

# Monetary Policy and the Redistribution Channel\*

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## Abstract

This paper evaluates the role of redistribution in the transmission mechanism of monetary policy to consumption. Three channels affect aggregate spending when winners and losers have different marginal propensities to consume: an *earnings heterogeneity channel* from unequal income gains, a *Fisher channel* from unexpected inflation, and an *interest rate exposure channel* from real interest rate changes. Sufficient statistics from Italian and U.S. data suggest that all three channels are likely to amplify the effects of monetary policy.

**JEL Classification:** D31, D52, E21, E52.

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There is a conventional view that redistribution is a side effect of monetary policy changes, separate from the issue of aggregate stabilization which these changes aim to achieve. Most models of the monetary policy transmission mechanism implicitly adopt this view by featuring a representative agent. By contrast, in this paper I argue that redistribution is a *channel* through which monetary policy affects macroeconomic aggregates, because those who gain from accommodative monetary policy have higher marginal propensities to consume (MPCs) than those who lose. The simple argument goes back to [Tobin \(1982\)](#):

Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and for debtors. But [...] the population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth or from current income.

In this paper, I use consumer theory to refine Tobin's intuitions about aggregation. My analysis clarifies who gains and who loses from monetary policy changes, as well as the effect on aggregate consumption. Monetary expansions tend to increase real incomes, to raise inflation and to lower real interest rates. Not everyone is equally affected by these changes. This generates three distinct sources of redistribution.

First, monetary expansions increase labor and profit earnings. The distribution of these gains is unlikely to be equal: some agents tend to benefit disproportionately, and conversely, some tend to lose in relative terms. This is the *earnings heterogeneity channel* of monetary policy.

Second, unexpected inflation revalues nominal balance sheets, with nominal creditors losing and nominal debtors gaining: this is the *Fisher channel*, which has a long history in the literature since [Fisher \(1933\)](#). This channel has been explored by [Doepke and Schneider \(2006\)](#), who measure the balance sheet exposures of various sectors and groups of households in the United States to different inflation scenarios. Net nominal positions (NNPs) quantify the exposures to unexpected increases in the price level.

Real interest rate falls create a third, more subtle form of redistribution. These falls increase financial asset prices. But it is incorrect to claim that asset holders generally benefit: instead, we have to consider whether their assets have longer durations than their liabilities. Importantly, liabilities include consumption plans, and assets include human capital. Unhedged interest rate exposures (UREs)—the difference between all maturing assets and liabilities at a point in time—are the correct measure of households' balance-sheet exposures to real interest rate changes, just like net nominal positions are for price level changes. For example, agents whose financial wealth is primarily invested in short-term certificates of deposit tend to have positive UREs, while those with large long-term

bond investments or adjustable-rate mortgage liabilities tend to have negative UREs. Real interest rate falls redistribute away from the first group towards the second group: this is what I call the *interest rate exposure channel*.

In this paper, I show how these three redistribution channels affect the transmission mechanism of monetary policy to consumption. My main theoretical result decomposes the consumption effect of a transitory change in monetary policy into a contribution from each of these channels, together with an *aggregate income* and a *substitution channel*. Representative-agent models only feature the latter two. My theorem shows that redistribution amplifies these effects, provided that winners from monetary expansions have higher MPCs than losers. The rest of the paper argues that this appears to be the case in the data. In brief, the redistributive effects of monetary policy are important to understand its aggregate effects.<sup>1</sup>

In the first part of the paper, I establish my main decomposition by studying a general aggregation problem. In partial equilibrium, I consider an optimizing agent with a given initial balance sheet, who values nondurable consumption and leisure, and is subject to a transitory change in income, inflation and the real interest rate. I decompose his consumption response into a substitution effect and a wealth effect, and show that the latter is the product of his MPC out of income and a balance-sheet revaluation term in which NNPs and UREs appear. This result is robust to the presence of durable goods, incomplete markets, idiosyncratic risk, and (certain kinds of) borrowing constraints. In other words, the MPC out of a windfall income transfer is a key determinant of the response of optimizing consumers to inflation- or real interest rate-induced changes in their balance sheets. This result generalizes previous findings by [Kimball \(1990\)](#) on the importance of MPCs in incomplete-markets consumption models.

I then aggregate these individual-level predictions and exploit the fact that financial assets and liabilities net out in general equilibrium to obtain the first-order response of aggregate consumption to simultaneous transitory shocks to output, inflation, and the real interest rate. This response is the sum of five terms, reflecting the contributions from the two aggregate and the three redistributive channels mentioned above. Moreover, the magnitudes of the redistributive channels are given by sufficient statistics: the cross-sectional covariances between MPCs and exposures to each aggregate shock. Since the pioneering work of [Harberger \(1964\)](#), sufficient statistics have been used in public finance to evaluate

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<sup>1</sup>My theorem applies to a broad class of general equilibrium models with heterogeneous agents, so it can be used to understand consumption in other contexts than that of monetary policy. At the same time, I am leaving a number of redistributive channels out of my analysis. First, I abstract away from aggregate risk, so cannot handle changes in risk premia, as in [Brunnermeier and Sannikov \(2016\)](#). Second, I do not model limited participation, so monetary policy cannot differentially affect participants and nonparticipants, as in the studies of [Grossman and Weiss \(1983\)](#), [Rotemberg \(1984\)](#) and others. Finally, since I assume that all assets are remunerated at the risk-free rate, my analysis does not address the unequal incidence of inflation due to larger cash holdings by the poor ([Erosa and Ventura 2002](#); [Albanesi 2007](#)). These are all interesting dimensions along which the theory could be extended.

the welfare effect of hypothetical policy changes in a way that is robust to the specifics of the underlying structural model (see [Chetty 2009](#) for a survey). Mine are useful to evaluate the impact of hypothetical changes in macroeconomic aggregates on *aggregate consumption* in a similarly robust way. All that is required is information on household balance sheets, income and consumption levels, and their MPCs.

By further assuming that the elasticity of intertemporal substitution  $\sigma$  and the elasticity of relative income to aggregate income  $\gamma$  are constant in the population, I obtain a set of five estimable moments that summarize all we need to know about agents' heterogeneity to recover the aggregate elasticities of consumption to the real interest rate, the price level, and aggregate income. Contrary to  $\sigma$  (and perhaps  $\gamma$ ), these sufficient statistics are not structural parameters: they are likely to vary over time and across countries.<sup>2</sup> I set out to measure them in three separate surveys, covering different time periods, countries, and methods from the literature. I use a 2010 Italian survey containing a self-reported measure of MPC ([Jappelli and Pistaferri 2014](#)); the 1999-2013 waves of the U.S. Panel Survey of Income Dynamics, together with semi structural approach to identify the MPC out of transitory income shocks ([Blundell, Pistaferri and Preston 2008](#)); and the 2001-2002 waves of the U.S. Consumer Expenditure Survey, together with a method that exploits the randomized timing of tax rebates as a source of identification for MPC ([Johnson, Parker and Souleles 2006](#)).

Consider first the elasticity of consumption to the real interest rate. In a representative-agent world, this elasticity is due to intertemporal substitution. It is negative, and its magnitude depends on  $\sigma$ . I define a method for measuring UREs, and show that, in each of my three datasets, their covariance with MPCs is also negative. Through the lens of my theorem, this implies that the interest rate exposure channel acts in the same direction as the substitution channel, and with comparable magnitude provided that  $\sigma$  is between 0.1 and 0.4. Hence representative-agent analyses that abstract from redistribution may fail to capture an important reason why real interest rates affect consumption, especially if  $\sigma$  is small.<sup>3</sup>

Similarly, across datasets, the covariance between MPCs and NNPs is negative on average. This implies that consumption tends to increase with inflation as a result of the Fisher channel. However, when cast in terms of elasticities, the magnitude is small: an unexpected 1% permanent increase in the price level raises consumption today by no more than 0.1%. This suggests that, while changes in monetary policy can entail significant nom-

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<sup>2</sup>For example, typical incomplete market models imply that they should vary over time, as aggregate shocks affect the extent to which households' borrowing limits are binding, and that they should vary across countries depending on the maturity structure of financial contracts and the degree to which contracts are indexed to inflation.

<sup>3</sup>Macroeconomists tend to assume that  $\sigma$  is around 0.5 (see e.g. [Hall 1988](#) or [Havránek 2015](#)). By contrast, financial economists tend to assume values around 2 (see e.g. [Bansal et al. 2016](#)). If  $\sigma$  is large, the substitution effect plays a dominant role in the overall consumption elasticity.

inal redistribution, the aggregate effect of this redistribution on consumption is likely to be modest.

Finally, in line with previous literature, I estimate the covariance between MPCs and incomes to be negative in the data. If, in addition, low-income agents disproportionately benefit from increases in aggregate income—as suggested, for example, by [Coibion et al. \(2017\)](#)—the earnings heterogeneity channel also amplifies the effects of monetary policy.

Future work can build on these empirical results in two ways: by providing more precise measures of exposures across groups of agents or regions to inform the debate on the winners and losers from changes in monetary policy, and by estimating the sufficient statistics more precisely in administrative data to help quantify the aggregate effect of this redistribution.<sup>4</sup>

A rapidly growing literature analyzes the effects of monetary policy in dynamic stochastic general equilibrium models with rich heterogeneity, matching various aspects of the cross-section such as the wealth distribution. Prominent examples include [Gornemann, Kuester and Nakajima \(2016\)](#), [McKay, Nakamura and Steinsson \(2016\)](#), and [Kaplan, Moll and Violante \(2018\)](#). These structural models overcome a number of important limitations of my sufficient statistics approach. They can study the role of investment, analyze the precise interaction between monetary and fiscal policy, and explore the effect of shocks that are persistent and/or announced in advance. My paper makes two contributions to this literature. First, I introduce a decomposition of the monetary policy transmission mechanism into its various sources of effects on consumption that is useful to shed light on the underlying mechanisms in any such model (see [Kaplan, Moll and Violante 2018](#) for an influential application.) Second, I argue that sufficient statistics can discipline the construction of these models. By making sure that the model’s sufficient statistics match the data, researchers can ensure that, even if the model is misspecified, its predictions for the response of consumption to shocks are consistent with the empirical evidence.

This paper is motivated by an extensive empirical literature documenting that MPCs are large and heterogeneous in the population (see [Jappelli and Pistaferri 2010](#) for a survey), and that they depend on household balance sheet positions.<sup>5</sup> Recently, [Di Maggio et al. \(2017\)](#) have measured the consumption response of households to changes in the interest rates they pay on their mortgages. My theory shows that their paper quantifies an important leg of the redistribution channel of monetary policy.

Several papers have focused on the redistributive channels of monetary policy I highlight in isolation. [Coibion et al. \(2017\)](#) propose an empirical evaluation of the earnings

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<sup>4</sup>See [Tzamourani \(2018\)](#) for a quantification of unhedged interest rate exposures in the Euro Area, and [Fagereng, Holm and Natvik \(2018\)](#) for estimates of sufficient statistics using Norwegian administrative data and the MPCs of lottery winners. The results in both papers are broadly consistent with mine.

<sup>5</sup>See for example [Mian, Rao and Sufi \(2013\)](#); [Mian and Sufi \(2014\)](#); [Baker \(2018\)](#); [Jappelli and Pistaferri \(2014\)](#) and [Cloyne, Ferreira and Surico \(2018\)](#).

heterogeneity channel by measuring how identified monetary policy shocks affect income inequality in the Consumer Expenditure Survey. The Fisher channel has received a great deal of attention in the literature following the work of [Doepke and Schneider \(2006\)](#). For example, on the normative side, [Sheedy \(2014\)](#) asks when the central bank should exploit its influence on the price level to ameliorate market incompleteness over the business cycle. On the positive side, [Sterk and Tenreyro \(2018\)](#) show that the Fisher channel can be a source of effects of monetary policy under flexible prices in a non-Ricardian model. The interest rate exposure channel has, by contrast, not received much attention in the context of monetary policy.<sup>6</sup>

The importance of MPC differences in the determination of aggregate demand is well understood by the theoretical literature on fiscal transfers.<sup>7</sup> MPC differences between borrowers and savers, in particular, have been explored as a source of aggregate effects from shocks to asset prices or to borrowing constraints.<sup>8</sup> In [Farhi and Werning \(2016b\)](#), MPCs enter as sufficient statistics for optimal macro-prudential interventions under nominal rigidities. None of these studies, however, focus on the role of MPC differences in generating aggregate effects of monetary policy.

The remainder of the paper is structured as follows. Section [I](#) presents a partial equilibrium decomposition of consumption responses to shocks into substitution and wealth effects. Section [II](#) provides my aggregation result and discusses the monetary policy transmission mechanism with and without heterogeneity. Section [III](#) contains my measurement exercise. Section [IV](#) concludes.

## **I Household balance sheets and wealth effects**

In this section, I show how households' balance sheets shape their consumption and labor supply adjustments to a transitory macroeconomic shock. I first highlight the forces at play in a life-cycle labor supply model ([Modigliani and Brumberg 1954](#); [Heckman 1974](#)) featuring perfect foresight and balance sheets with an arbitrary maturity structure. Balance sheet revaluations and marginal propensities to consume and work play a crucial role in determining both the welfare and the wealth effects of the shock ([theorem 1](#)). Under certain conditions, the positive results from [theorem 1](#) survive the addition of idiosyncratic income uncertainty ([theorem 2](#)) and therefore apply to a large class of microfounded models of consumption behavior.

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<sup>6</sup>Redistribution through real interest rates does play a prominent role, for example, in [Bassetto \(2014\)](#)'s study of optimal fiscal policy or in [Costinot, Lorenzoni and Werning \(2014\)](#)'s study of dynamic terms of trade manipulation.

<sup>7</sup>See [Galí, López-Salido and Vallés \(2007\)](#); [Oh and Reis \(2012\)](#); [Farhi and Werning \(2016a\)](#); [McKay and Reis \(2016\)](#).

<sup>8</sup>See [King \(1994\)](#); [Eggertsson and Krugman \(2012\)](#); [Guerrieri and Lorenzoni \(2017\)](#); [Korinek and Simsek \(2016\)](#).

## A Perfect-foresight model

Consider a household with separable preferences over nondurable consumption  $\{c_t\}$  and hours of work  $\{n_t\}$ .<sup>9</sup> I assume no uncertainty for simplicity: the same insights obtain when markets are complete, except with respect to the unanticipated initial shock. The household is endowed with a stream of real unearned income  $\{y_t\}$ . He has perfect foresight over the general level of prices  $\{P_t\}$  and the path of his nominal wages  $\{W_t\}$ , and holds long-term nominal and real contracts. Time is discrete, but the horizon may be finite or infinite, so I do not specify it in the summations. The agent solves the following utility maximization problem:

$$\begin{aligned} \max \quad & \sum_t \beta^t \{u(c_t) - v(n_t)\} \\ \text{s.t.} \quad & P_t c_t = P_t y_t + W_t n_t + ({}_{t-1}B_t) + \sum_{s \geq 1} ({}_t Q_{t+s}) ({}_{t-1}B_{t+s} - {}_t B_{t+s}) \\ & \quad + P_t ({}_{t-1}b_t) + \sum_{s \geq 1} ({}_t q_{t+s}) P_{t+s} ({}_{t-1}b_{t+s} - {}_t b_{t+s}) \end{aligned} \quad (1)$$

The flow budget constraint (1) views the consumer, in every period  $t$ , as having a portfolio of zero coupon bonds inherited from period  $t - 1$ , and determining consumption  $c_t$ , labor supply  $n_t$ , as well as a portfolio of bonds to carry into the next period.<sup>10</sup> Specifically,  ${}_t Q_{t+s}$  is the time- $t$  price of a nominal zero-coupon bond paying at  $t + s$ ,  ${}_t q_{t+s}$  the price of a real zero-coupon bond, and  ${}_{t-1}B_{t+s}$  (respectively  ${}_t b_{t+s}$ ) denote the quantities purchased. This asset structure is the most general one that can be written for this dynamic environment with no uncertainty. To keep the problem well-defined, I assume that the prices of nominal and real bonds prevent arbitrage profits. This implies a Fisher equation for the nominal term structure:

$${}_t Q_{t+s} = ({}_t q_{t+s}) \frac{P_t}{P_{t+s}} \quad \forall t, s$$

I focus on the period  $t = 0$ . The environment allows for a very rich description of the household's initial holdings of financial assets, denoted by the consolidated claims, nominal  $\{{}_{-1}B_t\}_{t \geq 0}$  and real  $\{{}_{-1}b_t\}_{t \geq 0}$ , due in each period. The former could represent deposits, long-term bonds and most typical mortgages. The latter could represent stocks (which here pay a riskless real dividend stream and therefore are priced according to the risk-free discounted value of this stream), inflation-indexed government bonds, and price-level adjusted mortgages. I write the real wage at  $t$  as  $w_t \equiv \frac{W_t}{P_t}$ , the initial real term structure as  $q_t \equiv {}_0 q_t$ , the initial nominal term structure as  $Q_t \equiv {}_0 Q_t$ , and impose the present-value

<sup>9</sup>I present results for separable preferences because expressions for substitution elasticities take simple and familiar forms in this case, but many of my results extend to arbitrary non satiable preferences (see appendix A.3). I assume that both  $u$  and  $v$  are increasing and twice continuously differentiable, with  $u$  concave and  $v$  convex.

<sup>10</sup>He may, of course, just decide to roll over his position from the previous period. This corresponds to the costless trade that sets  ${}_{t-1}b_{t+s} = {}_t b_{t+s}$  and  ${}_t B_{t+s} = {}_{t-1} B_{t+s}$  for all  $s$ .



normalization  $q_0 = Q_0 = 1$ .

Using either a terminal condition if the economy has finite horizon, or a transversality condition if the economy has infinite horizon, the flow budget constraints consolidate into an intertemporal budget constraint:

$$\sum_{t \geq 0} q_t c_t = \underbrace{\sum_{t \geq 0} q_t (y_t + w_t n_t)}_{\omega^H} + \underbrace{\sum_{t \geq 0} q_t \left( (-1b_t) + \left( \frac{-1B_t}{P_t} \right) \right)}_{\omega^F} \equiv \omega \quad (2)$$

Equation (2) states that the present value of consumption must be equal to wealth  $\omega$ : the sum of human wealth  $\omega^H$  (the present value of all future income) and financial wealth  $\omega^F$ . Since  $\{-1B_t\}$  and  $\{-1b_t\}$  only enter (2) through  $\omega^F$ , it follows that financial assets with the same initial present value deliver the same solution to the consumer problem. For instance, this framework predicts that a household with an adjustable-rate mortgage (ARM), with  $-1B_0 = -L$ , chooses the same plan for consumption and labor supply as an otherwise identical household with a fixed-rate mortgage (FRM),  $-1B_t = -M$  for  $t = 0 \dots T$ , provided the two mortgages have the same outstanding principal, i.e.  $L = \sum_{t=0}^T Q_t M$ . In this sense, the composition of balance sheets is irrelevant. But this composition matters following a shock, as the next section shows.

## B Adjustment after a transitory shock

I now consider an exercise where, keeping balance sheets fixed at  $\{-1B_t\}_{t \geq 0}$  and  $\{-1b_t\}_{t \geq 0}$ , the paths of variables relevant to the consumer choice problem change in the following way:

- all nominal prices rise in proportion,  $\frac{dP_t}{P_t} = \frac{dP}{P}$ , for  $t \geq 0$
- all present-value real discount rates rise in proportion,  $\frac{dq_t}{q_t} = -\frac{dR}{R}$ , for  $t \geq 1$
- the Fisher equation holds at the new sequence of prices:  $\frac{dQ_t}{Q_t} = -\frac{dR}{R}$  for  $t \geq 1$
- the agent's unearned income at  $t = 0$  rises by  $dy$ , and his real wage by  $dw$ .

This particular variation, depicted in figure 1, captures in a stylized way the major changes in a consumer's environment that usually follow a temporary change in monetary policy: over a period labelled  $t = 0$ , incomes and wages increase, the price level rises due to inflation between  $t = -1$  and  $t = 0$ , and the real interest rate  $R_0 = \frac{q_0}{q_1}$  falls.<sup>11</sup> As I show formally in appendix A.1, these are the changes that occur in the standard representative-agent New Keynesian model following a one-period change in monetary policy. Hence

<sup>11</sup>The assumption that balance sheets are fixed implies that coupon payments are not contingent on the macroeconomic changes  $dw$ ,  $dy$ ,  $dP$  or  $dR$ . This is an incomplete markets assumption. If assets payoffs are state contingent, my results go through provided insurance payments are counted as part of  $dy$ .



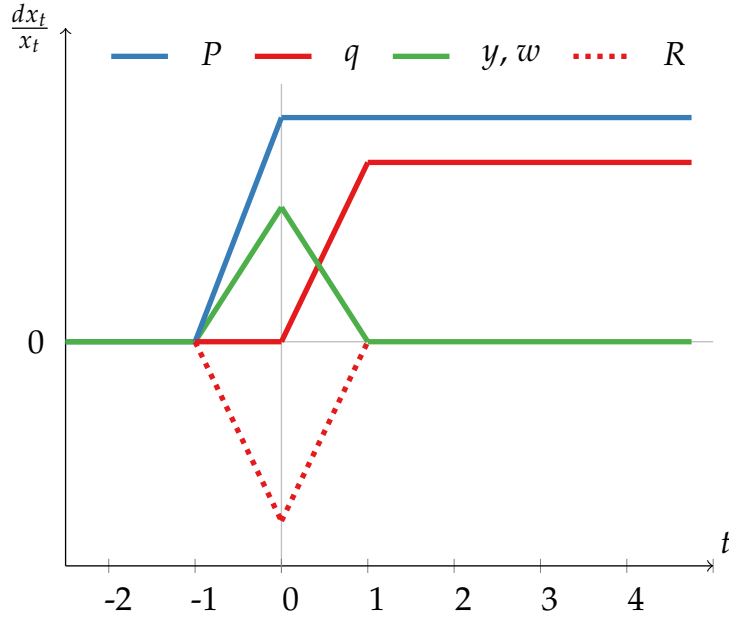


Figure 1: The experiment

this variation is a natural starting point for an analysis of the effects of monetary policy on individual households.

I am interested in the first-order change in initial consumption  $dc \equiv dc_0$ , labor supply  $dn \equiv dn_0$ , and welfare  $dU$  that results from this change in the environment.

Let  $\sigma$  and  $\psi$  be the local Frisch elasticities of substitution in consumption and hours.<sup>12</sup> Define the marginal propensity to consume as  $MPC = \frac{\partial c_0}{\partial y_0}$  along the initial path. When a consumer exogenously receives an extra dollar of income, he increases consumption by  $MPC$  dollars, but, to the extent that labor supply is elastic ( $\psi > 0$ ), he also reduces hours by  $MPN = \frac{\partial n_0}{\partial y_0} < 0$ , leaving only  $MPS = 1 - MPC + w_0 MPN$  dollars for saving.<sup>13</sup>

These behavioral responses to income changes turn out to also matter for the response to the real interest rate, wage, and price level changes, as the following theorem shows.

**Theorem 1.** *To first order, dropping  $t = 0$  subscripts whenever unambiguous,*

$$dc = MPC (d\Omega + \psi ndw) - \sigma c MPS \frac{dR}{R} \quad (3)$$

$$dn = MPN (d\Omega + \psi ndw) + \psi n MPS \frac{dR}{R} + \psi n \frac{dw}{w} \quad (4)$$

$$dU = u'(c) d\Omega \quad (5)$$

<sup>12</sup>Formally,  $\sigma \equiv -\frac{u'(c_0)}{c_0 u''(c_0)} > 0$  and  $\psi \equiv \frac{v'(n_0)}{n_0 v''(n_0)} \geq 0$ .

<sup>13</sup>Separable utility guarantees that  $MPC \in (0, 1)$ ,  $MPS \in (0, 1)$  and  $MPN \leq 0$ : in other words, consumption, saving and leisure are 'normal'. Below I provide an alternative definition of the marginal propensity to consume that corresponds to the more familiar split between consumption and savings alone.

where  $d\Omega$ , the net-of-consumption wealth change, is given by

$$d\Omega = dy + ndw - \sum_{t \geq 0} Q_t \left( \frac{-1B_t}{P_0} \right) \frac{dP}{P} + \left( y + wn + \left( \frac{-1B_0}{P_0} \right) + (-1b_0) - c \right) \frac{dR}{R} \quad (6)$$

The theorem, proved in appendix A.2, follows from an application of Slutsky's equations—separating the wealth and the substitution effects that result from the shock. The relative price changes  $dR$  and  $dw$  generate substitution effects on consumption and labor supply with familiar signs, and magnitudes given by a combination of Frisch elasticities and marginal propensities. All wealth effects are aggregated into a net revaluation term,  $d\Omega$ , which affects consumption and labor supply after multiplication by the marginal propensity to consume and work, respectively.

Note that theorem 1 makes no assumption on horizon or the form of  $u$  and  $v$ . In appendix A.3, I show that it extends to general utility functions and to persistent shocks.

**Unpacking the net wealth revaluation.** The net wealth change  $d\Omega$  in (6) is the key expression determining the sign and the magnitude of the welfare and the wealth effects in theorem 1. This term is a sum of products of *balance-sheet exposures* by *changes in aggregates*. I now describe the terms entering  $d\Omega$  one by one.

The first term,  $dy + ndw$ , is the traditional effect from the change in the present value of income. This is the sum of the unearned income gain,  $dy$ , and the change in earned income holding hours fixed,  $ndw$ . When the aggregate wage increases by  $dw$ , a worker gains more when he initially works more hours  $n$ : we say that  $n$  represents his exposure to the wage change. (The substitution effect on labor supply from the change in  $dw$  is not first-order relevant for welfare, so it does not enter  $d\Omega$ .)

The second term in  $d\Omega$  represents the effect from the immediate and permanent increase in the level of nominal prices, which matters here because of the nominal denomination of assets and liabilities. Define the household's *net nominal position* ( $NNP$ ) as the present value of his nominal assets, i.e.

$$NNP \equiv \sum_{t \geq 0} Q_t \left( \frac{-1B_t}{P_0} \right)$$

We can then rewrite the second term in  $d\Omega$  as  $-NNP \frac{dP}{P}$ , the product of exposure  $-NNP$  by inflation  $\frac{dP}{P}$ . Suppose for example that nominal prices unexpectedly rise by  $\frac{dP}{P} = 1\%$ . A nominal saver with  $NNP = \$100k$  experiences a wealth effect of  $-NNP \frac{dP}{P}$ , so loses the equivalent of \$1000.<sup>14</sup> Conversely, a nominal borrower with  $NNP = -\$100k$  gains

<sup>14</sup>If prices adjust more sluggishly, the Fisher exposure measure changes. For example, if prices adjust only after  $T$  (so that  $\frac{dP_t}{P_t} = \frac{dP}{P}$  for  $t \geq T$ ), the formulas hold if  $NNP$  is replaced by  $\sum_{t \geq T} Q_t \left( \frac{-1B_t}{P_0} \right)$ , the present value of assets maturing after  $T$ . In this case, short-maturity nominal assets maintain constant value, while long-maturity assets decline in value due to the increase in nominal discount rates that follows the expected rise in inflation. The general expression for any given path of price adjustment is given by formula (A.37) in appendix A.3.

the equivalent of \$1000. These net nominal positions can be computed directly from a survey of household finances. Doepke and Schneider (2006) conduct this exercise for various groups of U.S. households and show that *NNPs* are large and heterogenous in the population: they are very positive for rich, old households and negative for the young middle class with mortgage debt. Theorem 1 shows that these numbers are not only relevant for welfare, but also for the consumption response to this inflation scenario. Clearly, the composition of balance sheets matters. Exposures to changes in the level of nominal prices can be avoided by investing all wealth in inflation-indexed instruments, that is, by letting  ${}_{-1}B_t = 0$  for all  $t$ .

