

**Time limit:** 15 minutes.

**Instructions:** This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

**No calculators.**

1. If  $a, b, c$  are real numbers with  $a - b = 4$ , find the maximum value of  $ac + bc - c^2 - ab$ .
2. If  $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$  and  $\frac{1}{x+1} + \frac{1}{y+1} = \frac{3}{8}$ , compute  $\frac{1}{x-1} + \frac{1}{y-1}$ .
3. Let  $F_n$  denote the  $n$ -th term of the Fibonacci sequence defined recursively as  $F_1 = F_2 = 1$  and  $F_{n+2} = F_{n+1} + F_n$  for all  $n \geq 0$ . Compute the sum

$$\sum_{n=1}^{\infty} \frac{F_n}{2^n}$$