# A note on multipliers in NK models with GHH preferences

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**Abstract**

GHH preferences (from Greenwood, Hercowitz and Huffman 1988) are a common modeling device to eliminate wealth effects on labor supply. We show that in New Keynesian models, this specification can generate very large fiscal and monetary multipliers due to a feedback from consumption-labor complementarity. These multipliers are closely connected to the labor wedge and can be infinite in an undistorted economy. We argue that alternatives to GHH, particularly for the growing literature that merges nominal rigidities and incomplete markets, are needed.

Greenwood, Hercowitz and Huffman (1988) introduced a functional form for within-period household utility over consumption $c$ and labor hours $n$

$$U(c, n) = u(c - v(n)) \tag{1}$$

that has since become a staple of the macro literature. These *GHH preferences* eliminate wealth effects on labor supply, by making the marginal rate of substitution between $c$ and $n$ independent of changes in $c$. This feature is popular both for its theoretical tractability and for its perceived empirical advantages, and has been used particularly heavily in a recent incomplete markets literature, for reasons we discuss further in section 2 below.[1]

In this note we show that GHH preferences can generate, as an undesirable byproduct, extremely large multipliers on fiscal and monetary policy in New Keynesian models with flexible wages. In our benchmark specification, where real interest rates are held constant, the fiscal multiplier under GHH preferences is

$$\frac{\partial y_t}{\partial g_s} = \frac{1}{\tau}\mathbf{1}_{s=t}$$

where $y$ is output, $g$ is government spending, and $\tau \leq 1$ is the labor wedge. Here, a dollar of government spending increases output in the same period by $\frac{1}{\tau}$ dollars. This contrasts with the result in Woodford (2011) under separable preferences, where a dollar of spending increases output in the same period by only a dollar.

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[1] GHH preferences are ubiquitous in many other literatures, including those on news shocks (Jaimovich and Rebelo 2009), international real business cycles (e.g. Mendoza 1991), optimal income taxation (Diamond 1998), or liquidity traps (Korinek and Simsek 2016).

We show that the impact of monetary shocks under GHH preferences is, similarly, amplified by a factor of $\frac{1}{\tau}$ relative to separable preferences. Furthermore, under typical calibrations this factor can be quite large. For instance, if the only steady-state distortion in the model is the markup from monopolistically competitive intermediate goods producers, then $\frac{1}{\tau} = \epsilon$, where $\epsilon$ is the elasticity of substitution between intermediate goods. Since calibrations of NK models usually target $\epsilon \geq 5$, this in turn implies a multiplier on current government spending of 5 or more. Such multipliers, well outside the range of typical estimates, come as the inadvertent consequence of seemingly unrelated assumptions—such as the choices of $\epsilon$, or the steady-state labor tax.

This amplification is due to the complementarity between consumption and labor in (1). If a shock increases demand for output, wages rise until enough labor is supplied in equilibrium to produce that output. As they work more, households also want to consume more, and this additional consumption increases demand for output even further—prompting another increase in labor effort, and so on. With GHH preferences, this feedback is especially powerful: as households increase labor enough to produce 1 more unit of output, they increase consumption demand by $1 - \tau$ additional units. The ultimate effect is to amplify the initial impulse to output by $1 + (1 - \tau) + (1 - \tau)^2 + \cdots = 1/\tau$.

The importance of non-separability between consumption and labor for fiscal multipliers in sticky price models has been understood for some time, including in important work by Bilbiie (2011) and Monacelli and Perotti (2008). Our perspective makes clear how large these multipliers can be, and shows that distortions in the economy—for which the labor wedge is a sufficient statistic—play the central quantitative role. We show that these forces equally amplify monetary policy, demonstrating the consequences of nonseparable preferences for a large and distinct literature.

The rest of this short note is organized as follows: in section 1, we lay out a representative-agent New Keynesian model and provide explicit expressions for separable and GHH multipliers under various assumptions about monetary policy. In section 2, we draw out the implications of these results for the incomplete markets literature, and argue briefly for sticky wages as an alternative.

## 1 GHH multipliers in a representative agent NK model

Consider a representative agent New Keynesian model. Households have utility $\mathbb{E}\left[\sum \beta^t U(c_t, n_t)\right]$, where $U(c, n)$ is given either by GHH preferences or more traditional separable preferences:

$$U(c,n) = \begin{cases} u(c - v(n)) & \text{GHH} \\ u(c) - v(n) & \text{separable} \end{cases}$$

The final good is produced competitively as a CES aggregate, with elasticity $\epsilon$, of a continuum of intermediate goods $i$. These goods are produced with the technology $y_{it} = f(n_{it})$ by monopolistically competitive firms, which set prices à la Calvo. Wages are flexible and the labor market clears.

The fiscal authority has some rule for government spending $g_t$. The monetary authority has some rule for the nominal interest rate $i_t$. Market clearing for final goods in each period requires that $y_t = c_t + g_t$. Gross inflation on final goods is $\Pi_t$.