The final term in  $d\Omega$  is the wealth effect from the change in the real interest rate. If we define the household's *unhedged interest rate exposure*, or *URE*, as

$$URE \equiv y + wn + \left( \frac{{}_{-1}B_0}{P_0} \right) + ({}_{-1}b_0) - c$$

then this final term is equal to  $URE \frac{dR}{R}$ . Observe that *URE* is the difference between *all* maturing assets (including income) and liabilities (including planned consumption) at time 0. It represents the net saving requirement of the household at time 0, from the point of view of date  $-1$ . Because it includes the stocks of financial assets that mature at date 0 rather than interest flows, it can significantly diverge from traditional measures of savings, in particular if investment plans have short durations.

Why is *URE* the correct measure of exposure following a temporary real interest rate change  $dR$  at time 0? To fix ideas, suppose  $dR < 0$ . This is a decline in the discount rate, which results in an increase in the present value of assets (the traditional capital gains effect). But the present value of liabilities also increases, and consumption is one such liability. Overall, consumers experience a net wealth gain only if their future assets exceed their future liabilities which, in turn, can only happen if their currently-maturing liabilities exceed their currently-maturing assets, i.e. if  $URE < 0$ . Indeed, equation (2) implies that the difference between future assets and liabilities is

$$\sum_{t \geq 1} q_t (y_t + w_t n_t) + \sum_{t \geq 1} q_t \left( ({}_{-1}b_t) + \left( \frac{{}_{-1}B_t}{P_t} \right) \right) - \sum_{t \geq 1} q_t c_t = -URE$$

The intuition here is that a rise in the price of future consumption relative to current consumption (an increase in  $q_t$  for  $t \geq 1$ ) is the same as a decline in the price of current consumption relative to future consumption (a decline in  $q_0$  holding future  $q_t$  fixed). But a fall in the price of current goods benefits those consumers that are demanding more goods than they supply at that date, and conversely, it hurts the net sellers of current goods. *URE* is the measure of the net exposure to this price change. As I will argue in section III, *URE* is *also* measurable from a survey of household finances that has information on income and

consumption.<sup>15</sup>

This observation has the important implication that the *duration of asset plans* matters to determine what happens after a change in real interest rates. Fixed rate mortgage holders and annuitized retirees usually have income and outlays roughly balanced, and hence a *URE* of about zero. By contrast, ARM holders tend to have negative *URE*, and savers with large amounts of wealth invested at short durations tend to have positive *URE*. Hence the theory predicts that the former tend to gain and the latter tend to lose from a temporary decline in real interest rates.<sup>16</sup> In response, consumption increases whenever the substitution effect dominates the wealth effect. Equation 3 allows us to quantify these two effects, and shows that this happens whenever  $\sigma cMPS \geq MPC \cdot URE$ .

**Monetary policy and household welfare.** Theorem 1 shows that asset value changes give incomplete information to understand the effects of monetary policy on household welfare. In the model just presented, monetary policy can be thought of as influencing asset values through three channels: a risk-free real discount rate effect ( $dR$ ), an inflation effect ( $dP$ ), and an effect on dividends ( $dy$ ). But these asset value changes do not enter  $d\Omega$  directly, so they are not relevant *on their own* to understand who gains and who loses from monetary policy, contrary to what popular discussions sometimes imply. For example, it is sometimes argued that accommodative monetary policy benefits bondholders by increasing bond prices. Yet theorem 1 shows that, while increases in dividends do raise welfare, lower real risk-free rates have ambiguous effects on savers. They have no effect on bondholders whose dividend streams initially match the difference between their target consumption and other sources of income. They benefit households who hold long-term bonds to finance short-term consumption, through the capital gains they generate. And they hurt households who finance a long consumption stream with short-term bonds, by lowering the rates at which they reinvest their wealth. Unhedged interest rate exposures, not asset price changes, constitute the welfare-relevant metric for the impact of real interest rate changes on households. This is why it is important to measure them.

**The response of consumption to overall income changes.** Theorem 1 draws a distinction between exogenous changes in income and changes in wages, since the latter have

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<sup>15</sup>By contrast, measuring the exposure to real interest rate changes at any future date requires the knowledge of future income and consumption plans.

<sup>16</sup>One way to understand the importance of duration is as follows. Consider an agent with financial wealth  $\omega^F = \$100k$  that is currently consuming his income  $c = y$ . Suppose first that this agent has invested all his wealth  $\omega^F$  in one-period bonds, so that  $URE = \omega^F$ . Then a temporary one-year decline of 1% in the real interest rate requires him to reinvest his wealth at this lower rate, causing a net wealth loss of  $URE \frac{dR}{R} = -1000\$$ . Suppose instead that his wealth is entirely invested in coupon bonds maturing after the first year, so that  $URE = 0$ . In this case, the high interest rate on assets is 'locked in'. The net wealth effect is zero because the present value of assets and liabilities both increase by the same amount.

substitution effects on consumption. However, since preferences are separable, it is possible to rewrite the consumption response as a function of the *total* income change, inclusive of the labor supply response, as I show in appendix A.4.

**Corollary 1.** *Given an overall change in income  $dY = dy + ndw + wdn$ , the household's consumption response is given by*

$$dc = M\hat{P}C \left( dY - NNP \frac{dP}{P} + URE \frac{dR}{R} \right) - \sigma c (1 - M\hat{P}C) \frac{dR}{R} \quad (7)$$

where  $M\hat{P}C = \frac{MPC}{MPC+MPS} = \frac{MPC}{1+wMPN} \geq MPC$ .

Hence, once we have factored in the endogenous response of income to transfers, the relevant marginal propensity to consume becomes  $M\hat{P}C$ , the number between 0 and 1 that determines how the remaining amount of income is split between consumption and savings. This corresponds more closely to the textbook measure of the marginal propensity to consume. It is also what empirical measures tend to pick up, since these are usually regressions of observed consumption on observed income.<sup>17</sup>

**Durable goods.** So far I have restricted my analysis to nondurable consumption. However, durable expenditures tend to account for a substantial share of the overall consumption response to monetary policy shocks, so they are important to consider. Understanding how durable goods fit into the theory also helps deliver an accurate map to consumption data. As I show formally in appendix A.5, adding durable goods to the model does not alter the substantive conclusions from Theorem 1, but there are some subtleties.

The most straightforward case is the one in which the relative price of durable goods and nondurable goods is constant. In this case, formulas (3) or (7) continue to hold, provided that  $c$  is interpreted as overall expenditures,  $MPC$  is the marginal propensity to spend on all goods,  $URE$  counts all durable expenditures as part of  $c$ , and  $\sigma$  is adjusted upwards to reflect the fact that durable goods allow more opportunities for intertemporal substitution.

In multi-sector New Keynesian models with durable goods, a constant relative price of durable goods obtains when the prices of durables and nondurables are equally sticky (Barsky, House and Kimball 2007). However, there is some evidence that durables have more flexible prices (e.g. Klenow and Malin 2010), in which case these models imply a negative comovement between the relative price of durables  $p$  and the nondurable real interest rate  $R$ . Let  $\epsilon = -\frac{\partial p}{p} \frac{R}{\partial R}$  be the corresponding elasticity. When  $\epsilon \neq 0$ , nondurables

<sup>17</sup>When hours affect the marginal utility of consumption, it is generally not possible to obtain an expression such as (7). Instead,  $dw$  enters separately, with a sign reflecting the degree of complementarity between consumption and labor supply.

and durables matter separately, so there no longer exists a straightforward notion of aggregate demand. Instead, in appendix A.5 I derive separate expressions for the change in nondurable and durable consumption as a function of  $\epsilon$ . These resemble equations (3) or (7), except for the fact that the expression for  $c$  in *URE* only includes a share  $1 - \epsilon$  of durable expenditures.<sup>18</sup>

For the purpose of measuring the size of the interest rate exposure channel, I do not have to take a stand on the value of  $\epsilon$ . In the empirical section, I will assume  $\epsilon = 0$  as a benchmark from computing *UREs*,<sup>19</sup> but I will also show that my empirical results are robust to considering alternative values for  $\epsilon$ .

Even though all the results presented in this section assume no uncertainty and perfect foresight, they apply directly to environments with uncertainty provided that markets are complete, *except* for the shock that is unexpected (all summations are then over states as well as dates). An important feature of all these environments is that the marginal propensity to consume, *MPC*, is the same out of all forms of wealth ( $\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial \omega}$ ). The next section relaxes this assumption.

## C The consumption response to shocks under incomplete markets

I now consider a dynamic, incomplete-market partial equilibrium consumer choice model. Relative to the previous environment, I introduce idiosyncratic income uncertainty, restrict the set of assets that can be traded, and consider borrowing constraints. Specifically, the consumer now faces an idiosyncratic process for real wages  $\{w_t\}$  and unearned income  $\{y_t\}$ . He chooses consumption  $c_t$  and labor supply  $n_t$  to maximize the separable expected utility function

$$\mathbb{E} \left[ \sum_t \beta^t \{u(c_t) - v(n_t)\} \right] \quad (8)$$

The horizon is still not specified in the summation: as in the previous section, it will only influence behavior through its impact on the *MPC*. To model market incompleteness in a general form, I assume that the consumer can trade in  $N$  stocks as well as in a nominal long-term bond. In period  $t$ , stocks pay real dividends  $\mathbf{d}_t = (d_{1t} \dots d_{Nt})$  and can be purchased at real prices  $\mathbf{S}_t = (S_{1t} \dots S_{Nt})$ ; the consumer's portfolio of shares is denoted by  $\theta_t$ . Following the standard formulation in the literature, I assume that the long-term bond can be bought at time  $t$  at price  $Q_t$  and is a promise to pay a geometrically declining nominal

<sup>18</sup>When  $\epsilon = 1$ , durable purchases are not counted at all in *URE*, for the same reason that purchases of bonds or shares are not: in this case, durables completely hedge real interest rate movements.

<sup>19</sup>This is a natural benchmark since an  $\epsilon$  close to 0 is consistent with positive comovement of durables and non-durables after monetary policy shock (see Barsky, House and Kimball 2007), and would arise endogenously, for example, if wages or intermediate goods prices are sticky, or if there are frictions to the reallocation of labor between sectors in the short run.

coupon with pattern  $(1, \delta, \delta^2, \dots)$  starting at date  $t + 1$ . The current nominal coupon, which I denote  $\Lambda_t$ , then summarizes the entire bond portfolio, so it is not necessary to separately keep track of future coupons. The household's budget constraint at date  $t$  is now

$$P_t c_t + Q_t (\Lambda_{t+1} - \delta \Lambda_t) + \theta_{t+1} \cdot P_t \mathbf{S}_t = P_t y_t + P_t w_t n_t + \Lambda_t + \theta_t \cdot (P_t \mathbf{S}_t + P_t \mathbf{d}_t) \quad (9)$$

A borrowing constraint limits trading. This constraint specifies that real end-of-period wealth cannot be too negative: specifically,

$$\frac{Q_t \Lambda_{t+1} + \theta_{t+1} \cdot P_t \mathbf{S}_t}{P_t} \geq -\frac{\bar{D}}{R_t} \quad (10)$$

for some  $\bar{D} \geq 0$ , where  $R_t$  is the real interest rate at time  $t$ . The constraint in (10) is a standard specification for borrowing limits<sup>20</sup> and we will see that it generates reactions of constrained agents to balance sheet revaluations that are closely related to those of unconstrained agents. Given that the extent to which borrowing constraints react to macroeconomic changes is an open question, (10) provides an important benchmark.

Provided that the portfolio choice problem just described has a unique solution at date  $t - 1$ , the household's net nominal position and his unhedged interest rate exposure are both uniquely pinned down in each state at time  $t$ . This contrasts with the environment in section I.B, where the consumer was indifferent between all portfolio choices. Here, these quantities are defined as

$$\begin{aligned} NNP_t &\equiv (1 + Q_t \delta) \frac{\Lambda_t}{P_t} \\ URE_t &\equiv y_t + w_t n_t + \frac{\Lambda_t}{P_t} + \theta_t \cdot \mathbf{d}_t - c_t \end{aligned}$$

As before,  $NNP_t$  is the real market value of nominal wealth: the sum of the current coupon,  $\Lambda_t$ , and the value of the bond portfolio if it were sold immediately,  $Q_t \delta \Lambda_t$ . Similarly,  $URE_t$  is maturing assets (including income, real coupon payments and dividends) net of maturing liabilities (including consumption).

Consider the predicted effects on consumption resulting from a simultaneous unexpected change in his current unearned income  $dy$ , his current real wage  $dw$ , the general price level  $dP$  and the real interest rate  $dR$ , for one period only. Assume that this variation leads asset prices to adjust to reflect the change in discounting alone:  $\frac{dQ}{Q} = \frac{dS_j}{S_j} = -\frac{dR}{R}$  for  $j = 1 \dots N$ .<sup>21</sup> If  $MPC = \frac{\partial c}{\partial y}$ , and both  $MPN$  and  $MPS$  are similarly defined as the responses to current *income* transfers, then the positive results from theorem 1 carry through.

<sup>20</sup>For example, with short-term debt and no stocks ( $N = \delta = 0$ ),  $Q_t = \frac{1}{R_t} \frac{P_t}{P_{t+1}}$  and (10) reads  $\frac{\Lambda_{t+1}}{P_{t+1}} \geq -\bar{D}$ , as in Eggertsson and Krugman (2012).

<sup>21</sup>This is a natural assumption that obtains if asset prices are determined in a general equilibrium with incomplete markets. Absence of arbitrage in such a model implies the existence of a probability measure  $Q$  such that the price of each stock  $j$  at date 0 is  $S_{0j} = \frac{1}{R_0} \mathbb{E}^Q \left[ \sum_{t \geq 1} \frac{1}{R_{1 \dots t-1}} d_{jt} \right]$ , where  $R_t$  is the sequence of risk-free rates. My variation affects  $R_0$  but does not affect future interest rates, dividends, or risk-neutral probabilities, so results in  $\frac{dS_{0j}}{S_{0j}} = -\frac{dR}{R}$ .



**Theorem 2.** Assume that the consumer is at an interior optimum, at a binding borrowing constraint, or unable to access financial markets (in the latter two cases, let  $MPS=0$ ). Then his first order change in consumption  $dc$  and labor supply  $dn$  continue to be given by equations (3) and (4). In particular, writing  $\hat{MPC} \equiv \frac{MPC}{MPC+MPS}$ , the relationship between  $dc$  and the total change in income  $dY = dy + ndw + wdn$  is still given by equation (7).

The proof is given in appendix A.6. The intuition for why  $MPC$ ,  $MPN$  and  $MPS$  are relevant to understand the response of all agents to changes in the real interest rate and the price level is simple: when the consumer is locally optimizing, these quantities summarize the way in which he reacts to *all* balance-sheet revaluations, income being only one such revaluation. When the borrowing limit *is* binding, consumption and labor supply adjustments depend on the way the borrowing limit changes when the shock hits. Under the specification in (10), the changes in  $dR$  and  $dP$  free up borrowing capacity<sup>22</sup> exactly in the amount  $-NNP \frac{dP}{P} + URE \frac{dR}{R}$ . Finally, when the consumer is unable to access financial markets, he lives hand-to-mouth so  $NNP = URE = 0$ . In these latter two cases,  $\hat{MPC} = 1$  and we can interpret the consumption response as a pure wealth effect.

By showing that the marginal propensity to consume out of transitory income shocks, which has been the focus of a large empirical literature, remains a key sufficient statistic for predicting behavior with respect to other changes in consumer balance sheets, theorem 2 provides important theoretical restrictions. The rest of the paper takes these restrictions as given and uses them to predict aggregate consumption responses to changes in  $R$  or  $P$ . But these restrictions are also directly testable empirically: given independent variation in  $dP$ ,  $dy$  and  $dR$  as well as individual balance sheet information, one could check that individual consumption responds in accordance with equations (3) or (7). This provides an interesting avenue for future empirical work on consumption behavior.

## II Aggregation and the redistribution channel

This section shows how the microeconomic demand responses derived in section I aggregate in general equilibrium to explain the economy-wide response to shocks in a large class of heterogenous-agent models (theorem 3).

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The argument for  $\frac{dQ_0}{Q_0} = -\frac{dR}{R}$  is identical.

<sup>22</sup>The form of the borrowing constraint in (10), which imposes a bound on the real value of wealth in period  $t+1$ , is clearly important for this result. For example, if (10) is replaced by a constraint on the flow of income received from financial markets,  $\frac{Q_t \Lambda_{t+1} + \theta_{t+1} \cdot P_t S_t}{P_t} - \frac{\delta Q_t \Lambda_t + \theta_t \cdot P_t S_t}{P_t} \geq -\bar{D}$ , then the result collapses to  $dc = dY$ .

## A Environment

Consider a closed economy populated by  $I$  heterogenous types of agents with separable preferences (8). Each agent type  $i$  has its own discount factor  $\beta_i$ , period utility functions  $u_i$  and  $v_i$ , and time horizon. To accommodate idiosyncratic uncertainty, assume that within each type  $i$  there is a mass 1 of individuals, each in an idiosyncratic state  $s_{it} \in S_i$ . I write  $\mathbb{E}_I [z_{it}]$  for the cross-sectional average of any variable  $z_{it}$ , taken over individual types  $I$  and idiosyncratic states  $S_i$ . I write all aggregate variables in per capita units, so for example aggregate (per capita) consumption  $C_t$  is equal to average individual consumption  $\mathbb{E}_I [c_{it}]$ .

**Agents and asset structure.** Each agent type  $i$  in state  $s_{it}$  has a stochastic endowment of  $e_i(s_{it})$  efficient units of work, and receives a wage of  $w_{it} = e_i(s_{it}) w_t$  per hour, where  $w_t$  is the real wage per efficient hour. By choosing  $n_{it}$  hours of work, he therefore receives  $w_t e_{it} n_{it}$  in earned income. The agent also receives unearned income  $y_{it} = d_{it} - t_{it}$ , the total dividends on the trees he owns  $d_{it}$  net of taxes from the government  $t_{it}$ . Let the agent's overall gross-of-tax income be

$$Y_{it} \equiv w_t e_{it} n_{it} + d_{it}. \quad (11)$$

The economy has a fixed supply of aggregate capital  $K$ . A set of  $N$  trees constitute claims to firm profits and the capital stock. Each tree delivers dividends which, in the aggregate, add up to the sum of aggregate capital income and profits:  $\mathbb{E}_I [d_{it}] = \rho_t K + \pi_t$ . Agents can also trade nominal government bonds in net supply  $B_t$ , as well as a set of  $J - 1$  additional assets in zero net supply that can be nominal or real. Each agent of type  $i$  can trade a subset  $N_i$  of the trees and a subset  $J_i$  of the other assets. If both  $N_i$  and  $J_i$  are empty, agents of type  $i$  live hand-to-mouth. In other cases, I assume that trading is subject to a type-specific borrowing constraint  $\overline{D}_i$ , which takes the form in (10) and may be infinite.

**Firms.** There exists a competitive firm producing the unique final good in this economy, in quantity  $Y_t$  and nominal price  $P_t$ , by aggregating intermediate goods with a constant-returns technology. These intermediate goods are produced by a unit mass of firms  $j$  under constant returns to scale, using the production functions  $X_{jt} = A_{jt} F(K_{jt}, L_{jt})$ . Markets for inputs are perfectly competitive, so firms take the real wage  $w_t$  and the real rental rate of capital  $\rho_t$  as given. These firms sell their products under monopolistic competition and their prices can be sticky. Firm  $j$  therefore sets its price  $P_{jt}$  at a markup over marginal cost, and it makes real profits  $\pi_{jt}$ .<sup>23</sup> Summing across firms  $j \in J$ , aggregate production is equal

<sup>23</sup>Specifically, if  $\mu_{jt}$  is firm  $j$ 's markup at time  $t$ , then  $\pi_{jt} = (\mu_{jt} - 1) (w_t L_{jt} + \rho_t K_{jt})$ .

to aggregate income:

$$Y_t = \mathbb{E}_J \left[ \frac{P_{jt}}{P_t} X_{jt} \right] = w_t \mathbb{E}_J [L_{jt}] + \rho_t \mathbb{E}_J [K_{jt}] + \mathbb{E}_J [\pi_{jt}] \quad (12)$$

**Government.** A government has nominal short-term debt  $B_t$ , spends  $G_t$ , and runs the tax-and-transfer system. Its nominal budget constraint is therefore:

$$Q_t B_{t+1} = P_t G_t + B_t - P_t \mathbb{E}_I [t_{it}] \quad (13)$$

where  $Q_t = \frac{1}{R_t} \frac{P_t}{P_{t+1}}$  is the one-period nominal discount rate. The consequences of price-induced redistributive effects between households and the government depend crucially on the fiscal rule. I assume a simple rule in which the government targets a constant real level of debt  $\frac{B_t}{P_t} = \bar{b} > 0$  and spending  $G_t = \bar{G} > 0$ . I also assume that the government balances its budget at the margin by adjusting all transfers in a lump-sum manner. Hence, unexpected increases in  $P_t$  (which create ex-post deviations of  $\frac{B_t}{P_t}$  from  $\bar{b}$ ) and reductions in the real interest rate  $R_t$  result in immediate lump-sum rebates.

**Market clearing.** In equilibrium, the markets for capital, labor and goods all clear. This implies that at all times  $t$

$$\mathbb{E}_J [K_{jt}] \equiv K \quad (14)$$

$$\mathbb{E}_I [e_{it} n_{it}] = \mathbb{E}_J [L_{jt}] \quad (15)$$

$$\mathbb{E}_I [Y_{it}] = Y_t = C_t + G_t \quad (16)$$

Equilibrium also implies market clearing in all  $J + N$  asset markets. This environment nests a large class of one-good, closed economy general equilibrium models. It can accommodate many assumptions about population structure, asset market structure and participation, heterogeneity in preferences, endowments and skills, as well as the nature of price stickiness. With some minor modifications, it would accommodate wage stickiness as well. Note that the assumptions made here imply that all agents in this economy essentially solve either the problem in section I.A or that in section I.C.

## B Aggregation result

I am interested in the aggregate consumption response to a perturbation of this environment in which individual gross incomes  $dY_i$ , nominal prices  $dP$  and the real interest rate  $dR$  change at  $t = 0$  only. This exercise is useful to understand the effect of an *unexpected shock* that has *no persistence*. Let  $dY \equiv \mathbb{E}_I [dY_i]$  be the aggregate change in gross income. Assuming labor market clearing after the shock, this is also the aggregate output change.

Aggregation is simplified by several restrictions from market clearing at  $t = 0$ . Market clearing for nominal assets implies that all nominal positions net out except for that of the

government,

$$\mathbb{E}_I [NNP_{it}] = \bar{b} = -NNP_{gt} \quad \forall t \quad (17)$$

and market clearing for all assets, combined with (11)—(16) implies<sup>24</sup> that

$$\mathbb{E}_I [URE_{it}] = Y_t - \mathbb{E}_I [t_{it}] + \frac{B_t}{P_t} - C_t = G_t + \frac{B_t}{P_t} - \mathbb{E}_I [t_{it}] = -URE_{gt} \quad (18)$$

where  $NNP_{gt}$  and  $URE_{gt}$  are naturally defined as the net nominal position and the unhedged interest rate exposure of the government sector. Equations (17) and (18) are crucial restrictions from general equilibrium: since one agent's asset is another's liability, net nominal positions and interest rate exposures must net out in a closed economy. Aggregation of consumer responses as described by theorem 2 shows that the per capita aggregate consumption change can be decomposed as the sum of five channels:

**Theorem 3.** *To first order, in response to  $dY_i$ ,  $dY$ ,  $dP$  and  $dR$ , aggregate consumption changes by*

$$\begin{aligned} dC = & \underbrace{\mathbb{E}_I \left[ \frac{Y_i}{Y} M\hat{P}C_i \right] dY}_{\text{Aggregate income channel}} + \underbrace{\text{Cov}_I \left( M\hat{P}C_i, dY_i - Y_i \frac{dY}{Y} \right)}_{\text{Earnings heterogeneity channel}} - \underbrace{\text{Cov}_I (M\hat{P}C_i, NNP_i)}_{\text{Fisher channel}} \frac{dP}{P} \\ & + \left( \underbrace{\text{Cov}_I (M\hat{P}C_i, URE_i)}_{\text{Interest rate exposure channel}} - \underbrace{\mathbb{E}_I [\sigma_i (1 - M\hat{P}C_i) c_i]}_{\text{Substitution channel}} \right) \frac{dR}{R} \end{aligned} \quad (19)$$

The proof is given in appendix A.7. The key step is to aggregate predictions from theorem 2, decomposing  $i$ 's individual income change as  $dY_i = \frac{Y_i}{Y} dY + dY_i - \frac{Y_i}{Y} dY$  (the sum of an aggregate component and a redistributive component), and using market clearing conditions, the fiscal rule, and the fact that  $\mathbb{E}_I [dY_i - \frac{Y_i}{Y} dY] = 0$  to transform expectations of products into covariances.

Theorem 3 shows that, in the class of environments I consider, a small set of sufficient statistics is enough to understand and predict the first-order response of aggregate consumption to a macroeconomic shock. Equation (19) holds irrespective of the underlying model generating MPCs and exposures at the micro level, as well as the relationship between  $dY$ ,  $dP$  and  $dR$  at the macro level. Most of the bracketed terms are cross-sectional moments that are measurable in household level micro-data and are informative about the economy's macroeconomic response to a shock, no matter the source of this shock. The two exceptions are the EISs  $\sigma_i$ , which need to be obtained from other sources, and  $dY_i - Y_i \frac{dY}{Y}$ , which in general depends on the driving force behind the change in output.

I now use this theorem to discuss the channels of monetary policy transmission un-

<sup>24</sup>To see this, note that if  $b_{it}$  denotes the asset coupons that mature at time  $t$  for household  $i$ , we have  $URE_{it} = Y_{it} - t_{it} + b_{it} - c_{it}$ . Using market clearing in the  $J - 1$  zero net supply assets, all these coupons net out except for the government coupon, which here is  $\mathbb{E}_I [b_{it}] = \frac{B_t}{P_t}$ . The result then follows from goods market clearing and the government budget constraint.

der heterogeneity. Alternative applications, for example to short-term redistributive fiscal policy or open-economy models, are also possible.

### C Monetary policy shocks with and without a representative agent

Consider a transitory, accommodative monetary policy shock that, as in figure 1, lowers the real interest rate and raises aggregate income for one period ( $dR < 0$ ,  $dY > 0$ ), and permanently raises the price level ( $\frac{dP}{P} > 0$ ). Since these are the changes implied by the textbook New Keynesian model with sticky prices and flexible wages after a transitory monetary policy shock, we can apply theorem 3 to understand the consumption response in that model.

The textbook model features a representative agent ( $I = 1$ ) with separable preferences and EIS  $\sigma$ . Hence all covariance terms in (19) are zero, and we are left with

$$dC = M\hat{P}C dY - \sigma (1 - M\hat{P}C) C \frac{dR}{R} \quad (20)$$

The first term in (20) is a general-equilibrium income effect, and the second term is a substitution effect.<sup>25</sup> Solving out for  $dC = dY$  gives the textbook response,  $\frac{dC}{C} = -\sigma \frac{dR}{R}$ . Intuitively, a Keynesian multiplier  $\frac{1}{1 - M\hat{P}C}$  amplifies the initial 'first-round' effect from intertemporal substitution. Here this multiplier is entirely microfounded, and in particular takes into account the substitution and wealth effects on labor supply that play out in the background.

Heterogeneity implies a role for redistributive channels in the monetary transmission mechanism, except under special conditions. For example, if aggregate income is distributed proportionally to individual income, so that  $dY_i = \frac{Y_i}{Y} dY$ ; if no equilibrium asset trade is possible, so that agents consume all their incomes  $Y_i = c_i$  and  $NNP_i = URE_i = 0$ ; and if all agents have the same elasticity of intertemporal substitution  $\sigma_i = \sigma$ , then the representative-agent response  $\frac{dC}{C} = -\sigma \frac{dR}{R}$  obtains even under heterogeneity. [Werning \(2015\)](#) studies this important neutrality result, as well as several extensions.

