Demographics, Wealth, and Global Imbalances in the Twenty-First Century

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The world population is aging...

Source: 2019 United Nations World Population Prospects
...wealth-to-GDP ratios are increasing...

...real interest rates are falling...

Source: Laubach and Williams (2003), FRED, King and Low (2014)
...and “global imbalances” are rising

Source: International Monetary Fund (IMF), Penn World Table (PWT) 9.1
Q: How does population aging affect wealth-output ratios, real interest rates, and capital flows?

Given population projections, how will these macro trends likely evolve over the rest of the 21st century?

- **Our contribution:** discipline this exercise with a shift-share

\[
\left( \frac{W_t}{Y_t} \right)^{comp} = \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}} \quad t \geq 0
\]

- \(a_{jo}, h_{jo}\) are today’s asset and labor income profiles by age \(j\)
- \(\pi_{jt}\) are projections of the population share of age \(j\) in year \(t\)

Captures the **compositional effect** of aging on \(W/Y\)

We show how to use this for counterfactuals in general eqbm:

1. Exactly, when there is “balanced growth by age” (special SOE case)
2. Approximately, after demeaning, to forecast NFAs (general case)
What we find

$$\Delta_t^{comp} \equiv \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}} - \frac{W_o}{Y_o}$$

1. Measurement:
   - $\Delta^{comp}$ is positive, large and heterogeneous across countries
     [in 2100: 85pp in Germany vs 305pp in India]
     a) Older individuals hold more wealth and earn less income
     b) Timing of aging transition uneven across countries

2. Quantitative GE OLG model: across range of calibrations
   - $\Delta^{comp}$ closely approximates $W/Y$ transition of small open econ.
   - In integrated world, matching $\Delta^{comp}$ in each country implies large global imbalances by the end of the 21st century
     [2016-2100: $\Delta NFA/Y$ of -50pp in Germany vs 150pp in India]
   - Effects on interest rates and wealth-GDP ratios more uncertain
Related literature

Quantitative GE overlapping generations models:


We highlight an important moment that drives counterfactuals and can be measured in data

Shift share approaches to demographics:


We exhibit a shift-share that can be used for general equilibrium counterfactuals

Macroeconomic trends:

- Real interest rates [Eggertsson-Robbins-Wold 18, Farhi-Gourio 18, Gourinchas-Rey 18, Gomme-Ravikumar-Rupert 16, Marx-Mojon-Velde 19...]
- Global imbalances [Caballero-Farhi-Gourinchas 2008, Mendoza-Quadrini-Rios Rull 2009,...]

We isolate the role of demographics for these trends
1. An age shift-share for $W/Y$

2. Model

3. Small open economy

4. Global imbalances
1. An age shift-share for $W/Y$
Environment

- Economy with output $Y_t$ experiencing demographic change
- Population of age $j$ $N_{jt}$, total population $N_t \equiv \sum_j N_{jt}$
- Wealth
  \[ W_t = \sum_j N_{jt} A_{jt} \]  
- Effective labor supply
  \[ L_t = \sum_j N_{jt} h_{jt} \]
- Suppose there is growth in labor productivity $Y_t/L_t$
  - We expect $A_{jt}$ to scale with $Y_t/L_t$
  - Let $a_{jt} \equiv \frac{A_{jt}}{Y_t/L_t}$ denote productivity-normalized assets by age
Wealth-to-GDP ratio

• Rewrite wealth (1)

\[ W_t = \frac{Y_t}{L_t} \sum_j N_j a_j \]

• Wealth-to-GDP ratio using (2)

\[ \frac{W_t}{Y_t} = \frac{\sum_j \pi_j a_j}{\sum_j \pi_j h_j} \]

where \( \pi_{jt} \equiv \frac{N_{jt}}{N_t} \) is share of population age \( j \)

• Three reasons for changing \( W_t/Y_t \):
  1. Changing **population shares**: \( \pi_{jt} \)
  2. Changing **age profiles of productivity-normalized assets**: \( a_{jt} \)
  3. Changing **age profiles of labor efficiency**: \( h_{jt} \)
Age shift-share

• For any base year 0, define

\[ \Delta_t^{comp} = \frac{\sum_j \pi_j a_{j0}}{\sum_j \pi_j h_{j0}} - \frac{W_0}{Y_0} \]

• Can calculate \( \Delta^{comp} \) directly in from data and pop. projections

• Why is this a natural starting point for projections?