We assume that the economy is initially at a steady state $\left(\bar{c}, \bar{g}, \bar{n}, \bar{i}, \overline{\Pi} = 1\right)$. At this steady state, we define

$$\tau \equiv 1 + \frac{U_n(\bar{c}, \bar{n})}{U_c(\bar{c}, \bar{n})} \frac{1}{f'(\bar{n})} \quad \text{and} \quad \sigma \equiv -\frac{U_c(\bar{c}, \bar{n})}{U_{cc}(\bar{c}, \bar{n})\bar{c}} \tag{2}$$

where $\tau$ is the *labor wedge* and $\sigma$ is the *elasticity of intertemporal substitution* in consumption.

### 1.1 Multipliers under a constant-$R$ rule

Suppose that the fiscal and monetary authorities set exogenous paths for government spending $\{g_t\}$ and the real interest rate $\{R_t\}$.[2] We study the effects of perturbations to these paths, relative to the baseline steady state $\bar{g}$ and $\bar{R}$, assuming that no perturbations take place after some date $T$, at which point the economy reverts back to the steady state.

For the case of separable preferences, this echoes the analysis in Woodford (2011), where specifying monetary policy with a rule for the real interest rate facilitates a simple benchmark for the fiscal multiplier.

Our main results are summarized in the following proposition.

**Proposition 1.** *Under GHH preferences, the multipliers of date-s spending and real interest rates on date-t output, for any $s, t < T$, are*

$$\frac{\partial y_t}{\partial g_s} = \frac{1}{\tau}\mathbf{1}_{s=t} \tag{3}$$

$$\frac{\partial y_t}{\partial \log R_s} = -\frac{\sigma\bar{c}}{\tau}\mathbf{1}_{s\geq t} \tag{4}$$

*Under separable preferences, these multipliers are*

$$\frac{\partial y_t}{\partial g_s} = \mathbf{1}_{s=t} \tag{5}$$

$$\frac{\partial y_t}{\partial \log R_s} = -\sigma\bar{c}\,\mathbf{1}_{s\geq t} \tag{6}$$

The separable-preferences fiscal multiplier in (5) is from Woodford (2011), and the real interest rate multiplier in (6) is also standard. Relative to these, the fiscal and monetary multipliers (3) and (4) under GHH preferences are each amplified by a factor of $1/\tau$, where $\tau$ is the labor wedge.

*Proof.* For any $t$, we have an intertemporal Euler equation relating $U_c$ between periods $t$ and $T$; in

[2] The real interest rate target can be achieved under nominal rigidities by a monetary authority that adjusts the nominal interest rate in each period as needed given expected inflation. Specifically, it sets $1 + i_t = R_t \cdot \mathbb{E}_t[\Pi_{t+1}]$.

the latter, the economy has returned to steady state:

$$U_c(c_t, n_t) = \beta^{T-t} \left( \prod_{j=0}^{T-t-1} R_{t+j} \right) U_c(\bar{c}, \bar{n}) \equiv \Lambda_t \tag{7}$$

Evaluating $U_c(c_t, n_t)$ for GHH and separable preferences, this becomes

$$\Lambda_t = \begin{cases} u'(c_t - v(n_t)) & \text{GHH} \\ u'(c_t) & \text{separable} \end{cases}$$

which, imposing market clearing $y_t = c_t + g_t$ and applying $u'^{-1}$ to both sides, can be rearranged as

$$y_t - g_t - \lambda_t = \begin{cases} v(n_t) & \text{GHH} \\ 0 & \text{separable} \end{cases} \tag{8}$$

where $\lambda_t \equiv u'^{-1}(\Lambda_t)$.

Totally differentiating (8) for separable preferences is trivial. For GHH preferences, we also need to differentiate $v(n_t)$ on the right. The first-order relation $dy_t = f'(\bar{n})dn_t$ implies that $dv(n_t) = \frac{v'(\bar{n})}{f'(\bar{n})}dy_t$, and applying the labor wedge definition (2) to GHH we have $\frac{v'(\bar{n})}{f'(\bar{n})} = 1 - \tau$, so that

$$dy_t - dg_t - d\lambda_t = \begin{cases} (1-\tau)dy_t & \text{GHH} \\ 0 & \text{separable} \end{cases} \tag{9}$$

Rearranging to isolate $dy_t$, we conclude

$$dy_t = \begin{cases} \frac{1}{\tau}(dg_t + d\lambda_t) & \text{GHH} \\ dg_t + d\lambda_t & \text{separable} \end{cases} \tag{10}$$

The fiscal multipliers follow immediately from (10), while the monetary multipliers follow in conjunction with the observation that the Euler equation (7) implies $d\lambda_t = \sigma\bar{c} \cdot \sum_{j=0}^{T-t-1} d\log R_{t+j}$. $\square$

**Intuition and graphical illustration.** Equation (8) provides the key to understanding the results in proposition 1. For both types of preferences, government spending $g_t$ and impulses to consumption $\lambda_t$ from the Euler equation enter symmetrically on the left of (8). But for GHH preferences, consumption demand also includes labor disutility $v(n_t)$, which rises as additional output requires additional labor effort. Intuitively, in this case, equation (8) represents the intersection of two upward-sloping curves as a function of output $y_t$.