Away from this benchmark, the redistributive channels of monetary policy can be signed and quantified by measuring the covariance terms in equation (19), either directly in micro data or within a given model. In the next section, I follow the first route to obtain a sense of the plausible empirical magnitudes. As I will show, the data suggests that the following is true:

$$\text{Cov}_I (M\hat{P}C_i, URE_i) < 0 \quad (21)$$

$$\text{Cov}_I (M\hat{P}C_i, NNP_i) < 0 \quad (22)$$

$$\text{Cov}_I (M\hat{P}C_i, Y_i) < 0 \quad (23)$$

<sup>25</sup>Since the typical calibration of the representative-agent model implies a low  $M\hat{P}C$ , the substitution component is typically dominant in this decomposition, as noticed by [Kaplan, Moll and Violante \(2018\)](#).

These inequalities imply that redistribution *amplifies* the transmission mechanism of monetary policy.

Inequality (21) says that agents with unhedged borrowing requirements have higher marginal propensities to consume than agents with unhedged savings needs. Models with uninsured idiosyncratic risk tend to generate this as an endogenous outcome. Because of this interest rate exposure channel, aggregate consumption is more responsive to real interest rates than measures of intertemporal substitution alone would suggest. In other words, the first-round effect of monetary policy is larger than what the representative-agent model predicts.

Inequality (22) says that net nominal borrowers have higher marginal propensities to consume than net nominal asset holders. This is also an endogenous outcome of typical incomplete market models with nominal assets. It implies that, through its general equilibrium effect on inflation, monetary policy can increase aggregate consumption via a Fisher channel.<sup>26</sup>

Inequality (23) says low-income agents have high MPCs, echoing a finding in much of the empirical literature. On its own, this fact is not enough to sign the earnings heterogeneity channel: we need to know how increases in aggregate income affect agents at different levels of income. More specifically, let

$$\gamma_i \equiv \frac{\partial \left( \frac{Y_i}{Y} - 1 \right)}{\left( \frac{Y_i}{Y} - 1 \right)} \frac{Y}{\partial Y} \quad (24)$$

be the elasticity of agent  $i$ 's relative income to aggregate income. Assume that this is well approximated by a constant  $\gamma$ . Then the earnings heterogeneity channel term in equation (19) simplifies to  $\gamma \text{Cov}_I \left( M\hat{P}C_i, \frac{Y_i}{Y} \right) dY$ . There is empirical evidence that income risk is countercyclical (for example [Storesletten, Telmer and Yaron 2004](#) or [Guisar, Ozkan and Song 2014](#)) and that monetary policy accommodations reduce income inequality ([Coibion et al. 2017](#)). These studies suggest that  $\gamma$  is negative. Combining this fact with (23), it is likely that monetary expansions increase aggregate consumption because of their endogenous effect on the income distribution.<sup>27</sup>

Independently of the sign of the covariance terms in (19), theorem 3 provides an organizing framework for future research on the role of heterogeneity in the transmission mechanism of monetary policy.<sup>28</sup>

<sup>26</sup>Note that this effect from redistribution is conceptually distinct from the effect of future inflation lowering real interest rates, which has nothing to do with nominal redenomination and is present in representative-agent models with persistent shocks to inflation.

<sup>27</sup>Away from separable preferences, an additional *complementarity channel* of monetary policy can arise, even with a representative agent, when preferences are such that increases in hours worked increase the marginal utility of consumption.

<sup>28</sup>An early generation of papers in the heterogeneous agent New Keynesian literature analyzed the transmis-

## D Discussion

I now provide a discussion of my result, highlighting its limitations and possible generalizations.

**Interactions between the household and other sectors.** The market clearing equations (17) and (18) respectively state that the net nominal positions and the unhedged interest rate exposure of the combined household and government sectors are zero. This is a theoretical restriction that must hold in a closed economy, provided firms are correctly consolidated as part of the household sector. In practice there are two challenges: actual economies are open, and it is difficult to accurately take into account the indirect exposures through firms when measuring *NNPs* and *UREs*.

In an open economy, (17) and (18) are no longer true, so price-level and real interest rate changes redistribute between the domestic economy and the rest of the world. For example, [Doepke and Schneider \(2006\)](#) find that the net nominal position of the United States is negative, implying that unexpected inflation redistributes towards the U.S. Given a positive average *MPC*, consumption should rise by more than what equation (19) predicts. Similarly, [Gourinchas and Rey \(2007\)](#) find that the United States borrows short and lends long on its international portfolio, suggesting that it has a negative unhedged interest rate exposure. Hence, U.S. households benefit *on average* from lower real interest rates. This could contribute to the expansionary effects of monetary accommodations on consumption.<sup>29</sup>

The assumption that households and firms are consolidated is also important. For example, the household sector tends to be maturity mismatched, holding relatively short-term assets (deposits) and relatively long-term liabilities (fixed-rate mortgages). To a large extent, this is a counterpart to the reverse situation in the banking sector. An ideal measure of *UREs* and *NNPs* would take into account the indirect exposures that each household has through the firms it has a stake in. In practice, this is very challenging to do.

When we undercount household exposures to negative-*URE* sectors, we obtain a positive  $\mathbb{E}_I [URE_i]$ . This situation also arises in the model of section II, but there the negative-*URE* outside sector is the government. The logic of theorem 3 shows that, if marginal rebates from other sectors were immediate and lump-sum, this mismeasurement would be

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sion of monetary policy under limited heterogeneity. In ‘saver-spender’ models, such as [Bilbie \(2008\)](#), ‘spender’ agents live hand-to-mouth and consume their incomes, so they have  $\hat{MPC} = 1$ ; while ‘saver’ agents have access to financial markets, with a low  $\hat{MPC}$ . This has the effect of increasing the aggregate *MPC* in the economy, raising the importance of income effects relative to substitution effects in equation (19). In ‘borrower-saver’ models, as in [Iacoviello \(2005\)](#), the high-*MPC* agents are also borrowers. The literature usually assumes short-term debt, implying (21) and sometimes also nominal debt, implying (22). However, whether (23) holds crucially depends on the assumptions these papers make about the distribution of wages and profits across savers vs spenders.

<sup>29</sup>To the extent that these gains are evenly distributed across the population, these effects can be quantified, respectively, by evaluating  $\mathbb{E}_I [\hat{MPC}_i] \cdot NNP_{US}$  and  $\mathbb{E}_I [\hat{MPC}_i] \cdot URE_{US}$ .



irrelevant. In practice, rebates are likely to be delayed, and they could disproportionately affect higher or lower MPC agents, so that the numbers could depart from my benchmark covariance expression in either direction.

One way to assess the importance of all these effects is to directly measure in the data expressions such as  $\mathbb{E}_I [M\hat{P}C_i URE_i]$  and to compare them to the covariance numbers. These ‘no-rebate’ numbers replace the covariance terms in (19) under the assumption that none of the outside sectors rebate gains to the household sector. In this context, it is theoretically possible for the interest rate exposure term  $\mathbb{E}_I [M\hat{P}C_i URE_i]$  to be both positive and larger than the substitution term in (19). This suggests that, in a world in which outside rebates are highly delayed or benefit low-MPC agents, real interest rate cuts could *lower* aggregate consumption demand, significantly altering the conventional understanding of how monetary policy operates.<sup>30</sup>

**General equilibrium and persistent shocks.** Theorem 3 provides the response of consumption to a *transitory* shock to  $R$ ,  $P$  and  $Y$ . While this exercise provides an insightful decomposition that has the merit of involving measurable sufficient statistics, it has two major limitations.

First, the exercise is partial equilibrium in nature: in general, theorem 3 does not permit us to solve for the general equilibrium consumption effect of a given exogenous shock. This is because even transitory exogenous shocks tend to have long-lasting effects on agent behavior and the wealth distribution, which in general equilibrium tends to generate adjustments in future interest rates and/or income. Equation (19) does characterize the full equilibrium in my leading case of the benchmark New Keynesian model, but in more general heterogeneous-agent models it will typically only hold as an approximation of the consumption response to a transitory monetary policy shock.<sup>31</sup>

Second, empirically, monetary policy changes tend to be persistent. Persistent shocks make the derivation of sufficient statistics much more difficult: for example, to characterize the effect of future changes in  $R$ , one needs to know the distribution of future consumption and income plans.

In the context of a given structural model, it is possible to extend my decomposition in (19) to any degree of persistence, as shown by Kaplan, Moll and Violante (2018). As models grow in complexity and realism, the importance of the channels identified in Theorem 3 can be assessed and refined using such a procedure. I believe that my key finding that

<sup>30</sup>This theoretical possibility is sometimes mentioned in economic discussions of monetary policy. See Raghuram Rajan (“Interestingly [...] low rates could even hurt overall spending”), “**Money Magic**”, Project Syndicate, November 11, 2013

<sup>31</sup>For instance, the theorem cannot accommodate capital investment, where a current fall in the real interest rate  $dR < 0$  comes together with a future fall in capital income,  $d\rho_1 < 0$ . A previous version of this paper showed the quality of the approximation  $dC \simeq dY$  in the context of a model without investment.

Table 1: Seven cross-sectional moments that determine consumption in (25)

	Definition	Name	Channel
$\mathcal{E}_R$	$\text{Cov}_I \left( \text{MPC}_i, \frac{URE_i}{\mathbb{E}_I[c_i]} \right)$	Redistribution elasticity for $R$	Interest-rate exposure
$\mathcal{E}_R^{NR}$	$\mathbb{E}_I \left[ \text{MPC}_i \frac{URE_i}{\mathbb{E}_I[c_i]} \right]$	—, No Rebate	—
$\hat{S}$	$\mathbb{E}_I \left[ \left( 1 - \text{MPC}_i \right) \frac{c_i}{\mathbb{E}_I[c_i]} \right]$	Hicksian scaling factor	Substitution
$\mathcal{E}_P$	$\text{Cov}_I \left( \text{MPC}_i, \frac{NNP_i}{\mathbb{E}_I[c_i]} \right)$	Redistribution elasticity for $P$	Fisher
$\mathcal{E}_P^{NR}$	$\mathbb{E}_I \left[ \text{MPC}_i \frac{NNP_i}{\mathbb{E}_I[c_i]} \right]$	—, No Rebate	—
$\mathcal{E}_Y$	$\text{Cov}_I \left( \text{MPC}_i, \frac{Y_i}{\mathbb{E}_I[c_i]} \right)$	Redistribution elasticity for $Y$	Earnings heterogeneity
$\mathcal{M}$	$\mathbb{E}_I \left[ \text{MPC}_i \frac{Y_i}{\mathbb{E}_I[c_i]} \right]$	Income-weighted $\text{MPC}$	Aggregate income

redistribution amplifies the effects of monetary policy is likely to remain robust, but it will certainly need to be qualified. In particular, the work of [Christiano, Eichenbaum and Evans \(2005\)](#) and many others suggests that the empirical consumption response to identified monetary policy shocks builds up over time. Whether redistribution channel mechanisms can explain this persistence, and not just the impact response, remains an open question.

## E Estimable moments

Some of the terms in equation (19) require knowledge of additional information before they can be taken to the data. I make two further assumptions on these structural parameters so as to turn the equation into a full set of estimable moments. For convenience, I also rewrite the decomposition in terms of elasticities.

**Corollary 2.** *Assume that individuals have common elasticity of intertemporal substitution,  $\sigma_i = \sigma$ , and common elasticity of relative income to aggregate income,  $\gamma_i = \gamma$  for all  $i$ . Then,*

$$\frac{dC}{C} = (\mathcal{M} + \gamma\mathcal{E}_Y) \frac{dY}{Y} - \mathcal{E}_P \frac{dP}{P} + (\mathcal{E}_R - \sigma S) \frac{dR}{R} \quad (25)$$

where  $\mathcal{M}$ ,  $\mathcal{E}_Y$ ,  $\mathcal{E}_P$ ,  $\mathcal{E}_R$  and  $S$  are measurable cross-sectional moments summarized in table 1.

The proof is in appendix A.8. The assumption of a constant  $\gamma$  parametrizes the incidence of increases in aggregate output  $dY$  using a convenient functional form.<sup>32</sup> As is clear from equation (24), when  $\gamma > 0$ , agents with income above the mean benefit disproportionately from such an increase. The opposite happens when  $\gamma < 0$ . As discussed

<sup>32</sup>Such a specification appears, for example, if labor supply is inelastic ( $\psi = 0$ ) and all income is labor income ( $d = 0$ ). In this case, agent  $i$ 's gross earnings are  $e_i Y$ , the product of his skills  $e_i$  and aggregate output  $Y$ . Suppose that the government taxes these earnings at a rate  $\tau(Y)$  and rebates them lump-sum. Then post-redistribution earnings are  $Y_i = ((1 - \tau(Y)) e_i + \tau(Y) \mathbb{E}[e_i]) Y$ . A constant  $\gamma_i$  follows if the net-of-tax rate has constant elasticity with respect to output, i.e.  $\frac{\tau'(Y)}{1 - \tau(Y)} = -\gamma$ .

above, the evidence on the cyclical of income risk tends to suggest that the latter case is plausible, though a constant  $\gamma$  is clearly a strong assumption.

Table 1 summarizes the definitions of the moments entering equation (25). I call  $\mathcal{E}_P$ ,  $\mathcal{E}_R$  and  $\mathcal{E}_Y$  the *redistribution elasticities* of consumption with respect to the price level, the real interest rate and income, since these terms enter explicitly as elasticities in equation (25).<sup>33</sup> The next section measures these numbers in the data.

### III Measuring the redistribution elasticities of consumption

This section turns to data from three surveys to get a sense of the empirical magnitudes of each of the terms in table 1. This exercise is not intended as definitive and will need to be refined in future work. Yet we will see that it paints a fairly consistent picture, one in which inequalities (21)–(23) are satisfied. With these moment estimates in hand, only two parameters in equation (25) remain unknown.  $\sigma$  can be obtained from the vast literature studying the elasticity of intertemporal substitution, and  $\gamma$  can be obtained from studies on the cyclical of income distribution.

#### A Three surveys, three identification strategies

In order to compute my key cross-sectional moments, I need household-level information on income, consumption, and balance sheets. This information is available in household surveys from various countries. I also need information on  $M\hat{P}C$ , the marginal propensity to consume out of transitory income shocks.<sup>34</sup> The literature has used various techniques to estimate these MPCs (see Jappelli and Pistaferri 2010 for a survey). Three of the most influential approaches are implementable using public survey data. I compute my moments using all three approaches, each in a different survey. These surveys cover two countries and three different time periods. Given that sufficient statistics are likely to vary over time and across countries, this exercise gives a sense of robustness to the fundamental setting as well as the estimation method. Since I build on standard references in the literature, I restrict myself to a brief description of these methods, and refer the reader to appendix C and to the original sources for further detail.

My first source of data is the Italian Survey of Household Income and Wealth (SHIW).<sup>35</sup> In 2010, the survey asked households to self-report the part of any hypothetical windfall

<sup>33</sup>Calling  $\mathcal{E}_Y$  an elasticity is a slight abuse of terminology, since the actual elasticity is  $\gamma\mathcal{E}_Y$ .

<sup>34</sup>Recall that the theory makes a distinction between  $M\hat{P}C$ , which takes into account the endogenous response of labor supply, and  $MPC$  which does not. The methods used to compute MPC either regress observed consumption on observed income, or ask a question to respondents without mentioning a potential labor supply adjustment, so from now on I assume that they measure  $M\hat{P}C$ , and I sometimes write it  $MPC$  for convenience.

<sup>35</sup>Bank of Italy (2010).

that they would immediately spend (Jappelli and Pistaferri 2014). The benefit of this approach is that the windfall can be taken as exogenous for all agents, so in principle this empirical measure of *MPC* is the number that matters for the theory. Another benefit of this survey measure is that it provides *MPCs* at the household level, making it easy to compute covariances with individual balance-sheet information. On the other hand, a concern with self-reported answers to hypothetical situations is that they may not be informative about how households would actually behave in these situations. The other two measures I consider estimate *MPCs* from actual behavior instead.

My second source of data is the U.S. Panel Study of Income Dynamics (PSID),<sup>36</sup> where I use a 'semi-structural' approach to compute *MPCs* out of transitory income shocks. The procedure is due to Blundell, Pistaferri and Preston (2008) and has since been popularized by Kaplan, Violante and Weidner (2014) and others. The idea is to postulate an income process and a consumption function, and to use restrictions from the theory to back out the *MPC* out of transitory shocks from the joint cross-sectional distribution of consumption changes and income changes. Since this procedure can only recover an estimate at the group level, I compute my redistribution elasticities by first grouping households into different bins, then estimating *MPCs* within bins and covariances across bins. One drawback of such a procedure is that it generates large error bands.

My third source of data is the U.S. Consumer Expenditure Survey (CE),<sup>37</sup> in which *MPC* is identified using exogenous income variation following Johnson, Parker and Souleles (2006). These authors estimate the *MPC* out of the 2001 tax rebate by exploiting random variation in the timing of the receipt of this rebate across households. Since the policy was announced ahead of time, they identify the *MPC* out of an increase in income that is expected in advance. This is, in general, different from the theoretically-consistent *MPC* out of an unexpected increase. However, to the extent that borrowing constraints are important, or if households are surprised by the receipt despite its announcement, the resulting estimate may be close to the *MPC* that is important for the theory. This procedure also yields an *MPC* at a group level, so I again estimate covariances across groups, and this also delivers large error bands.

Each of these three techniques has its own limitations, and no survey contains perfect information on all components of household balance sheets. Notably, consumption in the SHIW and the PSID is imperfectly measured, as are income and assets in the CE. In addition, none of these surveys samples very rich households whose consumption behavior may be an important determinant of aggregate expenditures. Hence, the exercise in this section is tentative and intended to give a sense of magnitudes based on the current state

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<sup>36</sup>Survey Research Center, Institute for Social Research, University of Michigan (1999–2013).

<sup>37</sup>U.S. Department of Labor, Bureau of Labor Statistics (2000–2002).

of knowledge in the field. As administrative data on consumption, income and wealth become available and more sophisticated identification methods for *MPCs* develop, a priority for future work is to refine the estimates I provide here.

## B Measurement

Even though my analysis is in terms of elasticities, which are unitless numbers, the choice of temporal units is important: *MPC* needs to be measured over a period of time consistent with the time unit for income, consumption, and maturing elements of the balance sheet. To maximize comparability across surveys, I conduct all my measurement at an annual rate. While this is generally straightforward to do, *MPCs* require special treatment. Specifically, in the CE, the *MPC* identification strategy yields a quarterly estimate  $MPC^Q$ . I convert these to an annual *MPC* number  $MPC^A$  using the simple formula  $MPC^A = 1 - (1 - MPC^Q)^4$ . In appendix B, I provide a formal justification for this procedure.<sup>38</sup>

**MPC.** I choose a benchmark of  $\epsilon = 0$  for the elasticity of the relative price of durables to the real interest rate. Accordingly, my ideal measure of *MPC* includes total expenditures on nondurable and durable goods. The question in the SHIW refers to ‘spending’ without distinguishing between types of purchases, so it is safe to assume that it refers to both durables and nondurables. For my U.S. exercises, I prefer to follow the baseline estimates from [Blundell, Pistaferri and Preston \(2008\)](#) and [Johnson, Parker and Souleles \(2006\)](#), neither of which include durable goods in *MPC* estimation. Hence, my PSID estimate only includes nondurables, while my main CE estimate only includes food. In appendix C.4.1 I consider robustness to using total expenditures to estimate *MPC* instead. This makes the point estimates more negative, but also increases the confidence intervals. In appendix C.4.2, I consider robustness to alternative values of  $\epsilon$ , which has a similar effect.

**URE.** As defined in section I.B,  $URE_i$  measures the total resource flow that a household  $i$  needs to invest over the first period of his consumption plan. In each survey, I construct  $URE_i$  as

$$URE_i = Y_i - T_i - C_i + A_i - L_i \quad (26)$$

where  $Y_i$  is gross income,  $T_i$  is taxes net of transfers,  $C_i$  is consumption, and  $A_i$  and  $L_i$  represent, respectively, assets and liabilities that mature over the period, over and above the amounts already included in  $Y_i$  or  $C_i$ . I now describe what I include in these terms in detail. Table 2 provides a summary of the discussion that follows.

<sup>38</sup>In appendix C.4.4 I measure *MPC* and *URE* at a quarterly rate instead. This delivers similar results.

Table 2: Mapping model to data objects

Exposure measure: URE		Duration assumptions by scenario (in years)				
Data		Quarterly	Short	Benchmark	Long	Annual
$Y_i$	Gross income (all sources)					
$T_i$	Taxes net of transfers					
$C_i$	Nondurables + $(1 - \epsilon) \times$ Durables					
$A_i$	Deposits	0.25	0.25	0.5	0.75	1
	Bonds	4 (US), 7 (Italian government), 3.5 (Italian corporate)				
$L_i$	Adjustable Rate Mortgages	0.25	0.5	0.75	1	1
	Credit Cards	0.25	0.25	0.5	0.75	1
Exposure measure: NNP		Data				
	Nominal assets	Deposits + Bonds				
	Nominal liabilities	Mortgages + Consumer debt				

$Y_i$  includes gross income from all sources: labor, dividend, and interest income, as well as realized capital gains. This counts the maturing portion of equities, provided that we assume that equities have infinite maturity.<sup>39</sup>  $Y_i$  also counts bond coupons, with the remainder of maturing bonds included in  $A_i$  instead.  $T_i$  counts all taxes net of all transfers, so  $Y_i - T_i$  represents disposable income.

Given my benchmark of  $\epsilon = 0$ , I include in  $C_i$  all expenditures including rents and interest payments, as well as expenditure on durable goods including housing purchases and maintenance. In robustness exercises with respect to  $\epsilon$ , I only include in  $C_i$  a fraction  $1 - \epsilon$  of durable expenditures. In addition, I include all amortization payments in  $C_i$ . This accounts for the maturing portions of installment debt as well as fixed rate mortgages.

These choices leave me to account for four remaining categories of maturing assets and liabilities: deposits, bonds, adjustable rate mortgages,<sup>40</sup> and credit cards. Since I only observe very coarse maturity information in the data, I need to make assumptions on durations to convert stocks to flows. I define a benchmark scenario based on the limited external information I have, as well as four other scenarios to reflect uncertainty regarding true durations in the data. Table 2 summarizes these assumptions.

For remaining maturing assets  $A_i$ , I assume in my benchmark that time and savings deposits have a duration of two quarters. I assume that all bonds have a duration of

<sup>39</sup>I do not include unrealized capital gains in  $Y_i$ , consistent with an interpretation of these unrealized capital gains as resulting from real interest rates movements to which UREs summarize the exposure.

<sup>40</sup>In the U.S., fixed-rate mortgages carry a low-cost refinancing option. One possibility is to treat them as adjustable rate mortgages for rate cuts. Each household then has a different URE for rate increases vs rate cuts. Estimated in this way, the aggregate redistribution elasticity  $\widehat{\epsilon}_R$  for rate cuts is similar in the PSID, and it almost doubles in the CE.

four years in the U.S., matching the average duration of assets calculated by [Doepke and Schneider \(2006\)](#). For Italy, where I have separate information on holdings of government and corporate bonds, I use the average duration of 2010 government debt documented by the Italian Department of the Treasury (seven years), and assume that the maturity of corporate bonds is half as long.

For remaining maturing liabilities  $L_i$ , I assume a duration of three quarters for ARMs based on the results of [Stanton and Wallace \(1999\)](#). For credit cards, I assume a duration of two quarters. [Table 2](#) shows my assumptions for shorter and longer duration scenarios.

**NNP and income.** I compute net nominal positions as the difference between directly held nominal assets (deposits and bonds) and directly held nominal liabilities (mortgages and consumer credit). When assets are clearly indicated as shares of a financial intermediary that mostly owns nominal assets (for example, money market mutual funds), I also include the value of these shares in the households' nominal position. However, relative to [Doepke and Schneider \(2006\)](#), I do not calculate the indirect nominal positions arising from holdings of equity or other financial intermediaries, since my data is not sufficiently detailed for this purpose. For my income exposure measure, in keeping with the theory, I use pre-tax income ( $Y_i$ ) in all three surveys.

**Measurement error.** Measurement error is a very important issue in this exercise. These errors can stem from many sources: poor data quality, imperfect coverage, underreporting of consumption, or timing differences in the reporting of consumption and income. Each survey has its own strengths and weaknesses. The CE has excellent information on consumption and liabilities, but limited information on assets. Both the PSID and the SHIW appear to significantly undermeasure consumption. My covariance estimates are unbiased provided that the measurement errors in  $MPC$  and its cross-term ( $URE$ ,  $NNP$  or  $Y$ ) are additive and uncorrelated. Economically, this assumption corresponds to the presence of a 'mismeasurement' sector that rebates gains and losses lump-sum, just as the government does in the setting of [theorem 3](#).<sup>41</sup> This is certainly a strong assumption. The difference between my benchmark elasticities and their no-rebate counterpart can give a sense of the magnitude of this mismeasurement problem.

**Summary statistics.** [Table 3](#) reports the main summary statistics from each survey. Each line is normalized by average consumption in the survey, which facilitates comparability and corresponds to the normalization behind my elasticities in [table 1](#). Note that

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<sup>41</sup>For example, by abstracting away from indirect exposures to the banking sector, I tend to overstate the aggregate  $URE$ . If gains to the banking sector disproportionately favor low- $MPC$  households, my estimate of the  $MPC/URE$  correlation would be biased downwards.



Table 3: Main summary statistics from the three surveys

Survey	SHIW		PSID		CE	
Variable	mean	s.d.	mean	s.d.	mean	s.d.
Net income ( $Y_i - T_i$ )	1.19	0.83	1.42	1.64	1.14	1.01
Consumption ( $C_i$ )	1.00	0.94	1.00	2.58	1.00	0.90
Maturing assets ( $A_i$ )	0.93	2.48	1.30	5.06	0.53	1.77
Maturing liabilities ( $L_i$ )	0.31	1.40	0.51	1.44	0.52	1.47
Unhedged interest rate exposure ( $URE_i$ )	0.81	3.04	1.21	6.06	0.18	2.43
Nominal assets	0.74	2.36	1.18	3.66	0.47	1.84
Nominal liabilities	0.50	1.49	1.73	2.75	1.22	1.90
Net nominal position ( $NNP_i$ )	0.24	2.64	-0.55	4.43	-0.68	2.47
Gross income ( $Y_i$ )	1.27	1.04	1.69	2.51	1.23	1.09
Marginal propensity to consume ( $MPC_i$ )	0.47	0.35				
Number of households	7,951		7,287		4,833	

In each survey, 'mean' and 's.d.' represent the sample mean and standard deviation.

All statistics are computed using sample weights.

