Developing a Star Tracker Algorithm for Small Satellites

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Oct. 12, 2022
SCPNT 2022 PNT Symposium
Outline

- Star tracker
- Star catalog
- Lost-in-Space mode
  - Robust to non-star objects
  - Robust to noise
- Tracking mode
  - Stray light elimination
- Night sky test
- Conclusions
Attitude sensors for SmallSats

- Earth sensor
- Sun sensor
- IMU
- Magnetometer
- Star tracker
Star tracker

- Low mass, volume, power consumption
- Attitude determination with extremely high accuracy
- Autonomously without the aid of external navigation sensors
- Any orbit and wide angular range
- Long period without accumulating error
Interstar angles
Star tracker

Star catalog

Lost-in-Space mode

Tracking mode

Baffle
Optical system
Detector

Processing unit
Star catalog

Celestial Coordinate System on J2000

Vernal equinox

Celestial equator

North celestial pole

Declination (DEC)

Right ascension (RA)

epoch J2000
January 1st, 2000, 12:00 Terrestrial Time
Star catalog

Celestial Coordinate System at a certain time

- Proper motion
- Precession
- Nutation
- Annual aberration

- SKY2000 Master Catalog
  (http://tdc-www.harvard.edu/catalogs/sky2k.html)
  - RA, DEC, and proper motion
Star tracker

- Star catalog
  - Lost-in-Space mode
  - Tracking mode

- Baffle
- Optical system
- Detector
- Processing unit
Lost-in-Space mode

- Centroid Extraction
- Star Identification
- Attitude Estimation
Lost-in-Space mode

\[ I'(u, v) = \begin{cases} 
I_0(u, v) - T, & I_0(u, v) \geq T \\
0, & I_0(u, v) < T 
\end{cases} \]

\[ T = \text{global threshold} = \mu + 5\sigma \]

\[ I(u, v) = \begin{cases} 
I'(u, v), & \sum_{u' = u \pm w}^{u' = u \pm w} \sum_{v' = v \pm w}^{v' = v \pm w} 1\{I'(u', v') \neq 0\} \geq (w + 1)^2 \\
0, & \sum_{u' = u \pm w}^{u' = u \pm w} \sum_{v' = v \pm w}^{v' = v \pm w} 1\{I'(u', v') \neq 0\} < (w + 1)^2 
\end{cases} \]

\[ u_c = \frac{\sum_{u_{\text{start}}}^{u_{\text{end}}} \sum_{v_{\text{start}}}^{v_{\text{end}}} I(u, v)u}{\sum_{u_{\text{start}}}^{u_{\text{end}}} \sum_{v_{\text{start}}}^{v_{\text{end}}} I(u, v)} \quad v_c = \frac{\sum_{u_{\text{start}}}^{u_{\text{end}}} \sum_{v_{\text{start}}}^{v_{\text{end}}} I(u, v)v}{\sum_{u_{\text{start}}}^{u_{\text{end}}} \sum_{v_{\text{start}}}^{v_{\text{end}}} I(u, v)} \]
Lost-in-Space mode

Centroid Extraction
Star Identification
Attitude Estimation
Lost-in-Space mode

Pyramid Method

- Identify a basic triangle
- Confirm the basic triangle with 3 legs from the point to the basic triangle
- Identify remaining points with 4 legs from the point to the basic quadrilateral
- High confidence with 6 interstar angles
**Lost-in-Space mode**

### Simulation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mag. limit</td>
<td>6.0</td>
</tr>
<tr>
<td>Number of stars / points</td>
<td>21 / 231</td>
</tr>
</tbody>
</table>

### Results

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified / Detected</td>
<td>18 / 225</td>
</tr>
<tr>
<td>Matching time</td>
<td>0.12 sec</td>
</tr>
<tr>
<td>Attitude error</td>
<td>x    y    z</td>
</tr>
<tr>
<td>Mean (arcsec)</td>
<td>0.456 0.448 0.367</td>
</tr>
<tr>
<td>Std (arcsec)</td>
<td>1.277 0.875 0.823</td>
</tr>
</tbody>
</table>

### Test

- **1000 cases containing spikes which are 10 times as numerous as actual stars**
- Success rate: 100%
- Maximum matching time: 1.97 sec (600 points)
- Attitude error:
  - Mean (arcsec): 0.456 0.448 0.367
  - Std (arcsec): 1.277 0.875 0.823

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*Lost in Space mode*
### Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mag. limit</td>
<td>6.0</td>
</tr>
<tr>
<td>Number of stars</td>
<td>21</td>
</tr>
<tr>
<td>Noise STD</td>
<td>10 (gray value)</td>
</tr>
</tbody>
</table>

### Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified / Detected</td>
<td>12 / 14</td>
</tr>
<tr>
<td>Matching time</td>
<td>1 ms</td>
</tr>
<tr>
<td>Attitude error</td>
<td>x: 1.830</td>
</tr>
<tr>
<td></td>
<td>y: 2.164</td>
</tr>
<tr>
<td></td>
<td>z: -2.169</td>
</tr>
</tbody>
</table>

