

1. No math tournament exam is complete without a self referencing question. What is the product of the smallest prime factor of the number of words in this problem times the largest prime factor of the number of words in this problem?
2. King Midas spent  $\frac{100}{x}\%$  of his gold deposit yesterday. He is set to earn gold today. What percentage of the amount of gold King Midas currently has would he need to earn today to end up with as much gold as he started?
3. Find all integer pairs  $(a, b)$  such that  $ab + a - 3b = 5$ .
4. Find all values of  $x$  for which  $f(x) + xf\left(\frac{1}{x}\right) = x$  for any function  $f(x)$ .
5. Find the minimum possible value of  $2x^2 + 2xy + 4y + 5y^2 - x$  for real numbers  $x$  and  $y$ .
6. The dollar is now worth  $\frac{1}{980}$  ounce of gold. After the  $n^{\text{th}}$  \$7001 billion “No Bank Left Behind” bailout package passed by congress, the dollar gains  $\frac{1}{2^{2^n-1}}$  of its  $(n-1)^{\text{th}}$  value in gold. After four bank bailouts, the dollar is worth  $\frac{1}{b}\left(1 - \frac{1}{2^c}\right)$  in gold, where  $b, c$  are positive integers. Find  $b + c$ .
7. Evaluate  $\sum_{k=1}^{2009} \lfloor \frac{k}{60} \rfloor$
8. “Balanced tertiary” is a positional notation system in which numbers are written in terms of the digits  $\bar{1}$  (negative one), 0, and 1 with the base 3. For instance,  $10\bar{1}1 = (1)3^0 + (-1)3^1 + (0)3^2 + 1(3)^3 = 25_{10}$ . Calculate  $(1\bar{1}00)(\bar{1}1) + (1\bar{1}1)$  and express your answer in balanced tertiary.
9. All the roots of  $x^3 + ax^2 + bx + c$  are positive integers greater than 2, and the coefficients satisfy  $a + b + c + 1 = -2009$ . Find  $a$ .
10. Let  $\delta(n)$  be the number of 1s in the binary expansion of  $n$  (e.g.  $\delta(1) = 1, \delta(2) = 1, \delta(3) = 2, \delta(4) = 1$ ). Evaluate:

$$10 \left( \frac{\sum_{n=1}^{\infty} \frac{\delta(n)}{n^2}}{\sum_{n=0}^{\infty} \frac{(-1)^{n-1} \delta(n)}{n^2}} \right).$$