1. It can be a sufficient statistic in a demographic transition
   • Small open economy special case: \( a_{jt} \) and \( h_{jt} \) are constant
   • We say the economy ages without “behavioral effects”

2. It is always a component of the total change in \( W/Y \):

\[
\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \Delta_t^{comp} + \underbrace{\frac{\sum_j \pi_j a_{jt}}{\sum_j \pi_j h_{jt}} - \frac{\sum_j \pi_j a_{j0}}{\sum_j \pi_j h_{j0}}}_{\Delta_t^{beh}}
\]

→ Benchmark to evaluate transition dynamics in any GE model
Measuring $\Delta^{comp}$

- Calculate shift-share $\Delta^{comp}_t$ for US and 24 other countries

- Implementation:
  - Normalize labor supply so that $\sum \pi_j h_j = 1$
  - Then $a_j$ is average wealth by age normalized by GDP per capita
  - Can measure relative $h_j$ from relative labor income

- Data
  - $\pi_{jt}$: demographic projections by age
    - 2019 UN World Population Prospects, SSA and Gagnon et al. (2016)
  - $a_j$, $h_j$: age-wealth and labor income profiles in base year
    - Luxembourg Wealth/Income Study (LWS/LIS), European Household Finance and Consumption Survey (HFCS), China Household Finance Survey (CHFS), Indian National Sample Survey (NSS), National Survey of Family Income and Expenditure (JPN), Statistics Denmark
$\Delta^{comp}$ in the United States: 1950-2100

- Low fertility
- Baseline
- High fertility
- Data (WID)

Base year

Historical
Decomposing $\Delta^{\text{comp}}$

- Large effects. Where do they come from?
- Study separately effect on $W$ and effect on $Y$
- Separate into respective contributions using:

$$
\Delta_t^{\text{comp}} \approx \sum_j (\pi_{jt} - \pi_{jo}) \times a_{jo} + \left( -\frac{W_o}{Y_o} \sum_j (\pi_{jt} - \pi_{jo}) \times h_{jo} \right)
$$

$\Delta_t^{\text{comp},a}$ and $\Delta_t^{\text{comp},h}$

[recall $\sum_j \pi_{jo} h_{jo} = 1$]
Contribution of $W$ to $\Delta^{comp}$

Alternative base years
Contribution of $W$ to $\Delta^{comp}$

Alternative base years
Contribution of $W$ to $\Delta^{comp}$

Alternative base years
Summary contribution of $W$ to $W/Y$: $\Delta_t^{\text{comp,a}}$
Contribution of $Y$ to $\Delta^{comp}$

Alternative base years
Contribution of $Y$ to $\Delta^{comp}$
Contribution of $Y$ to $\Delta^{comp}$

Alternative base years

![Graph showing normalized labor income by age and population shares.](image)
Summary contribution of Y to W/Y: $\Delta_t^{\text{comp},h}$
Global trends in \( \Delta^{\text{comp}} \) for the 21st century
Δ^comp around the world in 2100

21
2. Model
• Standard multi-country GE OLG model featuring idiosyncratic income risk, intergenerational transmission of skills, bequests, and a social security system [eg Krueger-Ludwig 2007]

• **Final output** is produced out of capital and effective labor

\[ Y_t = F(K_t, Z_t L_t) \]

where \( Z_t \) is exogenous labor augmenting technology

• Perfect competition and free capital adjustment:

\[ r_t + \delta = F_L \left( \frac{K_t}{Z_t L_t}, 1 \right), \quad w_t = Z_t F_L \left( \frac{K_t}{Z_t L_t}, 1 \right) \]

• **Labor market clearing**

\[ L_t = \sum_j N_{jt} h_{jt} \]

where \( h_{jt} \) is average labor supply per person at age \( j \)
• Heterogeneous households with ages $j = 0 \ldots T$

Born Have children Retire Die for certain
0 $T^w$ $T^r_t$ $T$

• **Income** at age $j$ in time $t$:

$$y_{jt} = w_t \rho_{jt} \tilde{h}_{jt} \ell(s_j)$$

where

• $w_t$ is the wage per unit of effective labor supply
• $\rho_{jt} \in [0, 1]$ is fraction of agents still working at age $j$ (others retire)
• $\tilde{h}_{jt}$ is exogenous age-efficiency profile
• $\ell(s_j)$ is a stochastic labor supply shifter
Asset demand: household problem 2/2

