Iteration and Recursion

- Definitions
- Some familiar examples

Definitions

Iteration is when the same procedure is repeated multiple times. Take a moment to solve this division problem by hand:

$$6\overline{)347{,}861}$$

Definitions

Iteration is when the same procedure is repeated multiple times. Take a moment to solve this division problem by hand:

$$6\overline{)347{,}861}$$

The answer is not really important.

Hopefully you noticed what you were doing: how many times does 6 go in? Write that there. Multiply. Subtract. Bring down the next digit. Repeat. Repeat. Repeat, 5 or 6 total times. Thus, you completed 5 or 6 iterations of this procedure.

Definitions

Iteration is when the same procedure is repeated multiple times.

Recursion is when the output of one iteration becomes the input of the next.

Write down a list of the first 15 powers of 2:

1, 2, 4, ….

Definitions

Iteration is when the same procedure is repeated multiple times.

Recursion is when the output of one iteration becomes the input of the next.

Write down a list of the first 15 powers of 2:

1, 2, 4, ... (again, the answer is not important)

You may have the first few memorized, but after a while you just double: take your answer and double it, take that answer and double it. Each output became the next input. That's recursion.

Definitions

Iteration is when the same procedure is repeated multiple times.

Recursion is when the output of one iteration becomes the input of the next.

Another example of iteration

How would you determine if a number n is prime?

One way is to divide n by 2, then by 3, then by 4, then by everything up to n-1 to see if it divides evenly. If it doesn't, it's prime. This would require n-2 iterations.

There are ways that would require fewer iterations. What are they?

Prime numbers, step by step

How would you determine if a number n is prime?

0. Start with the number 2.
1. Divide.
2. Check if it divides evenly.
    a) if yes, then stop. n is not prime.
    b) if no, go to step 3.
3. If that was the last number, stop. n is prime. Or,
4. Choose the next number and go to step 1.

An example of iteration with recursion

Grab a calculator. Type in the number 325.

Here's the rules: if a number is even, divide it by 2. If it's odd, multiply by 3 and add 1.

Repeat until your number becomes 1.

Count the number of iterations it takes. ("divide by 2" is an iteration and "times 3 plus 1" is an iteration.)

A familiar example of recursion

The Fibonacci numbers are a number pattern that looks like this:

1, 1, 2, 3, 5, 8, 13, …

If you've seen them before, don't give it away! If you haven't, see if you can figure out how they grow...

How is this pattern recursive? How is it iterative?

Practice Problem 1

1a. Using an iterative procedure and your calculator, determine if the number 761 is prime in as few iterations as you can. Write down all the numbers you tested and whether they divided evenly into 761.

b. Repeat for the number 391.

Practice Problem 2

2. Suppose we changed the rules for Fibonacci numbers as follows:

a) Start with 2, 5 (currently they start 1, 1)

b) End when you get over 1000 (currently they go on forever – no end condition)

Write the sequence that would result:

Practice Problem 3

3. Suppose we changed the rules for Fibonacci numbers as follows:

a) Start with 2, 5, 4

b) End after 12 iterations, or if you get over 1000 or under -1000

c) Using the previous three numbers, multiply the first two and subtract the third.

d) Write the sequence that would result:

Practice Problem 4

4. Using the recursive calculator game (even ÷ 2, odd · 3 + 1), write down the formal procedure for a starting number n:

- Start with ___
- End when…
- Test: if ____ then _____,

  or else if _____ then _____.
- Describe the loop: using _____, return to ____.