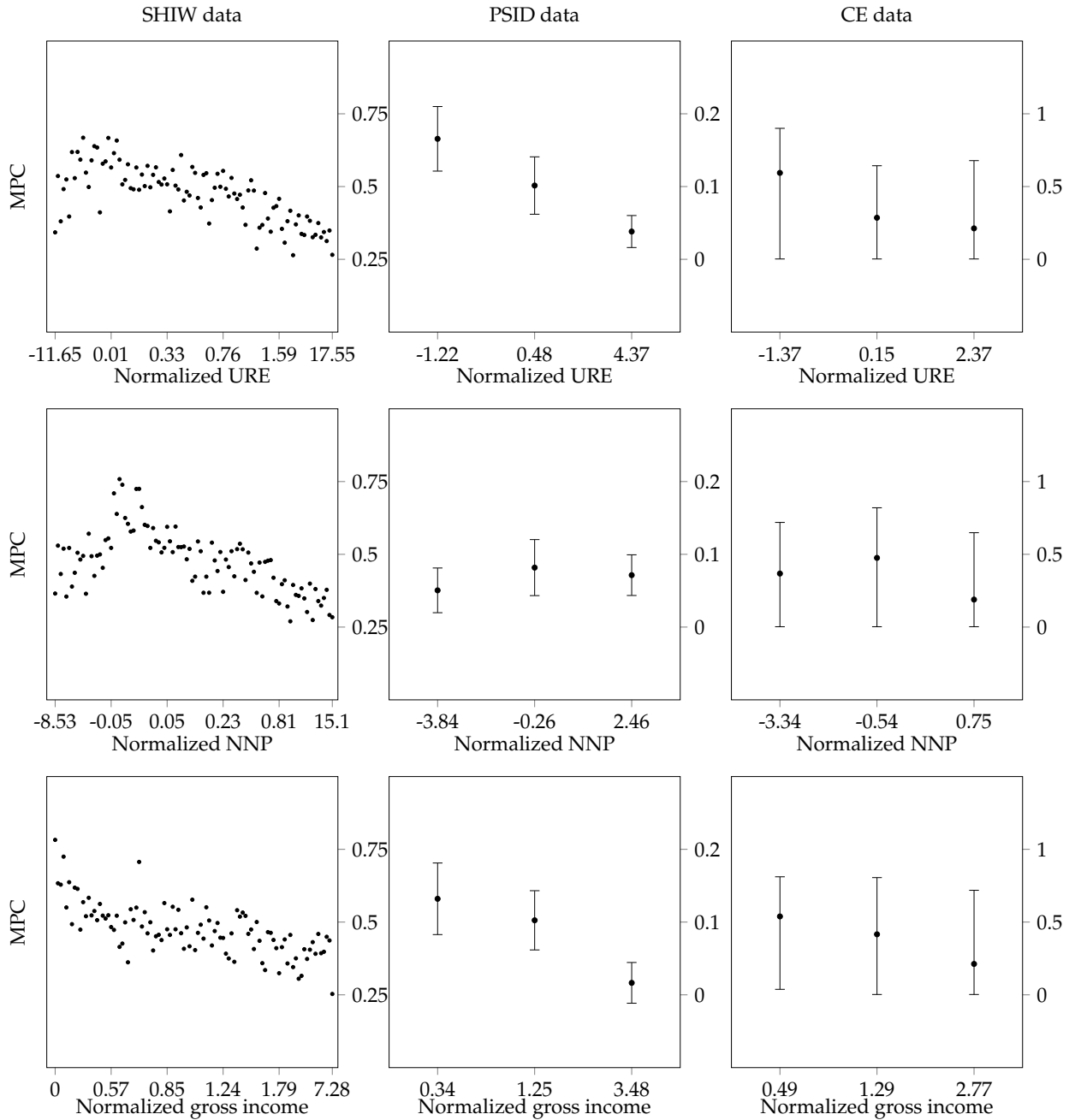
All variables except for MPC are normalized by average consumption in the sample.

the average  $URE$  is positive all three surveys. One reason, in addition to those highlighted in section II.D, is that consumption is below income at the mean, especially in the PSID and the SHIW—likely because of underreporting and coverage issues. The average net nominal position is quite negative in CE and PSID—possibly reflecting a poor measure of assets—and moderately positive in the Italian survey, where few households have a mortgage.

### C Redistribution elasticities in the data

I now turn to my main empirical results. Figure 2 reports the distribution of MPC by  $URE$ ,  $NNP$  and income across the three surveys. Columns correspond to datasets, and rows to exposure measures. The first column displays data from the SHIW, where individual MPC information is available. The three graphs report the average value of MPC in each percentile of the  $x$ -axis variable. In the PSID (second column) and the CE (third column), I estimate the MPC by stratifying the population in terciles of the  $x$ -axis variable, and then report the point estimate together with confidence intervals within each bin.

Starting with the interest exposure channel, looking across the first row, all three surveys show a negative correlation between MPC and  $URE$ . This is particularly apparent in the SHIW and the PSID data, but the pattern is there in the CE as well. A direct implication is that  $\mathcal{E}_R < 0$  in each of these datasets: falls in interest rates increase consumption demand



This graphs shows average annual marginal propensities to consume by exposure bin. The top row groups households by un-hedged interest rate exposure (URE), the middle row by net nominal position (NNP), and the third row by gross (pre-tax) income. The x axes report mean exposure per bin (all exposure measures are normalized by average consumption). The left column uses 100 bins in the SHIW. The middle and right column uses 3 bins in the PSID and the CE, respectively, and estimate MPC within bin. See the main text for details on MPC estimation.

Figure 2: Marginal propensities to consume and the redistribution channels.

Table 4: Estimates of table 1’s cross-sectional moments in three surveys

Survey	SHIW		PSID		CE	
	Estimate	95% C.I.	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}}_R$	-0.10	[-0.15,-0.05]	-0.12	[-0.16,-0.08]	-0.23	[-0.60,0.15]
$\widehat{\mathcal{E}}_R^{NR}$	0.28	[0.23,0.33]	0.00	[-0.03,0.04]	-0.09	[-0.48,0.31]
$\widehat{S}$	0.55	[0.53,0.58]	0.90	[0.88,0.92]	0.64	[0.36,0.92]
$\widehat{\mathcal{E}}_P$	-0.07	[-0.11,-0.02]	0.02	[-0.02,0.07]	-0.09	[-0.51,0.33]
$\widehat{\mathcal{E}}_P^{NR}$	0.05	[0.01,0.09]	-0.01	[-0.06,0.03]	-0.45	[-0.94,0.04]
$\widehat{\mathcal{E}}_Y$	-0.05	[-0.07, -0.03]	-0.06	[-0.09,-0.04]	-0.13	[-0.36,0.10]
$\widehat{M}$	0.55	[0.53, 0.57]	0.08	[0.04,0.12]	0.46	[-0.06,0.98]

All statistics are computed using survey weights. In the CE and the PSID, confidence intervals are bootstrapped by resampling households 100 times with replacement.

via the redistribution channel.

Turning to the Fisher channel, we also observe an overall negative correlation in the SHIW, though it is somewhat less pronounced. This weaker pattern is apparent in the PSID and the CE as well: in particular, MPCs tend to be slightly higher in the center of the NNP distribution than at the extremes. This could be consistent with a ‘wealthy hand-to-mouth’ explanation as in [Kaplan and Violante \(2014\)](#). Overall, the slight diminishing pattern suggests that  $\mathcal{E}_P < 0$ , consistent with Fisher’s hypothesis—unexpected increases in nominal prices tend to increase consumption overall, but this effect tends to be quantitatively small.

Finally, across all three surveys, the covariance between MPCs and gross incomes is also negative, confirming previous findings in the literature. Combined with  $\gamma < 0$ , a negative  $\mathcal{E}_Y$  implies an amplification role for the earnings heterogeneity channel in the transmission of monetary policy.

Moving on to magnitudes, table 4 computes my seven key cross-sectional moments, together with 95% confidence intervals. For the PSID and the CE, the estimation is done across bins by using three bins, just as in figure 2.<sup>42</sup>

Confirming the visual impression from figure 2, the point estimates for the redistribution elasticities  $\widehat{\mathcal{E}}_R$ ,  $\widehat{\mathcal{E}}_P$  and  $\widehat{\mathcal{E}}_Y$  are negative in all three surveys, except for a slight positive number for  $\widehat{\mathcal{E}}_P$  in the PSID. However, the magnitudes are relatively small—in particular, the confidence bands in the CE always include zero.<sup>43</sup>

To put these numbers in the context of standard representative-agent analyses, consider that many macroeconomists believe 0.1 to 0.5 as plausible values for the elasticity of

<sup>42</sup>Appendix C.4.3 reports a sensitivity analysis using four to eight bins. The results are little changed.

<sup>43</sup>Moreover, the estimated value of  $\mathcal{E}_R^{NR}$  is usually positive, implying that the negative covariance is not strong enough to overwhelm the positive value of URE at the mean.

Table 5: Estimated redistribution elasticity  $\mathcal{E}_R$  for five duration scenarios

		Duration scenario				
		Quarterly	Short	Benchmark	Long	Annual
$\widehat{\mathcal{E}}_R$	SHIW	-0.16 [-0.27,-0.06]	-0.20 [-0.27,-0.12]	-0.10 [-0.15,-0.05]	-0.07 [-0.11,-0.04]	-0.06 [-0.09,-0.02]
	PSID	-0.14 [-0.21,-0.07]	-0.13 [-0.20,-0.07]	-0.12 [-0.16,-0.08]	-0.11 [-0.15,-0.08]	-0.11 [-0.14,-0.08]
	CE	-0.55 [-1.35,0.24]	-0.48 [-1.10,0.14]	-0.23 [-0.60,0.15]	-0.22 [-0.50,0.06]	-0.23 [-0.49,0.03]

intertemporal substitution  $\sigma$ , though financial economists typically consider  $\sigma$  to be above one. (In his meta-analysis, [Havránek 2015](#) finds a mean of  $\sigma = 0.5$  but argues that it is pushed up by publication bias, while [Bansal et al. 2016](#)'s preferred estimate is  $\sigma = 2.2$ .) Equation (25) shows that  $\sigma$  should be compared to  $-\mathcal{E}_R/S$  to gauge the relative strength of the redistribution effect. According to the point estimates from table 4, this number is between 0.1 and 0.4. Hence the data suggests that, if  $\sigma$  is as small as macroeconomists think, the redistribution effect may be as important as the substitution effect in explaining why aggregate consumption responds to changes in real interest rates. On the other hand, the magnitudes of  $\widehat{\mathcal{E}}_P$  and  $\widehat{\mathcal{E}}_Y$  are fairly small, so that (unless  $\gamma$  is very negative) neither channel can account on its own for very large movements in consumption. But their combined effect may nevertheless be substantial, and further research is needed to refine the precision of these estimates.

As more sources of joint consumption, income and asset data become available, a better empirical understanding of *UREs* and *NNPs* will become possible, helping to shape our understanding of the winners and losers from changes in real interest rates and inflation. Real-time estimates of the redistribution covariances could also provide useful information about the dynamic evolution of the monetary policy transmission mechanism.

**The role of asset and liability durations.** Table 5 considers the sensitivity of my estimates of  $\mathcal{E}_R$  to my maturity assumptions listed in table 2. In all three surveys, shortening durations makes the redistribution elasticity more negative, while lengthening durations makes it approach zero. This finding illustrates the importance of durations in determining the magnitude of the interest rate exposure channel. As I discuss below, this finding has a simple structural interpretation in incomplete market models.

## D Empirical drivers of the redistribution covariances

While the sufficient statistic approach suggests that only the population-level redistribution elasticities matter to determine an overall effect, in practice it is interesting to understand the empirical drivers of these covariances. For example, is the covariance between  $MPC$  and  $URE$  negative because older households tend to have lower  $MPC$ s and higher  $URE$ s? In order to shed light on this and related questions, I perform a covariance decomposition, projecting each covariance onto observable components such as age or education. This procedure is inspired by the law of total covariance: focusing on  $URE$  for ease of notation, for any covariate  $Z_i$  we know that

$$\text{Cov}(MPC_i, URE_i) = \underbrace{\text{Cov}(\mathbb{E}[MPC_i|Z_i], \mathbb{E}[URE_i|Z_i])}_{\text{Explained fraction of covariance}} + \underbrace{\mathbb{E}[\text{Cov}(MPC_i, URE_i|Z_i)]}_{\text{Unexplained fraction of covariance}} \quad (27)$$

We can then implement this decomposition using an OLS regression, which performs a linear approximation to the conditional expectation function.<sup>44</sup> For any observable covariate  $Z_i$ , I run two OLS regressions

$$\begin{aligned} MPC_i &= \alpha_M + \beta_M Z_i + \epsilon_{Mi} \\ URE_i &= \alpha_R + \beta_R Z_i + \epsilon_{Ri} \end{aligned}$$

and compute the covariance between the fitted values  $\widehat{MPC}_i$  and  $\widehat{URE}_i$  to get an empirical counterpart of the explained component in (27). This gives me the part of the covariance that can be explained by  $Z_i$ , since

$$\begin{aligned} \text{Cov}(MPC_i, URE_i) &= \text{Cov}(\widehat{MPC}_i + \widehat{\epsilon}_{Mi}, \widehat{URE}_i + \widehat{\epsilon}_{Ri}) \\ &= \text{Cov}(\widehat{\beta}_M Z_i + \widehat{\epsilon}_{Mi}, \widehat{\beta}_R Z_i + \widehat{\epsilon}_{Ri}) \\ &= \text{Var}(Z_i) \widehat{\beta}_M \widehat{\beta}_R + \text{Cov}(\widehat{\epsilon}_{Mi}, \widehat{\epsilon}_{Ri}) \end{aligned} \quad (28)$$

where the last line follows because, by construction,  $\text{Cov}(\widehat{\epsilon}_{Mi}, Z_i) = \text{Cov}(\widehat{\epsilon}_{Ri}, Z_i) = 0$ . For example, in table 6, when  $Z_i$  is age,  $\widehat{\beta}_M$  is negative and  $\widehat{\beta}_R$  is positive, so older agents do tend to have lower  $MPC$  and higher  $URE$ . However, on its own, age can only explain 9% of the total covariance.

This procedure is straightforward to implement in the SHIW, where  $MPC$  is available at the individual level. Table 6 reports these results using Jappelli and Pistaferri (2014)'s control variables for  $MPC$ , one covariate at a time. For each of my three redistributive channels, I report each of the terms in the decomposition (28), as well as the fraction of the variance explained. In appendix C.5, I generalize this approach to multiple covariates, and I also report estimates of  $MPC$ ,  $URE$  and  $NNP$  by age and income bins in each survey. All of these give a consistent message: age, education and income tend to be negatively

<sup>44</sup>This is similar to implementing the law of total variance using  $R^2$ .

Table 6: Covariance decomposition for URE, NNP and income in the SHIW

$Z_i$	Var( $Z_i$ )	$\widehat{\beta}_M$	$\mathcal{E}_R$		$\mathcal{E}_P$		$\mathcal{E}_Y$	
			$\widehat{\beta}_R$	% expl.	$\widehat{\beta}_P$	% expl.	$\widehat{\beta}_Y$	% expl.
Age bins	0.77	-0.027	0.467	10%	0.472	15%	-0.008	-0%
Male	0.24	-0.055	0.352	5%	0.258	5%	0.271	7%
Married	0.18	-0.016	0.069	0%	-0.063	-0%	0.449	2%
Years of education	18.8	-0.005	0.052	5%	0.028	4%	0.097	19%
Family size	1.71	0.023	-0.094	4%	-0.194	12%	0.149	-11%
Resident of the South	0.22	0.198	-0.443	18%	-0.231	15%	-0.567	48%
City size	1.21	0.037	0.013	-0%	0.048	-3%	0.058	-5%
Unemployed	0.04	0.189	-0.610	5%	-0.278	3%	-0.637	10%

correlated with MPC and positively correlated with URE and NNP, so they help explain the negative covariance overall.

## E Sufficient statistics: model vs data

In a previous version of this paper (Auclert 2017), I considered the sufficient statistics generated by a standard partial-equilibrium incomplete markets model, similar to the one used as a building block by the heterogeneous-agent New Keynesian literature. The model is a Bewley-Huggett-Aiyagari model with nominal, long-term, circulating private IOUs (as in Huggett 1993). Such a model features rich heterogeneity in MPCs, UREs, NNPs and incomes. I calibrated it to the U.S. economy and quantitatively evaluated, in its steady state, the size of my sufficient statistics. This exercise delivered three main insights.

First, in the model, the interest rate exposure channel has the same sign and comparable magnitude as it does in the data. Moreover, as durations shorten, the redistribution elasticity becomes more negative, consistent with my findings in table 5. In the limit where all assets are short term, changes in real interest rates have large redistributive effects. The intuition is as follows: the shorter asset maturities are, the less capital gains expansionary monetary policy generates. Since capital gains accrue to low MPC agents, monetary policy is more potent in affecting consumption with short-term assets than with long-term assets. This role for asset durations is consistent with the results of Calza, Monacelli and Stracca (2013), who find that consumption reacts much more strongly to identified monetary policy shocks in countries where mortgages predominantly have adjustable rates.<sup>45</sup>

Second, I find that a calibration of the model in which all assets are nominal features a Fisher channel with the same sign as in the data, but a much larger magnitude. This is because inflation redistributes along the asset dimension, which in this class of models is highly correlated with MPC. As a result, Bewley models with nominal assets tend to

<sup>45</sup>See also Rubio (2011) and Garriga, Kydland and Šustek (2017).

overstate the correlation between MPCs and NNPs that exists in the data. A model with real assets, or in which assets have a high degree of inflation indexation, is more consistent with the empirical evidence.

Finally, in the model with short-term debt, changes in real interest rates have asymmetric effects. The sufficient statistic approach correctly predicts the effect of any increase in the real rate, but it overpredicts the effect of a large decline. This asymmetry comes from the differential response of borrowers at their credit limit to rises and falls in income: while these borrowers save an important fraction of the gains they get from low interest rates, they are forced to cut spending steeply when interest rates rise. This could help explain the empirical finding that interest rate hikes tend to lower output by more than falls increase it (Cover 1992; de Long and Summers 1988; Tenreyro and Thwaites 2016). My explanation, which has to do with asymmetric MPC differences in response to policy rate changes, provides an alternative to the traditional Keynesian interpretation of this fact, which relies on downward nominal wage rigidities.<sup>46</sup>

## IV Conclusion

This paper contributes to our understanding of the role of heterogeneity in the transmission mechanism of monetary policy. I identified three important dimensions along which monetary policy redistributes income and wealth, and argued that each of these dimensions was likely to be a source of aggregate effects on consumption. My classification holds in many environments and provides a simple, reduced-form approach to computing aggregate magnitudes. Hence it can guide future work on the topic, both theoretical and empirical.

An important finding of my paper is that capital gains and losses, both nominal and real, matter for understanding monetary policy transmission. This finding has broad implications for monetary policy. A change in the inflation target can create large redistribution in favor of high MPC agents and be expansionary over and beyond its effect on real interest rates. With long asset maturities, lower real interest rates can benefit asset holders with lower MPCs and make interest rate cuts less effective at increasing aggregate demand than they would otherwise be. Monetary policy becomes intertwined with fiscal policy, but also with government debt maturity management and mortgage design policies.

These are just some of the macroeconomic consequences of the presence of large and heterogeneous marginal propensities to consume, which are a robust feature of household

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<sup>46</sup>In practice, the refinancing option embedded in fixed rate mortgages in the U.S. is likely to create an asymmetric effect in the opposite direction from the one I stress here. See Wong (2018) for theory and empirical evidence along these lines.



micro data. My investigation opens up many avenues for future research on monetary policy with heterogeneous agents.

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# Monetary Policy and the Redistribution Channel

## Online Appendix

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### A Proofs for sections I and II

#### A.1 The standard New Keynesian model

This section shows that, in the standard New Keynesian model with sticky Calvo prices, the impulse response to the path for prices  $P_t$ , real discount rates  $q_t$ , real wages  $w_t$  and unearned income are those given by my main experiment in figure 1. I only outline the elements of the model relevant to my argument, the reader is referred to the textbook treatments of Woodford (2003) or Galí (2008) for details.

I consider the model in its ‘cashless limit’, with no aggregate uncertainty. The model features a representative agent with separable utility trading in one-period nominal bonds and holding a fixed stock of capital  $k$ , so equation (1) simplifies to

$$\sum \beta^t \{u(c_t) - v(n_t)\}$$

$$P_t c_t + ({}_t Q_{t+1}) B_{t+1} = P_t \pi_t + W_t n_t + B_t + P_t \rho_t k$$

Here  $\rho_t$  denotes the real rental rate of capital, so  $\rho_t k$  are total real rents, and  $\pi_t$  are real firm profits. Together, rents and profits make up the unearned income in this economy. Consumption  $c_t$  is an aggregate of intermediate goods, with constant elasticity of substitution  $\epsilon$ . Hence the price index, aggregating the individual goods prices  $p_{jt}$ , is  $P_t = \left( \int_0^1 p_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ .

Each good  $j$  is produced under monopolistic competition with constant returns to scale and unit productivity. The production function is

$$y_{jt} = F(k_{jt}, l_{jt}) = k_{jt}^\alpha l_{jt}^{1-\alpha}$$

Firms can only adjust their price with probability  $\theta$  each period, independent across firms and periods (the Calvo assumption). Nominal wages  $W_t$  and nominal rents are flexible. Cost minimization by the firm therefore implies

$$\begin{aligned} \rho_t P_t &= \Lambda_{jt} F_k(k_{jt}, l_{jt}) \\ W_t &= \Lambda_{jt} F_l(k_{jt}, l_{jt}) \end{aligned}$$

for some scalar  $\Lambda_{jt}$  representing the nominal marginal cost of production for firm  $j$ . Hence

$$\frac{F_k(k_{jt}, l_{jt})}{F_l(k_{jt}, l_{jt})} = \frac{F_k\left(\frac{k_{jt}}{l_{jt}}, 1\right)}{F_l\left(\frac{k_{jt}}{l_{jt}}, 1\right)} = \frac{\rho_t}{w_t}$$

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<sup>†</sup> Last update: September 2019. I thank Thomas Hintermaier for carefully reading through the proof and pointing out typos in the derivation of Theorem 1. (These do not affect the final result as stated in the paper.)



so all firms have the same capital-labor ratio  $\frac{k_{jt}}{l_{jt}} = \frac{k_t}{l_t}$ , and hence all firms have the same nominal marginal cost of production  $\Lambda_t$ .

As is well-known, a first-order approximation to the equilibrium equations of this model is given by the system of three equations

$$\log\left(\frac{c_t}{\bar{c}}\right) = \log\left(\frac{c_{t+1}}{\bar{c}}\right) - \sigma\left(i_t - \log\left(\frac{P_{t+1}}{P_t}\right) - \varrho\right) \quad (\text{A.1})$$

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \beta \log\left(\frac{P_{t+1}}{P_t}\right) + \kappa \log\left(\frac{c_t}{\bar{c}}\right) \quad (\text{A.2})$$

$$i_t = \varrho + \phi_\pi \log\left(\frac{P_t}{P_{t-1}}\right) + \epsilon_t \quad (\text{A.3})$$

where  $\bar{c}$  is the level of consumption that would prevail under flexible prices, which (normalizing  $k = 1$ ) solves

$$\frac{v'\left(\bar{c}^{\frac{1}{1-\alpha}}\right)}{u'(\bar{c})} = \frac{\epsilon - 1}{\epsilon} \frac{(1-\alpha)}{\bar{c}} \equiv \bar{w}$$

$\varrho = \beta^{-1} - 1$  is the steady-state net real interest rate,  $\sigma = -\frac{u'(\bar{c})}{cu''(\bar{c})}$  is the elasticity of substitution around  $\bar{c}$ , and  $\kappa$  is the slope of the Phillips curve (a function of model parameters). Equation (A.3) is a Taylor rule describing the behavior of monetary policy. We assume that  $\phi_\pi > 1$ , which guarantees equilibrium uniqueness. We consider the effects of a time-0 monetary policy loosening,  $\epsilon_0 < 0$  and  $\epsilon_t = 0$  for  $t \geq 1$ , assuming the system was at steady-state at  $t = -1$ , with constant price level  $\bar{P}$ .

It is easy to guess and verify that the equilibrium features  $i_t = \rho$ ,  $P_t = P_{t-1}$  and  $c_t = \bar{c}$  for  $t \geq 1$ . Solving backwards, this implies that

$$\begin{aligned} i_0 &= \rho + \frac{1}{1 + \kappa\sigma\phi_\pi} \epsilon_0 \\ \log\left(\frac{c_0}{\bar{c}}\right) &= -\frac{\sigma}{1 + \kappa\sigma\phi_\pi} \epsilon_0 \\ \log\left(\frac{P_0}{\bar{P}}\right) &= -\frac{\kappa\sigma}{1 + \kappa\sigma\phi_\pi} \epsilon_0 \end{aligned}$$

In other words, a monetary loosening raises  $c_t$  at  $t = 0$  only, and raises  $P_t$  immediately and permanently. (Firms that get an opportunity to reset at  $t = 0$  all increase their price above  $\bar{P}$ , pulling up the price level to  $P_0$ . Thereafter, all firms that get a chance reset their price to  $P_0$ , so there is no inflation.) To a first-order approximation, the real wage satisfies

$$w_t = \frac{v'\left(c_t^{\frac{1}{1-\alpha}}\right)}{u'(c_t)}$$

so  $w_t$  increases at  $t = 0$  only and then reverts to  $\bar{w}$ . Moreover, real rents are

$$\rho_t = \frac{\alpha}{1-\alpha} w_t c_t^{\frac{1}{1-\alpha}}$$

so they also increase at  $t = 0$  and then revert to  $\bar{\rho} = \frac{\alpha}{1-\alpha} \bar{w} (\bar{c})^{\frac{1}{1-\alpha}}$ .<sup>47</sup> Date-0 nominal and real state prices

<sup>47</sup>Since price dispersion rises as a result of the monetary policy shock, the nonlinear solution features a real wage that is different from steady state even beyond  $t \geq 1$ , but the difference is second order in  $\epsilon_0$ .

are  $Q_0 = q_0 = 1$  and, for  $t \geq 1$ , given that  $P_t = P_0$ ,

$$q_t = Q_t = \prod_{s=0}^{t-1} ({}_s Q_t) = \frac{1}{1+i_0} \beta^{t-1}$$

Hence, the path of  $q_t$  and  $Q_t$  for  $t \geq 1$  is shifted upwards by  $\frac{dq_t}{q_t} = \frac{dQ_t}{Q_t} = -\frac{dR}{R}$  where the proportional real interest rate change is  $\frac{dR}{R} = \frac{d\epsilon_0}{(1+\kappa\sigma\phi\pi)} \frac{1}{(1+\rho)}$ . Finally, aggregate profits are, to first-order, given by

$$\pi_t = c_t - w_t n_t - \rho_t k = c_t \left( 1 - \frac{1}{1-\alpha} \frac{v' \left( (c_t)^{\frac{1}{1-\alpha}} \right)}{u'(c_t)} c_t \right) \quad (\text{A.4})$$

Hence they also deviate only at  $t = 0$  from their steady state value of  $\bar{d} = \frac{\bar{c}}{\bar{c}}$ . The first term in (A.4) is volume, which rises with  $c_0$ . The second term is the markup, which falls with  $c_0$ . In typical calibrations, the markup effect dominates and profits fall in response to an expansionary monetary shock  $\epsilon_0 < 0$ .

Collecting results, the timing of changes for  $w_t$ ,  $P_t$  and  $q_t$ , as well as unearned income  $\rho_t k + \pi_t$ , is exactly that depicted in figure 1, as claimed in the main text.

## A.2 Proof of theorem 1

The proof is greatly simplified by first applying a simple renormalization of discount factors. Instead of the present value normalization  $q_0 = 1$ , I normalize  $q_1 = 1$  and let  $q_0$  vary. Then, setting

$$\frac{dq_0}{q_0} = \frac{dR}{R} \quad (\text{A.5})$$

yields the experiment in figure 1. Intuitively, a rise in the relative price of future goods relative to a current good is the same as a fall in the price of that current good relative to all future goods. This renormalization is innocuous since there is a degree of freedom in choosing discount factors.

Given the experiment, we can hold  $q_t$  fixed for  $t \geq 1$ . Hence, only three parameters  $y_0$ ,  $w_0$  and  $q_0$  vary, together with the sequence  $\{P_t\}$ .

With this renormalization, the proof has three steps: first, I apply Slutsky's theorem to break down  $dc$  and  $dn$  into income and substitution effects. Second, I work out explicit expressions for MPC and MPN. Finally, I calculate compensated derivatives, and use my expressions from the second step to simplify their expressions.

