# Applications of the Gradient

- Evaluating the Gradient at a Point
- Meaning of the Gradient

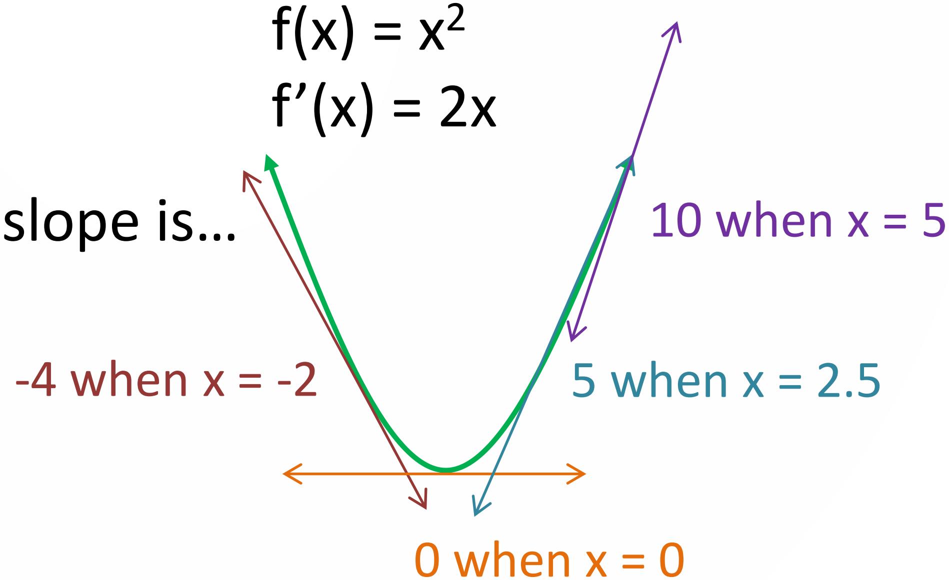
# Evaluating the Gradient

In 1-variable calculus, the derivative gives you an equation for the slope at any x-value along f(x). You can then plug in an x-value to find the actual slope at that point.

$f(x) = x^2$

$f'(x) = 2x$

Actual tangent line slope is…



# Evaluating the Gradient

Similarly, the gradient gives you an equation for the slope of the tangent plane at any point (x, y) or (x, y, z) or whatever. You can then plug in the actual values at any point to find the slope of the tangent plane at that point.

The slope of the tangent plane will be written as a vector, composed of the slopes along each dimension.

# Evaluating the Gradient

As an example, given the function $f(x, y) = 3x^2y - 2x$ and the point (4, -3), the gradient can be calculated as:

$$[6xy - 2 \quad 3x^2]$$

Plugging in the values of x and y at (4, -3) gives

$$[-74 \quad 48]$$

which is the value of the gradient at that point.

# Practice Problems 1 and 2

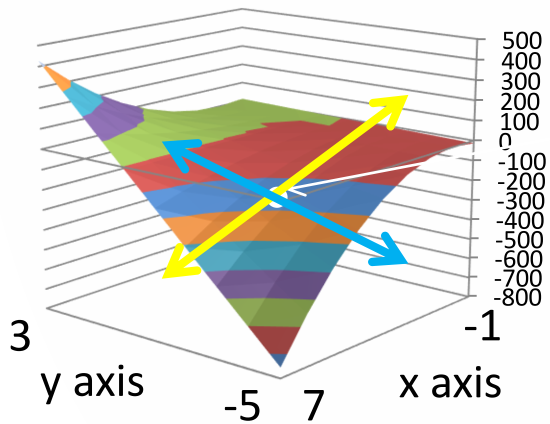
Evaluate the gradient of…

1. $f(x, y) = x^2 + y^2$ at
   a) (0, 0)   b) (1, 3)   c) (-1, -5)

2. $f(x, y, z) = x^3z - 2y^2x + 5z$ at
   a) (1, 1, -4)   b) (0, 1, 0)   c) (-3, -2, 1)

# Meaning of the Gradient

In the previous example, the function $f(x, y) = 3x^2y - 2x$ had a gradient of $[6xy - 2 \quad 3x^2]$, which at the point (4, -3) came out to [-74  48].