### Test

1000 cases containing noise with STD 10

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success rate</td>
<td>98.6%</td>
</tr>
<tr>
<td>Avg. matching time</td>
<td>0.33 ms</td>
</tr>
<tr>
<td>Attitude error</td>
<td>Mean (arcsec) x: 6.811 y: 6.432 z: 4.628</td>
</tr>
<tr>
<td></td>
<td>Std (arcsec) x: 9.273 y: 7.063 z: 5.891</td>
</tr>
</tbody>
</table>

14 failed cases: detected stars<7, identified stars<4
Star tracker

Star catalog

Lost-in-Space mode

Tracking mode

Baffle
Optical system
Detector
Processing unit
Tracking mode

- Extended Kalman Filter
- Local dynamic background template
- Updating reference stars
Tracking mode

Extended Kalman Filter

\[
q^*_k = Q_{\text{tran}} q_{k-1}
\]

\[
P^*_k = \Phi_{k-1} P_{k-1} \Phi_{k-1}^T + Q_{k-1}
\]

\[
k_k = P^*_k H_k^T (H_k P^*_k H_k^T + R_k)^{-1}
\]

\[
q_k = q^*_k + k_k (z_k - h(q^*_k))
\]

\[
P_k = (I - k_k H_k) P^*_k
\]
Tracking mode

○: stars inside the FOV
△: mapped stars

mapped: 35
Tracking mode

- Extended Kalman Filter
- Local dynamic background template
- Updating reference stars
Tracking mode

Updating reference stars

previous attitude and angular rate

estimated attitude and angular rate

EKF

prediction

correction

predicted attitude

expect centroid locations

predicted centroids

Star catalog

partitioning

reference stars

mapped stars

mapping

measured centroids
Tracking mode

Updating reference stars

Partitioning

Partition Table:
- Center vector
- Number of members
- Member’s indices
- Max distance to center
Tracking mode

○: stars inside the FOV
△: mapped stars
Tracking mode

- Extended Kalman Filter
  - Updating reference stars
  - Local dynamic background template
Tracking mode

Local dynamic background template

uneven background
Tracking mode

- Low-pass filter to estimate background template
Tracking mode

First filter

Background template by a low-pass filter

\[
l'(u, v) = \begin{cases} 
I_0(u, v) - T, & I_0(u, v) \geq T' \\
0, & I_0(u, v) < T'
\end{cases}
\]

\[T' = T + 4\sigma\]

Threshold

Second filter

\[
I(u, v) = \begin{cases} 
I'(u, v), & \sum_{u' = u \pm a, v' = v} 1 \{I'(u', v') \neq 0\} \geq (a + 1)^2 \\
0, & \sum_{u' = u \pm a, v' = v} 1 \{I'(u', v') \neq 0\} < (a + 1)^2
\end{cases}
\]
Tracking mode
Tracking mode

- ○: stars inside the FOV
- △: mapped stars

**Simulation**
- Noise STD: 5 (gray value)
- Diagonal FOV (deg): 24.5
- Angular rate (rad/s): [0.04, −0.03, −0.025]

**Results**
- Attitude error
  - Mean (arcsec): x = 3.18, y = 3.01, z = 1.54
  - Std (arcsec): x = 3.63, y = 3.61, z = 1.55
Night sky test

- Location: rooftop of NCKU DAA (urban area)
- Camera and Lens: Opto Engineering

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution (px)</td>
<td>1280 x 1024</td>
</tr>
<tr>
<td>Pixel size (μm)</td>
<td>4.8</td>
</tr>
<tr>
<td>Dimensions (mm)</td>
<td>29 x 29 x 30</td>
</tr>
<tr>
<td>FOV (deg)</td>
<td>21.3 x 17.1</td>
</tr>
<tr>
<td>Mass (g)</td>
<td>56+76=132</td>
</tr>
<tr>
<td>Focal length (mm)</td>
<td>16.3363</td>
</tr>
<tr>
<td>Aperture</td>
<td>f/1.4 - f/22</td>
</tr>
<tr>
<td>Dimensions (mm)</td>
<td>37.9(L) x 34(ϕ)</td>
</tr>
</tbody>
</table>

- Verification
  alt-azimuth mount: iOptron AZ Mount Pro
### Results

<table>
<thead>
<tr>
<th>Identified / Detected</th>
<th>11 / 22</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Attitude error (arcminute)</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.50</td>
<td>4.43</td>
<td>4.68</td>
</tr>
</tbody>
</table>

**Detected but not identified**

**Detected and identified**

Exposure time: 200 μs

**Markers**
- △: detected stars
- ○: stars in catalog
- ✴: identified stars
Probable sources of error

- Camera calibration precision
- Camera mounting precision
- Alt-azimuth mount tracking precision
- Light pollution, cloud cover
Conclusions

- The software algorithm of a star tracker with high robustness to non-star objects, noise and stray light, was developed.
- It achieved attitude estimation at arcsecond-level accuracy in the test with simulated star-field images.
- It reached arcminute-level accuracy with real night-sky photos.
- The precise camera calibration and the hardware designing are the works in the next stage to improve the accuracy and the capability.


Thank you for listening.