Operation Planning for Electrical Power Networks

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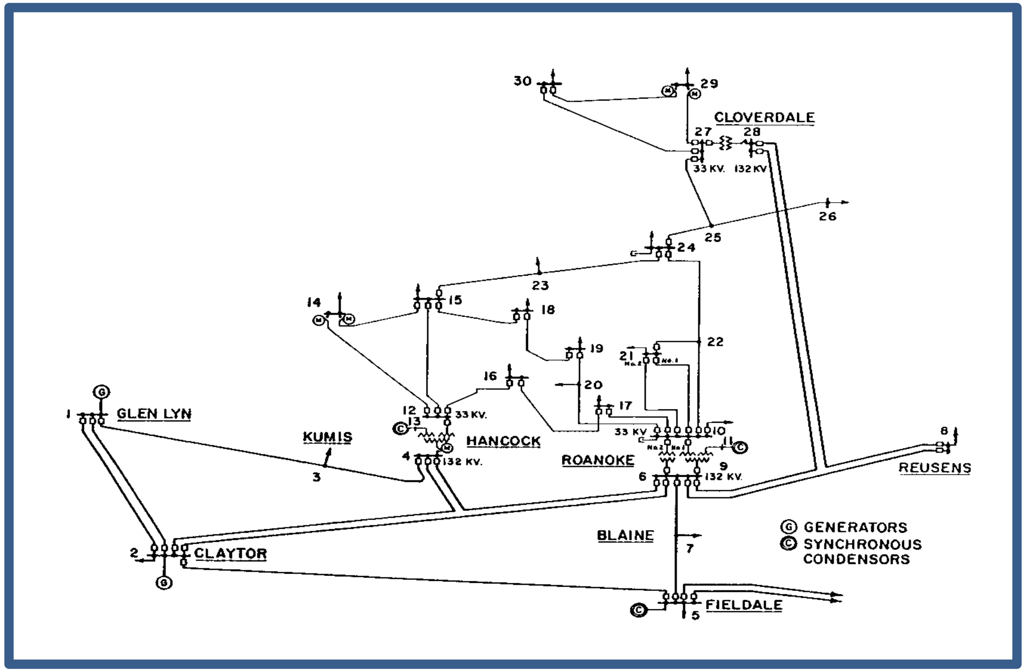
Power Networks *(CDC 10, Allerton 10, ACC 11, TPS 11, ACC 12, PGM 12)*

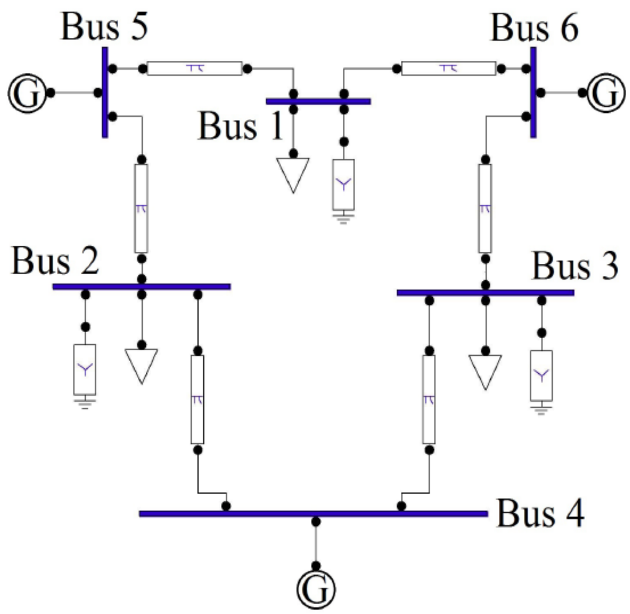
- **Optimizations:**
  - Resource allocation
  - State estimation
  - Scheduling



- **Issue:** Nonlinearities (power being quadratic in voltage)

- **Transition from traditional grid to smart grid:**
  - More variables (10X)
  - Time constraints (100X)

Resource Allocation: Optimal Power Flow (OPF)



OPF: Given constant-power loads, find optimal *P*'s subject to:

- Demand constraints
- Constraints on *V*'s, *P*'s, and *Q*'s.

Broad Interest in Optimal Power Flow

- Interested companies: ISOs, TSOs, RTOs, Utilities, FERC

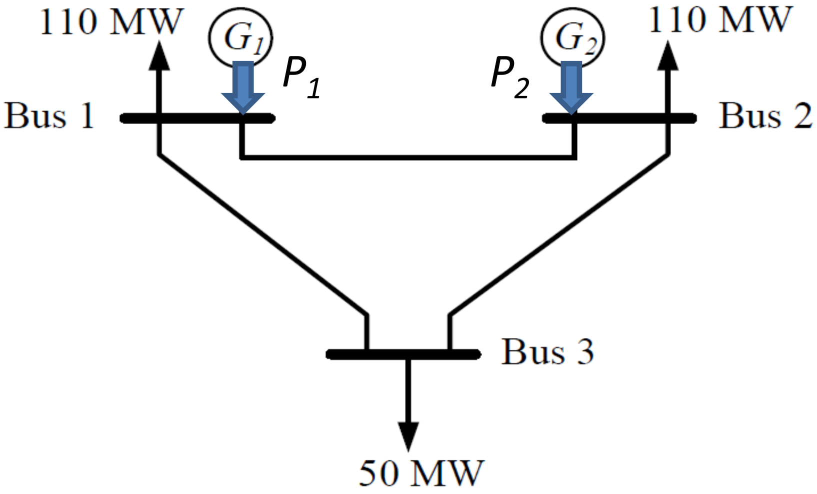
- OPF solved on different time scales:
  - Electricity market
  - Real-time operation
  - Security assessment
  - Transmission planning