- A household born at time $k$ solves

$$\max_{c, a, \psi} \mathbb{E} \left[ \sum_{j=T}^{T} \beta^j \Phi_j^k \left( \psi_j, k+j \right) u(c_j, k+j) + \gamma (1 - \phi_j^k) v_t \left( a_{j+1, k+j+1} \right) \right]$$

- $\Phi_j^k = \phi_j^k \Phi_{j-1}^k$: survival rate for cohort $k$
- $\psi_j$: utility modifier due to children (can microfound func. form)
- $v_t$ captures nonhomotheticities in bequests

- **Constraints**: $a_{j+1, t+1} \geq -\bar{a}Z_t$ and

$$c_{jt} + a_{j+1, t+1} \leq \left(1 - \tau_t^y\right) y_{jt}(s^j) + \left(1 + r_t^a\right) a_{jt} + tr_{jt}(s^j) + b_{jt}^r(s^j)$$

- **Government** adjusts $\{\rho_{jt}, \tau_t^y, tr_{jt}(\cdot), G_t, B_t\}$, follows a fiscal rule
Asset market clearing

\[ A_t + A_{t}^{\text{mig.net}} = K_t + B_t + NFA_t \]

where

- \( A_t \) is household wealth
- \( A_{t}^{\text{mig.net}} \) is net wealth coming from migrants
- \( K_t \) is the capital stock
- \( B_t \) is domestic government bonds
- \( NFA_t \) is net foreign asset position
- Small open economy has exogenous \( r_t \)
- World economy has \( \sum_c NFA_{c,t} = 0 \) (endogenous \( r_t \)
Steady state asset market equilibrium

• Divide by $Y$ and consider a steady state for a given country

• Let $\Theta$ index demographic parameters:

$$\frac{NFA}{Y} (r, \Theta) = \frac{W}{Y} (r, \Theta) - \frac{A^s}{Y} (r)$$

• $A^s/Y$ independent of demographics! (at unchanged $B/Y$)

• Hence, change between two steady states

$$\Delta \left( \frac{NFA}{Y} \right) \simeq \Delta^{comp} + \Delta^{beh|r} + \epsilon^d \Delta r + \epsilon^s \Delta r$$

where $\epsilon^d, \epsilon^s$ are interest sensitivities of $W/Y$ and $A^s/Y$

• If bars denote averages across countries

$$\Delta r \simeq - \frac{\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}}{\bar{\epsilon}^d + \bar{\epsilon}^s}$$

$\Rightarrow$ SOE effect $\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}$ quantifies net asset demand shift

Key determinant of equilibrium $\Delta r$, so $\Delta \frac{NFA}{Y}$ and $\Delta \frac{W}{Y}$
3. Small open economy
Proposition

Consider small open economy with constant $r$ and assume:

1. Constant efficiency-profile $\tilde{h}_j$ and TFP growth $\gamma$
2. Constant mortality profiles $\phi_j$
3. Constant valuation of children’s consumption $\psi_j$
4. Constant tax and retirement policies: $\tau^Y$, $\rho_j$ and $tr_j(s^i) + b'_j(s^i)$

Then there exists an equilibrium with $a_{jt}(s^i) = (1 + \gamma)^t a_j(s^i)$, $\forall t$.

In this equilibrium, the wealth-to-GDP ratio is

$$\frac{W_t}{Y_t} = \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}, \forall t$$

where $h_{jo} = \rho_j \tilde{h}_j$ and $a_{jo} = \frac{E_{jo}[A_{ijo}]}{Y_o/N_o}$ are age profiles in a base year 0.