Figure 1 represents both sides of this relation as a function of $y$, after substituting the equilibrium relationship $v(n) \simeq v\left(f^{-1}(y)\right)$ in (8).[3] Point $S$ represents the initial steady state under

[3] The exact relationship is $n_t = \Delta_t f^{-1}(y_t)$, where $\Delta_t$ is a parameter summarizing price dispersion, and whose deviations from its steady-state value of $\overline{\Delta} = 1$ are second-order.

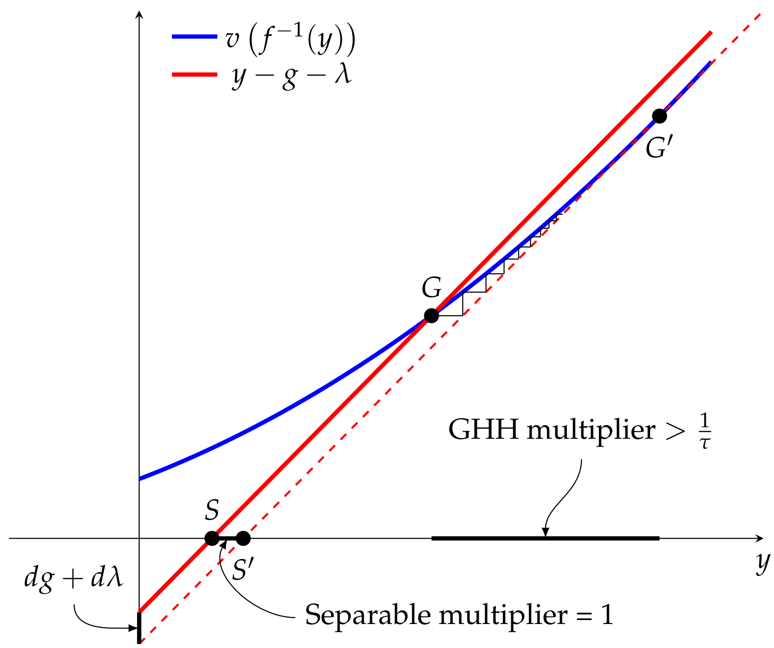


Figure 1: Equilibrium output after a change in monetary or fiscal policy

separable preferences. A change $dg + d\lambda$ in monetary or fiscal policy moves the equilibrium to point $S'$, in which output is higher by exactly $1 \times (dg + d\lambda)$, hence a multiplier of 1.

By contrast, under GHH preferences, the initial steady-state is represented by point $G$. The same change in monetary or fiscal policy leads to a new equilibrium at point $G'$, which is visually much further away from $G$ than $S'$ is from $S$. The reason is that increasing output requires increasing labor disutility, increasing consumption demand further.

The role of the labor wedge can be understood as follows. At the margin, increasing output by $dy$ requires adding $dn = (1/f'(\bar{n}))dy$ to labor. This adds $dv(n) = (v'(\bar{n})/f'(\bar{n}))dy$ to labor disutility, and therefore $dc = dv\,(n) = (1-\tau)\,dy$ to consumption demand, by definition of the labor wedge $\tau$. In turn, this leads to a further increase in consumption, this time by $(1-\tau)^2\,dy$. This process can be visualized via the solid black lines away from point $G$ in figure 1. It converges to an amplification factor of of $1 + (1-\tau) + (1-\tau)^2 + \cdots = 1/\tau$ of the initial impulse. This explains the multiplier in equation (10). Figure 1 also shows that the nonlinear multiplier for a positive shock is actually higher than this, because the labor wedge endogenously becomes smaller as output increases.

If $\tau = 0$, then the amplification process $1 + (1-\tau) + (1-\tau)^2 + \cdots$ diverges to $1/\tau = \infty$. The problem here is that any increment $dy$ in output implies an exactly equal increment $dv(n)$ in labor disutility. This is no coincidence: it is exactly the condition for efficiency under GHH preferences, since an economy maximizing $c - v(n)$ in each period will efficiently trade off consumption $c$ and the disutility $v(n)$ from the labor needed to produce consumption goods. Only when $\tau > 0$, and

$n$ is inefficiently low in the steady state, do we obtain $dv(n) < dy$ and finite multipliers.[4,5]

**Quantitative implications.** The $1/\tau$ factor in (3) and (4) can be very large in practical applications, since the steady state in New Keynesian models is often close to undistorted.

