Introduction to Mathematical Optimization

- Prerequisites
- Information and Vocabulary
- Course Outline

Course prerequisites

- First three units: math content around Algebra 1 level, analytical skills approaching Calculus. Students at the Pre-Calculus level should feel comfortable. Talented students in Algebra 1 can certainly give it a shot.
- Last two units: Calculus required – know how to take derivatives and be familiar with their implications for finding maxima and minima.
- Computer programming skills will be taught from the ground up. Previous experience is not necessary.

Equipment Needed

For much of the first unit, a scientific calculator is sufficient, though a graphing calculator will make your life easier.

Towards the end of the first unit, when we get into coding, a computer able to download and install software (specifically, the programming language Julia) is necessary. Julia is written for Mac, Windows and Linux systems.

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But "best" can vary. If you're a football player, you might want to maximize your running yards, and also minimize your fumbles. Both maximizing and minimizing are types of optimization problems.

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- Manufacturing
- Production
- Inventory control
- Transportation
- Scheduling
- Networks
- Finance
- Engineering
- Mechanics
- Economics
- Control engineering
- Marketing
- Policy Modeling

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- Constraints, which are equations that place limits on how big or small some variables can get. Equality constraints are usually noted $h_n(x)$ and inequality constraints are noted $g_n(x)$.

Optimization Vocabulary

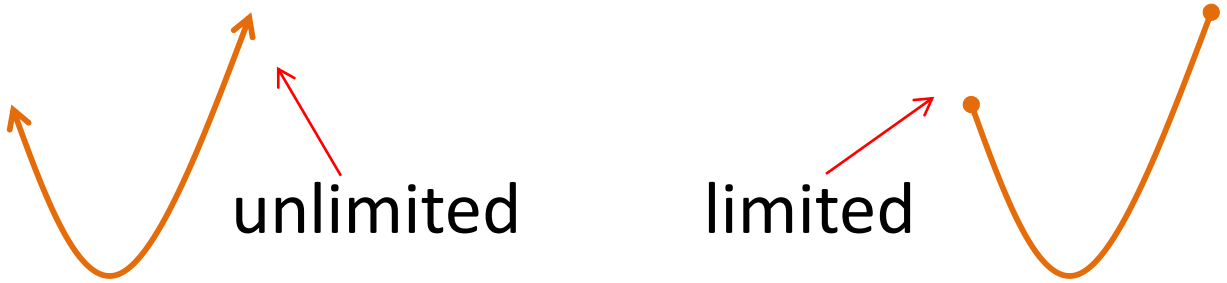
A football coach is planning practices for his running backs.

- His main goal is to maximize running yards – this will become his objective function.
- He can make his athletes spend practice time in the weight room; running sprints; or practicing ball protection. The amount of time spent on each is a variable.
- However, there are limits to the total amount of time he has. Also, if he completely sacrifices ball protection he may see running yards go up, but also fumbles, so he may place an upper limit on the amount of fumbles he considers acceptable. These are constraints.

Note that the variables influence the objective function and the constraints place limits on the domain of the variables.

Types of Optimization Problems

- Some problems have constraints and some do not.



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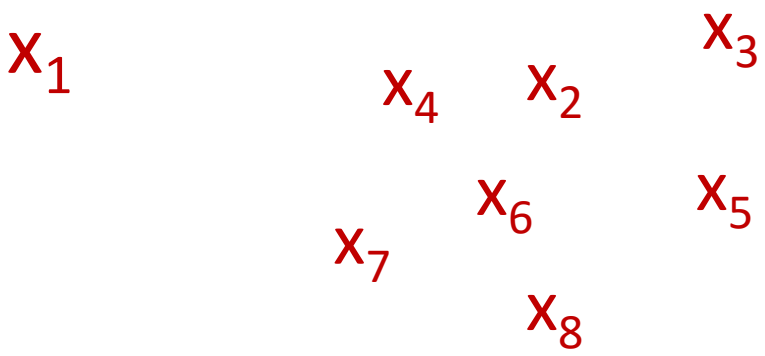