- Sufficient statistic: $a_{jo}$ is all we need to know about savings motives
- Irrelevant: time vs cohort effects, type of savings motives, timing of $G$ adjustment, ...
Behavioral responses

• Special case above has $\Delta_{t}^{\text{beh}|r} = 0$, all $t$

• Full model has 5 forces for non-zero behavioral effects:
  1. **Labor supply** effect (changing $\tilde{h}_{jt}$ or retirement policy $\rho_{jt}$)
  2. **Declining mortality** effect ($\Phi_{j}^{k}$: allowed to vary by cohort $k$)
  3. **Cost of children** effect ($\psi_{j}$: utility modifier due to children)
  4. **Bequest dilution** effect (changing ratio of givers to receivers)
  5. **Social security balance** effect (adjust $\tau_{t}^{y}$, $tr_{jt}$, $\rho_{jt}$ rather than $G_{t}$)

• **Next**: evaluate quant. magnitude of $\Delta_{t}^{\text{beh}|r}$ when relaxing 1–5
US calibration and counterfactual

- U.S. as laboratory. External calibration of
  - Elasticity of intertemporal substitution $\sigma^{-1}$
  - Income process, production side
  - Social security
  - Demographics: start from **observed 2016 age distribution**

- Estimated parameters:
  - Discount factor $\beta$
  - Bequest preferences: $\gamma$ and $\nu$
  - Weight and exponent on altruism towards children: $\lambda$ and $\varphi$

- Targets:
  - **Shift-share 2016-2100**: $\Delta^{comp} = 1.27$ [from Section 1]
  - Age-consumption profile [CEX]
  - Lorenz curve for bequests [Hurd-Smith 2002]
  - Bequest-to-GDP: 5% [Hendricks 2001, Alvaredo-Garbinti-Piketty 2017]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T^w, T^r, T )</td>
<td>Age structure</td>
<td>20, 65, 100</td>
<td>Standard values</td>
</tr>
<tr>
<td>( n_t, \pi_{jt}, \phi_{jt} )</td>
<td>Demographics</td>
<td>Data</td>
<td>Gagnon et al. (2016)</td>
</tr>
<tr>
<td>( \frac{W}{Y} )</td>
<td>Household wealth</td>
<td>504%</td>
<td>2016 US value (SCF)</td>
</tr>
<tr>
<td>( \frac{B}{Y} )</td>
<td>Government debt</td>
<td>42%</td>
<td>2016 US value (FoF)</td>
</tr>
<tr>
<td>( \frac{G}{Y} )</td>
<td>Government expenditures</td>
<td>17.6%</td>
<td>2016 US value (BEA)</td>
</tr>
<tr>
<td>( \frac{I}{Y} )</td>
<td>Investment</td>
<td>17%</td>
<td>2016 US value (BEA)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>TFP growth rate</td>
<td>0.73%</td>
<td>Average 2010-17 (Fernald)</td>
</tr>
<tr>
<td>( s^L )</td>
<td>Labor share</td>
<td>62%</td>
<td>2016 US Value (NIPA)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation rate</td>
<td>2.5%</td>
<td>( \frac{K}{Y} )</td>
</tr>
<tr>
<td>( r )</td>
<td>Real interest rate</td>
<td>6.1%</td>
<td>( \frac{I}{Y}, \frac{K}{Y} = \frac{W}{Y} - \frac{B}{Y}, s^L )</td>
</tr>
<tr>
<td>( \bar{d} )</td>
<td>Benefits</td>
<td>47%</td>
<td>2016 benefits over GDP</td>
</tr>
<tr>
<td>( \tau^{ss} )</td>
<td>Social security tax</td>
<td>7.6%</td>
<td>Balance SS system</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Total income tax</td>
<td>31%</td>
<td>Balance budget</td>
</tr>
<tr>
<td>((\sigma_{\epsilon}, \rho_{\epsilon}))</td>
<td>Idiosyncratic risk</td>
<td>(0.92, 0.91)</td>
<td>Auclert and Rognlie (2018)</td>
</tr>
<tr>
<td>((\sigma_{\theta}, \rho_{\theta}))</td>
<td>Intergenerational transmission</td>
<td>(0.61, 0.677)</td>
<td>De Nardi (2014)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Inverse EIS</td>
<td>1</td>
<td>Standard value</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Subjective discount factor</td>
<td>0.96</td>
<td>Calibrated value</td>
</tr>
<tr>
<td>((\nu, \bar{\gamma}))</td>
<td>Preference for bequests</td>
<td>(0.81, 2.7)</td>
<td>Calibrated values</td>
</tr>
<tr>
<td>((\lambda, \varphi))</td>
<td>Preference for children</td>
<td>(0.07, 1.8)</td>
<td>Calibrated values</td>
</tr>
</tbody>
</table>
Fitted age profiles

![Graph 1: Wealth-over-GDP vs Age](image1)

