1. **Answer: 10**

   Label three consecutive vertices of the polygon A, B, and C. Let BP be the common side to the pentagons placed on sides AB and BC. Then $m\angle ABP = m\angle PBC = 108°$. Since $m\angle ABP + m\angle PBC + m\angle ABC = 360°$, this gives $m\angle ABC = 144°$. So the exterior angle of this polygon is 36, and thus it has 10 sides.

2. **Answer: $8\sqrt{2}$**

   Notice that the ball travels the length of the room twice and the width of the room twice, so it's traveled a total of 8 meters in the horizontal direction and 8 meters in the vertical direction. Because the ball is bouncing (and thus its path after a bounce is the same as its path before the bounce, but reflected), we can rearrange the four segments of its path into a straight line by only reflection and translation. This line travels 8 meters horizontally and 8 meters vertically, so its length, which is the total length of the ball's path, is $8\sqrt{2}$.

3. **Answer: $\frac{\sqrt{3}\pi}{2}$**

   Since the space diagonal of the cube is a diameter of the sphere, we have $s\sqrt{3} = 2r$. The ratio is then

$$\frac{\frac{4}{3}\pi r^3}{\left(\frac{2r}{\sqrt{3}}\right)^3} = \frac{\sqrt{3}\pi}{2}$$

4. **Answer: $288\sqrt{3} - 432$**

   Let r be the radius of a small circle. The centers of the small circles form an equilateral triangle of side length $2r$. The length of the median of such a triangle is $\sqrt{3}r$, so the distance from the center of the triangle (which is also the center of the large circle) to a vertex is $\frac{2\sqrt{3}}{3}r$. Since each vertex of the triangle is distance r from the edge of the large circle, the radius of the large circle is $\frac{2\sqrt{3}}{3}r + r = 144$. This gives $(2\sqrt{3}+3)r = 432$, so $r = \frac{432}{2\sqrt{3}+3} \cdot \frac{2\sqrt{3}-3}{2\sqrt{3}-3} = 144(2\sqrt{3}-3) = 288\sqrt{3} - 432$.

5. **Answer: $15° + \tan^{-1} x$ or $\frac{\pi}{12} + \tan^{-1} x$**

   From basic trigonometry, we have $\tan(m\angle B) = \frac{2-\sqrt{3}+x}{1-(2-\sqrt{3})x}$. This is the tangent angle addition identity, for angles with tangents x and $2-\sqrt{3}$. Since $\tan(15°) = 2-\sqrt{3}$, $m\angle B$, the inverse tangent, is therefore $15° + \tan^{-1} x$.

6. **Answer: $\frac{44}{13}$**

   Let $\alpha = m\angle E$ and $\beta = m\angle F$. Note that D is a right angle. Therefore, $\sin\alpha = \frac{5}{13}$. $[CBF] = \frac{1}{2} \cdot 11^2 \sin\alpha = \frac{1}{2} \cdot 121 \cdot \frac{5}{13}$. Similarly, $[ABE] = \frac{1}{2} \cdot 2^2 \sin\beta = \frac{1}{2} \cdot 4 \cdot \cos\alpha = \frac{1}{2} \cdot 4 \cdot \frac{12}{13}$. Finally, $[ACD] = \frac{3 \cdot 1}{2}$. Subtracting these three areas from that of $\triangle DEF$ gives the result.

7. **Answer: $2008^2\pi$ or 4032064π**

   Note that we can scale the triangle down by a factor of 2008 to a 3,4,5 right triangle. Let $\overline{AB}$, $\overline{AC}$ be the legs of the triangle. The incircle splits $\overline{AB}$ into two segments of lengths x and y. It similarly splits $\overline{AC}$ into segments of lengths x and z and $\overline{BC}$ into segments of lengths y and z. Thus, we get:

$$x + y = 3$$
$$x + z = 4$$
$$y + z = 5$$

   Thus, $x = 1$, $y = 2$, $z = 3$. Thus, the incircle has a radius of 1, and so an area of π. Scaling back up will increase the incircle's radius by a factor of 2008, giving us an area of $2008^2\pi$.

8. **Answer:** $\frac{\sqrt{30}}{3}$

   Let O be the center of the circle, and X be the center of the rhombus (the intersection of $\overline{AC}$ and $\overline{BD}$). Let $m\angle ABC = \theta = \cos^{-1}\left(-\frac{2}{3}\right)$. Considering $\triangle OBX$ and $\triangle ABX$, using triangle angle sums and the fact that an inscribed angle has half the measure of the intercepted arc, we have $OX = \cos(\pi - \theta)$, so $AX = 1 + \cos(\pi - \theta)$. Also, $BX = \sin(\pi - \theta)$. The Pythagorean theorem then gives $l = \sqrt{2(1 + \cos(\pi - \theta)} = \sqrt{2(1 + \frac{2}{3})}$.

9. **Answer: 12**

   Let the trapezoid be $ABCD$ with $AB = 10$, $CD = 15$. Let P be the intersection of the diagonals, and let $\overline{XY}$ be the segment through P parallel to the bases with X on $\overline{AD}$ and Y on $\overline{BC}$. Note that $\triangle PYC \sim \triangle ABC$, so $\frac{PY}{AB} = \frac{YC}{BC}$. Also, $\triangle PYB \sim \triangle DCB$, so $\frac{PY}{CD} = \frac{BY}{BC}$. Adding these equations gives $\frac{PY}{AB} + \frac{PY}{CD} = \frac{BY+YC}{BC} = 1$, so $PY(\frac{1}{10} + \frac{1}{15}) = PY \cdot \frac{1}{6} = 1$, hence $PY = 6$.

   The same argument shows that $PX = 6$, so $XY = 12$.

10. **Answer: 3**

    The polygon has angles of 171°, and the smallest triangle has two adjacent sides of the original polygon as two of its sides. The area of this triangle is $\frac{1}{2} \cdot 1 \cdot 1 \cdot \sin(171) = \frac{1}{2}\sin(9)$. So the question is, how many square roots do we need to express $\sin(9)$? Conveniently enough, $\sin(18) = \frac{\sqrt{5}-1}{4}$, so $\cos(18) = \sqrt{1 - \sin^2(18)}$, which requires two square roots to express. Then by the half-angle formula, $\sin(9) = \sqrt{\frac{1-\cos(18)}{2}}$, which requires three square roots.