

Time limit: 15 minutes.

Instructions: This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

No calculators.

1. Mary is painting her five fingernails. She has four different colors of nail polish: red, green, blue, and pink. If she chooses the color of each nail independently and uniformly at random, compute the probability that at least four of her nails end up the same color.
2. Compute the smallest positive integer that can be written as both the sum of 16 consecutive positive integers and as the sum of 17 consecutive positive integers.
3. Twenty guests sit at twenty seats around a round table according to the following process. One guest, who is selected uniformly at random from the unseated guests, selects an empty seat uniformly at random and sits in that seat. This process repeats until all guests are seated. Alice is one of the guests. Compute the probability that when Alice sits down the both the seat to her left and the seat to her right are already occupied.