

Time limit: 15 minutes.

Instructions: This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will be informed if an answer submission is correct or incorrect upon submission. Resubmissions are allowed, but incorrect submissions incur a penalty if the question is ultimately solved correctly. In addition, to prevent excessive guessing, after making an incorrect submission, you may not make another submission for 30 seconds.

No calculators.

1. Let $p(n)$ be the smallest digit that is part of the decimal writing of a natural number n .
Compute $p(100) + p(101) + p(102) + \dots + p(998) + p(999)$.
2. We define n to be a squarefree integer if, for every prime p , p^2 does not divide n . Let $f(n)$ be the sum of the reciprocals of all the divisors of n . We define n to be an amazing integer if $f(n) = 2$. How many squarefree amazing integers are there?
3. There are 7 cages in a row in an animal shelter and an ample supply of three different kind of animals: dogs, cats, and golden bears. Since golden bears do not like each other, they cannot be placed in adjacent cages. Cats also do not like each other (but not as much so as golden bears), so there cannot be more than two cats in a row. How many ways are there to fill the cages with animals?