- **Model**
- **Data (SCF)**

![Graph 2: Normalized labor income vs Age](image2)

- **Model (net)**
- **Model (gross)**
- **Data (LIS)**
Evaluating behavioral responses: change in $W/Y$ in the U.S.
Evaluating behavioral responses: transitions of $W/Y$
4. Global imbalances
• Solve for integrated world equilibrium
  • 12 countries that are at least 1% of GDP among our 25

• Parameters remain the same, except:
  • Demographics $n_t^c, \pi_{jt}^c$
  • Social security system parameters $\tau_t^{ss,c}, \tau_t^c, \bar{d}_t^c$
  • Discount and bequests factors $\beta^c, \gamma^c$ to hit empirical $\frac{w_t^c}{\gamma_t^c}, \Delta_{comp,c}^c$
  • Technology $s_t^{L,c}$ to hit empirical $\frac{NFA_t^c}{\gamma_t^c}$

• We vary, within range from literature:
  • Elasticity of intertemporal substitution $\sigma^{-1}$
  • Elasticity of capital-labor substitution $\eta$
World change in $r$ for alternative $\sigma$ and $\eta$
World change in $W/Y$ for alternative $\sigma$ and $\eta$
Change in $NFA/Y$ for fast aging countries for alternative $\sigma$ and $\eta$
Role of interest rate sensitivities for $\Delta NFA$

- Recall
  \[ \Delta \left( \frac{NFA_c}{Y_c} \right) = \Delta_{c}^{\text{comp}} + \Delta_{c}^{\text{beh} | r} + \epsilon_{c}^{d} \Delta r + \epsilon_{c}^{s} \Delta r \]

- Since average NFA is 0:
  \[ \Delta \left( \frac{NFA_c}{Y_c} \right) = \Delta_{c}^{\text{comp}} + \Delta_{c}^{\text{beh} | r} - \left( \bar{\Delta}^{\text{comp}} + \bar{\Delta}^{\text{beh} | r} \right) + \left[ \epsilon_{c}^{d} + \epsilon_{c}^{s} - (\bar{\epsilon}^{d} + \bar{\epsilon}^{s}) \right] \Delta r \]

1. $\Delta_{c}^{\text{comp}}$ is large and heterogeneous across countries [Section 1]
2. $\Delta_{c}^{\text{beh} | r}$ is small in comparison [Section 3]
3. $\sigma$ and $\eta$ affect level of $\epsilon_{c}^{d}$ and $\epsilon_{c}^{s}$, not differences across countries

- This suggests
  \[ \Delta \left( \frac{NFA_c}{Y_c} \right) \approx \Delta_{c}^{\text{comp}} - \bar{\Delta}^{\text{comp}} \]

$\Rightarrow$ Can approximately forecast GE NFAs with demeaned $\Delta_{c}^{\text{comp}}$
Change in NFA vs shift share in our model: 2016-2100

\[
\frac{\Delta^{NFA}_c}{Y_c} \quad \Delta^c_{comp}
\]

---

45° line
Change in NFA vs shift share historically: 1970-2011 data
Predicted global imbalances using $\Delta_{c}^{comp} - \bar{\Delta}^{comp}$
Conclusion

• How does population aging affect wealth-output ratios, real interest rates, and capital flows?

• Use compositional effect $\Delta^{comp}$ as starting point for forecasts

• $\Delta^{comp}$ are large and heterogeneous in the data

• Going forward, our approach suggests demographics will cause:
  1. real interest rates to fall, with uncertain magnitude (40-120 bp)
  2. wealth-GDP ratios to rise, with effects attenuated relative to $\Delta^{comp}$
  3. global imbalances to substantially increase from today’s levels

• Global savings glut has just begun!
Thank you!
Additional slides
Source: World Inequality Database (WID), Survey of Consumer Finances (SCF)
Share of the population aged 65+

Source: 2019 United Nations World Population Prospects
Countries by income group

Source: 2019 United Nations World Population Prospects
National Wealth over GDP

Source: World Inequality Database (WID)
National Wealth over GDP

Source: World Inequality Database (WID)
Figure 4: Post-Tax Real Yields on Short-Term 1-Year Treasuries
Age-wealth profiles