**Step 1: Slutsky's theorem.** Recall that the sequences  $\{q_t\}$  and  $\{w_t\}$  are fixed in the experiment, except for  $q_0$  and  $w_0$ . Define the following expenditure function

$$e(q_0, w_0, U) = \min \left\{ \sum_t q_t (c_t - w_t n_t) \quad \text{s.t.} \quad \sum_t \beta^t \{u(c_t) - v(n_t)\} \geq U \right\} \quad (\text{A.6})$$

and let  $c_0^h, n_0^h$  be the resulting compensated (Hicksian) demands for time-0 consumption and hours. Applying the envelope theorem, we obtain a version of Shephard's lemma:

$$e_{q_0} = c_0 - w_0 n_0 \quad (\text{A.7})$$

$$e_{w_0} = -q_0 n_0 \quad (\text{A.8})$$

Define 'unearned' wealth as

$$\tilde{\omega} \equiv \sum_{t \geq 0} q_t \left( y_t + (-1)b_t + \left( \frac{-1B_t}{P_t} \right) \right)$$

and note that, given the variation we consider,

$$d\tilde{\omega} = \left( y_0 + (-1)b_0 + \left( \frac{-1B_0}{P_0} \right) \right) dq_0 + q_0 dy_0 - \sum_{t \geq 0} q_t \left( \frac{-1B_t}{P_t} \right) \frac{dP_t}{P_t} \quad (\text{A.9})$$

Using the Fisher equation  $\frac{q_t}{P_t} = \frac{Q_t}{P_0}$ , and the fact that  $\frac{dP_t}{P_t} = \frac{dP}{P}$  is a constant, the last term rewrites

$$\sum_{t \geq 0} q_t \left( \frac{-1B_t}{P_t} \right) \frac{dP_t}{P_t} = \sum_{t \geq 0} Q_t \left( \frac{-1B_t}{P_0} \right) \frac{dP}{P} = q_0 NNP \frac{dP}{P}$$

where we have defined the household's net nominal position as the present value of his nominal assets

$$q_0 NNP \equiv \sum_{t \geq 0} Q_t \left( \frac{-1B_t}{P_0} \right)$$

Moreover, defining

$$URE \equiv w_0 n_0 + y_0 + (-1)b_0 + \left( \frac{-1B_0}{P_0} \right) - c_0$$

we can rewrite (A.9) as

$$d\tilde{\omega} = (URE + c_0 - w_0 n_0) dq_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \quad (\text{A.10})$$

Next, define the indirect utility function that attains  $\tilde{\omega}$  as

$$V(q_0, w_0, \tilde{\omega}) = \max \left\{ \sum_t \beta^t \{u(c_t) - v(n_t)\} \quad \text{s.t.} \quad \sum_t q_t (c_t - w_t n_t) = \tilde{\omega} \right\} \quad (\text{A.11})$$

Let  $c_0, n_0$  denote the resulting Marshallian demands. Applying the envelope theorem, we find

$$\frac{\partial V}{\partial q_0} = -\frac{u'(c_0)}{q_0} (c_0 - w_0 n_0) \quad (\text{A.12})$$

$$\frac{\partial V}{\partial w_0} = \frac{u'(c_0)}{q_0} q_0 n_0 \quad (\text{A.13})$$

$$\frac{\partial V}{\partial \tilde{\omega}} = \frac{u'(c_0)}{q_0} \quad (\text{A.14})$$

As in the proof of Slutsky's theorem, we next differentiate along the identities

$$\begin{aligned} c_0^h(q_0, w_0, U) &= c_0(q_0, w_0, e(q_0, w_0, U)) \\ n_0^h(q_0, w_0, U) &= n_0(q_0, w_0, e(q_0, w_0, U)) \end{aligned}$$

to find that Marshallian and Hicksian derivatives are related via

$$\frac{\partial c_0^h}{\partial q_0} = \frac{\partial c_0}{\partial q_0} + \frac{\partial c_0}{\partial \tilde{w}} e_{q_0} \quad \frac{\partial c_0^h}{\partial w_0} = \frac{\partial c_0}{\partial w_0} + \frac{\partial c_0}{\partial \tilde{w}} e_{w_0} \quad (\text{A.15})$$

$$\frac{\partial n_0^h}{\partial q_0} = \frac{\partial n_0}{\partial q_0} + \frac{\partial n_0}{\partial \tilde{w}} e_{q_0} \quad \frac{\partial n_0^h}{\partial w_0} = \frac{\partial n_0}{\partial w_0} + \frac{\partial n_0}{\partial \tilde{w}} e_{w_0} \quad (\text{A.16})$$

Next, define

$$MPC \equiv q_0 \frac{\partial c_0}{\partial \tilde{w}} \quad (\text{A.17})$$

$$MPN \equiv q_0 \frac{\partial n_0}{\partial \tilde{w}} \quad (\text{A.18})$$

these express the dollar-for-dollar (or hour-for-dollar) marginal propensities to consume and work at date 0: indeed,

$$\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial y_0} = \frac{MPC}{q_0} q_0 = MPC$$

and similarly  $\frac{\partial n_0}{\partial y_0} = MPN$ .

Totally differentiating the Marshallian consumption function and using (A.10), we find

$$dc_0 = \frac{\partial c_0}{\partial q_0} dq_0 + \frac{\partial c_0}{\partial w_0} dw_0 + \frac{\partial c_0}{\partial \tilde{w}} \left( (URE + c_0 - w_0 n_0) dq_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \right)$$

Using (A.15)–(A.16),

$$\begin{aligned} dc_0 &= \left( \frac{\partial c_0^h}{\partial q_0} - \frac{\partial c_0}{\partial \tilde{w}} e_{q_0} \right) dq_0 + \left( \frac{\partial c_0^h}{\partial w_0} - \frac{\partial c_0}{\partial \tilde{w}} e_{w_0} \right) dw_0 \\ &\quad + \frac{\partial c_0}{\partial \tilde{w}} \left( (URE + c_0 - w_0 n_0) dq_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \right) \\ &= \frac{\partial c_0}{\partial \tilde{w}} \left( -e_{w_0} dw_0 + q_0 dy_0 + (-e_{q_0} + URE + c_0 - w_0 n_0) dq_0 - NNP \frac{dP}{P} \right) + \frac{\partial c_0^h}{\partial q_0} dq_0 + \frac{\partial c_0^h}{\partial w_0} dw_0 \end{aligned}$$

and using (A.7), (A.8) and (A.17) to replace  $e_{w_0}$ ,  $e_{q_0}$  and  $\frac{\partial c_0}{\partial \tilde{w}}$ , we find

$$\begin{aligned} dc_0 &= \frac{MPC}{q_0} \left( q_0 n_0 dw_0 + q_0 dy_0 + URE dq_0 - q_0 NNP \frac{dP}{P} \right) + \frac{\partial c_0^h}{\partial q_0} dq_0 + \frac{\partial c_0^h}{\partial w_0} dw_0 \\ &= MPC \left( n_0 dw_0 + dy_0 + URE \frac{dq_0}{q_0} - NNP \frac{dP}{P} \right) + c_0 \left( \frac{q_0}{c_0} \frac{\partial c_0^h}{\partial q_0} \frac{dq_0}{q_0} + \frac{w_0}{c_0} \frac{\partial c_0^h}{\partial w_0} \frac{dw_0}{w_0} \right) \end{aligned}$$

Finally, dropping time subscripts for ease of notation, using (A.5), and defining compensated elasticities

by

$$\begin{aligned}\epsilon_{c,q}^h &\equiv \frac{q_0}{c_0} \frac{\partial c_0^h}{\partial q_0} \\ \epsilon_{c,w}^h &\equiv \frac{w_0}{c_0} \frac{\partial c_0^h}{\partial w_0}\end{aligned}$$

we obtain

$$dc = MPC \left( ndw + dy + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + c \left( \epsilon_{c,q}^h \frac{dR}{R} + \epsilon_{c,w}^h \frac{dw}{w} \right) \quad (\text{A.19})$$

In a completely analogous way, we also find

$$dn = MPN \left( ndw + dy + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + n \left( \epsilon_{n,q}^h \frac{dR}{R} + \epsilon_{n,w}^h \frac{dw}{w} \right) \quad (\text{A.20})$$

The rest of the proof calculates the compensated elasticities and relates them to  $MPC$  and  $MPN$ , which will yield our expressions for consumption and labor supply. To get my expression for welfare, totally differentiate the indirect utility function and use (A.12)–(A.14) and (A.10) to obtain

$$\begin{aligned}dU &= \frac{\partial V}{\partial q_0} dq_0 + \frac{\partial V}{\partial w_0} dw_0 + \frac{\partial V}{\partial \tilde{\omega}} d\tilde{\omega} \\ &= \frac{u'(c_0)}{q_0} \cdot \left( URE dq_0 + q_0 n_0 dw_0 + q_0 dy_0 - q_0 NNP \frac{dP}{P} \right)\end{aligned}$$

This yields my expression in (5),

$$dU = u'(c) \cdot \left( dy + ndw + URE \frac{dR}{R} - NNP \frac{dP}{P} \right)$$

**Step 2: Marginal propensities.** I now derive explicit expressions for marginal propensities to consume, that is, the Marshallian derivatives of the consumption and labor supply functions that are solutions to (A.11). Inverting the first-order conditions

$$u'(c_t) = \beta^{-t} \left( \frac{q_t}{q_0} \right) u'(c_0) \quad (\text{A.21})$$

$$v'(n_t) = \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v'(n_0) \quad (\text{A.22})$$

and inserting the resulting values for  $c_t$  and  $n_t$  into the budget constraint (redefining  $W = \frac{\tilde{\omega}}{q_0}$  as present-value wealth for simplicity)

$$\sum_{t \geq 0} \frac{q_t}{q_0} (c_t - w_t n_t) = W$$

we obtain

$$c_0 + \sum_{t \geq 1} \frac{q_t}{q_0} (u')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) u'(c_0) \right] - w_0 \left( n_0 + \sum_{t \geq 1} \frac{q_t}{q_0} \frac{w_t}{w_0} (v')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v'(n_0) \right] \right) = W \quad (\text{A.23})$$

Recall that  $MPC = \frac{\partial c_0}{\partial W}$  and  $MPN = \frac{\partial n_0}{\partial W}$ . Differentiating (A.23) with respect to  $W$ , we obtain

$$MPC \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} \right) - w_0 MPN \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \frac{w_t}{w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \right) = 1 \quad (\text{A.24})$$

moreover, the intratemporal first order condition

$$v'(n_0) = w_0 u'(c_0) \quad (\text{A.25})$$

implies

$$\begin{aligned} v''(n_0) MPN &= w_0 u''(c_0) MPC \\ \frac{v''(n_0)}{v'(n_0)} MPN &= \frac{u''(c_0)}{u'(c_0)} MPC \end{aligned}$$

so, using the definition of the local elasticities of substitution,

$$-\sigma(c_t) c_t u''(c_t) = u'(c_t) \quad (\text{A.26})$$

$$\psi(n_t) n_t v''(n_t) = v'(n_t) \quad (\text{A.27})$$

we see that  $MPC$  and  $MPN$  are related through

$$MPN = -\frac{\psi(n_0) n_0}{\sigma(c_0) c_0} MPC$$

Inserting into (A.24), this gives

$$MPC = \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} + \frac{\psi(n_0) w_0 n_0}{\sigma(c_0) c_0} \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \frac{w_t}{w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \right) \right)^{-1} \quad (\text{A.28})$$

as well as

$$\begin{aligned} MPS &= 1 - MPC + w_0 MPN \\ &= MPC \left( \sum_{t \geq 1} \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} \right. \\ &\quad \left. + \frac{\psi(n_0) w_0 n_0}{\sigma(c_0) c_0} \sum_{t \geq 1} \frac{q_t}{q_0} \frac{w_t}{w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \right) \end{aligned} \quad (\text{A.29})$$

Expressions (A.28) and (A.29) can also be rewritten using the fact that (A.21)-(A.22) together with (A.26)-(A.27) yield

$$\beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} = \frac{\sigma(c_t) c_t}{\sigma(c_0) c_0} \quad \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} = \frac{\psi(n_t) n_t}{\psi(n_0) n_0}$$

So, we also have

$$MPC = \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \frac{\sigma(c_t) c_t}{\sigma(c_0) c_0} + \frac{\psi(n_0) w_0 n_0}{\sigma(c_0) c_0} \left( 1 + \sum_{t \geq 1} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{\psi(n_t) n_t}{\psi(n_0) n_0} \right) \right)^{-1}$$

**Step 3: Hicksian elasticities.** The solution to the expenditure minimization problem in (A.6) also involves the first-order conditions (A.21)-(A.22), from which we obtain

$$u(c_t) = u \left( (u')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) u'(c_0) \right] \right) \quad v(n_t) = v \left( (v')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v'(n_0) \right] \right)$$

attaining utility  $U$  requires that the initial values  $c_0, n_0$  satisfy

$$\begin{aligned} & u(c_0) + \sum_{t \geq 1} \beta^t u \left( (u')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) u'(c_0) \right] \right) - v(n_0) \\ & - \sum_{t \geq 1} \beta^t v \left( (v')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v'(n_0) \right] \right) = U \end{aligned} \quad (\text{A.30})$$

Differentiating with respect to  $q_0$  along the indifference curve (A.30) results in

$$\begin{aligned} & \frac{\partial c_0}{\partial q_0} \left( u'(c_0) + \sum_{t \geq 1} \beta^t u'(c_t) \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} \right) \\ & - \frac{\partial n_0}{\partial q_0} \left( v'(n_0) + \sum_{t \geq 1} \beta^t v'(n_t) \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \right) \\ & - \sum_{t \geq 1} \beta^t \frac{u'(c_t)}{u''(c_t)} \left( \beta^{-t} \frac{q_t}{q_0^2} u'(c_0) \right) - \sum_{t \geq 1} \beta^t \frac{v'(n_t)}{v''(n_t)} \left( \beta^{-t} \frac{q_t}{q_0^2} \left( \frac{w_t}{w_0} \right) v'(n_0) \right) = 0 \end{aligned}$$

dividing by  $u'(c_0)$  and using (A.21), (A.25), (A.26) and (A.27) we find

$$\begin{aligned} & \frac{\partial c_0}{\partial q_0} \left( 1 + \sum_t \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} \right) - \frac{\partial n_0}{\partial q_0} w_0 \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \frac{w_t}{w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \right) \\ & = \frac{1}{u'(c_0)} \left( \sum_{t \geq 1} \beta^t \frac{u'(c_t)}{u''(c_t)} \left( \beta^{-t} \frac{q_t}{q_0^2} u'(c_0) \right) + \sum_{t \geq 1} \beta^t \frac{v'(n_t)}{v''(n_t)} \left( \beta^{-t} \frac{q_t}{q_0^2} \left( \frac{w_t}{w_0} \right) v'(n_0) \right) \right) \end{aligned}$$

moreover, differentiating (A.25) we also find

$$\frac{\partial n_0}{\partial q_0} = - \frac{\psi(n_0) n_0}{\sigma(c_0) c_0} \frac{\partial c_0}{\partial q_0}$$



Gathering results, we recognize, on the left-hand-side, the MPC expression in (A.28). We then use first-order conditions on the right hand side to obtain

$$\begin{aligned}\frac{\partial c_0}{\partial q_0} MPC^{-1} &= \frac{1}{u'(c_0)} \left\{ \sum_{t \geq 1} \beta^t \frac{u'(c_t)}{u''(c_t)} \left( \beta^{-t} \frac{q_t}{q_0^2} u'(c_0) \right) - \sum_{t \geq 1} \beta^t \frac{v'(n_t)}{v''(n_t)} \left( \beta^{-t} \frac{q_t}{q_0^2} \left( \frac{w_t}{w_0} \right) v'(n_0) \right) \right\} \\ &= \frac{1}{q_0} \left( \sum_{t \geq 1} \frac{u'(c_t)}{u''(c_t)} \frac{q_t}{q_0} - w_0 \sum_{t \geq 1} \frac{v'(n_t)}{v''(n_t)} \frac{q_t}{q_0} \left( \frac{w_t}{w_0} \right) \right)\end{aligned}$$

Manipulating the right-hand side, we recognize the expression for (A.29) as

$$\begin{aligned}\frac{\partial c_0}{\partial q_0} MPC^{-1} &= -\frac{1}{q_0} \sigma(c_0) c_0 \left\{ \sum_{t \geq 1} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u''(c_0)}{u''(c_t)} \frac{q_t}{q_0} \right. \\ &\quad \left. + \frac{w_0 n_0}{c_0} \frac{\psi(n_0)}{\sigma(c_0)} \sum_{t \geq 1} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \frac{q_t}{q_0} \left( \frac{w_t}{w_0} \right) \right\} \\ &= -\frac{1}{q_0} \sigma(c_0) c_0 \frac{MPS}{MPC}\end{aligned}$$

and therefore, we finally simply have

$$\left. \frac{\partial c_0}{\partial q_0} \right|_U = -\frac{c_0}{q_0} \sigma(c_0) MPS$$

which corresponds to a Hicksian elasticity of

$$\epsilon_{c_0, q_0}^h = -\sigma(c_0) MPS \tag{A.31}$$

A similar procedure can be used to differentiate with respect to  $w_0$ : from (A.25) we obtain

$$\frac{\partial n_0}{\partial w_0} = -\frac{\psi(n_0)}{\sigma(c_0)} \frac{n_0}{c_0} \frac{\partial c_0}{\partial w_0} + \psi(n_0) \frac{n_0}{w_0}$$

and differentiating along (A.30) we therefore obtain

$$\begin{aligned}\frac{\partial c_0}{\partial w_0} u'(c_0) MPC^{-1} - \psi(n_0) \frac{n_0}{w_0} \left( v'(n_0) + \sum_{t \geq 1} \beta^t v'(n_t) \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v''(n_0)}{v''(n_t)} \right) \\ = \sum_{t \geq 1} \beta^t \frac{v'(n_t)}{v''(n_t)} \beta^{-t} \frac{q_t}{q_0} \left( \frac{w_t}{w_0} \right) v'(n_0)\end{aligned}$$

We conclude by noticing that  $v'(n_0) = \psi(n_0) n_0 v''(n_0)$ , so

$$\left. \frac{\partial c_0}{\partial w_0} \right|_U = MPC \psi(n_0) n_0$$

and

$$\epsilon_{c_0, w_0}^h = MPC \left( \psi(n_0) \frac{w_0 n_0}{c_0} \right) \tag{A.32}$$

Finally, elasticities for  $n_0$  result from a final differentiation of (A.25):

$$\epsilon_{n_0, q_0}^h = -\frac{\psi(n_0)}{\sigma(c_0)} \epsilon_{c_0, q_0}^h \quad (\text{A.33})$$

$$\begin{aligned} \epsilon_{n_0, w_0}^h &= \psi(n_0) \left( 1 - \frac{1}{\sigma(c_0)} \epsilon_{c_0, w_0}^h \right) \\ &= \psi(n_0) \left( 1 - \frac{\psi(n_0)}{\sigma(c_0)} \frac{w_0 n_0}{c_0} MPC \right) \\ &= \psi(n_0) (1 + w_0 MPN) \end{aligned} \quad (\text{A.34})$$

**Step 4: Putting all expressions together.** For consumption, equations (A.31)–(A.32) can be inserted into (A.19) to yield

$$dc = MPC \left( ndw + dy + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + c \left( -\sigma MPS \frac{dR}{R} + \psi MPC \frac{wn}{c} \frac{dw}{w} \right)$$

The first term is the wealth effect, and the last two terms the substitution effects with respect to interest rates and wages. We then simplify the expression to

$$dc = MPC \left( dy + n(1 + \psi) dw + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) - \sigma c MPS \frac{dR}{R} \quad (\text{A.35})$$

which is our equation (3).

Similarly, equations (A.33)–(A.34) can be inserted into (A.20) to yield

$$dn = MPN \left( ndw + dy + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + n \left( \psi MPS \frac{dR}{R} + \psi(1 + w MPN) \frac{dw}{w} \right)$$

and we again naturally separate the latter piece to obtain

$$dn = MPN \left( dy + n(1 + \psi) dw + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + \psi n MPS \frac{dR}{R} + \psi n \frac{dw}{w} \quad (\text{A.36})$$

which is equation (4).

### A.3 Extension of Theorem 1 to general preferences and persistent changes

Theorem 1 in the main text is a special case of a general decomposition that holds for arbitrary nonsatiable preferences  $U$  over  $\{c_t\}$  and  $\{n_t\}$  and for any change in the price level  $\{P_0, P_1 \dots\}$ , the real term structure  $\{q_0 = 1, q_1, q_2 \dots\}$ , the agent's unearned income sequence  $\{y_0, y_1 \dots\}$  and the stream of real wages  $\{w_0, w_1 \dots\}$ , with the nominal term structure adjusting instantaneously to make the Fisher equation hold at the post-shock sequences of interest rates and prices. The utility maximization problem is then

$$\begin{aligned} \max \quad & U(\{c_t, n_t\}) \\ \text{s.t.} \quad & P_t c_t = P_t y_t + W_t n_t + ({}_{t-1}B_t) + \sum_{s \geq 1} ({}_t Q_{t+s}) ({}_{t-1}B_{t+s} - {}_t B_{t+s}) \\ & + P_t ({}_{t-1}b_t) + \sum_{s \geq 1} ({}_t q_{t+s}) P_{t+s} ({}_{t-1}b_{t+s} - {}_t b_{t+s}) \end{aligned}$$

and the first order date-0 responses of consumption, labor supply and welfare to the considered change are, in this case, given by

$$\begin{aligned} dc_0 &= MPCd\Omega + c_0 \left( \sum_{t \geq 0} \epsilon_{c_0, q_t}^h \frac{dq_t}{q_t} + \sum_{t \geq 0} \epsilon_{c_0, w_t}^h \frac{dw_t}{w_t} \right) \\ dn_0 &= MPNd\Omega + n_0 \left( \sum_{t \geq 0} \epsilon_{n_0, q_t}^h \frac{dq_t}{q_t} + \sum_{t \geq 0} \epsilon_{n_0, w_t}^h \frac{dw_t}{w_t} \right) \\ dU &= U_{c_0} d\Omega \end{aligned}$$

where  $\epsilon_{x_0, y_t}^h = \frac{\partial x_0^h}{\partial y_t} \frac{y_t}{x_0}$  for  $x \in \{c, n\}$  and  $y \in \{q, w\}$  are Hicksian elasticities and  $d\Omega = dW - \sum_{t \geq 0} c_t dq_t$ , the net-of-consumption wealth change, is given by

$$\begin{aligned} d\Omega &= \underbrace{\sum_{t \geq 0} (q_t y_t) \frac{dy_t}{y_t}}_{\text{Real unearned income change}} + \underbrace{\sum_{t \geq 0} (q_t w_t n_t) \frac{dw_t}{w_t}}_{\text{Real earned income change}} \\ &+ \underbrace{\sum_{t \geq 0} q_t \left( y_t + w_t n_t + \left( \frac{-1B_t}{P_t} \right) + (-1b_t) - c_t \right) \frac{dq_t}{q_t}}_{\text{Revaluation of net savings flows}} - \underbrace{\sum_{t \geq 0} Q_t \left( \frac{-1B_t}{P_0} \right) \frac{dP_t}{P_t}}_{\text{Revaluation of net nominal position}} \quad (\text{A.37}) \end{aligned}$$

The proof is a generalization of that in section A.2. I omit it here in the interest of space.

**Values of all elasticities with separable preferences in a steady-state with no growth.** Following once more the steps of section A.2, it is possible to derive the value of Hicksian elasticities for a change at any horizon. Here I just report the values of these elasticities in the case of an infinite horizon model where  $\frac{q_s}{q_0} = \beta^s$  and  $w_s = w^*$ ,  $\forall s$ . These prices correspond to those prevailing in a steady-state with no growth of any such model, and the resulting elasticities are relevant, for example, to determine the impulse responses in many RBC and DSGE models. The first order conditions imply that consumption and labor supply are constant. Let us call the solutions  $c^*$  and  $n^*$ , respectively. Writing  $\vartheta \equiv \frac{w^* n^*}{c^*}$  for the share of earned income in consumption and  $\kappa \equiv \frac{\frac{\psi}{\sigma} \vartheta}{1 + \frac{\psi}{\sigma} \vartheta} \in (0, 1)$ , obtain values of elasticities summarized in table A.1.

Table A.1: Steady-state moments, separable preferences

$\epsilon^h$	$q_0$	$q_{s, s \geq 1}$	$w_0$	$w_{s, s \geq 1}$	Marg. propensity
$c_0$	$-\sigma\beta$	$\sigma(1-\beta)\beta^s$	$\sigma\kappa(1-\beta)$	$\sigma\kappa(1-\beta)\beta^s$	$MPC \quad (1-\kappa)(1-\beta)$
$n_0$	$\psi\beta$	$-\psi(1-\beta)\beta^s$	$\psi(1-\kappa(1-\beta))$	$-\psi\kappa(1-\beta)\beta^s$	$MPN \quad -\frac{1}{w^*}\kappa(1-\beta)$
					$MPS \quad (1-\beta)$

## A.4 Proof of corollary 1

Rewrite equations (A.35) and (A.36) as

$$\begin{aligned} dc &= MPC \left( dY + \psi dw - wdn + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) - \sigma c MPS \frac{dR}{R} \\ wdn - \psi ndw &= wMPN \left( dY + \psi dw - wdn + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + \psi wn MPS \frac{dR}{R} \end{aligned}$$

Hence

$$wdn - \psi ndw = \frac{1}{1 + wMPN} \left\{ wMPN \left( dY + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) + \psi wn MPS \frac{dR}{R} \right\}$$

which, inserted into the expression for  $dc$  yields

$$dc = MPC \left( 1 - \frac{wMPN}{1 + wMPN} \right) \left( dY + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) - \sigma c MPS \left( 1 + MPC \frac{\psi wn}{\sigma c} \frac{1}{1 + wMPN} \right) \frac{dR}{R}$$

But  $MPC \frac{\psi n}{\sigma c} = -MPN$  so this is

$$dc = \left( \frac{MPC}{1 + wMPN} \right) \left( dY + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) - \sigma c \frac{MPS}{1 + wMPN} \frac{dR}{R}$$

and noting that

$$1 + wMPN = MPC + MPS$$

we can finally rewrite this in terms of  $M\hat{P}C = \frac{MPC}{MPC + MPS}$  as

$$dc = M\hat{P}C \left( dY + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) - \sigma c (1 - M\hat{P}C) \frac{dR}{R}$$

as claimed.

## A.5 Adding durable goods

This section shows the consequences of adding durable goods to the model.

I consider a standard durable goods problem. For simplicity, I ignore labor supply and nominal assets, neither of which interacts with the conclusions below. A consumer maximizes a separable intertemporal utility function

$$\begin{aligned} \max \quad & \sum \beta^t \{u(C_t) + w(D_t)\} \\ \text{s.t.} \quad & C_t + p_t I_t = Y_t + ({}_{t-1}b_t) + \sum_{s \geq 1} ({}_t q_{t+s}) ({}_{t-1}b_{t+s} - {}_t b_{t+s}) \\ & D_t = I_t + D_{t-1} (1 - \delta) \\ & D_{-1}, \{{}_{-1}b_t\} \quad \text{given} \end{aligned}$$

where  $C_t$  is now *nondurable* consumption,  $D_t$  is the consumer's stock of durables, and  $p_t$  is the relative price of durable goods in period  $t$ .

I am interested in the response of the demand for nondurable goods  $C_t$  and durables goods  $I_t$ , as well as that of total expenditures

$$X_t \equiv C_t + p_t I_t \quad (\text{A.38})$$

to a change in the time-0 nondurable real interest rate  $R_0$  and (potentially) a simultaneous change in the price of durables  $p_0$ . As I argue below, the notion of *aggregate demand* makes most sense when the relative price of durables does not change with  $R_0$ , but I start by covering the general case in which  $p_0$  can change.

