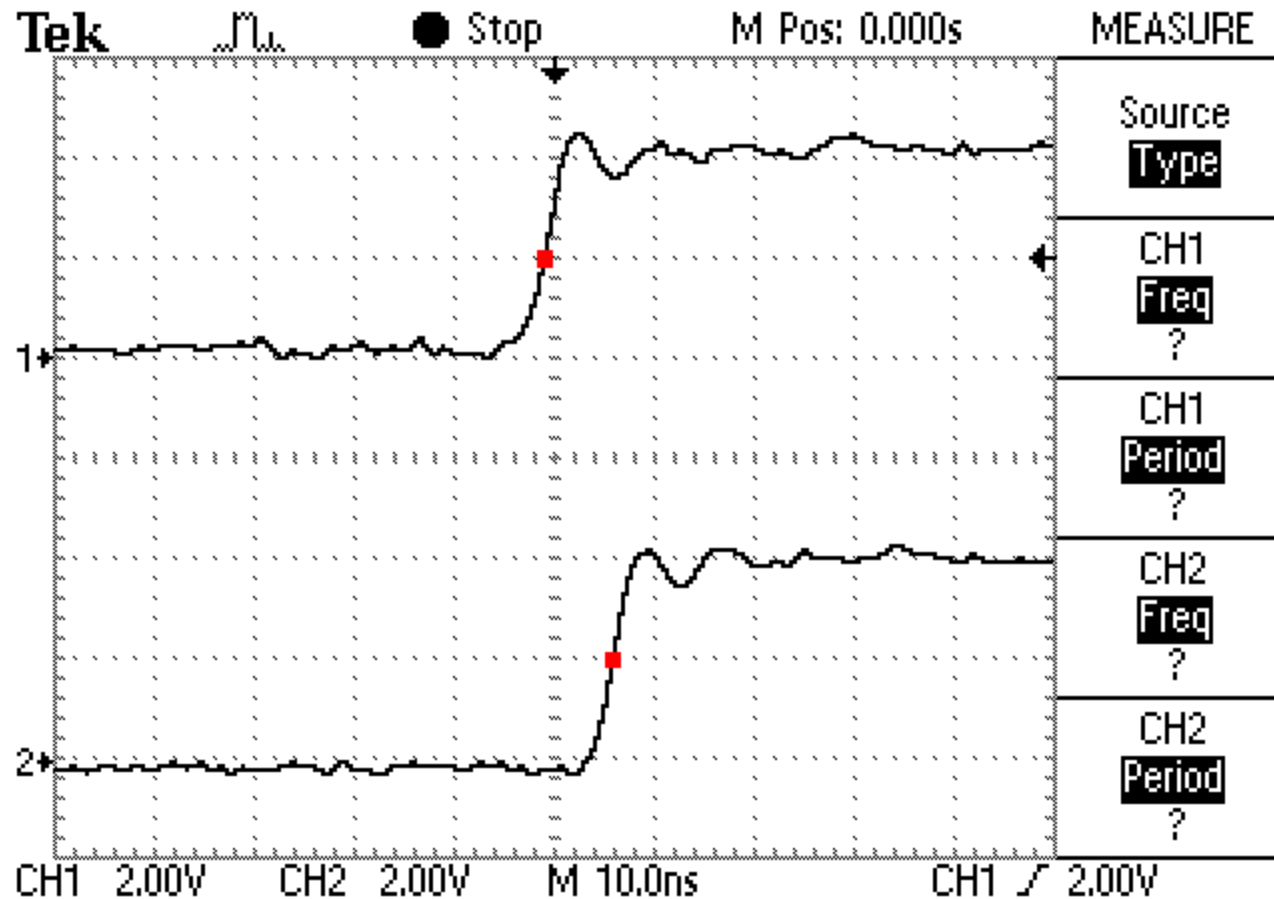


# Atomic clock 1PPS – 10 $\mu\text{s}/\text{div}$





# Atomic clock 1PPS – 10 ns/div



# Cesium 9,192,631,770 Hz

