1. Find $\sin 18^\circ$.

2. The quarter-imaginary positional number system has radix $2i$ and uses the digits $0, 1, 2$, and 3. It can be used to represent any complex number without using a negative sign. For example, $1032_{2i} = 1(2i)^3 + 0(2i)^2 + 3(2i) + 1(2i)^0 = 1 - 2i$. Compute the base-10 representation of $3.\overline{0123}_{2i}$.

3. What is the smallest number of people that can be invited to the 2010 Stanford Mathematics Tournament such that either three of them met each other last year, or three of them did not meet each other last year?

4. Express $\cos^5 \theta$ in the form $\sum_{i=1}^{n} a_i \cos(k\theta)$ for some positive integers k and n and real numbers a_i.

5. Compute
$$\sum_{j=0}^{2010} \sum_{i=j}^{2010} \binom{i}{j}.$$

6. In an n-by-m grid, 1 row and 1 column are colored blue, the rest of the cells are white. If precisely $\frac{1}{2010}$ of the cells in the grid are blue, how many values are possible for the ordered pair (n, m)?

7. A bug either splits into two perfect copies of itself or dies. If the probability of splitting is $p > \frac{1}{2}$ (and is independent of the bug's ancestry), what is the probability that a bug's descendants die out? Express your answer as a function in terms of p.

8. Find all pairs of positive integers (x, y) such that $2^x + 1 = 3^y$, and y is not divisible by 4.

9. How many ordered pairs of complex numbers (x, y) satisfy
$$x^2 + y^2 = \frac{1}{x} + \frac{1}{y} = 9?$$

10. Compute the product of all positive integers such that $\lfloor \frac{n^2}{5} \rfloor$ is prime.