- Existing methods based on local search algorithms

- Can save **$$$** if solved efficiently

- Huge literature since 1962 by power, OR and Econ people

Local Solutions



OPF

$$\text{minimize } f_1(P_1) + f_2(P_2)$$

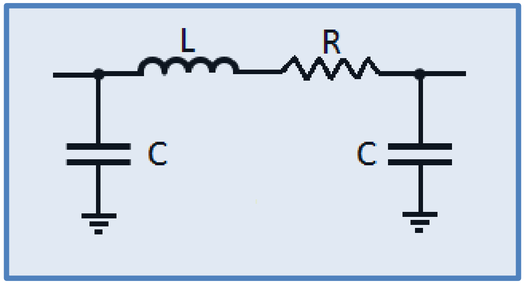
Local solution: $1502

Summary of Results

Project 1: How to solve a given OPF in polynomial time?

- A sufficient condition to globally solve OPF:
  - Numerous randomly generated systems
  - IEEE systems with 14, 30, 57, 118, 300 buses
  - European grid
  - California grids with several thousand load profiles

- **Various theories:** It holds widely in practice



- Generalizable to many optimizations in smart grids

Summary of Results

Project 2: Find network topologies over which optimization is easy?

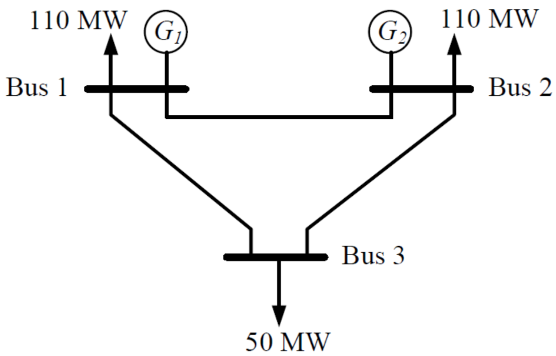
- ❑ Distribution networks are fine.
- ❑ Every transmission network can be turned into a good one.

Project 3: How to design a parallel algorithm for solving OPF?

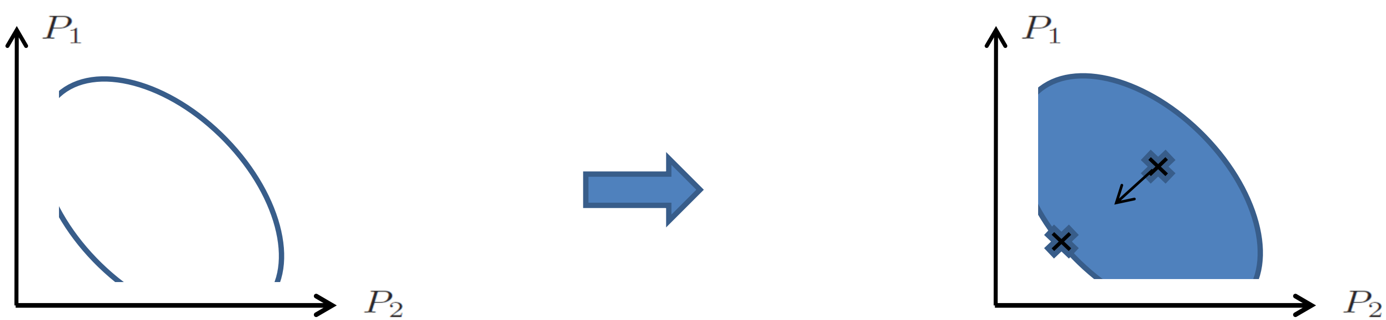
- ❑ A practical (infinitely) parallelizable algorithm
- ❑ It solves 10,000-bus OPF in 0.85 seconds on a single core machine.

