

**Time limit:** 50 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

**No calculators.**

1. In triangle  $ABC$ ,  $AC = 7$ .  $D$  lies on  $AB$  such that  $AD = BD = CD = 5$ . Find  $BC$ .
2. What is the perimeter of a rectangle of area 32 inscribed in a circle of radius 4?
3. Robin has obtained a circular pizza with radius 2. However, being rebellious, instead of slicing the pizza radially, he decides to slice the pizza into 4 strips of equal width both vertically and horizontally. What is the area of the smallest piece of pizza?
4.  $ABCD$  is a regular tetrahedron with side length 1. Find the area of the cross section of  $ABCD$  cut by the plane that passes through the midpoints of  $AB$ ,  $AC$ , and  $CD$ .
5. In square  $ABCD$  with side length 2, let  $P$  and  $Q$  both be on side  $AB$  such that  $AP = BQ = \frac{1}{2}$ . Let  $E$  be a point on the edge of the square that maximizes the angle  $PEQ$ . Find the area of triangle  $PEQ$ .
6.  $ABCD$  is a rectangle with  $AB = CD = 2$ . A circle centered at  $O$  is tangent to  $BC$ ,  $CD$ , and  $AD$  (and hence has radius 1). Another circle, centered at  $P$ , is tangent to circle  $O$  at point  $T$  and is also tangent to  $AB$  and  $BC$ . If line  $AT$  is tangent to both circles at  $T$ , find the radius of circle  $P$ .
7.  $ABCD$  is a square such that  $\overline{AB}$  lies on the line  $y = x + 4$  and points  $C$  and  $D$  lie on the graph of parabola  $y^2 = x$ . Compute the sum of all possible areas of  $ABCD$ .
8. Let equilateral triangle  $ABC$  with side length 6 be inscribed in a circle and let  $P$  be on arc  $AC$  such that  $AP \cdot PC = 10$ . Find the length of  $BP$ .
9. In tetrahedron  $ABCD$ ,  $AB = 4$ ,  $CD = 7$ , and  $AC = AD = BC = BD = 5$ . Let  $I_A$ ,  $I_B$ ,  $I_C$ , and  $I_D$  denote the incenters of the faces opposite vertices  $A$ ,  $B$ ,  $C$ , and  $D$ , respectively. It is provable that  $AI_A$  intersects  $BI_B$  at a point  $X$ , and  $CI_C$  intersects  $DI_D$  at a point  $Y$ . Compute  $XY$ .
10. Let triangle  $ABC$  have side lengths  $AB = 16$ ,  $BC = 20$ ,  $AC = 26$ . Let  $ACDE$ ,  $ABFG$ , and  $BCHI$  be squares that are entirely outside of triangle  $ABC$ . Let  $J$  be the midpoint of  $EH$ ,  $K$  be the midpoint of  $DG$ , and  $L$  the midpoint of  $AC$ . Find the area of triangle  $JKL$ .