1. Compute $\lim_{x\to 0} \dfrac{\tan x - x - \frac{x^3}{3}}{x}$.

2. For how many integers n is $\frac{n}{20-n}$ equal to the square of a positive integer.

3. Find all possible solutions $(x_1, x_2, \ldots, x_n)$ to the following equations, where

$$\begin{aligned}
x_1 &= \frac{1}{2}\left(x_n + \frac{x_{n-1}^2}{x_n}\right)\\
x_2 &= \frac{1}{2}\left(x_1 + \frac{x_n^2}{x_1}\right)\\
x_3 &= \frac{1}{2}\left(x_2 + \frac{x_1^2}{x_2}\right)\\
x_4 &= \frac{1}{2}\left(x_3 + \frac{x_2^2}{x_3}\right)\\
x_5 &= \frac{1}{2}\left(x_4 + \frac{x_3^2}{x_4}\right)\\
&\vdots\\
x_n &= \frac{1}{2}\left(x_{n-1} + \frac{x_{n-2}^2}{x_{n-1}}\right) = 2010.
\end{aligned}$$

4. Let $ABCDEF$ be a convex hexagon, whose opposite angles are parallel, satisfying $AF = 3$, $BC = 4$, and $DE = 5$. Suppose that AD, BE, CF intersect in a point. Find CD.

5. Rank the following in decreasing order:

$$A = \frac{1\sqrt{1} + 2\sqrt{2} + 3\sqrt{3}}{\sqrt{1} + \sqrt{2} + \sqrt{3}}, B = \frac{1^2 + 2^2 + 3^2}{1 + 2 + 3}, C = \frac{1 + 2 + 3}{3}, D = \frac{\sqrt{1} + \sqrt{2} + \sqrt{3}}{\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}}.$$

6. What is the least m such that for any m integers we can choose 6 integers such that their sum is divisible by 6?

7. Find all positive integers n such that $\phi(n) = 16$, where $\phi(n)$ is defined to be the number of positive integers less than or equal to n that are relatively prime to n.

8. Suppose that for an infinitely differentiable function f,

$$\lim_{x\to 0} \frac{f(4x) + af(3x) + bf(2x) + cf(x) + df(0)}{x^4}$$

exists. Find $1000a + 100b + 10c + d$.

9. Minimize $x^3 + 4y^2 + 9z$ under the constraints that $xyz = 1$, $x, y, z \geq 0$.

10. Positive real numbers x, y, and z satisfy the equations

$$\begin{aligned}
x^2 + y^2 &= 9\\
y^2 + \sqrt{2}yz + z^2 &= 16\\
z^2 + \sqrt{2}zx + x^2 &= 25.
\end{aligned}$$

Compute $\sqrt{2}xy + yz + zx$.

11. Find the volume of the region given by the inequality

$$|x + y + z| + |x + y - z| + |x - y + z| + |-x + y + z| \leq 4.$$

12. Suppose we have a polyhedron consisting of triangles and quadrilaterals, and each vertex is shared by exactly 4 triangles and one quadrilateral. How many vertices are there?

13. Rank the following in increasing order:

$$A = \frac{\sqrt{2011}+\sqrt{2009}}{2},\ B = \frac{2011\sqrt{2011}-2009\sqrt{2009}}{3},$$
$$C = \frac{\sqrt{2011}+2\sqrt{2010}+\sqrt{2009}}{4},\ D = \sqrt{2010},\ E = \sqrt{2011} - \frac{1}{2\sqrt{2011}}$$

14. Suppose f and g are continuously differentiable functions satisfying

$$f(x+y) = f(x)f(y) - g(x)g(y)$$
$$g(x+y) = f(x)g(y) + g(x)f(y)$$

Also suppose that $f'(0) = 1$ and $g'(0) = 2$. Find $f(2010)^2 + g(2010)^2$.

15. Find the number of n-tuples $(a_1, \ldots a_n)$ that maximize

$$a_1a_2a_3 + a_2a_3a_4 + \cdots + a_{n-2}a_{n-1}a_n$$

under the constraints that $n \geq 3$ and

$$a_1 + a_2 + \cdots + a_n = 3m$$

for a fixed integer m, where a_i are positive integers.