If the only steady-state distortion is the markup $\epsilon/(\epsilon-1)$ from monopolistic competition, then $1-\tau = v'(n)/f'(n) = (\epsilon-1)/\epsilon$, and

$$\frac{1}{\tau} = \epsilon \tag{11}$$

The government spending multiplier is therefore identical to the elasticity of substitution $\epsilon$ between intermediate goods, a seemingly unrelated parameter. This has played a role in applications, although its analytical origin has not previously been clarified—for instance, Nakamura and Steinsson (2014) report a constant-$R$ multiplier under GHH of 7.00, equal to their assumed $\epsilon = 7$.

If additionally there is a tax $\tau^w$ on wages, then

$$\frac{1}{\tau} = \frac{1}{1-(1-\tau^w)\cdot\frac{\epsilon-1}{\epsilon}} \tag{12}$$

Adding a labor tax can reduce the multiplier to high but more empirically realistic levels: for instance, $\tau^w = 1/3$ combined with $\epsilon = 7$ gives $1/\tau = 2.33$. If, on the other hand, a negative $\tau^w$ is chosen to offset the monopolistic distortion—a common tactic to obtain an efficient steady state in New Keynesian models—then the multiplier $1/\tau$ becomes infinite.

**Generalized formula.** As discussed above, the crucial aspect of GHH preferences that drives amplification is the complementarity between consumption and labor in the utility function. For more general preferences, fiscal and monetary multipliers can be stated explicitly in terms of second derivatives of the utility function.

**Proposition 2.** *For arbitrary preferences $U(c,n)$, the multipliers of date-$s$ spending and real interest rates on date-$t$ output, for any $s,t < T$, are*

$$\frac{\partial y_t}{\partial g_s} = \frac{1}{1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}}\mathbf{1}_{s=t} \tag{13}$$

$$\frac{\partial y_t}{\partial \log R_s} = -\frac{\sigma\bar{c}}{1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}}\mathbf{1}_{s\geq t} \tag{14}$$

[4]Another way to think about the singularity when $\tau = 0$ is via the Euler equation. A decline in interest rates, holding the future constant, will push down $u'(c - v(n))$, implying a rise in $c - v(n)$. But if $\tau = 0$, $c - v(n)$ is already at its maximum level, and it can rise no further. Point $G'$ in figure 1 represents this situation, in which the blue and the red dashed curves are tangent, so there is no equilibrium point with $\lambda$ any higher.

[5]Formally, our results imply negative multipliers when $\tau < 0$, although the infinite series that we use to interpret them diverges. This is true of multipliers in other contexts: after reaching a singularity, they switch to being negative. We think these cases should be treated with caution, since are hard to interpret and all comparative statics are flipped.

The results in proposition 1 are special cases of (13) and (14). The crucial feature of preferences is the ratio $\frac{U_{cn}/U_n}{U_{cc}/U_c}$, which is the ratio of the elasticities of $U_n$ and $U_c$ to $c$. This ratio is a measure of consumption-labor complementarity, relative to the magnitude of diminishing returns from consumption.

For separable preferences, this ratio is 0, since $U_n$ is unaffected by $c$. For GHH preferences, this ratio is 1, which is exactly enough to eliminate wealth effects on labor supply: changes in $c$ affect $U_n$ and $U_c$ proportionately and do not disturb the optimal labor supply condition $U_n = -U_c w$.

More generally, $\frac{U_{cn}/U_n}{U_{cc}/U_c}$ parametrizes the extent to which fiscal and monetary impulses are amplified. It is worth noting that if consumption-labor complementarity is increased beyond GHH, so that $\frac{U_{cn}/U_n}{U_{cc}/U_c} > 1$, then (13) and (14) can blow up to infinity even for strictly positive $\tau > 0$.

### 1.2 Multipliers under a Taylor rule for monetary policy

In this section, we depart from the constant-$R$ benchmark to characterize multipliers under a more conventional specification in which monetary policy follows a Taylor rule.

For tractability, we look at local perturbations to fiscal and monetary policy that follow AR(1) processes. In particular, assume that perturbations $dg_t = g_t - \bar{g}$ to the path of government spending follow the process

$$dg_t = \rho dg_t + d\epsilon_{gt}$$

where $d\epsilon_{gt}$ is an iid disturbance, and that monetary policy follows a simple Taylor rule perturbed with AR(1) monetary policy shocks

$$\begin{aligned} i_t &= \bar{i} + \phi \pi_t + d\nu_t \\ d\nu_t &= \rho d\nu_{t-1} + d\epsilon_{mt} \end{aligned}$$

where $\pi_t = \log(\Pi_t)$ and $d\epsilon_{mt}$ is another iid disturbance, independent of $d\epsilon_{gt}$. Linearizing all equations around the steady state, we can show that

$$dy_t = \omega_g dg_t + \omega_m d\nu_t$$

where, as above, $\omega_g$ and $\omega_m$ can be interpreted as multipliers for fiscal and monetary policy.

