Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. For some positive integer n, $2021 - 2(5^n)$ can be expressed as the sum and difference of distinct integer powers of 5. Compute 5^n.

2. Find the smallest integer $n \geq 2021$ such that $30n^3 + 143n^2 + 117n - 56$ is divisible by 13.

3. Suppose that a positive integer n has 6 positive divisors where the 3^{rd} smallest is a and the a^{th} smallest is $\frac{n}{3}$. Find the sum of all possible value(s) of n.

4. A positive integer n has 4 positive divisors such that the sum of its divisors is $\sigma(n) = 2112$. Given that the number of positive integers less than and relative prime to n is $\phi(n) = 1932$, find the sum of the proper divisors of n.

5. $15380 - n^2$ is a perfect square for exactly four distinct positive integers. Given that $13^2 + 37^2 = 1538$, compute the sum of these four possible values of n.

6. Find the sum of all possible values of abc where a, b, c are positive integers that satisfy

$$\begin{aligned} a &= \gcd(b, c) + 3, \\ b &= \gcd(a, c) + 3, \\ c &= \gcd(a, b) + 3. \end{aligned}$$

7. Let a be the positive integer that satisfies the equation

$$1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + ... + \frac{29}{30} = \frac{a}{30!}.$$

   What is the remainder when a is divided by 17?

8. Compute the remainder when

$$2018^{2019^{2020}} + 2019^{2020^{2021}} + 2020^{2020^{2020}} + 2021^{2020^{2019}} + 2022^{2021^{2020}}$$

   is divided by 2020.

9. Find the least positive integer k such that there exists a set of k distinct positive integers $\{n_1, n_2, \ldots, n_k\}$ that satisfy the equation

$$\prod_{i=1}^{k} \left(1 - \frac{1}{n_i}\right) = \frac{72}{2021}.$$

10. Compute the smallest positive integer n such that $n^{44}+1$ has at least three distinct prime factors less than 44.