![Graph showing age-wealth profiles with data points for different years: a1989, a1998, a2004, a2010, and a2016.](image-url)
Age-labor income profiles
Contribution of fertility to aging in the 21st century

Percentage of population aged 50+

Year

1960 1980 2000 2020 2040 2060 2080 2100

Actual

Fixed 2016 mortality

Fixed 1950 mortality
Measuring income and wealth profiles

- **Measuring age-labor income profiles** $h_{jt}$
  - Data from the Luxembourg Income Study (LIS)
  - $h_{jt}$ is proportional to total labor income per person
  - In 2016: normalize aggregate effective labor per person
    \[ 1 = L_{2016} = \sum_{j} \pi_{j,2016} h_{j,2016} \]
  - In $t$: $L_t$ grows as aggregate labor input from the BLS $\frac{L_t^{BLS}}{L_{2016}^{BLS}}$

- **Measuring age-wealth profiles** $a_{jt}$
  - Data from the Survey of Consumer Finances (SCF)
  - Provide net worth by age at the household level
  - $A_{jt}$ is aggregate household net worth over total individuals
  - Divide by $Y_t/L_t^{BLS}$ to obtain $a_{jt}$
Retrospective U.S. exercise

- To first order:
  \[
  \frac{W_t}{Y_t} - \frac{W_0}{Y_0} \equiv \Delta_t = \sum_i \frac{\pi_{it} a_{io}}{\pi_{it} h_{io}} - \sum_i \frac{\pi_{io} a_{io}}{\pi_{io} h_{io}} + \sum_i \pi_{io} (a_{it} - a_{io}) - \sum_i \pi_{io} \frac{W_0}{Y_0} (h_{it} - h_{io}) + \Delta_{t}^{err}
  \]

\[
\Delta_t = \Delta_t^{\pi} + \Delta_t^{a} + \Delta_t^{h} + \Delta_{t}^{err}
\]

Graph showing percentage of GDP from 1990 to 2015 with different categories labeled:

- $W_t/Y_t$
- $\pi_t$ only
- $a_t$ only
- $h_t$ only

Bar charts with values:

147.0
78.0
110.0
-27.0
-14.0

% of GDP
Robustness to baseline year for age profiles (past)

<table>
<thead>
<tr>
<th>Age-wealth profile (SCF)</th>
<th>Change in W/Y: 1950 to 2016</th>
</tr>
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<tbody>
<tr>
<td>1989</td>
<td>0.70 0.71 0.73 0.73 0.71 0.70 0.73 0.69 0.68 0.65 0.65 0.66</td>
</tr>
<tr>
<td>1992</td>
<td>0.63 0.64 0.66 0.66 0.66 0.65 0.64 0.66 0.63 0.62 0.60 0.59 0.60</td>
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<tr>
<td>1995</td>
<td>0.70 0.72 0.74 0.73 0.72 0.71 0.73 0.71 0.70 0.67 0.67 0.67</td>
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<tr>
<td>1998</td>
<td>0.83 0.85 0.87 0.87 0.85 0.84 0.87 0.83 0.82 0.79 0.79 0.80</td>
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<tr>
<td>2001</td>
<td>1.02 1.04 1.07 1.07 1.05 1.04 1.07 1.03 1.01 0.98 0.97 0.98</td>
</tr>
<tr>
<td>2004</td>
<td>1.02 1.05 1.08 1.08 1.06 1.05 1.08 1.04 1.03 0.99 0.99 1.00</td>
</tr>
<tr>
<td>2007</td>
<td>1.14 1.17 1.20 1.20 1.18 1.17 1.20 1.16 1.15 1.11 1.10 1.11</td>
</tr>
<tr>
<td>2010</td>
<td>0.93 0.95 0.98 0.98 0.97 0.96 0.98 0.95 0.94 0.91 0.91 0.92</td>
</tr>
<tr>
<td>2013</td>
<td>0.89 0.91 0.94 0.94 0.93 0.92 0.94 0.91 0.90 0.87 0.87 0.88</td>
</tr>
<tr>
<td>2016</td>
<td>1.17 1.20 1.24 1.24 1.23 1.22 1.25 1.21 1.20 1.16 1.16 1.17</td>
</tr>
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Age-labor income profile (LIS)
Robustness to baseline year for age profiles (future)