**Proposition 3.** *There exists two parameters $\nu^h > 0$ and $\nu^s > 0$, independent of $\phi$, such that under GHH preferences, the multipliers for fiscal and monetary policy are*

$$\omega_g^h = \frac{1}{\tau + \nu^h (\phi - \rho)} \qquad \omega_m^h = -\frac{\sigma \bar{c}}{(1-\rho) R} \frac{1}{\tau + \nu^h (\phi - \rho)}$$

*while under separable preferences they are*

$$\omega_g^s = \frac{1+\nu^s(\phi-\rho)\Gamma^s}{1+\nu^s(\phi-\rho)} \qquad \omega_m^s = -\frac{\sigma\bar{c}}{(1-\rho)R}\frac{1}{1+\nu^s(\phi-\rho)}$$

*where* $\Gamma^s \in (0,1)$ *is the flexible-price government spending multiplier for those preferences.*

Proposition 3 generalizes the results of proposition 1 to the case where real interest rates respond endogenously given a monetary rule. Under the assumed AR(1) structure for shocks, all variables endogenously decay at the same rate, so that the effect of monetary and fiscal policy on output can still be summarized by a single multiplier. Moreover, this structure implies that expected inflation is $\mathbb{E}_t[\pi_{t+1}] = \rho\pi_t$, so the case of $\phi = \rho$ reduces to the $R$-constant case studied in proposition 1. Even away from this benchmark, the labor wedge $\tau$ remains crucial in determining the GHH multipliers, as the intercept in a denominator that is an affine function of $\phi - \rho$.

When $\phi < \rho$, as in the constant nominal rate case $\phi = 0$ (often encountered at the zero lower bound), the real interest rate falls endogenously in response to inflationary shocks, leading to larger and potentially infinite multipliers. A large literature has documented this effect under separable preferences—see, for instance, Eggertsson and Woodford (2003) and Christiano, Eichenbaum and Rebelo (2011)—but the multipliers under GHH are even higher because the intercept term $\tau$ in the denominators of $\omega_g^h$ and $\omega_m^h$ may start close to 0. .

By contrast, when $\phi > \rho$, monetary policy becomes more responsive to the inflationary effects of fiscal and monetary innovations, and multipliers are therefore smaller. Note also that, under sufficiently responsive monetary policy, the GHH multipliers *can* become smaller than the separable multipliers. In particular, in the limit of infinite responsiveness, ($\phi \to \infty$, so $\nu \to \infty$), we recover a GHH flexible price multiplier of $\omega_g = 0$, reflecting the absence of wealth effects on labor supply. By contrast, under separable preferences the flexible price multiplier is $\Gamma^s > 0$, reflecting the negative wealth effect on labor supply, as is well known from Baxter and King (1993) or Woodford (2011).

Proposition 3 is a special case of a proposition that covers generic preferences.

**Proposition 4.** *For arbitrary preferences* $U(c,n)$, *there exists a parameter* $\nu^u > 0$, *independent of* $\phi$ *and given in appendix equation (25), such that the multipliers for fiscal and monetary policy are*

$$\omega_g^u = \frac{1+\nu^u(\phi-\rho)\Gamma^u}{1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}+\nu^u(\phi-\rho)} \qquad \omega_m^u = -\frac{\sigma\bar{c}}{(1-\rho)R}\frac{1}{1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}+\nu^u(\phi-\rho)}$$

*where* $\Gamma^u$ *is the flexible-price government spending multiplier for those preferences, given in equation (24).*

Proposition 4 provides monetary and fiscal policy multipliers for New Keynesian models with arbitrary preferences and an arbitrarily distorted state, as summarized by the labor wedge $\tau$. To the best of our knowledge, this expression is new to the literature. For fiscal multipliers, Bilbiie (2011) and Monacelli and Perotti (2008) provide close precedents in a special case where $\tau = 1/\epsilon$.[6]

[6] See the expression below Proposition 3 in Bilbiie (2011), and equation (77) in Monacelli and Perotti (2008).

# 2 Implications for incomplete markets models

To avoid the large fiscal and monetary multipliers that follow from GHH preferences, one natural alternative is to use separable preferences when writing New Keynesian models. This leads to wealth effects on labor supply, but these wealth effects are often of manageable size, and may be desirable from an economic standpoint.

In this section, we demonstrate that these wealth effects on labor supply become very large in incomplete markets models with households that have high marginal propensities to consume (MPCs). We further show that if household labor supply is frictionless, there is no specification of preferences that can fix this problem without reintroducing the large fiscal and monetary multipliers we have documented for GHH. To resolve this dilemma, we conclude that it may be necessary to introduce frictions in household labor supply.