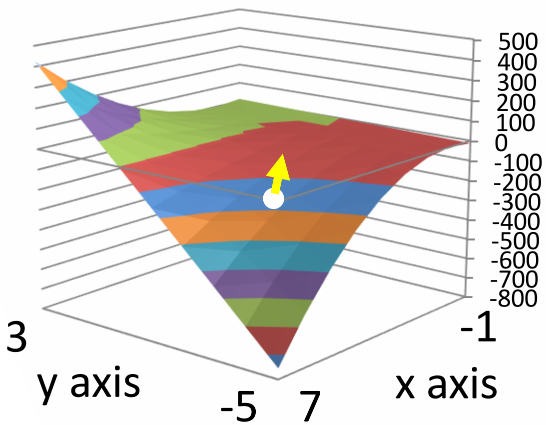

(4, -3)

The tangent plane at that point will have a slope of -74 in the x direction and +48 in the y direction.

Even more important is the vector itself, [-74  48].

# Meaning of the Gradient

Here is the graph again, with the vector drawn in as a vector rather than two sloped lines:

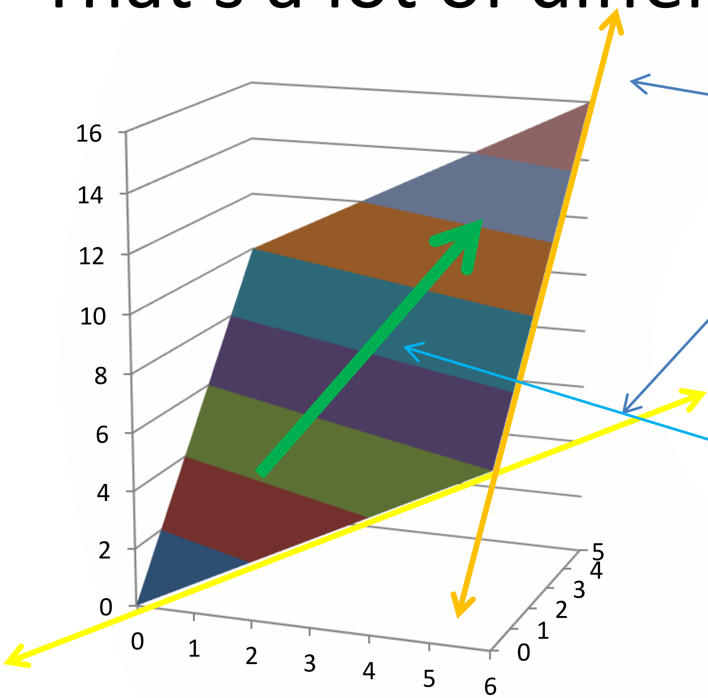



Recall that vectors give us direction as well as magnitude. The direction of the gradient vector will always point in the direction of steepest increase for the function.

And, its magnitude will give us the slope of the plane in that direction.

# Meaning of the Gradient

That's a lot of different slopes!




Each component of the gradient vector gives the slope in one dimension only.

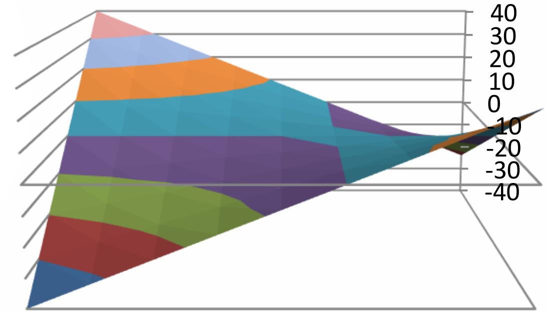
The magnitude of the gradient vector gives the steepest possible slope of the plane.

Recall that the magnitude can be found using the Pythagorean Theorem, $c^2 = a^2 + b^2$, where c is the magnitude and a and b are the components of the vector.

# Practice Problem 3

Find the gradient of f(x, y) = 2xy – 2y, and the magnitude of the gradient, at…

a) (0, 0)
b) (5, -3)
c) (20, 10)
d) (-5, 4)




e) Find where the gradient = 0.

# Practice Problem 4

Find the gradient of

$$f(x, y, z, w) = 3xy - 2xw + 5xz - 2yw$$

and the magnitude of the gradient at (0, 1, -1, 2).

# Practice Problem 5

Suppose we are maximizing the function

$$f(x, y) = 4x + 2y - x^2 - 3y^2$$

Find the gradient and its magnitude from

a) (1, 5)

b) (3, -2)

c) (2, 0)

d) (-4, -6)

e) Find where the gradient is 0.

# Practice Problem 6

Suppose you were trying to *minimize* $f(x, y) = x^2 + 2y + 2y^2$. Along what vector should you travel from (5, 12)?