The intertemporal budget constraint reads

$$\sum_{t \geq 0} q_t (C_t + p_t I_t) = \sum_{t \geq 0} q_t Y_t + \sum_{t \geq 0} q_t (-_1 b_t)$$

Defining  $R_t \equiv \frac{q_t}{q_{t+1}}$ , the first-order conditions of this problem are, for all  $t \geq 0$

$$u'(C_t) = \beta R_t u'(C_{t+1}) \quad (\text{A.39})$$

$$w'(D_t) = u'(C_t) \left[ p_t - \frac{(1-\delta)p_{t+1}}{R_t} \right] \quad (\text{A.40})$$

Equation (A.39) is the standard Euler equation for nondurable consumption. Equation (A.40) shows that the consumer equates the marginal rate of substitution between the stock of durables and consumption to the user cost of durables,  $p_t - \frac{(1-\delta)p_{t+1}}{R_t}$ . A fall in the nondurable real interest rate at date 0,  $R_0$ , increases the desired level of nondurable consumption and of the stock of durables (an intertemporal substitution effect). Holding  $p_1$  constant, it also reduces the user cost of durables, increasing the desired stock of durables relative to nondurable consumption. A fall in  $p_0$  has the same effect of reducing the durable user cost, but it does not affect intertemporal substitution in consumption.

Suppose that the path for interest rates  $\{R_t\}$ , relative prices  $\{p_t\}$  and income  $\{Y_t\}$  delivers the solution  $\{C_t, D_t\}$ . Consider the solution under the alternative paths  $\{\bar{R}_0, R_1, R_2 \dots\}$ ,  $\{\bar{p}_0, p_1, p_2 \dots\}$ , and  $\{\bar{Y}_0, Y_1, Y_2 \dots\}$ . Let  $dR = \bar{R}_0 - R_0$ ,  $dp = \bar{p}_0 - p_0$  and  $dY = \bar{Y}_0 - Y_0$ . I am interested in the response of the paths of nondurable and durable expenditures to these changes. To obtain this, I find the paths for consumption  $\{C_t\}$  and durables  $\{D_t\}$ , and then find the implied path for durable expenditures  $\{p_t I_t\}$ .

**Marshallian demand.** In order to determine the Marshallian demands, I could follow the same proof as that of section A.2, but here I follow an alternative and somewhat more intuitive procedure. The procedure is in two steps. First, I determine a variation that respects all the first-order conditions (A.39)–(A.40) at the new prices. This gives  $dC^*$  and  $dD^*$ , which result in a budgetary cost  $d\Omega^*$  at the old prices. Second, I determine the change in net wealth  $d\Omega$  that results from the change in prices. The Marshallian demands are then

$$dC = dC^* + MPC (d\Omega - d\Omega^*) \quad (\text{A.41})$$

$$dD = dD^* + MPD (d\Omega - d\Omega^*) \quad (\text{A.42})$$

where  $MPD = \frac{\partial D}{\partial Y}$  is the increase in the stock of date-0 durables that results from a date-0 increase in income. Note that  $MPC$  and  $MPD$  are related: differentiating (A.40), we find

$$w''(D_0) MPD = u''(C_0) MPC \left[ p_0 - \frac{(1-\delta)p_1}{R_0} \right]$$

so

$$MPD = \frac{\sigma_D}{\sigma_C} \frac{D_0}{C_0} MPC$$

where  $\sigma_C \equiv -\frac{u'(C_0)}{u''(C_0)C_0}$  and  $\sigma_D \equiv -\frac{w'(D_0)}{w''(D_0)D_0}$  are the elasticities of intertemporal substitution in consumption and in the stock of durables. Since  $D_0 = I_0 + D_{-1}(1 - \delta)$  and the initial stock  $D_{-1}$  is fixed, the total constant- $p$  marginal propensity to spend at date 0 is

$$\begin{aligned} MPX &\equiv \frac{\partial(C + pI)}{\partial Y} = \frac{\partial C}{\partial Y} + p \frac{\partial D}{\partial Y} = MPC + pMPD \\ &= MPC \left( 1 + \frac{\sigma_D}{\sigma_C} \frac{pD}{C} \right) \end{aligned}$$

**Step 1: variation respecting FOCs.** The simplest variation that respects all FOCs holds the paths  $\{C_t\}$  and  $\{D_t\}$  fixed for all  $t \geq 1$  and adjusts  $C_0$  and  $D_0$  by  $dC$  (respectively  $dD$ ) such that (A.39) and (A.40) are satisfied at  $t = 0$ . Differentiating these equations, I obtain

$$\begin{aligned} -\frac{1}{\sigma_C} \frac{dC}{C} &= \frac{dR}{R} \\ -\frac{1}{\sigma_D} \frac{dD}{D} &= -\frac{1}{\sigma_C} \frac{dC}{C} + \frac{p_1 \frac{1-\delta}{R}}{p_0 - p_1 \frac{1-\delta}{R}} \frac{dR}{R} + \frac{p_0}{p_0 - p_1 \frac{1-\delta}{R}} \frac{dp}{p} \end{aligned}$$

Hence we find

$$dC^* = -\sigma_C C \frac{dR}{R} \tag{A.43}$$

and

$$dD^* = -\sigma_D D \left[ \frac{p_0}{p_0 - p_1 \frac{1-\delta}{R}} \right] \left( \frac{dR}{R} + \frac{dp}{p} \right) \tag{A.44}$$

These responses are very intuitive: one way to respond to a fall in real interest rates is to raise nondurable consumption and the stock of durables. The relevant elasticity for durables is higher than  $\sigma_D$  because of the additional substitution effect coming from the change in the user cost. A lower current relative price of durables has a symmetric effect on the demand for durables as that of a lower real interest rate (in other words, it is the real interest rate in terms of durables that matters for durables demand).

We are now ready to determine the net cost of this variation. Since

$$\begin{aligned} D_0 &= (1 - \delta) D_{-1} + I_0 \\ D_1 &= (1 - \delta) D_0 + I_1 \end{aligned}$$

the sequence of investment that achieves this variation consists naturally in an increase of  $dD^*$  followed by a subsequent decrease:

$$\begin{aligned} dI_0^* &= dD^* \\ dI_1^* &= -(1 - \delta) dD^* \end{aligned}$$

Hence the total budgetary cost of this 'star' variation at the old prices  $p$  and  $R$  has the simple form

$$\begin{aligned}
d\Omega^* &= dC^* + p_0 dI_0^* + p_1 \frac{dI_1^*}{R} \\
&= dC^* + \left( p_0 - p_1 \frac{1-\delta}{R} \right) dD^* \\
&= -(\sigma_C C + p_0 \sigma_D D) \frac{dR}{R} - \sigma_D p_0 D \frac{dp}{p}
\end{aligned}$$

**Step 2: change in net wealth.** Let  $\Omega$  be defined as

$$\Omega \equiv \sum_{t \geq 0} q_t \{Y_t + (-_1 b_t) - C_t - p_t I_t\}.$$

At the initial prices, the intertemporal budget constraint implies  $\Omega = 0$ . The exogenous variation  $dR$ ,  $dp$  and  $dY$  yields

$$\begin{aligned}
d\Omega &= dY - Idp + \sum_{t \geq 0} dq_t \{Y_t + (-_1 b_t) - C_t - p_t I_t\} \\
&= dY - Idp - \sum_{t \geq 1} q_t \{Y_t + (-_1 b_t) - C_t - p_t I_t\} \frac{dR}{R} \\
&= dY - pI_0 \frac{dp}{p} + \left( \underbrace{Y_0 + (-_1 b_0) - C_0 - p_0 I_0}_{URE} \right) \frac{dR}{R}
\end{aligned} \tag{A.45}$$

The intuition is as follows. Suppose that the nondurable real interest rate falls at date 0. As before, this benefits consumers that have a negative  $URE$ , that is, maturing liabilities  $C_0 + p_0 I_0$  in excess maturing assets  $Y_0 + (-_1 b_0)$ . Note that, for this effect, total expenditures *including expenditures on durables* are counted as part of  $URE$ . In that sense,  $URE$  measures the true balance-sheet exposure to a change in the real interest rate. In particular, ceteris paribus, when investment is higher today the consumer benefits more from a fall in real interest rates.

Suppose however that, in parallel, the relative price of durables rises. In the general equilibrium model of [Barsky, House and Kimball \(2007\)](#), for example, this happens in response to an accommodative monetary policy shock when durable goods prices are more flexible than nondurable goods prices. In that case, equation (A.45) shows that there is an additional capital loss on wealth due to the rise in the durable relative price. While conceptually distinct, these two effects could be consolidated into a single one, if we restrict ourselves to variations that feature a constant elasticity of the durable-good price to the nondurable real interest rate

$$\epsilon_{pR} \equiv -\frac{\partial p}{p} \frac{R}{\partial R} \tag{A.46}$$

The benchmark case where  $p$  is constant corresponds to  $\epsilon_{pR} = 0$ , the case where the durable real interest rate is constant to  $\epsilon_{pR} = 1$ . Then,

$$d\Omega = dY + \left( \underbrace{Y_0 + (-_1 b_0) - C_0 - p_0 I_0 (1 - \epsilon_{pR})}_{URE^\epsilon} \right) \frac{dR}{R} \tag{A.47}$$



In other words, once we net out the capital revaluation effect, an alternative measure of  $URE$  becomes  $URE^\epsilon$ , which subtracts a fraction  $(1 - \epsilon_{pR})$  of durable expenditures.

**Step 3: demand for durables and nondurables.** Combining (A.41)–(A.42) with (A.43), (A.44) and (A.45), I obtain the Marshallian demands (recall that  $dI = dD$  at time 0)

$$\begin{aligned} dC &= MPC \left( dY + URE \frac{dR}{R} + (\sigma_C C + p\sigma_D D) \frac{dR}{R} + (p\sigma_D D - pI_0) \frac{dp}{p} \right) - \sigma_C C \frac{dR}{R} \\ dD &= MPD \left( dY + URE \frac{dR}{R} + (\sigma_C C + p\sigma_D D) \frac{dR}{R} + (p\sigma_D D - pI_0) \frac{dp}{p} \right) \\ &\quad - \sigma_D D \left[ \frac{p_0}{p_0 - p_1 \frac{1-\delta}{R}} \right] \left( \frac{dR}{R} + \frac{dp}{p} \right) \end{aligned}$$

This separates out the separate effects from changing  $R$  and  $p$ . Given the elasticity  $\epsilon_{pR}$  in (A.46), we can also rewrite this as

$$\begin{aligned} dC &= MPC \left( dY + URE^\epsilon \frac{dR}{R} \right) - \sigma_C C (1 - MPC) \frac{dR}{R} \\ &\quad + \sigma_D \cdot pD \cdot MPC \cdot (1 - \epsilon_{pR}) \cdot \frac{dR}{R} \end{aligned} \tag{A.48}$$

$$\begin{aligned} dD &= MPD \left( dY + URE^\epsilon \frac{dR}{R} \right) + \sigma_C \cdot MPD \cdot C \cdot \frac{dR}{R} \\ &\quad - \sigma_D \cdot pD \cdot (1 - \epsilon_{pR}) \cdot (1 - MPD) \cdot \left[ \frac{1}{p_0 - p_1 \frac{1-\delta}{R}} \right] \frac{dR}{R} \end{aligned} \tag{A.49}$$

Where  $URE^\epsilon$  is defined in (A.47).

**Special case with constant durable real interest rate ( $\epsilon_{pR} = 1$ ).** When  $\epsilon_{pR} = 1$ , equations (A.48)–(A.49) simplify to

$$\begin{aligned} dC &= MPC \left( dY + URE^1 \frac{dR}{R} \right) - \sigma_C C (1 - MPC) \frac{dR}{R} \\ dD &= MPD \left( dY + URE^1 \frac{dR}{R} \right) + \sigma_C \cdot MPD \cdot C \cdot \frac{dR}{R} \end{aligned}$$

which are simple extensions of expressions in the main text, with  $URE^1$  (which does not subtract durable expenditures) replacing  $URE$ . Note that to the extent that  $URE^1 \geq 0$ , the expression for  $dD$  implies a contraction in durable goods from an increase in real interest rates, as in Barsky, House and Kimball (2007). This is counterfactual, suggesting that  $\epsilon_{pR} = 1$  may be too high an elasticity in practice.

**Special case with constant relative price ( $\epsilon_{pR} = 0$ ).** While the cases where  $\epsilon_{pR} \neq 0$  are interesting in principle, they prevent a straightforward definition of *aggregate demand*  $X = C + pI$ : if the relative price of two goods can change, then the relative demands for these two goods (as well as their relative supplies) will matter for general equilibrium. Therefore, the case where  $\epsilon_{pR} = 0$  is the most relevant for my purposes. Assume then that  $p_0 = p_1 = p$ . In this case, we can combine (A.48) and (A.49) to obtain an expression for the change in aggregate demand  $dX = dC + pdD$  as a function of the marginal

propensity to spend  $MPX = MPC + pMPD$  and other variables

$$dX = MPX \left( dY + URE \frac{dR}{R} + \sigma_C C + \sigma_D pD \right) - \left( \sigma_C C + \frac{\sigma_D pD}{1 - \frac{1}{R\delta}} \right) \frac{dR}{R}$$

This can further be simplified to yield an expression with the same form as the expression in the main text,

$$dX = MPX \left( dY + URE \frac{dR}{R} \right) - \sigma_X (1 - MPX) X \frac{dR}{R} \quad (\text{A.50})$$

where  $\sigma_X$  is defined as

$$\sigma_X \equiv \frac{C}{X} \cdot \sigma_C + \left( 1 - \frac{C}{X} \right) \cdot \sigma_D \cdot \frac{pD}{pI} \cdot \frac{\frac{1}{1 - \frac{1}{R\delta}} - MPX}{1 - MPX} \quad (\text{A.51})$$

In other words,  $\sigma_X$  is a weighted average of  $\sigma_C$  and the relevant elasticity of substitution in durable expenditures: the product of  $\sigma_D$  by the stock-flow ratio  $\frac{pD}{pI}$ , multiplied by a term that increases in the elasticity of the user cost to the real interest rate.

Quantitatively, the second term is likely to be much larger than the first. If initially durable expenditures cover replacement costs  $I = D\delta$ , then the stock-flow ratio is  $\frac{1}{\delta}$ . Hence, with  $\delta = 5\%$  and  $R = 1.05$  at annual rates, the second term in (A.51) is at least as large as  $\frac{1}{20} \times \frac{1}{10} \times \sigma_D = 200\sigma_D$ . This makes aggregate demand very sensitive to given changes in the real interest rate because of the large substitution effect that results from the presence of long-lived durables, a point made by Barsky et al. (2007).

## A.6 Proof of theorem 2

After dividing through by  $P_t$ , defining the real bond position as  $\lambda_t \equiv \frac{\Lambda_t}{P_{t-1}}$  and writing  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  for the inflation rate between  $t - 1$  and  $t$ , the budget constraint (9) becomes

$$c_t + Q_t \left( \lambda_{t+1} - \delta \frac{\lambda_t}{\Pi_t} \right) + (\theta_{t+1} - \theta_t) \cdot \mathbf{S}_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot \mathbf{d}_t$$

In this notation, the consumer's date- $t$  net nominal position is

$$NNP_t = (1 + Q_t \delta) \frac{\lambda_t}{\Pi_t}$$

while his unhedged interest rate exposure is:

$$URE_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot \mathbf{d}_t - c_t = Q_t \left( \lambda_{t+1} - \delta \frac{\lambda_t}{\Pi_t} \right) + (\theta_{t+1} - \theta_t) \cdot \mathbf{S}_t$$

His optimization problem can be represented using the recursive formulation

$$\begin{aligned} & \max_{c,n,\lambda',\theta'} u(c) - v(n) + \underbrace{\beta \mathbb{E} [V(\lambda', \theta'; y', w', Q', \Pi', \mathbf{d}', \mathbf{S}')] ]}_{\equiv W(\lambda', \theta')} \\ \text{s.t.} \quad & c + Q \left( \lambda' - \delta \frac{\lambda}{\Pi} \right) + (\theta' - \theta) \mathbf{S} = y + wn + \frac{\lambda}{\Pi} + \theta \mathbf{d} \\ & Q\lambda' + \theta' \mathbf{S} \geq \frac{\bar{D}}{R} \end{aligned} \quad (\text{A.52})$$

The function  $V$  corresponds to the value from optimizing given a starting real level of bonds  $\lambda'$  and shares  $\theta'$ , and includes the possibility of hitting future borrowing constraints.

I consider the predicted effects on  $c$  and  $n$  resulting from a simultaneous unexpected change in unearned income  $dy$ , the real wage  $dw$ , the price level  $\frac{dP}{P} = \frac{d\Pi}{\Pi}$  and the real interest rate  $dR$ , which result in a change in asset prices  $\frac{dQ}{Q} = \frac{dS_j}{S_j} = -\frac{dR}{R}$  for  $j = 1 \dots N$ . By leaving the future unaffected, this purely transitory change does not alter the value from future optimization starting at  $(\lambda', \theta')$ — that is, the function  $W$  is unchanged. I claim that, provided the consumption and labor supply functions are differentiable, their first order differentials are

$$dc = \text{MPC} \left( dy + n(1 + \psi) dw + \text{URE} \frac{dR}{R} - \text{NNP} \frac{dP}{P} \right) - \sigma c \text{MPS} \frac{dR}{R} \quad (\text{A.53})$$

$$dn = \text{MPN} \left( dy + n(1 + \psi) dw + \text{URE} \frac{dR}{R} - \text{NNP} \frac{dP}{P} \right) + \psi n \text{MPS} \frac{dR}{R} + \psi n \frac{dw}{w} \quad (\text{A.54})$$

where  $\sigma \equiv -\frac{u'(c)}{cu''(c)}$  and  $\psi = \frac{v'(n)}{nv''(n)}$  are the local elasticities of intertemporal substitution and labor supply, respectively,  $\text{MPC} = \frac{\partial c}{\partial y}$ ,  $\text{MPN} = \frac{\partial n}{\partial y}$  and  $\text{MPS} = 1 - \text{MPC} + w\text{MPN}$ .

In order to prove (A.53) and (A.54), there are two cases to consider. In the first case, the consumer is at a binding borrowing limit or lives hand-to-mouth. The problem is then a static choice between  $c$  and  $n$ . In the second case, the consumer is at an interior optimum. The result then follows from application of the implicit function theorem to the set of  $N + 2$  first-order conditions which, together with the budget constraint, characterize the solution to the problem in (A.52). Here, to simplify the notation and the proof, I first prove the statement in the case where all variables are changing but  $N = 0$ , and then consider the case with stocks ( $N > 0$ ) but without bonds and assuming only  $R$  is changing.

### Case 1. Binding borrowing limit and hand-to-mouth agents.

*Proof.* The consumption of an agent at the borrowing limit is given by

$$c = wn + Z \quad (\text{A.55})$$

where

$$Z = z + (1 + Q\delta) \frac{\lambda}{\Pi} + \theta \cdot (\mathbf{d} + \mathbf{S}) + \frac{\bar{D}}{R}$$

Similarly, the consumption of an agent that lives hand to mouth is

$$c = wn + z$$

Given that  $d\mathbf{S} = -\frac{\mathbf{S}}{R}dR$ ,  $dQ = -\frac{Q}{R}dR$  and  $d\left(\frac{1}{\Pi}\right) = -\frac{1}{\Pi^2}d\Pi = -\frac{1}{\Pi} \frac{dP}{P}$ , we have, if the agent is at the

borrowing limit

$$dZ = dz - \underbrace{(1 + Q\delta) \frac{\lambda}{\Pi} \frac{dP}{P}}_{\text{NNP}} + \underbrace{\left( Q\delta \frac{\lambda}{\Pi} + \theta \cdot \mathbf{S} + \frac{\bar{D}}{R} \right)}_{\text{-URE}} \left( -\frac{dR}{R} \right) \quad (\text{A.56})$$

and, if the agent lives hand to mouth,

$$dZ = dz$$

but since that agent also has

$$\text{NNP} = \text{URE} = 0$$

equation (A.56) still applies. In both cases, the consumer is making a static choice between  $c$  and  $n$  given the budget constraint (A.55), and hence has  $\text{MPS} = 0$ . We can then apply the results of section A.2 to find

$$\begin{aligned} dc &= \text{MPC} (dZ + w(1 + \psi)) \\ dn &= \text{MPN} (dZ + w(1 + \psi)) + \psi ndw \end{aligned}$$

which yields the desired result. □

**Case 2a).  $N = 0$ , all variables changing** I first prove the following lemma.

**Lemma A.1.** *Let  $c(z, w, q, b)$  and  $n(z, w, q, b)$  be the solution to the following separable consumer choice problem under concave preferences over current consumption  $u(c)$  and assets  $V(a)$ , and convex preferences over hours worked  $v(n)$ :*

$$\begin{aligned} \max \quad & u(c) - v(n) + V(a) \\ \text{s.t.} \quad & c + q(a - b) = wn + z \end{aligned}$$

Assume  $c(\cdot)$  and  $n(\cdot)$  are differentiable. Then the first order differentials are

$$\begin{aligned} dc &= \text{MPC} (dz + n(1 + \psi)dw - (a - b)dq + qdb) - \sigma c \text{MPS} \frac{dq}{q} \\ dn &= \text{MPN} (dz + n(1 + \psi)dw - (a - b)dq + qdb) + \psi n \text{MPS} \frac{dq}{q} + \psi n \frac{dw}{w} \end{aligned}$$

where  $\text{MPC} = \frac{\partial c}{\partial z}$ ,  $\text{MPN} = \frac{\partial n}{\partial z}$  and  $\text{MPS} = 1 - \text{MPC} + w\text{MPN} = 1 - \text{MPC} \left( 1 + \frac{wn}{c} \frac{\psi}{\sigma} \right)$ .

*Proof.* The following first-order conditions are necessary and sufficient for optimality:

$$u'(c) = \frac{1}{w} v'(n) = \frac{1}{q} V'(a) \quad (\text{A.57})$$

I first obtain the expression for  $\text{MPC}$  by considering an increase in income  $dz$  alone. Consider how that increase is divided between current consumption, leisure and assets. (A.57) implies

$$u''(c) dc = \frac{1}{w} v''(n) dn = \frac{1}{q} V''(a) da \quad (\text{A.58})$$

where the changes  $dc$ ,  $dn$  and  $da$  are related to  $dz$  through the budget constraint

$$dc + qda = wdn + dz \quad (\text{A.59})$$

Define  $MPC = \frac{\partial c}{\partial z}$ ,  $MPN = \frac{\partial n}{\partial z}$  and  $MPS = q \frac{\partial a}{\partial z}$ . Then (A.58) implies

$$\begin{aligned} \frac{MPN}{MPC} &= w \frac{u''(c)}{v''(n)} = \frac{u''(c)}{u'(c)} \frac{v'(n)}{v''(n)} = -\frac{n}{c} \psi \\ \frac{MPS}{MPC} &= \frac{q^2 u''(c)}{V''(a)} = \frac{q}{c} \frac{V'(a)}{\sigma V''(a)} \end{aligned}$$

where  $\sigma \equiv -\frac{u'(c)}{cu''(c)}$  and  $\psi \equiv \frac{v'(n)}{nv''(n)}$ . Hence the total marginal propensity to spend is

$$1 - MPS = \frac{\partial c}{\partial z} - w \frac{\partial n}{\partial z} = MPC \left( 1 + \frac{wn}{c} \frac{\psi(n)}{\sigma(c)} \right) = 1 - \frac{q^2 u''(c)}{V''(a)} MPC \quad (\text{A.60})$$

and the marginal propensity to consume is

$$MPC = \frac{1}{1 + q^2 \frac{u''(c)}{V''(a)} - w^2 \frac{u''(c)}{v''(n)}} = \frac{V''(a) v''(n)}{V''(a) v''(n) + q^2 u''(c) v''(n) - w^2 u''(c) V''(a)}$$

Consider now the overall effect on  $c$ ,  $n$  and  $a$  of a change in  $q$ ,  $w$ ,  $z$  and  $b$ . Applying the implicit function theorem to the system of equations

$$\begin{cases} v'(n) - wu'(c) = 0 \\ V'(a) - qu'(c) = 0 \\ c + q(a - b) - wn - z = 0 \end{cases}$$

results in the following expression for partial derivatives:

$$\begin{aligned} & \begin{bmatrix} \frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \\ \frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \\ \frac{\partial a}{\partial q} & \frac{\partial a}{\partial z} & \frac{\partial a}{\partial w} & \frac{\partial a}{\partial b} \end{bmatrix} \\ &= - \underbrace{\begin{bmatrix} -wu''(c) & v''(n) & 0 \\ -qu''(c) & 0 & V''(a) \\ 1 & -w & q \end{bmatrix}}_{\equiv A}^{-1} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix} \quad (\text{A.61}) \end{aligned}$$

now

$$\det(A) = v''(n) V''(a) - w^2 u''(c) V''(a) + q^2 u''(c) v''(n) = \frac{V''(a) v''(n)}{MPC}$$

and so

$$A^{-1} = \frac{MPC}{V''(a) v''(n)} \begin{bmatrix} wV''(a) & -v''(n)q & v''(n)V''(a) \\ q^2 u''(c) + V''(a) & -wqu''(c) & wu''(c)V''(a) \\ qu''(c) & w^2 u''(c) - v''(n) & qu''(c)v''(n) \end{bmatrix}$$

therefore, the first row of (A.61)

$$\begin{bmatrix} \frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \end{bmatrix} = MPC \begin{bmatrix} -\frac{w}{v''(n)} & \frac{q}{V''(a)} & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix} \quad (\text{A.62})$$

Using (A.60) we find

$$-q \frac{u'(c)}{V''(a)} MPC = \frac{\sigma c}{q} q^2 \frac{u''(c)}{V''(a)} MPC = \frac{\sigma c}{q} MPS$$

so that the first column of the matrix equation (A.62) reads

$$\frac{\partial c}{\partial q} = \frac{\sigma c}{q} MPS - (a-b) MPC$$

The second and fourth column of (A.62) yield directly

$$\begin{aligned} \frac{\partial c}{\partial z} &= MPC \\ \frac{\partial c}{\partial b} &= qMPC \end{aligned}$$

Finally, using (A.57) we have

$$w \frac{u'(c)}{v''(n)} = \frac{v'(n)}{v''(n)} = \psi n$$

so that the third column of (A.62) reads

$$\begin{aligned} \frac{\partial c}{\partial w} &= MPC\psi n + MPCn \\ &= MPC(1 + \psi)n \end{aligned}$$

The first-order total differential  $dc$  is then

$$\begin{aligned} dc &= \frac{\partial c}{\partial z} dz + \frac{\partial c}{\partial b} db + \frac{\partial c}{\partial q} dq + \frac{\partial c}{\partial w} dw \\ &= MPC(dz + qdb - (a-b)dq + (1 + \psi)ndw) + \sigma c MPS \frac{dq}{q} \end{aligned} \quad (\text{A.63})$$

as claimed. Similarly, after using  $MPN = MPCw \frac{u''(c)}{v''(n)}$ , the second row of (A.61) is

$$\begin{bmatrix} \frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \end{bmatrix} = MPN \begin{bmatrix} -\frac{q^2 + v''(a)/u''(c)}{wV''(a)} & \frac{q}{V''(a)} & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -u'(c) & 0 \\ -u'(c) & 0 & 0 & 0 \\ (a-b) & -1 & -n & -q \end{bmatrix} \quad (\text{A.64})$$

Using (A.60) we find

$$-q \frac{u'(c)}{V''(a)} MPN = \frac{\sigma c}{q} q^2 \frac{u''(c)}{V''(a)} MPC \left( \frac{-n\psi}{\sigma c} \right) = -\frac{n\psi}{q} MPS$$

Again the first column yields

$$\frac{\partial c}{\partial q} = -\frac{n\psi}{q}MPS - (a-b)MPN$$

The second and fourth column of (A.62) yield directly

$$\begin{aligned}\frac{\partial c}{\partial z} &= MPN \\ \frac{\partial c}{\partial b} &= qMPN\end{aligned}$$

Finally, since

$$\begin{aligned}\left(q^2 + \frac{V''(a)}{u''(c)}\right) \frac{u'(c)}{V''(a)} &= -\sigma c \left(q^2 \frac{u''(c)}{V''(a)} + 1\right) \\ &= \psi n \frac{MPC}{MPN} \left(\frac{MPS}{MPC} + 1\right)\end{aligned}$$

the third column yields

$$\frac{\partial n}{\partial w} = \frac{1}{w}\psi n(MPS + MPC) + MPNn = \frac{1}{w}\psi n(1 + wMPN) + MPNn = \psi n \frac{1}{w} + MPN(n + \psi n)$$

The first-order total differential  $dn$  is then

$$\begin{aligned}dn &= \frac{\partial n}{\partial z}dz + \frac{\partial n}{\partial b}db + \frac{\partial n}{\partial q}dq + \frac{\partial n}{\partial w}dw \\ &= MPN(dz + qdb - (a-b)dq + (1 + \psi)ndw) - \psi nMPS \frac{dq}{q} + \psi n \frac{dw}{w}\end{aligned}\tag{A.65}$$

□

*Proof of theorem 2 in case 2a).* If the policy functions are differentiable and the consumer is at an interior optimum, then the conditions of lemma A.1 are satisfied: the borrowing constraint is not binding so can be ignored, and the value function is concave per standard dynamic programming arguments. The notation of theorem 2 can be cast using that of the lemma by using the mapping

$$q \equiv Q \quad z \equiv y + \frac{\lambda}{\Pi} \quad a \equiv \lambda' \quad b \equiv \delta \frac{\lambda}{\Pi}$$

with  $\frac{dP}{P} = \frac{d\Pi}{\Pi}$  and  $\frac{dQ}{Q} = -\frac{dR}{R}$ . Hence  $dz = dy - \frac{\lambda}{\Pi} \frac{dP}{P}$ ,  $db = -\delta \frac{\lambda}{\Pi} \frac{dP}{P}$  and  $\frac{dq}{q} = -\frac{dR}{R}$ ; so

$$dz + qdb - (a-b)dq = dy - \underbrace{\left(1 + Q\delta\right) \frac{\lambda}{\Pi} \frac{dP}{P}}_{\text{NNP}} + \underbrace{\left(\lambda' - \delta \frac{\lambda}{\Pi}\right) Q \frac{dR}{R}}_{\text{URE}}$$

Inserting this equation into (A.63) and (A.65) yields the desired result. □

**Case 2b)  $N > 0$ , no bonds, only  $R$  changing.** Since we are not considering changes in wages, it is sufficient to restrict the analysis to a choice between consumption and assets. The following lemma

then proves the result for  $dc$ . The result for  $dn$  follows as a straightforward extension.