**Incomplete markets model.** For the following sections, consider a standard incomplete markets model with endogenous labor supply, as in Aiyagari and McGrattan (1998) or Pijoan-Mas (2006): households solve

$$\begin{aligned} \max_{c_t,n_t} \quad & \mathbb{E}\left[\sum \beta^t U\left(c_t, n_t\right)\right] \\ \text{s.t.} \quad & c_t + a_{t+1} = w_t e_t n_t + T_t + R a_t \\ & a_t \geq -\underline{a} \end{aligned} \tag{15}$$

where $e_t$ evolves stochastically, assets $a_t$ cannot be made contingent on the realization of $e_t$, and $\underline{a}$ represents a borrowing limit which can be infinite. This model constitutes the backbone of a recent literature that incorporates heterogeneity into the equilibrium analysis of monetary and fiscal policy.

## 2.1 Frictionless labor supply

With flexible wages and no other labor market frictions, a household solving the problem in (15) is on a labor supply curve at all times, so that

$$-U_n\left(c_t, n_t\right) = w_t e_t U_c\left(c_t, n_t\right) \tag{16}$$

Let $MPC = \frac{\partial c_0}{\partial T_0}$ be the consumption response of this household to a one-time transfer at date 0. Constrained agents have high $MPC$s, reflecting current or future borrowing constraints. Similarly let $MPN = \frac{\partial n_0}{\partial T_0}$ be the household's labor supply response to the transfer. Totally differentiating (16), we find

$$wMPN = -\frac{wen}{c} \frac{\frac{-cU_{cc}}{U_c} + \frac{cU_{nc}}{U_n}}{\frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}} MPC \tag{17}$$

To understand the quantitative implications of (17), consider first the case of separable preferences, where $U_{cc} = U_{nc} = 0$. Then

$$wMPN = -\frac{wen}{c}\frac{\psi}{\sigma}MPC \tag{18}$$

where $\psi \equiv \frac{U_n}{nU_{nn}}$ is the Frisch elasticity of labor supply and $\sigma \equiv \frac{-U_c}{cU_{cc}}$ the elasticity of intertemporal substitution in consumption. In typical calibrations, $\sigma \simeq \psi$, and asset and transfer income is small, so that $c \simeq wen$. (18) then implies that a constrained household receiving a transfer of 1 adjusts by increasing consumption by about 0.5 and reducing earned income by around 0.5. Such large labor supply responses of constrained agents to transfers are generally deemed to be counterfactual: while there exists good empirical evidence supporting the predictions of this model for MPCs, there is no similarly good evidence for MPN.

Alternatively, for GHH preferences, the numerator in (17) is 0: as discussed in section 1.1, GHH preferences imply $U_{cc}/U_c = U_{nc}/U_n$, thereby eliminating wealth effects on labor supply. GHH preferences have often been used in the incomplete markets literature—starting with Aiyagari (1995) or Heathcote (2005), and including early versions of Auclert (2016) and Kaplan, Moll and Violante (2016)—in part to avoid the large MPNs created by separable preferences given (18).

As we have documented, however, GHH preferences lead to very large fiscal and monetary multipliers in models with nominal rigidities. More generally, there is a tension between reducing wealth effects on labor supply and obtaining realistic multipliers. Attempting to calibrate $U$ such that the numerator in (17) is small will bring $\frac{U_{cn}/U_n}{U_{cc}/U_c}$ closer to 1, and thus push the amplification of fiscal and monetary shocks toward $\frac{1}{\tau}$, as is clear from equations (13)-(14).[7] Separable and GHH preferences are two options from a continuum of possible utility specifications—but it is impossible for *any* of these other specifications to bring down both wealth effects on labor supply and fiscal multipliers at the same time.

## 2.2 Frictional labor supply.

A natural solution to the dilemma from the previous section is to introduce frictions in labor supply that break equation (16). There are many possibilities, but our favorite way of doing so involves using wage instead of price rigidities. This takes households off their short-run labor supply curves and allows the model to flexibly match—with the right labor rationing rule—the observed incidence of aggregate income shocks. As a side benefit, it fixes some of the inaccurate distributional implications, such as strongly countercyclical firm profits, that are associated with price rigidities, and that are particularly problematic in heterogenous agent models where income distribution matters. See Auclert and Rognlie (2016) and Auclert, Rognlie and Straub (2017) for recent examples of this approach.

[7] More formally, conditional on the inverse IES, $-\frac{U_{cc}c}{U_c}$, and the inverse Frisch elasticity of labor supply, $\frac{U_{nn}n}{U_n}$, both of which tend to be pinned down separately by empirical evidence, the ratio $\frac{-cU_{cc}/U_c+cU_{nc}/U_n}{nU_{nn}/U_n-nU_{cn}/U_c}$ on the right of (17) is strictly increasing in the ratio $\frac{U_{cn}/U_n}{U_{cc}/U_c}$. Meanwhile, the multipliers in (13)-(14) are increasing in this ratio, and amplification relative to separable preferences equals $\frac{1}{\tau}$ when the ratio hits 1 (the GHH case).

