

1. Compute $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}}}}$.
2. Write $0.2010\overline{228}$ as a fraction.
3. Bob sends a secret message to Alice using her RSA public key $n = 400000001$. Eve wants to listen in on their conversation. But to do this, she needs Alice's private key, which is the factorization of n . Eve knows that $n = pq$, a product of two prime factors. Find p and q .
4. If $x^2 + 1/x^2 = 7$, find all possible values of $x^5 + 1/x^5$.
5. A series of lockers, numbered 1 through 100, are all initially closed. Student 1 goes through and opens every locker. Student 3 goes through and "flips" every 3rd locker ("flipping" a locker means changing its state: if the locker is open he closes it, and if the locker is closed he opens it. Student 5 then goes through and "flips" every 5th locker. This process continues with all students with odd numbers $n < 100$ going through and "flipping" every n^{th} locker. How many lockers are open after this process?
6. Consider the sequence 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, ... Find n such that the first n terms sum up to 2010.
7. Find all the integers x in $[20, 50]$ such that $6x + 5 \equiv -19 \pmod{10}$, that is, 10 divides $(6x + 15) + 19$.
8. Let $P(x)$ be a polynomial of degree n such that $P(k) = 3^k$ for $0 \leq k \leq n$. Find $P(n + 1)$.
9. Suppose $xy - 5x + 2y = 30$, where x and y are positive integers. Find the sum of all possible values of x .
10. Find the sum of all solutions of the equation

$$\frac{1}{x^2 - 1} + \frac{2}{x^2 - 2} + \frac{3}{x^2 - 3} + \frac{4}{x^2 - 4} = 2010x - 4.$$