**Lemma A.2.** Let  $c(\theta, Y, R)$  be the solution to the following consumer choice problem under concave preferences over current consumption  $u(c)$  and assets  $W(\theta')$

$$\begin{aligned} \max_{c, \theta'} \quad & u(c) + W(\theta') \\ \text{s.t.} \quad & c + (\theta' - \theta) \mathbf{S} = Y + \theta \mathbf{d} \end{aligned}$$

where  $\frac{d\mathbf{S}}{dR} = -\frac{\mathbf{S}}{R}$ . Then, to first order

$$dc = \text{MPC} \left( dY + \text{URE} \frac{dR}{R} \right) - \sigma(c) c (1 - \text{MPC}) \frac{dR}{R}$$

where  $\sigma(c) \equiv -\frac{u'(c)}{cu''(c)}$  is the local elasticity of intertemporal substitution,  $\text{MPC} = \frac{\partial c}{\partial Y}$ , and  $\text{URE} = Y + \theta \mathbf{d} - c$

*Proof.* The following first-order conditions characterize the solution

$$S^i u'(Y + \theta \mathbf{d} - (\theta' - \theta) \mathbf{S}) = W_{\theta^i}(\theta') \quad \forall i = 1 \dots N \quad (\text{A.66})$$

Consider first an increase in income  $dY$  alone. Differentiating along (A.66) we find

$$S^i u''(c) \left( 1 - \sum_j S^j \frac{d\theta'^j}{dY} \right) = \sum_j W_{\theta^i \theta^j}(\theta') \frac{d\theta'^j}{dY} \quad \forall i \quad (\text{A.67})$$

Define  $\eta^j \equiv S^j \frac{d\theta'^j}{dY}$ . Then (A.67) rewrites

$$\sum_j \left( \frac{1}{S^i S^j} W_{\theta^i \theta^j}(\theta') + u''(c) \right) \eta^j = u''(c) \quad \forall i$$

Defining the matrix  $M$  with elements

$$m_{ij} \equiv \frac{1}{S^i S^j} W_{\theta^i \theta^j}(\theta') + u''(c)$$

this system can also be written in matrix form as

$$M\eta = u''(c) \mathbf{1}$$

or

$$\eta = u''(c) M^{-1} \mathbf{1}$$

The budget constraint then implies that

$$\text{MPC} = \frac{dc}{dY} = 1 - \sum_j \eta^j = 1 - u''(c) m \quad (\text{A.68})$$

where  $m$  is defined as

$$m \equiv \mathbf{1} M^{-1} \mathbf{1} \quad (\text{A.69})$$



Next, consider an increase in the real interest rate  $dR$ . Differentiating along (A.66) we now have

$$\frac{dS^i}{dR} u'(c) + S^i u''(c) \left( -\sum_j S^j \frac{d\theta'^j}{dR} - \sum_j \frac{dS^j}{dR} (\theta'^j - \theta^j) \right) = \sum_j W_{\theta^i \theta^j} (\theta') \frac{d\theta'^j}{dR} \quad \forall i$$

Using  $\frac{dS^i}{S^i} = -\frac{dR}{R}$  this rewrites

$$-\frac{S^i}{R} u'(c) + S^i u''(c) \left( -\sum_j S^j \frac{d\theta'^j}{dR} + \sum_j \frac{S^j}{R} (\theta'^j - \theta^j) \right) = \sum_j W_{\theta^i \theta^j} (\theta') \frac{d\theta'^j}{dR} \quad \forall i \quad (\text{A.70})$$

Defining now  $\gamma^j \equiv S^j \frac{d\theta'^j}{dR}$ , (A.70) shows that  $\gamma^j$  solves

$$\sum_j m_{ij} \gamma^j = -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S^j}{R} (\theta'^j - \theta^j) \quad \forall i$$

which rewrites in matrix form

$$M\gamma = \left( -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S^j}{R} (\theta'^j - \theta^j) \right) \mathbf{1}$$

or

$$\gamma = \left( -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S^j}{R} (\theta'^j - \theta^j) \right) M^{-1} \mathbf{1} \quad (\text{A.71})$$

Differentiating with respect to  $R$  along the budget constraint  $c = Y + \theta \mathbf{d} - (\theta' - \theta) \mathbf{S}$ , we next see that

$$\frac{dc}{dR} = -\sum_j S^j \frac{d\theta'^j}{dR} + \sum_j \frac{S^j}{R} (\theta^j - \theta'^j) = -\sum_j \gamma^j + \sum_j \frac{S^j}{R} (\theta^j - \theta'^j)$$

inserting (A.71) and using the definition of  $m$ ,

$$\frac{dc}{dR} = -\left( -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S^j}{R} (\theta'^j - \theta^j) \right) m + \sum_j \frac{S^j}{R} (\theta^j - \theta'^j) \quad (\text{A.72})$$

rearranging terms and using  $u'(c) \equiv -\sigma(c) u''(c)$  we find

$$\frac{dc}{dR} = -\sigma(c) \frac{c}{R} u''(c) m + \sum_j \frac{S^j}{R} (\theta^j - \theta'^j) (1 - u''(c) m)$$

But using the expression for MPC in (A.68), this is simply

$$\frac{dc}{dR} = -\sigma(c) \frac{c}{R} (1 - MPC) + \sum_j \frac{S^j}{R} (\theta^j - \theta'^j) MPC$$

and using the budget constraint  $\sum_j S^j (\theta^j - \theta) = (\theta' - \theta) \cdot \mathbf{S}_t = URE$  we obtain

$$\frac{dc}{dR} = -\sigma(c) \frac{c}{R} (1 - MPC) + \frac{1}{R} URE \cdot MPC \quad (\text{A.73})$$

Finally, considering a simultaneous change in income and the real interest rate, combining (A.68) and (A.73) we obtain the first order differential

$$dc = MPC \left( dY + URE \frac{dR}{R} \right) - \sigma(c) c (1 - MPC) \frac{dR}{R}$$

as was to be shown. □

### A.7 Proof of theorem 3

Given the assumption of fixed balance sheets and purely transitory shocks, Theorem 2 shows that

$$dc_i = M\hat{P}C_i \left( dY_i - dt_i + URE_i \frac{dR}{R} - NNP_i \frac{dP}{P} \right) - \sigma_i c_i (1 - M\hat{P}C_i) \frac{dR}{R}$$

where, where  $dY_i = n_i e_i dw + w e_i dn_i + d(d_i)$  is the change in gross income at the individual level and  $dt_i$  the change in taxes. We can further decompose the change in gross income as

$$dY_i = \frac{Y_i}{Y} dY + dY_i - \frac{Y_i}{Y} dY$$

and note that, since  $\mathbb{E}_I [Y_i] = Y$ ,

$$\mathbb{E}_I \left[ dY_i - \frac{Y_i}{Y} dY \right] = dY - \frac{\mathbb{E}_I [Y_i]}{Y} dY = 0 \quad (\text{A.74})$$

Hence,

$$dc_i = M\hat{P}C_i \left( \frac{Y_i}{Y} dY + dY_i - \frac{Y_i}{Y} dY - dt_i + URE_i \frac{dR}{R} - NNP_i \frac{dP}{P} \right) - \sigma_i c_i (1 - M\hat{P}C_i) \frac{dR}{R}$$

and taking a cross-sectional average

$$\begin{aligned} dC &= \mathbb{E}_I \left[ \frac{Y_i}{Y} M\hat{P}C_i \right] dY + \mathbb{E}_I \left[ M\hat{P}C_i \left( dY_i - \frac{Y_i}{Y} dY \right) \right] - \mathbb{E}_I [M\hat{P}C_i (dt_i)] - \mathbb{E}_I [M\hat{P}C_i NNP_i] \frac{dP}{P} \\ &\quad + (\mathbb{E}_I [M\hat{P}C_i URE_i] - \mathbb{E}_I [\sigma_i (1 - M\hat{P}C_i) c_i]) \frac{dR}{R} \end{aligned} \quad (\text{A.75})$$

Now, the government budget (13) with the fiscal rule  $G_t = \bar{G}$  and target  $\frac{B_t}{P_t} = \bar{b}$  reads

$$\mathbb{E}_I [t_{it}] = \bar{G} + \frac{B_t}{P_t} - \frac{\bar{b}}{R_t}$$

Using the fact that at the margin, taxes are adjusted lump-sum, and the fact that  $NNP_g = -\bar{b}$  as well as  $URE_g = -\frac{\bar{b}}{R}$ , this implies

$$dt_i = dt = NNP_g \frac{dP}{P} - URE_g \frac{dR}{R}$$

In other words, taxes fall with unexpected increases in prices which reduce the government debt burden, and they fall with reductions in real interest rates which reduces the government's debt servicing costs. But the market clearing conditions (17) and (18) imply that these gains and losses have counterparts at the household level:

$$dt_i = dt = -\mathbb{E}_I [NNP_i] \frac{dP}{P} + \mathbb{E}_I [URE_i] \frac{dR}{R} \quad (\text{A.76})$$

Hence, (A.75) rewrites

$$\begin{aligned} dC &= \mathbb{E}_I \left[ \frac{Y_i}{Y} M\hat{P}C_i \right] dY + \mathbb{E}_I \left[ M\hat{P}C_i \left( dY_i - \frac{Y_i}{Y} dY \right) \right] - \mathbb{E}_I [M\hat{P}C_i] (dt) - \mathbb{E}_I [M\hat{P}C_i NNP_i] \frac{dP}{P} \\ &\quad + \left( \mathbb{E}_I [M\hat{P}C_i URE_i] - \mathbb{E}_I [\sigma_i (1 - M\hat{P}C_i) c_i] \right) \frac{dR}{R} \end{aligned}$$

so

$$\begin{aligned} dC &= \mathbb{E}_I \left[ \frac{Y_i}{Y} M\hat{P}C_i \right] dY + \mathbb{E}_I \left[ M\hat{P}C_i \left( dY_i - \frac{Y_i}{Y} dY \right) \right] + \mathbb{E}_I [M\hat{P}C_i] \mathbb{E}_I [NNP_i] \frac{dP}{P} - \mathbb{E}_I [M\hat{P}C_i NNP_i] \frac{dP}{P} \\ &\quad + \left( \mathbb{E}_I [M\hat{P}C_i URE_i] - \mathbb{E}_I [M\hat{P}C_i] \mathbb{E}_I [URE_i] - \mathbb{E}_I [\sigma_i (1 - M\hat{P}C_i) c_i] \right) \frac{dR}{R} \end{aligned}$$

and finally, using (A.74)

$$\begin{aligned} dC &= \mathbb{E}_I \left[ \frac{Y_i}{Y} M\hat{P}C_i \right] dY + \text{Cov}_I \left( M\hat{P}C_i, dY_i - Y_i \frac{dY}{Y} \right) - \text{Cov}_I (M\hat{P}C_i, NNP_i) \frac{dP}{P} \\ &\quad + \left( \text{Cov}_I (M\hat{P}C_i, URE_i) - \mathbb{E}_I [\sigma_i (1 - M\hat{P}C_i) c_i] \right) \frac{dR}{R} \end{aligned}$$

as claimed.

**Case with heterogeneous taxes.** If the taxes were not lump-sum, equation (A.76) would be replaced by

$$\mathbb{E}_I [dt_i] = -\mathbb{E}_I [NNP_i] \frac{dP}{P} + \mathbb{E}_I [URE_i] \frac{dR}{R}$$

we would therefore use the fact that

$$\mathbb{E}_I [M\hat{P}C_i (dt_i)] = \mathbb{E}_I [M\hat{P}C_i] \mathbb{E}_I [dt_i] + \text{Cov}_I (M\hat{P}C_i, dt_i)$$

to finally obtain

$$\begin{aligned} dC &= \mathbb{E}_I \left[ \frac{Y_i}{Y} M\hat{P}C_i \right] dY + \text{Cov}_I \left( M\hat{P}C_i, dY_i - Y_i \frac{dY}{Y} \right) - \text{Cov}_I (M\hat{P}C_i, NNP_i) \frac{dP}{P} \\ &\quad + \left( \text{Cov}_I (M\hat{P}C_i, URE_i) - \mathbb{E}_I [\sigma_i (1 - M\hat{P}C_i) c_i] \right) \frac{dR}{R} - \text{Cov}_I (M\hat{P}C_i, dt_i) \end{aligned}$$

The additional heterogeneous-taxation term is very natural. Suppose for example that, at the margin, gains from the government budget ( $\mathbb{E}_I [dt_i] < 0$ ) lead to disproportionate reductions of taxes on high-

MPC agents. Then  $\text{Cov}_I(M\hat{P}C_i, dt_i) < 0$ , so aggregate consumption increases by more than the benchmark from Theorem 1. The opposite happens when tax reductions fall disproportionately on low-MPC agents.

## A.8 Proof of corollary 2

From the definition of  $\gamma_i$  in (24), we have

$$d\left(\frac{Y_i}{Y}\right) = \gamma_i \left(\frac{Y_i}{Y} - 1\right) \frac{dY}{Y}$$

Moreover,

$$dY_i - Y_i \frac{dY}{Y} = Y d\left(\frac{Y_i}{Y}\right) = \gamma_i \left(\frac{Y_i}{Y} - 1\right) dY \quad (\text{A.77})$$

Next, rewrite equation (19) in elasticity terms by dividing by per-capita consumption  $C = \mathbb{E}_I[c_i]$  and using (A.77). We find

$$\begin{aligned} \frac{dC}{C} &= \underbrace{\mathbb{E}_I \left[ \frac{Y_i}{\mathbb{E}_I[c_i]} M\hat{P}C_i \right]}_{\text{Aggregate income channel}} \frac{dY}{Y} + \underbrace{\text{Cov}_I \left( M\hat{P}C_i, \gamma_i \frac{Y_i}{\mathbb{E}_I[c_i]} \right)}_{\text{Earnings heterogeneity channel}} \frac{dY}{Y} - \underbrace{\text{Cov}_I \left( M\hat{P}C_i, \frac{NNP_i}{\mathbb{E}_I[c_i]} \right)}_{\text{Fisher channel}} \frac{dP}{P} \\ &\quad + \left( \underbrace{\text{Cov}_I \left( M\hat{P}C_i, \frac{URE_i}{\mathbb{E}_I[c_i]} \right)}_{\text{Interest rate exposure channel}} - \underbrace{\mathbb{E}_I \left[ \sigma_i (1 - M\hat{P}C_i) \frac{c_i}{\mathbb{E}_I[c_i]} \right]}_{\text{Substitution channel}} \right) \frac{dR}{R} \end{aligned}$$

Imposing  $\gamma_i = \gamma$  and  $\sigma_i = \sigma$  for all  $i$ , this equation writes

$$\begin{aligned} \frac{dC}{C} &= \underbrace{\mathbb{E}_I \left[ \frac{Y_i}{\mathbb{E}_I[c_i]} M\hat{P}C_i \right]}_{\mathcal{M}} \frac{dY}{Y} + \underbrace{\gamma \times \text{Cov}_I \left( M\hat{P}C_i, \frac{Y_i}{\mathbb{E}_I[c_i]} \right)}_{\mathcal{E}_Y} \frac{dY}{Y} - \underbrace{\text{Cov}_I \left( M\hat{P}C_i, \frac{NNP_i}{\mathbb{E}_I[c_i]} \right)}_{\mathcal{E}_P} \frac{dP}{P} \\ &\quad + \left( \underbrace{\text{Cov}_I \left( M\hat{P}C_i, \frac{URE_i}{\mathbb{E}_I[c_i]} \right)}_{\mathcal{E}_R} - \underbrace{\sigma \times \mathbb{E}_I \left[ (1 - M\hat{P}C_i) \frac{c_i}{\mathbb{E}_I[c_i]} \right]}_S \right) \frac{dR}{R} \end{aligned}$$

which is equation (25).

## B From quarterly to annual MPCs

In this appendix I derive a simple theoretical relationship between quarterly MPC ( $MPC^Q$ ) and annual MPC ( $MPC^A$ ), namely

$$MPC^A = 1 - (1 - MPC^Q)^4 \quad (\text{B.1})$$

This relationship holds exactly in some models of consumption, and tends to be a good approximation in many others.

Time is discrete,  $t = 0, 1, 2, \dots, \infty$  and represents quarters. A consumer faces a constant real interest rate  $r$ , and chooses consumption  $c_t$  in each period  $t$ . His budget constraint along any realized path of income  $y_t$  is

$$\sum_{t \geq 0} \left( \frac{1}{1+r} \right)^t c_t = \sum_{t \geq 0} \left( \frac{1}{1+r} \right)^t y_t + \omega \quad (\text{B.2})$$

Denote by  $m_t = \frac{\partial E[c_t]}{\partial \omega}$  the average response of consumption response at date  $t$  following a transfer at date 0. By definition,  $MPC^Q \equiv m_0$ , while the annual MPC cumulates spending for the first four quarters,  $MPC^A \equiv \sum_{t=0}^3 m_t$ .

Consider now a simple model where the response of consumption is exponential

$$m_t = m_0 \lambda^t \quad \text{for } \lambda \in (0, 1), \quad t > 0 \quad (\text{B.3})$$

This rule is exact, for example, in any model in which agents have CRRA utility and consume only out of wealth  $\omega$  (so  $y_t = 0$ ), so that consumption is proportional to wealth. (B.2) implies that the discounted sum of the quarterly responses is 1:

$$\sum_{t \geq 0} \left( \frac{1}{1+r} \right)^t m_t = m_0 \frac{1}{1 - \frac{\lambda}{1+r}} = 1$$

Hence,  $m_0$  and  $\lambda$  are related via  $\lambda = (1+r)(1 - m_0)$ . The annual MPC is then

$$MPC^A = \sum_{t=0}^3 m_t = m_0 \sum_{t=0}^3 \lambda^t = m_0 \frac{1 - \lambda^4}{1 - \lambda} = MPC^Q \frac{1 - [(1+r)(1 - MPC^Q)]^4}{1 - [(1+r)(1 - MPC^Q)]}$$

In the special case where  $r = 0$ , this delivers equation (B.1). Given the simplicity and robustness<sup>48</sup> of this formula, I apply it to convert quarterly into annual MPCs in the CE, where annual MPCs are not available.<sup>49</sup>

<sup>48</sup>Equation (B.1) holds approximately in partial equilibrium Bewley models. For example, fix  $r = 0$ , consider a standard lognormal income process, and simulate the model implied mapping between  $MPC^Q$  and  $MPC^A$  for different values of the discount factor  $\beta$  and the elasticity of intertemporal substitution  $\sigma$ . The model-implied conversion gets very close to (B.1), especially for low values of  $\sigma$ . In general, the simplified model overstates the true  $MPC^A$  a little since quarterly MPCs decay faster than exponentially, but in all of my simulations this never accounted for no more than 10 annual MPC points.

<sup>49</sup>While Johnson et al. (2006) cannot estimate annual MPCs given their identification strategy and the nature of the panel component of the CE, they are able to estimate 6-month MPCs. The formula  $MPC^{6M} = 1 - (1 - MPC^Q)^2$  provides a good approximation to their findings. For example, for strictly nondurable goods they find  $MPC^Q = 0.248$  and  $MPC^{6M} = 0.34$  while my formula delivers 0.43. For nondurable goods they find  $MPC^Q = 0.386$  and  $MPC^{6M} = 0.69$  while my formula delivers 0.62.

## C Data appendix

This section starts out by providing more details about the data and the MPC identification strategies for the SHIW (section C.1), the PSID (section C.2), and the CE (section C.3).

Section C.4 then performs a sensitivity analysis along several dimensions. Section C.4.1 considers the consequence of using total consumption expenditure to estimate MPC in the PSID and CE. Section C.4.2 considers the effect of varying the fraction of durable expenditures included in URE, corresponding to different assumptions about the elasticity of the relative durable price to the real interest rate  $\epsilon$ . Section C.4.3 considers robustness to the number of bins used to stratify the population in the PSID and in the CE. Finally, section C.4.4 calculates redistribution elasticities for URE at a quarterly level in the CE.

Section C.5 cuts the data in various ways to examine the empirical drivers of the correlations I document in the data. Section C.5.1 looks at the influence of age, and section C.5.2 examines the role of income. Section C.5.3 generalizes my covariance decomposition procedure from section D to multiple covariates, and reports the decomposition when all of Jappelli and Pistaferri (2014)'s covariates are simultaneously used in this decomposition.

Section C.6 concludes this appendix by contrasting the financial asset and liability information available in the PSID and the CE, and comparing it to the same information reported in the Survey of Consumer Finances (SCF).

### C.1 SHIW

My first dataset comes from the 2010 wave of the Italian Survey of Household Income and Wealth, which is publicly available from the Bank of Italy's website. This is the data source employed by Jappelli and Pistaferri (2014), and it is very useful for my purposes because it contains a direct household-level measure of MPC, reported as part of a survey question.<sup>50</sup> An additional benefit of this dataset is that it presents detailed information on financial assets and liabilities, allowing a fairly precise measurement of URE and NNP for each household.

#### C.1.1 Exposure measures

The survey is annual, so I do not need to make adjustments to the raw data.<sup>51</sup> Table C.1 presents summary statistics in euros.

**URE:**  $Y - T - C + A - L$ . To construct my measure of unhedged interest rate exposure, I use net annual disposable income (which includes taxes, transfers, interest income and realized capital gains) as my measure of income net of taxes  $Y - T$ . My consumption measure  $C$  includes expenditures on both durables and non durables goods as well as interest and principal payments (the SHIW records up to three mortgages for each household). I also count house purchases and extraordinary maintenance towards  $C$ .

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<sup>50</sup>"Imagine you unexpectedly receive a reimbursement equal to the amount your household earns in a month. How much of it would you save and how much would you spend? Please give the percentage you would save and the percentage you would spend."

<sup>51</sup>Note that the time frame for  $MPC$  is not specified in the question, as issue that is left unresolved in Jappelli and Pistaferri (2014). A follow-up question in the 2012 SHIW separates durable and nondurable consumption, and specifies the time frame as a full year. The equivalent "MPC" out of both durable and nondurable consumption has close to the same distribution as that of  $MPC$  in the 2010 SHIW (respective means are 47 in 2010 and 45 in 2010) which suggests that households tended to assume that the question referred to the full year.

Table C.1: Summary statistics, SHIW

	N	mean	p5	p25	p50	p75	p95
Net Income	7,951	36,187	9,629	20,013	30,838	45,515	81,320
Consumption	7,951	30,442	10,800	17,200	24,200	34,500	65,603
Maturing assets	7,951	28,280	0	2,000	10,467	30,242	100,000
Maturing liabilities	7,951	9,440	0	0	0	305	49,000
URE	7,951	24,586	-43,958	1,903	15,622	38,984	115,403
Nominal assets	7,951	22,499	0	1,274	6,796	22,000	77,272
Nominal liabilities	7,951	15,133	0	0	0	4,285	99,000
Net nominal position	7,951	7,366	-81,712	-1	3,830	17,113	71,216
Gross income	7,951	38,691	7,907	19,102	31,059	48,377	92,193
MPC	7,951	0.47	0.00	0.20	0.50	0.80	1.00

Units: 2010 Euros. All statistics are computed using survey weights.

For remaining assets maturing in the year ( $A$ ), I consider as “deposits” the amounts held in checking accounts, savings accounts, certificates of deposits, and repurchase agreements. I consider as “bonds” government and corporate bonds, for which I make separate maturity assumptions, as reported in table 2. Given an assumed maturity of  $N_j$  years for a given asset or liability  $j$ , I scale the observed amounts by  $\frac{1}{N_j}$  to obtain an annual measure of maturing flows.

For liabilities maturing in the year ( $L$ ), I scale the principal balance outstanding on adjustable rate mortgages and on credit cards by  $\frac{1}{N_j}$ , given my assumptions for  $N_j$ .

**NNP and income.** To construct my measure of net nominal position, I include in nominal assets the full amount held in checking accounts, savings accounts, certificates of deposits and repurchase agreements. I also include the full amounts held in bonds from Italian banks and firms, with the exception of inflation-indexed BTP bonds. I assume that two-thirds of foreign bonds are denominated in euros, and count that amount in nominal assets. I then include all the shares of money market mutual funds and bonds mutual funds, in keeping with [Doepke and Schneider \(2006\)](#). For shares held at ‘mixed’ mutual funds, I assume that half of those are indirectly invested in bonds. Finally, I count all credit originating from commercials or private party loans.

For nominal liabilities, my measure includes all debt due to banks, other financial institutions, and other households, as well as commercial loans.

My results are not influenced in any meaningful way by altering the share of ‘mixed’ mutual funds invested in bonds, the share of foreign bonds that are euro-denominated, or by excluding commercials and private party loans from both nominal assets and liabilities.

For my income exposure measure  $Y$ , I use a measure of gross income from the Household Finance and Consumption Survey for Italy, which I merge into my main dataset.

