1. x is a base-10 number such that when the digits of x are interpreted as a base-20 number, the resulting number is twice the value as when they are interpreted as a base-13 number. Find the sum of all possible values of x.

   Answer: 198

   Solution: Clearly it can't be one-digit. If we try two digits, we get $20a + b = 2(13a + b)$ or $6a + b = 0$, which again has no solution.

   For three digits, we have $400a+20b+c = 2(169a+13b+c)$, or $62a = 6b+c$. If we want $1 \leq a \leq 9$ and $0 \leq b, c \leq 9$, there is only one solution, namely $a = 1$, $b = 9$, and $c = 8$. If we try four digits, we easily see that $20^4 - 2 \cdot 13^4$ is far too large for anything to come even close. Thus the only possible value for x is 198.

2. If f is a monic cubic polynomial with $f(0) = -64$, and all roots of f are non-negative real numbers, what is the largest possible value of $f(-1)$? (A polynomial is monic if it has a leading coefficient of 1.)

   Answer: -125

   Solution: If the three roots of f are r_1, r_2, r_3, we have $f(x) = x^3-(r_1+r_2+r_3)x^2+(r_1r_2+r_1r_3+r_2r_3)x - r_1r_2r_3$, so $f(-1) = -1-(r_1+r_2+r_3)-(r_1r_2+r_1r_3+r_2r_3)-r_1r_2r_3$. Since $r_1r_2r_3 = 64$, the arithmetic mean-geometric mean inequality reveals that $r_1+r_2+r_3 \geq 3(r_1r_2r_3)^{1/3} = 12$ and $r_1r_2+r_1r_3+r_2r_3 \geq 3(r_1r_2r_3)^{2/3} = 48$. It follows that $f(-1)$ is at most $-1-12-48-64 = \boxed{-125}$. We have equality when all roots are equal, i.e. $f(x) = (x-4)^3$.

3. Find the minimum of $f(x, y, z) = x^3 + 12\frac{yz}{x} + 16(\frac{1}{yz})^{\frac{3}{2}}$ where x, y, and z are all positive. [1]

   Answer: 24

   Solution:

$$f(x,y,z) = x^3+12\frac{yz}{x}+16\left(\frac{1}{yz}\right)^{\frac{3}{2}} \geq 6\sqrt[6]{x^3 \cdot 4\frac{yz}{x} \cdot 4\frac{yz}{x} \cdot 4\frac{yz}{x} \cdot 8\left(\frac{1}{yz}\right)^{\frac{3}{2}} \cdot 8\left(\frac{1}{yz}\right)^{\frac{3}{2}}} = 6\sqrt[6]{4^6} = \boxed{24}.$$

   This is attainable by setting $x = yz = \sqrt[3]{4}$.

[1] The problem as given in the tiebreaker did not specify that each of x, y, and z had to be positive. Without this constraint, the answer is $-\infty$, as x^3 can be an arbitrarily large negative value and dominate the expression.