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# A Proofs

## A.1 Proof of proposition 2

As in the proof of proposition 1, start with (7):

$$U_c(c_t, n_t) = \Lambda_t \tag{7}$$

Substitute in $c_t = y_t - g_t$ and totally differentiate to obtain

$$U_{cc}(dy_t - dg_t) + U_{cn} dn_t = d\Lambda_t$$

As before, we have $dn_t = \frac{1}{f'(\bar{n})} dy_t$. The definition of the labor wedge states that that $\tau = 1 - \frac{U_n}{U_c}\frac{1}{f'(\bar{n})}$, so that we can write rewrite this for the general case as

$$dn_t = (1-\tau)\frac{U_c}{U_n} dy_t$$

Substituting gives

$$\left(U_{cc} - (1-\tau)\frac{U_c}{U_n}U_{cn}\right) dy_t - U_{cc} dg_t = d\Lambda_t$$

Rearranging, we have

$$dy_t = \frac{1}{1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}}\left(dg_t + \frac{1}{U_{cc}} d\Lambda_t\right) \tag{19}$$

Now, noting that $d(\log \Lambda_t) = d\Lambda_t/\bar{\Lambda} = d\Lambda_t/U_c$, we write

$$\frac{1}{U_{cc}} d\Lambda_t = \frac{U_c}{U_{cc}\bar{c}}\bar{c} d(\log \Lambda_t) = -\sigma \bar{c} d(\log \Lambda_t)$$

and substitute into (19) to obtain

$$dy_t = \frac{1}{1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}}\left(dg_t - \sigma\bar{c}d(\log\Lambda_t)\right) \tag{20}$$

The fiscal multiplier result (13) follows immediately from (20). The monetary multiplier result (14) also follows from (20) given $d\log\Lambda_t = \sum_{j=0}^{T-t-1} d\log R_{t+j}$.

## A.2 Proof of propositions 3 and 4

We first prove proposition 4, and then prove proposition 3 as a special case.

Let us first define the parameters in the proposition in terms of underlying primitives. Let $\alpha \equiv \frac{\bar{n}f'(\bar{n})}{f(\bar{n})}$ be the elasticity of output to employment, and $\gamma \equiv -\frac{\bar{n}f''(\bar{n})}{f'(\bar{n})}$ be the elasticity of the marginal product of labor to employment. Let $\theta$ be the probability that a firm keeps its price fixed in a given period. Equilibrium dynamics are summarized by a three equation system in terms of $(dy_t, \pi_t, i_t)$ as a function of disturbances $(dg_t, d\nu_t)$: an the equation for the nominal interest rate, an Euler equation, and a New Keynesian Phillips curve. These equations are respectively

$$\begin{aligned}
i_t &= \bar{i} + \phi\pi_t + d\nu_t && (21)\\
\tau^u dy_t - dg_t &= \mathbb{E}_t\left[\tau^u dy_{t+1} - dg_{t+1}\right] - \frac{\sigma\bar{c}}{R}\mathbb{E}_t\left[i_t - \pi_{t+1} - \bar{i}\right] && (22)\\
\pi_t &= \beta\mathbb{E}_t\left[\pi_{t+1}\right] + \frac{\kappa^u}{\bar{y}}\left(dy_t - \Gamma^u dg_t\right) && (23)
\end{aligned}$$

where the parameters $\tau^u$, $\kappa^u$ and $\Gamma^u$ are defined as

$$\begin{aligned}
\tau^u &\equiv 1-(1-\tau)\frac{U_{cn}/U_n}{U_{cc}/U_c}\\
\kappa^u &\equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{1}{1+\epsilon\frac{\gamma}{\alpha}}\left(\left(1-\frac{U_{nc}/U_n}{U_{cc}/U_c}\right)\frac{1}{\sigma}\frac{\bar{y}}{\bar{c}} + \left(\frac{U_{nn}\bar{n}}{U_n} - \frac{U_{cn}\bar{n}}{U_c} + \gamma\right)\frac{1}{\alpha}\right)\\
\Gamma^u &\equiv \frac{\frac{1}{\sigma}\frac{\bar{y}}{\bar{c}}\left(1-\frac{U_{nc}/U_n}{U_{cc}/U_c}\right)}{\frac{1}{\sigma}\frac{\bar{y}}{\bar{c}}\left(1-\frac{U_{nc}/U_n}{U_{cc}/U_c}\right) + \frac{1}{\alpha}\left(\frac{U_{nn}\bar{n}}{U_n} - \frac{U_{cn}\bar{n}}{U_c} + \gamma\right)}
\end{aligned} \tag{24}$$