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<tbody>
<tr>
<td>1974</td>
<td>0.66</td>
<td>0.57</td>
<td>0.68</td>
<td>0.75</td>
<td>0.96</td>
<td>1.00</td>
<td>1.12</td>
<td>0.94</td>
<td>0.95</td>
<td>1.29</td>
</tr>
<tr>
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Change in W/Y: 2016 to 2100

Age-labor income profile (LIS)
Low and high fertility scenarios
W/Y from shift-share in 2016 and in 2100
Shift-share at common age profiles (rescaled)
Shift-share at common demographic change

![Bar chart showing Δn values for different countries with varying degrees of demographic change. The chart includes countries such as HUN, POL, EST, FIN, GRE, SVN, SVK, AUT, SWE, DNK, JPN, DEU, CAN, IRL, USA, FRA, NLD, GBR, LUX, AUS, ITA, BEL, ESP, CHN, and IND. The Δn values range from 0 to 300, with each country represented by a different color or shade.](chart.png)
• Population evolves as

\[ N_{jt} = (N_{j-1,t-1} + M_{j-1,t-1}) \phi_{j-1,t-1} \]

where

• \( N_{jt} \) denotes the numbers of individuals aged \( j \) in year \( t \)
• \( M_{j,t} \) is migration
• \( \phi_{j,t} \) are survival probabilities

• Total population is

\[ N_t = \sum_j N_{jt} \]

• Population converges to a stationary distribution in the long run
• Let \( c = c^P + nc^C \) be the total cons. of parent and children
• Assume flow utility function of a parent is
  \[
  U(c^P, c^C) = u(c^P) + \lambda n^\varphi u(c^C)
  \]
• Utility maximization implies:
  \[
  u'(c^P) = \lambda n^\varphi - u'(c^C)
  \]
  \( \Rightarrow \) total value of having children
  \[
  W(c) = u(c^P) + \lambda n^\varphi u(c^C) = \left( 1 + \lambda \frac{n}{\sigma} \frac{\varphi - 1}{\varphi} \right)^\sigma u(c)
  \]
• Hence \( \psi_i = \left( 1 + \lambda \frac{1}{\sigma} n_i \frac{\varphi - 1}{\varphi} \right)^\sigma \)
  • Children raise the m.u.c. if \( \lambda > 0 \) and \( \varphi > 1 - \sigma \)
  • \( n_i \) comes from empirical distribution of children for parent aged \( i \)
Retirement policy

• Retirement is phased at age $T'_t$

• At age $T'_t$, agents still work a fraction $\rho_t \in [0, 1]$ of total hours

• Retirement policy is therefore

$$\rho_{jt} = \mathbb{1}_{j<T'_t} + \rho_t \mathbb{1}_{j=T'_t}$$

• Effective labor supply is

$$L_t \equiv \sum_{j<T'_t} \pi_{jt} \tilde{h}_{jt} + \rho_t \pi_{T'_t t} \tilde{h}_{T'_t t}$$

• Effective share of retirees is

$$\mu_{t}^{ret} \equiv (1 - \rho_t) \pi_{T'_t t} + \sum_{j \geq T'_t} \pi_{jt}$$
Government policy

• Flow budget constraint

\[ B_t + T_t = (1 + r_{t-1}) B_{t-1} + G_t \]

where \( B_t \) is debt, \( G_t \) are expenditures, \( T_t \) are net taxes

\[ T_t = w_t N_t \left( (\tau_{t}^{SS} + \tau_t (1 - \tau_t^{SS})) L_t - (1 - \tau_t) \bar{d}_t \mu_{t}^{ret} \right) \]

• Government sets retirement policy \( \{\rho_{jt}\} \) and follows fiscal rules

\[ \tau_{t}^{SS} = \overline{\tau}^{SS} + \phi^{ss} (B_t/Y_t - \overline{b}) \]

\[ \tau_t = \overline{\tau} + \phi^{T} (B_t/Y_t - \overline{b}) \]

\[ G_t/Y_t = \overline{G}/Y - \phi^G (B_t/Y_t - \overline{b}) \]

\[ \overline{d}_t = \overline{d} - \phi^d (B_t/Y_t - \overline{b}) \]

where \( \overline{b} \) is the 2016 debt-to-GDP ratio

• Coefficients \( \phi \)'s regulate the aggressiveness of the adjustment
Extension 1: other sources of asset supply

