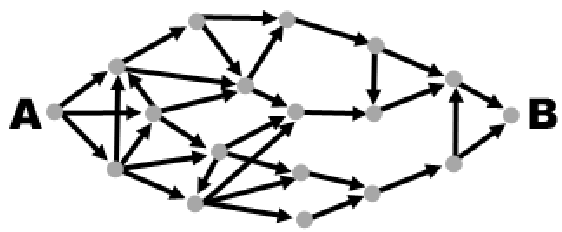
1. Given 8 coins, at most one of them is counterfeit. A counterfeit coin is lighter than a real coin. You have a free weight balance. What is the minimum number of weighings necessary to determine the identity of the counterfeit coin if it exists?
2. Find the smallest prime p such that the digits of p (in base 10) add up to a prime number greater than 10.
3. How many zeros are there at the end of $\binom{200}{124}$?
4. Compute $\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}}}$
5. Alice sends a secret message to Bob using her RSA public key $n = 400000001$. Eve wants to listen in on their conversation. But to do this, she needs Alice's private key, which is the factorization of n. Eve knows that $n = pq$, a product of two prime factors. Find p and q.
6. A triangle has side lengths 7, 9, and 12. What is the area of the triangle?
7. How many paths are there from A to B in the directed graph below?



8. Find all solutions of $\frac{a}{x} = \frac{x-a}{a}$ for x.
9. A straight line connects City A at $(0, 0)$ to City B, 300 meters away at $(300, 0)$. At time $t = 0$, a bullet train instantaneously sets out from City A to City B while another bullet train simultaneously leaves from City B to City A going on the same train track. Both trains are traveling at a constant speed of 50 meters/second. Also at $t = 0$, a super fly stationed at $(150, 0)$ and restricted to move only on the train tracks travels towards City B. The fly always travels at 60 meters/second, and any time it hits a train, it instantaneously reverses its direction and travels at the same speed. At the moment the trains collide, what is the total distance that the fly will have traveled? Assume each train is a point and that the trains travel at their same respective velocities before and after collisions with the fly.
10. Compute the base 10 value of 14641_{99}.
11. What is the area of the regular hexagon with perimeter 60?
12. Consider the sequence $1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, \ldots$ Find n such that the first n terms sum up to 2010.
13. Find all the integers x in $[20, 50]$ such that $6x + 5 \equiv -19 \mod 10$, that is, 10 divides $(6x + 15) + 19$.
14. A series of lockers, numbered 1 through 100, are all initially closed. Student 1 goes through and opens every locker. Student 3 goes through and "flips" every 3^{rd} locker ("flipping") a locker means changing its state: if the locker is open he closes it, and if the locker is closed he opens it). Thus, Student 3 will close the third locker, open the sixth, close the ninth. . . . Student 5 then goes through and "flips" every 5^{th} locker. This process continues with all students with odd numbers $n < 100$ going through and "flipping" every n^{th} locker. How many lockers are open after this process?
15. Find the best approximation of $\sqrt{3}$ by a rational number with denominator less than or equal to 15.

16. A wheel is rolled without slipping through 15 laps on a circular racecourse with radius 7. The wheel is perfectly circular and has radius 5. After the three laps, how many revolutions around its axis has the wheel been turned through?

17. An equilateral triangle is inscribed inside of a circle of radius R. Find the side length of the triangle.

18. In an n-by-m grid, 1 row and 1 column are colored blue, the rest of the cells are white. If precisely $\frac{1}{2010}$ of the cells in the grid are blue, how many values are possible for the ordered pair (n, m)?

19. Find the roots of $6x^4 + 17x^3 + 7x^2 - 8x - 4$

20. Given five circles of radii 1, 2, 3, 4, and 5, what is the maximum number of points of intersections possible (every distinct point where two circles intersect counts).

21. How many nonnegative integer solutions are there for $x^4 - 2y^2 = 1$?

22. We need not restrict our number system radix to be an integer. Consider the phinary numeral system in which the radix is the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$ and the digits 0 and 1 are used. Compute $1010100_\phi - .010101_\phi$.

23. Let $f(X, Y, Z) = X^5Y - XY^5 + Y^5Z - YZ^5 + Z^5X - ZX^5$. Find

$$\frac{f(2009, 2010, 2011) + f(2010, 2011, 2009) - f(2011, 2010, 2009)}{f(2009, 2010, 2011)}.$$

24. We are given a coin of diameter $\frac{1}{2}$ and a checkerboard of 1×1 squares of area 2010×2010. We toss the coin such that it lands completely on the checkerboard. If the probability that the coin doesn't touch any of the lattice lines is $\frac{a^2}{b^2}$ where $\frac{a}{b}$ is a reduced fraction, find $a + b$.

25. There are five balls that look identical, but their weights all differ by a little. We have a balance that can compare only two balls at a time. What is the minimum number of times, in the worst case, we have to use to balance to rank all balls by weight?