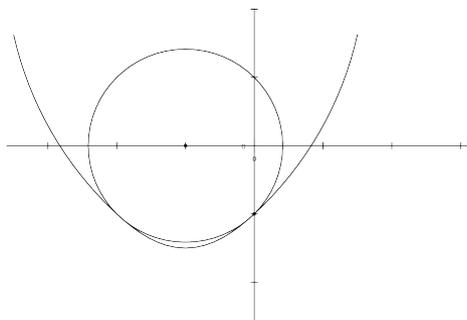


Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. Compute $\int_0^{2\pi} \theta^2 d\theta$.
2. Let $f(x) = x \ln x + x$. Solve $f'(x) = 0$ for x .
3. Compute $\int_0^{\pi/4} \cos x - 2 \sin x \sin 2x dx$.
4. Let $f_0(x) = (\sqrt{e})^x$, and recursively define $f_{n+1}(x) = f'_n(x)$ for integers $n \geq 0$. Compute $\sum_{i=0}^{\infty} f_i(1)$.
5. Consider the parabola $y = ax^2 + 2019x + 2019$. There exists exactly one circle which is centered on the x -axis and is tangent to the parabola at exactly two points. It turns out that one of these tangent points is $(0, 2019)$. Find a . (Diagram below does not picture the specified parabola.)



6. What is the smallest natural number n for which the limit

$$\lim_{x \rightarrow 0} \frac{\sin^n x}{\cos^2 x (1 - \cos x)^3}$$

exists?

7. Turn the graph of $y = \frac{1}{x}$ by 45° counter-clockwise and consider the bowl-like top part of the curve (the part above $y = 0$). We let a 2D fluid accumulate in this 2D bowl until the maximum depth of the fluid is $\frac{2\sqrt{2}}{3}$. What's the area of the fluid used?

8. Compute

$$\lim_{x \rightarrow \infty} \left(\left(1 + \frac{1}{x}\right)^x x - ex \right).$$

9. Magic liquid forms a cone whose circular base rests on the floor. Time is measured in seconds. At time 0, the cone has height and radius 1 cm. Let $R(t)$ be the rate at which liquid evaporates in cm^3/s at time t . As the liquid evaporates, the cone's radius remains the same but its height decreases. Let $S(t)$ be the surface area of the slanted part of the cone in cm^2 at time t . If $R(t) = S(t)^2$ (numerically in the specified units), how many seconds does it take for the liquid to evaporate entirely?

10. Compute

$$\int_0^2 \frac{\ln(1+x)}{x^2 - x + 1} dx.$$