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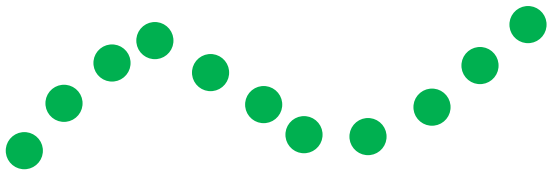

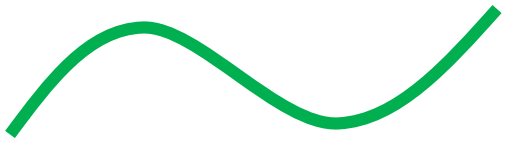

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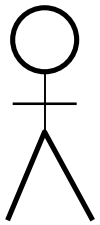

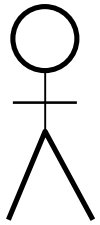

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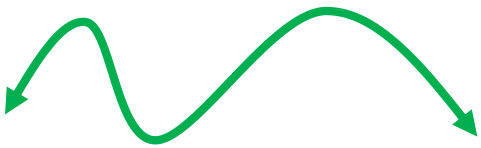
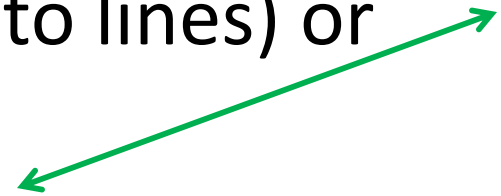
- Equations can be linear (graph to lines) or nonlinear (graph to curves)





Why Mathematical Optimization is Important

- Mathematical Optimization works better than traditional “guess-and-check” methods
- M. O. is a lot less expensive than building and testing
- In the modern world, pennies matter, microseconds matter, microns matter.

Why Mathematical Optimization is worth learning

Q: Which of these things is not like the others?

a) A degree in engineering

b) A degree in chemistry

c) A degree in pure mathematics

d) A large pepperoni pizza

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(With the others, you can feed a family of four)

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Joking aside, if you're interested in a career in mathematics (outside of teaching or academia), your best bet is applied mathematics with computers. Mathematical optimization is a powerful career option within applied math.

If you're not interested in a career in mathematics, you will probably run into optimization problems anyway.

Course Outline

- Unit 1: Introductions and Skills

  Optimization, vectors, iteration and recursion, foundational programming skills
- Unit 2: Non-calculus methods without constraints

  Methods in two dimensions using computers; extension to methods in three or more dimensions
- Unit 3: Non-calculus methods with constraints

  Linear programming
- Unit 4: Calculus methods without constraints

  Newton's method and review of derivative meaning; derivatives in 3D and above with implications for optimization
- Unit 5: Calculus methods with constraints

  Penalty functions; overview of other methods; Lagrange multipliers

Practice Question 1

Group the following into what might be maximized, minimized or cannot be optimized.

1. When choosing a new phone and plan, you might consider: minutes of talk time per month; how much is charged for overages; whether extra minutes roll over; amount of data allowed; cost per month; amount of storage/memory; how many phones are available; brands/types of available phones; cost of the phone; amount of energy used; time it takes to download apps or music; whether or not you get signal in your home.

Practice Question 2

2. An airplane designer is trying to build the most fuel-efficient airplane possible. Write one factor as an objective ("Minimize/maximize _____") and the rest as constraints ("_____ ≤ c_1", or ≥ or =). Delete any non-numerical factors:

speed, fuel consumption, range, noise, weight, type of propulsion, cost, ease of use, amount of lift, amount of drag, sonic boom volume, payload (how much it can carry).

Practice Questions 3-5

For each of the following tasks, write an objective function ("maximize ____") and at least two constraints ("subject to _____ ≤ c_1", or ≥ or =)

3. A student must create a poster project for a class.

4. A shipping company must deliver packages to customers.

5. A grocery store must decide how to organize the store layout.