The SUMO Speaker Series for Undergraduates

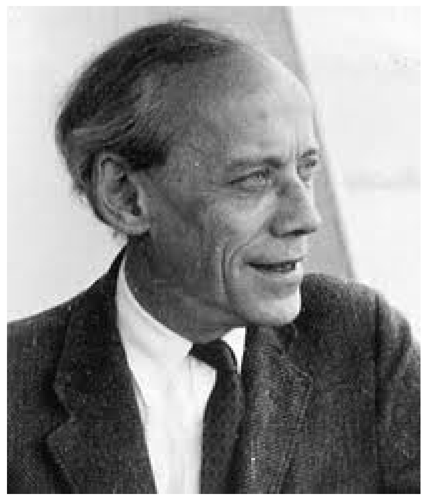
(Food Provided)

Wednesday, February 1st

4:15-5:05, room 380C

Hilbert's 17th Problem

Professor Gregory Brumfiel



ABSTRACT:

$$f(x,y) = 1 + x^2y^4 + x^4y^2 - 3x^2y^2 \geq 0 \text{ on } \mathbb{R}^2$$

$$f(x,y) = \frac{x^2y^2(x^2+y^2+1)(x^2+y^2-2)^2 + (x^2-y^2)^2}{(x^2+y^2)^2}$$

Among the 23 famous problems posed by Hilbert in 1900, the 17th Problem conjectured that if f is a polynomial in n variables with real coefficients that is nowhere negative as a function on R^n, then f is a sum of squares of *rational functions*. Hilbert knew (but had no explicit examples) that f need not be a sum of squares of *polynomials* if n >= 2.

Artin solved Hilbert's problem around 1927, and in so doing began a systematic study of ordered fields and their connections with real algebraic geometry. Nonetheless, modern real algebraic geometry did not really get off the ground until about 40 years later. The simple specific example f(x, y) above, which is not a sum of squares of polynomials, was only found around 1967 by Motzkin.

In this talk I will give a brief account of Artin's theory of ordered fields, Hilbert's 17th Problem, and some more recent developments.

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