$\Gamma^u$ is the flexible-price government spending multiplier for general preferences $U(c,n)$. Define

$$\nu^u \equiv \kappa^u \frac{\sigma\bar{c}}{R\bar{y}}\frac{1}{(1-\rho)(1-\beta\rho)} \tag{25}$$

then the multipliers for government spending and monetary policy are, respectively,

$$\omega_g^u = \frac{1+\nu^u(\phi-\rho)\Gamma^u}{\tau^u+\nu^u(\phi-\rho)} \qquad \omega_m^u = -\frac{\sigma\bar{c}}{(1-\rho)R}\frac{1}{\tau^u+\nu^u(\phi-\rho)} \tag{26}$$

which is proposition 4. Proposition 3 covers two special cases. For GHH preferences, $\frac{U_{cn}/U_n}{U_{cc}/U_c} = 1$, so $\tau^g = \tau$ and $\Gamma^g = 0$. For separable preferences, $\frac{U_{cn}/U_n}{U_{cc}/U_c} = 0$, so $\tau^s = 1$ and $\Gamma^s \in (0, 1)$.

**Equilibrium representation.** We first establish equations (22) and (23). The derivation of the Phillips curve in terms of marginal costs is standard and yields

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \lambda \left( \frac{dw_t}{\overline{w}} + \frac{\gamma}{\alpha} \frac{dy_t}{\overline{y}} \right)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1}{1+\epsilon\frac{\gamma}{\alpha}}$, $w_t$ is the equilibrium real wage, and $\frac{\gamma}{\alpha}$ is the output elasticity of the marginal product of labor. From the first-order condition for labor supply, $-\frac{U_n(c_t,n_t)}{U_c(c_t,n_t)} = w_t$, we find

$$\frac{dw_t}{\overline{w}} = \left( \frac{1}{\sigma} + \frac{U_{nc}\overline{c}}{U_n} \right) \frac{dc_t}{\overline{c}} + \left( \frac{U_{nn}\overline{n}}{U_n} - \frac{U_{cn}\overline{n}}{U_c} \right) \frac{dn_t}{\overline{n}}$$

after substituting for $dc_t = dy_t - dg_t$ and $\frac{dn_t}{\overline{n}} = \frac{1}{\alpha} \frac{dy_t}{\overline{y}}$, we obtain

$$\frac{dw_t}{\overline{w}} = \left( \left( 1 - \frac{U_{nc}/U_n}{U_{cc}/U_c} \right) \frac{1}{\sigma} \frac{\overline{y}}{\overline{c}} + \left( \frac{U_{nn}\overline{n}}{U_n} - \frac{U_{cn}\overline{n}}{U_c} \right) \frac{1}{\alpha} \right) \frac{dy_t}{\overline{y}} - \left( 1 - \frac{U_{nc}/U_n}{U_{cc}/U_c} \right) \frac{1}{\sigma} \frac{\overline{y}}{\overline{c}} \frac{dg_t}{\overline{y}}$$

Combining these two equations yields the Phillips curve in (23).

Equation (22) derives from the Euler equation, which in differential form reads

$$dc_t + \frac{U_{cn}}{U_{cc}} dn_t = -\frac{\sigma\overline{c}}{R} \mathbb{E}_t \left[ i_t - \pi_{t+1} - \overline{i} \right] + \mathbb{E}_t \left[ dc_{t+1} + \frac{U_{cn}}{U_{cc}} dn_{t+1} \right] \tag{27}$$

Imposing market clearing, $dc_t = dy_t - dg_t$ as well as $dn_t = \frac{1}{f'(\overline{n})} dy_t$, this is also

$$\left( 1 + \frac{U_{cn}}{U_{cc}} \frac{1}{f'(\overline{n})} \right) dy_t - dg_t = -\frac{\sigma\overline{c}}{R} \mathbb{E}_t \left[ i_t - \pi_{t+1} - \overline{i} \right] + \mathbb{E}_t \left[ \left( 1 + \frac{U_{cn}}{U_{cc}} \frac{1}{f'(\overline{n})} \right) dy_{t+1} - dg_{t+1} \right]$$

from which (22) follows, by definition of $\tau^u$ and the labor wedge $\tau$.

**Solution.** Our guess for $dy_t = \omega_g^u dg_t + \omega_m^u d\nu_t$ suggests that all variables decay at rate $\rho$ in expectation. Using this information, we can, for example, rewrite (22) as

$$\left( \tau^u \omega_g^u - 1 \right) (1 - \rho) = -\frac{\sigma\overline{c}}{R\overline{y}} \frac{\phi - \rho}{1 - \beta\rho} \kappa^u \left( \omega_g^u - \Gamma^u \right)$$

from which $\omega_g^u$ follows. The derivation for $\omega_m^u$ follows instead from rewriting (22) as

$$\tau^u \omega_m^u (1 - \rho) = -\frac{\sigma\overline{c}}{R} \left( \frac{\phi - \rho}{1 - \beta\rho} \frac{\kappa^u}{\overline{y}} \omega_m^u + 1 \right)$$

and solving for $\omega_m^u$.