• In simple cases, alternative assets just add to supply

• Allow for
  • Markups \( \mu \), capitalized monopoly profits
  • Government bonds with long-run rule \( \frac{B}{Y} = b(r) \)

• Then

\[
\frac{a(r, \theta)}{y(r)} = \frac{k(r)}{y(r)} + b(r) + \left(1 - \frac{1}{\mu}\right) \frac{1}{r - (n + \gamma)}
\]

• \( \theta \) directly affects both \( W \) and market cap. through discounting

• Extra terms on RHS affect elasticity of asset supply \( \varepsilon^s \)
  • Similar formula still determines \( dr \)
• Model housing by introducing Cobb-Douglas utility

\[ \frac{1}{1 - \sigma} \left( c^{1-\alpha_h} h^\alpha_h \right)^{1-\sigma} \]

• All households rent to a REIT who owns
  • fixed supply of land \( L \), equilibrium price \( P^L \)
  • stock of dwellings \( H \), depreciating at \( \delta^H \), investment price \( = 1 \)
  • \( \beta = \frac{p^L}{p^L + H} \) is s.s. share of land

• Households invest in mutual fund that owns the REIT

• Housing supply in steady state adjusts so that

\[
\frac{a(r, \theta)}{y(r)} = \frac{k(r)}{y(r)} + \frac{\alpha^h}{1 - \alpha^h} \left( \frac{\beta}{r - (n + \gamma)} + \frac{1 - \beta}{r + \delta^H} \right) \frac{\sum \pi_i(\theta) c_i(r, \theta)}{\sum \pi_i(\theta) h_i}
\]
Projected survival functions

![Projected survival functions graph]

Survival function (%) vs. Age

- 2016
- 2030
- 2050
- 2100
Projected population growth rate

![Graph showing projected population growth rate from 2020 to 2100. The graph indicates a gradual decrease in population growth rate over time, starting at around 0.8 in 2020 and decreasing towards 0.2 in 2100.](image-url)
Projected population shares
Distribution of children
Distribution of bequests received
Bequests distribution and consumption profile

- Model, $B/Y = 4.9\%$
- Data (Hurd and Smith; 2002)

- Model
- Data (CEX)
Robustness

Change in wealth-to-GDP

- Lower benefits only
- Higher SS taxes only
- No migration
- Baseline

Years: 2020 to 2100
Historical trends in wealth

• We’ll use our model primarily for prospective counterfactuals.
• But: can the model account for trends in wealth since 1960?
• Concurrent developments to demographics over the period:
  • Falling real rates
  • Falling productivity growth
• We feed the model with observed trends in $r$, $\gamma$, $B$ and $G$. 
Historical trends in wealth
Demographics: population growth rates

The graph shows the population growth rates for various countries from 2020 to 2090. Each line represents a different country, with labels for AUS, DEU, GBR, JPN, CAN, ESP, IND, NLD, CHN, FRA, ITA, and USA.
## World economy calibration

<table>
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<tr>
<th>Country</th>
<th>Parameters</th>
<th>Model</th>
<th>Data</th>
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World economy calibration

Graphs showing model and data comparisons for various countries:
- AUS
- CAN
- CHN
- DEU
- ESP
- FRA
- GBR
- IND
- ITA
- JPN
- NLD
- USA
Predicted NFA/Y from demographics

Historical (data)

Net Foreign Assets (% GDP)

Year

1980 1990 2000 2010

Predicted from demographics (model)

Year (Model)

2020 2040 2060 2080 2100
Elasticities by country

The diagram illustrates the elasticities of different countries. Each point represents a country, and the size of the circle indicates the magnitude of the elasticity. The x-axis represents \( \Delta_{\text{comp}, c} \), and the y-axis represents \( \varepsilon_d^c + \varepsilon_s^c \). The countries are labeled with their respective initials: AUS (Australia), ESP (Spain), USA (United States), ITA (Italy), GBR (United Kingdom), CAN (Canada), NLD (Netherlands), JPN (Japan), DEU (Germany), CHN (China), and IND (India).
Note: Response of wealth to a reduction in the wealth tax. We replicate the model experiments of Jakobsen et al. (2020). The first (Couples DD) analyzes a reduction of the wealth tax from 2.2% to 1.2% on the top 1%. The second (Ceiling DD) analyzes the a reduction of 1.56 percentage points on the top 0.3%.