Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. At the grocery store, 3 avocados and 2 pineapples cost \$8.80, while 5 avocados and 3 pineapples cost \$14.00. How much do 1 avocado and 1 pineapple cost in dollars?

2. Let a, b, c, d be an increasing sequence of numbers such that a, b, c forms a geometric sequence and b, c, d forms an arithmetic sequence. Given that $a = 8$ and $d = 24$, what is b?

3. Given that the roots of the polynomial $x^3 - 7x^2 + 13x - 7 = 0$ are r, s, t, compute the value of $\frac{1}{r} + \frac{1}{s} + \frac{1}{t}$.

4. Find all possible pairs of integers (m, n) which satisfy $m^2 + 2m - 35 = 2^n$.

5. Let $a_1, a_2, a_3, a_4, a_5, \ldots$ be a geometric progression with positive ratio such that $a_1 > 1$ and $(a_{1357})^3 = a_{34}$. Find the smallest integer n such that $a_n < 1$.

6. Let $a_k = \pm 1$ for all integers $1 \le k \le 2018$. The sum
$$\sum_{1 \le i < j \le 2018} a_i a_j$$
can take on both positive and negative values. Find the smallest positive value of the sum.

7. Let x, y, z be non-negative real numbers satisfying $xyz = \frac{2}{3}$. Compute the minimum value of
$$x^2 + 6xy + 18y^2 + 12yz + 4z^2.$$

8. Define $\{x\} = x - \lfloor x \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer not exceeding x. If $|x| \le 8$, find the number of real solutions to the equation
$$\{x\} + \{x^2\} = 1.$$

9. Let (a, b, c, d, e) be an integer solution to the system of equations
$$\begin{aligned} a + d &= 12 \\ b + ad + e &= 57 \\ c + bd + ae &= 134 \\ cd + be &= 156 \\ ce &= 72 \end{aligned}$$
Find all possible values of $b + d$.

10. Let $a_1, ..., a_{2018}$ be the roots of the polynomial
$$x^{2018} + x^{2017} + \cdots + x^2 + x - 1345 = 0.$$
Compute
$$\sum_{n=1}^{2018} \frac{1}{1 - a_n}.$$