

1. Let a , b , c , and d be the numbers that show when four fair dice, numbered 1 through 6 are rolled. What is the probability that $|(a - 1)(b - 2)(c - 3)(d - 6)| = 1$?
2. Find all possibilities for the second-to-last digit of a number whose square is of the form $1_2.3.4.5.6.7.8.9.0$ (each $_$ is a digit).
3. Ten gears are lined up in a single file and meshed against each other such that the i^{th} gear from the left has $5i + 2$ teeth. Gear $i = 1$ (counting from the left) is rotated 21 times. How many revolutions does gear 10 make?
4. In the game Pokemawn, players pick a team of 6 different Pokemawn creatures. There are 25 distinct Pokemawn creatures, and each one belongs to exactly one of four categories: 7 Pokemawn are plant-type, 6 Pokemawn are bug-type, 4 Pokemawn are rock-type, and 8 Pokemawn are bovine-type. However, some Pokemawn do not get along with each other when placed on the same team: bug-type Pokemawn will eat plant-type Pokemawn, plant-type Pokemawn will eat rock-type Pokemawn, and bovine-type Pokemawn will eat anything except other Bovines. How many ways are there to form a team of 6 different Pokemawn such that none of the Pokemawn on the team want to eat any of the other Pokemawn?
5. Four cards are drawn from a standard deck (52 cards) with suits indistinguishable (for example, $A\spadesuit$ is the same as $A\clubsuit$). How many distinct hands can one obtain?
6. Find all complex numbers z such that $z^5 = 16\bar{z}$, where if $z = a + bi$, then $\bar{z} = a - bi$.
7. Evaluate $\sqrt{\frac{1+\sqrt{3}i}{2}}$
8. Frank alternates between flipping a weighted coin that has a $\frac{2}{3}$ chance of landing heads and a $\frac{1}{3}$ chance of landing tails and another weighted coin that has a $\frac{1}{4}$ chance of landing heads and a $\frac{3}{4}$ chance of landing tails. The first coin tossed is the “ $2/3 - 1/3$ ” weighted coin. What is the probability that he sees two heads in a row before he sees two tails in a row?
9. The triangular numbers $T_n = 1, 3, 6, 10, \dots$ are defined by $T_1 = 1$ and $T_{n+1} = T_n + (n + 1)$. The square numbers $S_n = 1, 4, 9, 16, \dots$ are defined by $S_1 = 1$ and $S_{n+1} = T_{n+1} + T_n$. The pentagonal numbers $P_n = 1, 5, 12, 22, \dots$ are defined by $P_1 = 1$ and $P_{n+1} = S_{n+1} + T_n$. What is the 20th pentagonal number P_{20} ?
10. Evaluate $e^{i\pi/3} + 2e^{2i\pi/3} + 2e^{3i\pi/3} + 2e^{4i\pi/3} + e^{5i\pi/3} + 9e^{6i\pi/3}$.