

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. If $f(x) = x^2$ and $g(x) = \ln(x)$, compute $f'(1) + g'(1)$.

2. Given that $f(x) = x^2 + ax - 17$, find all real values of a such that $f(4) = f'(4)$.

3. Find the value of a such that

$$\int_1^a (3x^2 - 6x + 3) dx = 27.$$

4. Compute

$$\int_0^4 \frac{dx}{\sqrt{|x-2|}}.$$

5. Eric and Harrison are standing in a field, and Eric is 400 feet directly East of Harrison. Eric starts to walk North at a rate of 4 feet per second, while Harrison starts to walk South at the same time at a rate of 6 feet per second. After 30 seconds, at what rate is the distance between Eric and Harrison changing?

6. Compute

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x)^2}{x^2 - x^2 \cos^2 x}.$$

7. Compute

$$\int_{-2}^0 \frac{x^3 + 4x^2 + 7x - 20}{x^2 + 4x + 8} dx + \int_0^2 \frac{2x^3 - 7x^2 + 9x - 10}{x^2 + 4} dx.$$

8. Compute

$$\lim_{n \rightarrow \infty} n^2 \int_0^{1/n} x^{2018x+1} dx.$$

9. Compute

$$\int_0^\pi \frac{2x \sin x}{3 + \cos 2x} dx.$$

10. *Fact:* The value $\ln(2)$ is not the root of any polynomial with rational coefficients.

For any nonnegative integer n , let $p_n(x)$ be the unique polynomial with integer coefficients such that

$$p_n(\ln(2)) = \int_1^2 (\ln(x))^n dx.$$

Compute the value of the sum

$$\sum_{n=0}^{\infty} \frac{1}{p_n(0)}.$$