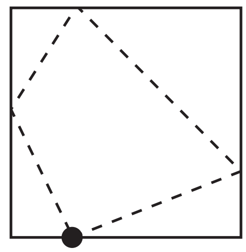
1. A regular polygon of side length 1 has the property that if regular pentagons of side length 1 are placed on each side, then each pentagon shares a side with the two adjacent ones. How many sides does such a polygon have?

2. John stands against one wall of a square room with walls of length 4 meters each. He kicks a frictionless, perfectly elastic ball in such a way that it bounces off the three other walls once each and returns to him (diagram not geometrically accurate). How many meters does the ball travel?



3. A cube is inscribed in a sphere of radius r. Find the ratio of the volume of the cube to that of the sphere.

4. A circle of radius 144 has three smaller circles inside it, all congruent. Each small circle is tangent to the other two and to the large circle. Find the radius of one of the smaller circles.

5. In $\triangle ABC$, $\angle C$ is right, $AC = 2 - \sqrt{3} + x$ and $BC = 1 - 2x + x\sqrt{3}$. Find $m\angle B$.

6. Points A, B, C lie on sides $\overline{DE}$, $\overline{EF}$, and $\overline{FE}$ of $\triangle DEF$, respectively. If $DA = 3$, $AE = 2$, $EB = 2$, $BF = 11$, $FC = 11$, and $CD = 1$, find the area of $\triangle ABC$.

7. What is the area of the incircle of a triangle with side lengths 10040, 6024, and 8032?

8. Rhombus ABCD has side length l, with $\cos(m\angle B) = -\frac{2}{3}$. The circle through points A, B, and D has radius 1. Find l.

9. A trapezoid has bases of length 10 and 15. Find the length of the segment that stretches from one leg of the trapezoid to the other, parallel to the bases, through the intersection point of the diagonals.

10. A regular polygon with 40 sides, all of length 1, is divided into triangles, with each vertex of each triangle being a vertex of the original polygon. Let A be the area of the smallest triangle. What is the minimum number of square root signs needed to express the exact value of A?