Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. A number is formed using the digits $\{2, 0, 1, 8\}$, using all 4 digits exactly once. Note that $0218 = 218$ is a valid number that can be formed. What is the probability that the resulting number is strictly greater than 2018?

2. Let $S(n)$ denote the sum of the digits of the integer n. If $S(n) = 2018$, what is the smallest possible value $S(n + 1)$ can be?

3. A dice is labeled with the integers $1, 2, ..., n$ such that it is 2 times as likely to roll a 2 as it is a 1, 3 times as likely to roll a 3 as it is a 1, and so on. Suppose the probability of rolling an odd integer with the dice is $\frac{17}{35}$. Compute n.

4. One of the six digits in the expression $435 \cdot 605$ can be changed so that the product is a perfect square N^2. Compute N.

5. A sequence is defined as follows. Given a term a_n, we define the next term a_{n+1} as

$$a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{if } a_n \text{ is even} \\ a_n - 1 & \text{if } a_n \text{ is odd} \end{cases}$$

The sequence terminates when $a_n = 1$. Let $P(x)$ be the number of terms in such a sequence with initial term x. For example, $P(7) = 5$ because its corresponding sequence is $7, 6, 3, 2, 1$. Evaluate $P(2^{2018} - 2018)$.

6. Elizabeth is at a candy store buying jelly beans. Elizabeth begins with 0 jellybeans. With each scoop, she can increase her jellybean count to the next largest multiple of 30, 70 or 110. (For example, her next scoop after 70 can increase her jellybean count to 90, 110, or 140). What is the smallest number of jellybeans Elizabeth can collect in more than 100 different ways?

7. Let S be the set of all 1000 element subsets of the set $\{1, 2, 3, ..., 2018\}$. What is the expected value of the minimum element of a set chosen uniformly at random from S?

8. Positive integer n can be written in the form $a^2 - b^2$ for at least 12 pairs of positive integers (a, b). Compute the smallest possible value of n.

9. Let

$$S = \sum_{k=1}^{2018102} \sum_{n=1}^{1008} n^k.$$

Compute the remainder when S is divided by 1009.

10. Morris plays a game using a fair coin. He starts with \$2 and proceeds using the following rules:

- If Morris flips a heads, he gains \$2.
- If Morris flips a tails, he loses half his money.
- If Morris flips two tails in a row, the game ends (but he doesn't lose any more money)

For instance, if Morris flips the sequence THTT, he will end up with \$1.50. What is the expected amount of money in dollars Morris will have after the game ends?