Companies interested in our SDP approach: FERC, EU grid operator, ISOs, SCE,…

Geometric Intuition: Two-Generator Network



$$\text{minimize } f_1(P_1) + f_2(P_2)$$
$$\text{subject to } (P_1, P_2) \in \mathcal{P}$$



$$\text{minimize } f_1(P_1) + f_2(P_2)$$
$$\text{subject to } (P_1, P_2) \in \mathcal{P}$$

$=$

$$\text{minimize } f_1(P_1) + f_2(P_2)$$
$$\text{subject to } (P_1, P_2) \in \text{conv}(\mathcal{P})$$

Optimal Power Flow

$$\min_{\mathbf{V},\mathbf{P}_G,\mathbf{Q}_G} \quad \sum_{k\in\mathcal{G}} f_k(P_{G_k}) \qquad (1a)$$

Subject to

$$P_k^{\min} \leq P_{G_k} \leq P_k^{\max} \qquad (1b)$$

$$Q_k^{\min} \leq Q_{G_k} \leq Q_k^{\max} \qquad (1c)$$

$$V_k^{\min} \leq |V_k| \leq V_k^{\max} \qquad (1d)$$

$$\mathrm{Re}\left\{V_l(V_l - V_m)^* y_{lm}^*\right\} \leq P_{lm}^{\max} \qquad (1e)$$

$$\mathrm{trace}\{\mathbf{V}\mathbf{V}^*\mathbf{Y}^* e_k e_k^*\} = P_{G_k} - P_{D_k} + (Q_{G_k} - Q_{D_k})\mathrm{i} \qquad (1f)$$

(1a)	Cost
(1b)–(1d)	Operation
(1e)	Flow
(1f)	Balance

- Extensions:
  - Other objective (voltage support, reactive power, deviation)
  - More variables, e.g. capacitor banks, transformers
  - Preventive or corrective contingency constraints
  - Multi-period OPF

- Conventional OPF captures common sources of non-convexity.

Zero Duality Gap

- OPF:
  - Real-valued (DC)
  - Complex-valued (AC)

- Networks:
  - Distribution (acyclic)
  - Transmission (cyclic)

Theorem

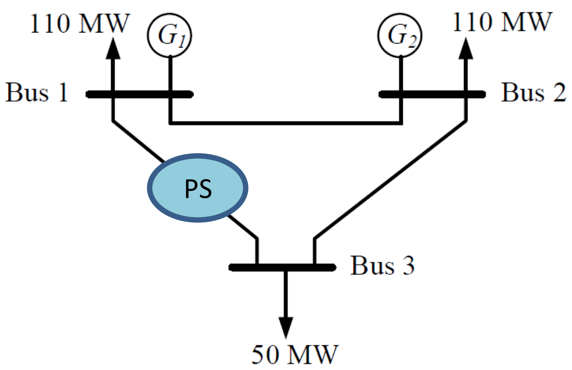
Convexification works in three cases:

- *DC/AC distribution networks*
- *DC transmission networks*

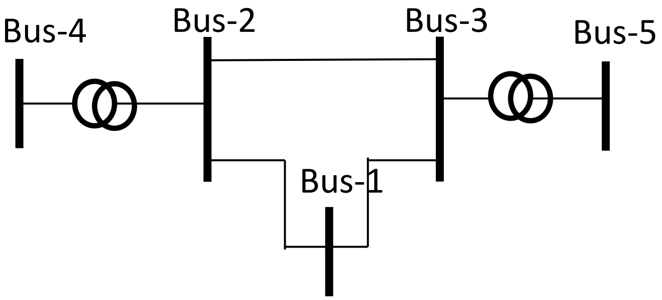
AC Transmission Networks

- ❑ **How about AC transmission networks?**
  - May not work exactly for every network
  - Various sufficient conditions

- ❑ **Result 1:** AC transmission network manipulation:
  - High performance (lower generation cost)
  - Easy optimization
  - Easy market



- ❑ **Result 2:** Reduced computational complexity



Simulations

Convexification (zero duality gap):

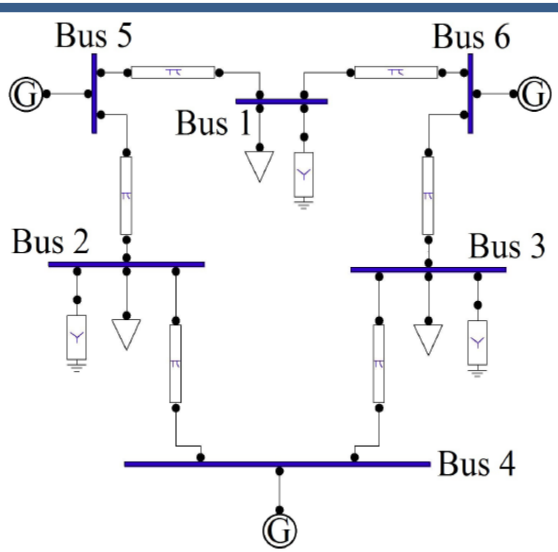
- Zero duality gap for IEEE 30-bus system
- Guarantee zero duality gap for all possible load profiles?
- **Theoretical side:** Add 12 phase shifters and get 11% improvement
- **Practical side:** 2 phase shifters are enough
- IEEE 118-bus system needs no phase shifters (power loss case)

Scalability:

- OPF-based problem with 10 million variables solved in 17 minutes

Conclusions

- **Focus:** OPF with a 50-year history
- **Goal:** Find a global solution efficiently



- Obtained provably global solutions for many practical OPFs
- Developed various theories for distribution and transmission networks
- Parallel implementation via ADMM

Companies interested in our approach: FERC, EU grid operator, ISOs, SCE,…

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