## C.2 Panel Study of Income Dynamics

The procedure to identify MPC out of transitory income shocks that I employ for the PSID closely follows [Blundell, Pistaferri and Preston \(2008\)](#) (BPP), [Kaplan, Violante and Weidner \(2014\)](#), and [Berger et al. \(2018\)](#). Since the PSID only starts recording detailed consumption information in 1999, my sample period starts with the 1999 wave, and ends in 2013. I use the core sample of the PSID (made up of the

Table C.2: Summary statistics, PSID

	N	mean	p5	p25	p50	p75	p95
Net Income	41,820	57,943	11,338	26,882	45,356	72,554	136,969
Consumption	41,820	40,906	8,104	16,047	24,380	36,889	114,037
Maturing assets	41,820	53,180	0	1,035	8,454	35,712	220,000
Maturing liabilities	41,820	20,776	0	74	8,290	20,410	67,690
URE	41,820	49,440	-89,630	10	18,009	57,797	256,536
Nominal assets	41,820	48,289	0	700	6,209	34,861	230,248
Nominal liabilities	41,820	70,728	0	0	17,000	106,839	282,000
Net Nominal Position	41,820	-22,438	-248,383	-80,228	-4,421	7,084	183,004
Gross income	41,820	69,131	899	23,169	50,212	89,994	184,603

Units: 2009 USD. All statistics are computed using survey weights.

SCR, SEO and Immigrant samples) and drop households with intermittent headship, those appearing only once, and those with missing information on the head's race, education or the state of residence. I then drop households whose income or consumption increases by more than 500% or falls by more than 80% over two consecutive surveys, as well as households whose consumption is below \$100 in any period. I treat top-coded income or consumption data as missing data.

While the literature usually restricts the sample to working-age households, in my benchmark scenario I keep all families whose head is between 20 and 90 years old, in order to have a more accurate picture of the cross-sectional distribution of UREs and NNPs by age.<sup>52</sup> This sample selection leaves me with 41,820 observations from 7,287 different households.

### C.2.1 Exposure measures

The PSID is annual, so I do not need to perform a frequency adjustment. I deflate all nominal variables to 2009 dollars using the CPI. Table C.2 reports summary statistics.

**URE:**  $Y - T - C + A - L$ . For URE, I use an annual measure of net disposable income for  $Y - T$  (which includes interest and capital gains), and an annual consumption measure  $C$  that includes only the consumption categories continuously available in the survey since 1999 (my first sample year). Those consists of expenditures on food, rent, property taxes, home insurance, utilities, telecommunications, transportations, education, childcare and healthcare. I also add a measure of housing expenditures, as well as interest and principal payments.

For assets maturing in the year ( $A$ ), the PSID contains a variable that groups together checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T-bills. I treat this category as "deposits", to which I apply the maturity assumptions of table 2. The PSID contains another variable that includes bonds, trusts, estates, cash value of life insurance and collection. I assume that half of this amount is "bonds" and that the rest is equity-like, with an infinite maturity.

<sup>52</sup>Figure C.2 shows that young and old households tend to have the largest net nominal positions, and with opposite signs (see also Doepke and Schneider 2006) Since households' income processes tend to change upon entering retirement, however, including older households could lead to noisier estimates of MPCs. However, I verified that my elasticity estimates are essentially unchanged when I restrict the PSID sample to households heads between the ages of 25 and 55.



For the remainder of liabilities ( $L$ ), the PSID reports up to two mortgages for each household, and reports whether they are ARMs or FRMs. The PSID also contains a variable that includes credit cards debt, student loans, medical bills, legal debt and loan from relatives. From 2011 onwards, a breakdown of categories is available, and credit cards account for an average of 40% of the total. I assume that this fraction has been constant over time to form my “credit cards” variable.

**NNP and income.** To construct a household’s net nominal position, I count as nominal assets all the amount held in checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T-bills, as well as half of “bonds, trusts, estates, cash value of life insurance and collection”, which I assume to be all nominal. I include the whole amount in IRAs invested in bonds, and half the amount in IRAs invested in a mix of stocks and bonds.

For nominal liabilities, I count the principal balance outstanding on each mortgage and the whole amount due in the form of credit cards debt, student loans, medical bills, legal debt and loan from relatives.

For my income exposure measure, I use the PSID measure of gross income before taxes and government transfers.

### C.2.2 Identification of MPC

As mentioned in main text, the literature exploits the panel dimension of the data in PSID in order to estimate the MPC out of transitory income shocks. I follow BPP and construct my consumption measure for MPC using all non durable consumption categories.<sup>53</sup> For my income measure, I use labor income plus government transfers, as in Kaplan, Violante and Weidner (2014). Following BPP and Kaplan, Violante and Weidner (2014), I first regress the log of consumption and the log of income on observables characteristics of the households, including dummy variables for year of birth, family size, number of children, and income coming from other members of the family, as well as dummies for interactions between year with education, race, employment status and region. I then use the residuals of these regressions (call them  $y_{it}$  and  $c_{it}$ ) to estimate the MPC out of transitory income shocks. Specifically, for each exposure measure, in each year, I stratify the population in  $J$  bins. I then estimate  $\psi_j = \frac{\text{Cov}_j(\Delta c_t, \Delta y_{t+1})}{\text{Cov}_j(\Delta y_t, \Delta y_{t+1})}$  as the pass-through coefficient of log income on log consumption, pooling all years together.<sup>54</sup> I finally recover a measure of the marginal propensity to consume  $MPC_j$  by multiplying  $\psi_j$  by the ratio of average consumption to average income in each bin  $j$ .

Next, for each exposure measure, I calculate the average value of exposure in each bin,  $EXP_j$ , normalized by average consumption in the sample. I finally compute my estimators as<sup>55</sup>

<sup>53</sup>This is also consistent with Kaplan et al. (2014) and Berger et al. (2018). In section C.4.1, I report instead an MPC calculated using all consumption expenditures available in the PSID.

<sup>54</sup>See Blundell et al. (2008) and Kaplan et al. (2014) for the structural assumptions under which this procedure correctly recovers the MPC out of transitory income shocks. The estimate can be recovered with an instrumental variable regression of  $\Delta c_t$  on  $\Delta y_t$ , using  $\Delta y_{t+1}$  as an instrument.

<sup>55</sup>Note that I simply take  $\hat{S}$  to be the sample counterpart to  $1 - \mathbb{E}_I [MPC]$ . The procedure cannot simultaneously recover an estimate of the covariance between MPC and consumption. In the SHIW data, the difference between average MPC and consumption-weighted MPC is small, so this is unlikely to significantly affect the value of  $S$ .

$$\widehat{\mathcal{E}}_{EXP}^{NR} = \frac{1}{J} \sum_{j=1}^J MPC_j EXP_j$$

$$\widehat{\mathcal{E}}_{EXP} = \widehat{\mathcal{E}}_{EXP}^{NR} - \left( \frac{1}{J} \sum_{j=1}^J MPC_j \right) \left( \frac{1}{J} \sum_{j=1}^J EXP_j \right)$$

$$\widehat{S} = 1 - \left( \frac{1}{J} \sum_{j=1}^J MPC_j \right)$$

In order to take into account sampling uncertainty, I compute the distribution of these estimators using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section C.4.3 considers robustness to using  $J = 3$  to 8 bins to stratify the sample.

### C.3 Consumer Expenditure Survey, 2001-2002 (JPS sample)

My data for the Consumer Expenditure Survey comes from the [Johnson, Parker and Souleles \(2006\)](#) (JPS) dataset, which I merge with the main survey data and detailed expenditure files to obtain additional information on households's consumption expenditures, financial assets and liabilities. The dataset covers households with interviews between February 2001 and March 2002. Relative to the full CE sample, JPS drop the bottom 1% of nondurable expenditure in levels, households living in student housing, those with age less than 21 or greater than 85, those with age changing by more than a unit or by a negative amount between quarters, and those whose family size changes by more than three members between quarters. Since the 2001 CE survey has several observations with missing values for income—which is a crucial component of URE and a measure of exposure in its own right—I do not consider observations with incomplete income information when analyzing the interest rate exposure or the earnings heterogeneity channel. My sample is therefore made of 9,983 observations from 4,833 different households when computing statistics relevant to these two channels, and contains 12,227 observations from 5,900 households when analyzing the Fisher channel.

#### C.3.1 Exposure measures

As discussed in the main text, I measure all variables at an annual rate, summing across quarterly survey observations when necessary and adjusting MPC measures using the formula from appendix B. Table C.3 presents summary statistics in dollars.

**URE:**  $Y - T - C + A - L$ . In order to construct my annual measure of URE, I use annual net disposable income as my measure of income  $Y - T$ . For  $C$ , I sum durables and non durables goods as well as house purchases, obtained from the CE's supplemental expenditure files.

I count checking accounts and savings accounts as “deposits”, and I assume that half of the “securities” variables is bonds (“securities” contains the amount held in stocks, mutual funds, private sector bonds, government bonds or Treasury notes).

I proceed as usual for liabilities. The CE also contains information on adjustable-rate home equity loans, which I add to my ARM liability measure.

**NNP and income.** To construct my NNP measure, I include in nominal assets all the amount in savings and checking accounts, half of the “securities” variable, and all the amount held in US savings

Table C.3: Summary statistics, CE

	N	mean	p5	p25	p50	p75	p95
Net income	9,983	46,482	5,780	18,200	35,980	62,572	118,824
Consumption	9,983	40,702	9,361	19,272	32,245	51,820	96,228
Maturing assets	9,983	21,721	0	0	400	9,000	100,000
Maturing liabilities	9,983	20,976	0	0	1,200	8,478	137,609
URE	9,983	7,464	-115,773	-11,408	2,588	22,355	120,033
Nominal assets	12,227	19,006	0	0	9	5,000	100,000
Nominal liabilities	12,227	49,671	0	0	12,786	73,951	200,794
Net Nominal Position	12,227	-27,859	-174,317	-58,440	-6,800	0	58,078
Gross income	9,983	50,082	6,923	19,257	38,000	67,000	130,000

Units: 2001 USD. All flow variables are annualized. All statistics are computed using survey weights.

bonds and in private party loans owed. Using the supplemental expenditure files, my measure of nominal liabilities is fairly detailed. I take the sum of principal balances outstanding on mortgages, home equity loans, home equity line of credit, loans on vehicles, personal debt and credit card debt. For my income exposure measure, I use the CE's annual measure of gross income before taxes.

### C.3.2 MPC identification strategy

JPS identified the propensity to consume out of the 2001 tax rebate by exploiting random variation in the timing of its receipt across households. I closely follow their procedure for analyzing responses to the rebate among different exposure groups. Specifically, for each of my redistribution channels, I rank households in equally-sized bins according to their measure of exposure as at the time of the first interview. I then regress changes in the level of consumption expenditures ( $\Delta C_{it}$  in JPS's notation) on the amount of the tax rebate ( $Rebate_{it}$ ). I follow their instrumental-variable specification, instrumenting  $Rebate_{it}$  with a dummy indicator for whether the debate was received. I include month effects and control for age and changes in family composition, and I allow both the intercept and the rebate coefficients to differ across households bins.

My benchmark estimate uses food consumption expenditures as dependent variable. This allows for substantially more precise estimates, as it does in JPS. Section C.4.1 below reports all results using total consumption expenditures as dependent variable instead.

The procedure to compute estimators is the same as the one I use for the PSID—confidence intervals are constructed using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section C.4.3 reports redistribution elasticities by stratifying the sample in 3 to 8 bins.

## C.4 Sensitivity analysis

In this section I perform several robustness checks. As a general matter, my results are remarkably stable across all scenarios.

### C.4.1 Using total expenditure to estimate MPC

Table C.4 replicates the right two columns of table 4 when *all* available consumption expenditures are used to estimate MPC in the PSID and in the CE, instead of my benchmark scenario (which uses non-

durable consumption in the PSID and food consumption in the CE). The results are intuitive: the confidence intervals get larger, so are the average MPCs, and all my point elasticity estimates for redistribution elasticities turn more negative. In particular, the point estimate  $\widehat{\mathcal{E}}_P$  turns negative, just as it is in the other two surveys. Interestingly, this is true despite the fact that the point estimate for the average income-weighted MPC is actually a little lower in the PSID than it is when using nondurable consumption alone.

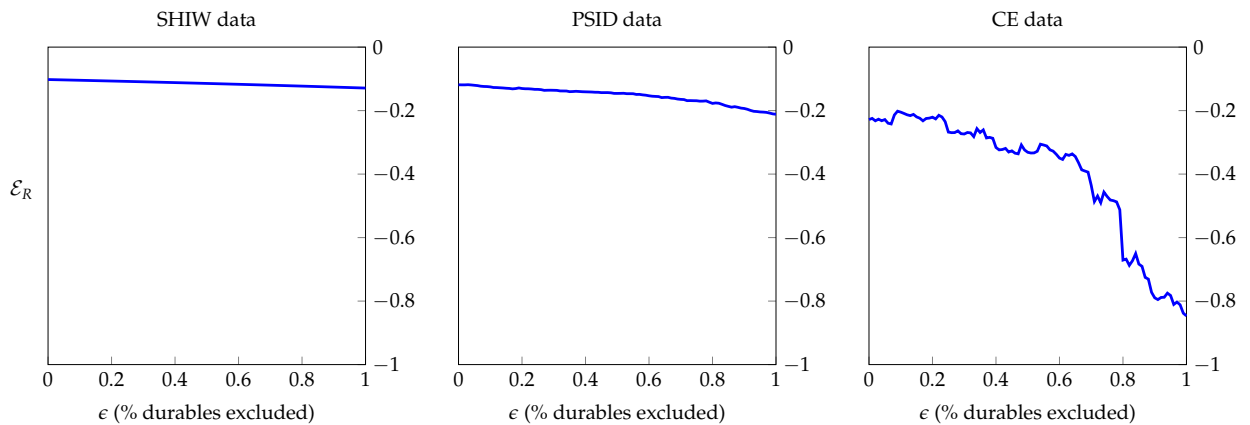
Table C.4: Using total expenditures to estimate MPC in the PSID and CE

Survey	PSID		CE	
	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}}_R$	-0.16	[-0.23,-0.10]	-0.59	[-1.34,0.17]
$\widehat{\mathcal{E}}_R^{NR}$	-0.01	[-0.07,0.04]	-0.45	[-1.21,0.30]
$\widehat{S}$	0.87	[0.84,0.91]	0.65	[0.15,1.16]
$\widehat{\mathcal{E}}_P$	-0.03	[-0.10,0.03]	-0.15	[-0.97,0.67]
$\widehat{\mathcal{E}}_P^{NR}$	-0.07	[-0.14,0.00]	-0.83	[-1.78,0.12]
$\widehat{\mathcal{E}}_Y$	-0.08	[-0.12,-0.04]	-0.25	[-0.72,0.22]
$\widehat{M}$	0.05	[-0.01,0.11]	0.51	[-0.34,1.36]

This figure recomputes the right two columns of table 4, but uses total expenditures to estimate MPC.

#### C.4.2 Excluding durable consumption from the URE calculation

Section B shows that, if relative durable goods prices have an elasticity  $\epsilon$  with respect to the real interest rate, then a theoretically-consistent measure of URE counts a fraction  $1 - \epsilon$  of nondurable expenditures. Figure C.1 plots my estimated  $\widehat{\mathcal{E}}_R$  against  $\epsilon$  in all three datasets. The left-most part of the graph corresponds to  $\epsilon = 0$ , which is my benchmark scenario. In all the surveys, excluding durable goods make the estimated value of  $\mathcal{E}_R$  more negative. This effect is most pronounced in the CE.



This figure plots the estimated covariance between MPCs and UREs ( $\widehat{\mathcal{E}}_R$ ) under various assumptions about the fraction of durable purchases, including house purchases, excluded from the computation of URE.  $\epsilon = 0$  is the benchmark from table 4 in which all durable purchases are included in the URE consumption measure.  $\epsilon = 1$  counts no durable purchase instead.

Figure C.1: Estimating  $\widehat{\mathcal{E}}_R$  assuming alternative values of  $\epsilon$ .

### C.4.3 Number of bins in the PSID and CE

Recall that my estimates of MPCs in the PSID and the CE are obtained by stratifying the population in three equally-sized groups. Table C.5 reports the full redistribution elasticities of all three channels by progressively increasing the number of bins from 3 to 8 bin in both samples. As is evident from the table, the number of bins used to stratify the sample does not have a meaningful impact on my main estimates, though magnitudes in the PSID tend to be a little larger with more bins.

Table C.5: Redistribution elasticities using 3 to 8 bins in the PSID and the CE

		Number of bins					
		3 (Benchmark)	4	5	6	7	8
PSID	$\widehat{\mathcal{E}}_R$	-0.12 [-0.16,-0.08]	-0.13 [-0.17,-0.09]	-0.12 [-0.16,-0.07]	-0.10 [-0.15,-0.06]	-0.11 [-0.16,-0.06]	-0.11 [-0.16,-0.06]
	$\widehat{\mathcal{E}}_P$	0.02 [-0.02,0.07]	0.03 [-0.01,0.08]	0.02 [-0.03,0.06]	0.02 [-0.03,0.06]	0.01 [-0.04,0.06]	0.02 [-0.03,0.07]
	$\widehat{\mathcal{E}}_Y$	-0.06 [-0.09,-0.04]	-0.07 [-0.10,-0.04]	-0.07 [-0.10,-0.04]	-0.07 [-0.10,-0.04]	-0.07 [-0.10,-0.04]	-0.07 [-0.10,-0.04]
CE	$\widehat{\mathcal{E}}_R$	-0.23 [-0.60,0.15]	-0.26 [-0.66,0.15]	-0.27 [-0.70,0.16]	-0.23 [-0.66,0.21]	-0.44 [-0.88,-0.01]	-0.21 [-0.65,0.23]
	$\widehat{\mathcal{E}}_P$	-0.09 [-0.51,0.33]	-0.14 [-0.56,0.28]	-0.18 [-0.61,0.25]	-0.14 [-0.58,0.30]	-0.22 [-0.67,0.23]	-0.41 [-0.82,-0.00]
	$\widehat{\mathcal{E}}_Y$	-0.13 [-0.36,0.10]	-0.13 [-0.34,0.08]	-0.17 [-0.36,0.02]	-0.14 [-0.33,0.05]	-0.15 [-0.31,0.01]	-0.17 [-0.35,0.02]

### C.4.4 Quarterly measurement in the CE

To ensure comparability across surveys, in the main text I measure MPCs and UREs at an annual level. In the CE, this requires me to use the formula from appendix B to map quarterly into annual MPCs. An alternative is to measure both UREs and MPCs at a quarterly level in that dataset. This requires adjusting UREs accordingly: I divide annual income by 4 to obtain  $Y - T$ , use quarterly consumption for  $C$ , and scale all maturities for assets and liabilities according to my benchmark assumptions from table 2. Table C.6 reports the outcome of this exercise. Calculating elasticities in this way cuts  $\widehat{\mathcal{E}}_R$  is cut in half but the point estimate remains negative.

Table C.6: Estimates with quarterly measurement in the CE

Survey	Benchmark		Quarterly	
	Estimate	95% C.I.	Estimate	95% C.I.
$\widehat{\mathcal{E}}_R$	-0.23	[-0.60,0.15]	-0.09	[-0.29,0.11]
$\widehat{\mathcal{E}}_R^{NR}$	-0.09	[-0.48,0.31]	-0.06	[-0.26,0.14]
$\widehat{S}$	0.64	[0.36,0.92]	0.90	[0.77,1.03]
$\widehat{\mathcal{E}}_P$	-0.09	[-0.51,0.33]	-0.11	[-0.82,0.60]
$\widehat{\mathcal{E}}_P^{NR}$	-0.45	[-0.94,0.04]	-0.54	[-1.31,0.23]
$\widehat{\mathcal{E}}_Y$	-0.13	[-0.36,0.10]	-0.05	[-0.15,0.06]
$\widehat{\mathcal{M}}$	0.46	[-0.06,0.98]	0.14	[-0.12,0.39]

## C.5 Correlates of MPCs and exposures

This section complements section D by providing other perspectives on the empirical drivers of my main objects of interest.

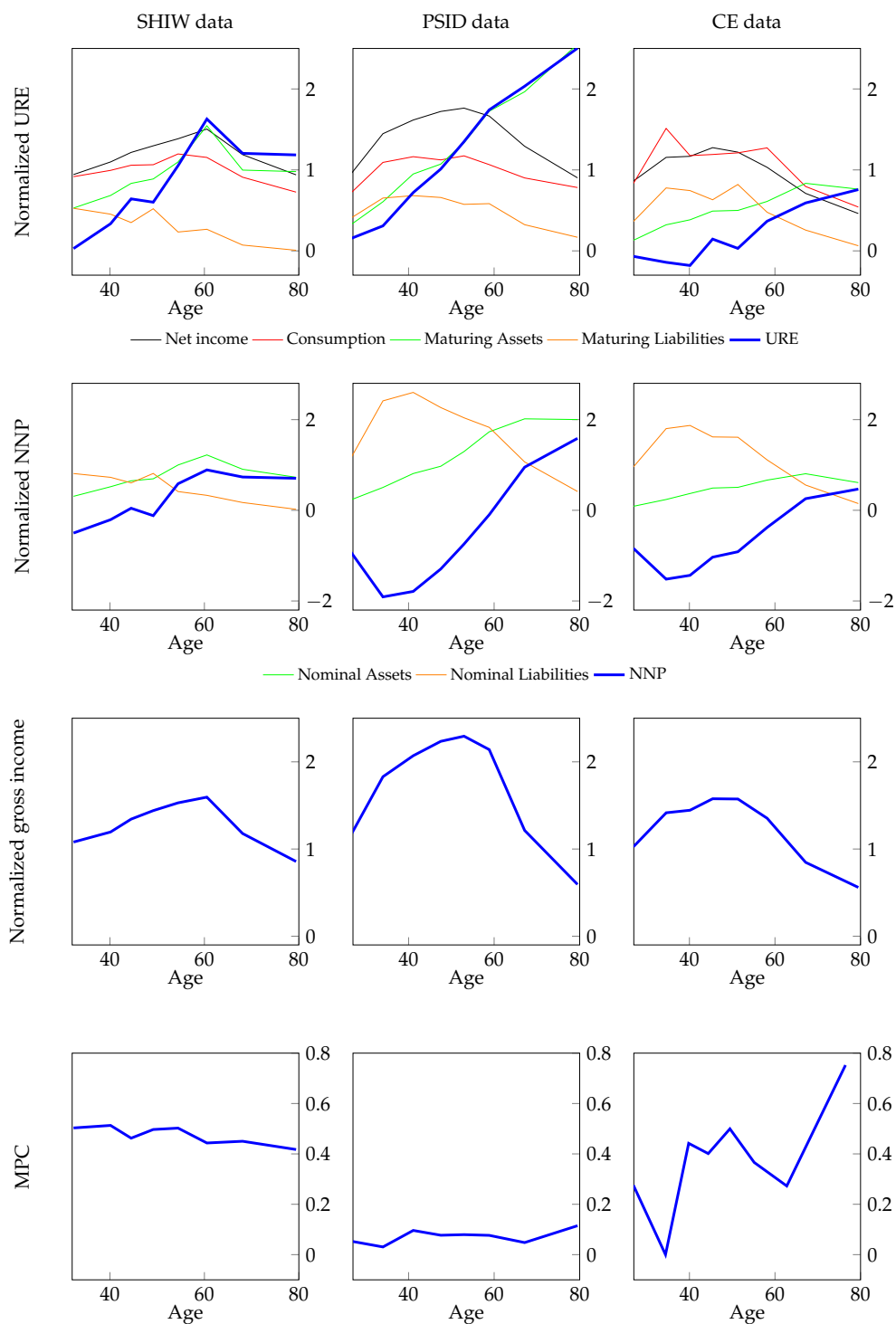
### C.5.1 The role of age

This section examines the distribution of exposures and MPC by age in each survey. I divide the population in eight equally-sized age bins. This allows me to assess life-cycle dynamics. It also helps to visualize clearly the relative strengths and weaknesses of each survey.

**Exposure measures.** Figure C.2 reports the average value of URE, NNP and income in each age bin, normalized by average consumption in the survey. Average URE (the blue line in the first row of graphs) is increasing in age across all three surveys, with a pattern of decline after retirement in the SHIW. This pattern is mostly due to a decumulation of financial assets in that survey (as represented by the green line). In terms of magnitudes, average URE is always positive in the SHIW and in the PSID, while in the CE average URE is negative for most working-age households. However, this is clearly driven by the different data flaws in each survey: the SHIW and the PSID greatly underreport consumption relative to income—notice the difference between the black and the red line. This tends to overestimate URE. By contrast, as documented above, the CE severely underreports assets, underestimating URE.

Regarding net nominal positions (the blue line in the second row of graphs), the life-cycle pattern in the SHIW is also increasing in age. By contrast, the PSID and the CE display an interesting U shape, with a minimum around age 40. In particular, in the SHIW, nominal liabilities are declining almost monotonically with age, while nominal assets are sharply increasing until age 60 and then decline rapidly. By contrast, in the PSID and in the CE, nominal liabilities are increasing in age for young households, and then start to decline steadily after age 40—while nominal assets are almost monotonically increasing in age. In terms of magnitudes, average NNP is negative for most of working age population in the SHIW, while it is very negative in the CE and PSID for all households cohorts except the oldest ones. This highlights, once again, the issue that these surveys cover liabilities better than they cover assets.

**MPC.** Figure C.2 also reports marginal propensities to consume by age bins in all three surveys. There is an overall declining pattern in age, except for a spike for the oldest cohort in the CE. Interestingly, all three surveys also suggest a rise in MPC around middle age. This pattern is not sensitive to the



This figure plots all three exposure measures and estimated MPCs by age in all three surveys. Households are grouped by 8 equally-sized age bins. The  $x$  axis reports the average age in each group, the  $y$  axis reports mean exposure as well as estimated MPC in each bin.

Figure C.2: Exposure measures by age bins in all three datasets

number of bins employed to stratify the population. Combining this graph with figure C.2, it appears that age is indeed a driver of the negative correlation between MPC and my exposure measures—as already apparent in table 6.

### C.5.2 The role of income

Figure C.3 examine the distribution of URE and NNP in all three surveys, when the population is grouped into eight income bins. Unsurprisingly, average URE is increasing in income, especially in the SHIW and the PSID. In these surveys, average URE increases more than one for one with income at the top of the distribution, owing an increase in maturing assets. Interestingly, maturing liabilities (the orange line) also increase in income across all three surveys.

For net nominal position, patterns are different in Italy and in the United States. In the SHIW, net nominal position is initially flat, and then increases with income, owing to an increase in assets at the top of the income distribution. By contrast, in the PSID and in the CE, net nominal position initially declines in income, and then flattens out. This is because nominal liabilities initially increase strongly with income, while nominal assets only increase mildly.

### C.5.3 A general covariance decomposition

In section D, I presented a covariance decomposition that projected observables on a single covariate. This approach can of course be generalized to include any number of covariates. The procedure is in two steps: first, run an OLS regression

$$\begin{aligned} MPC_i &= (\beta^m)' \mathbf{Z}_i + \epsilon_i^m \\ URE_i &= (\beta^u)' \mathbf{Z}_i + \epsilon_i^u \end{aligned}$$

where  $\mathbf{Z}_i = (1, Z_{i1}, \dots, Z_{ij})'$  is now a vector of covariates. Then, recover fitted values

$$\begin{aligned} \widehat{MPC}_i &= (\widehat{\beta}^m)' \mathbf{Z}_i \\ \widehat{URE}_i &= (\widehat{\beta}^u)' \mathbf{Z}_i \end{aligned}$$

and residuals  $\widehat{\epsilon}_i^m, \widehat{\epsilon}_i^u$ . The law of total covariance can now be expressed as

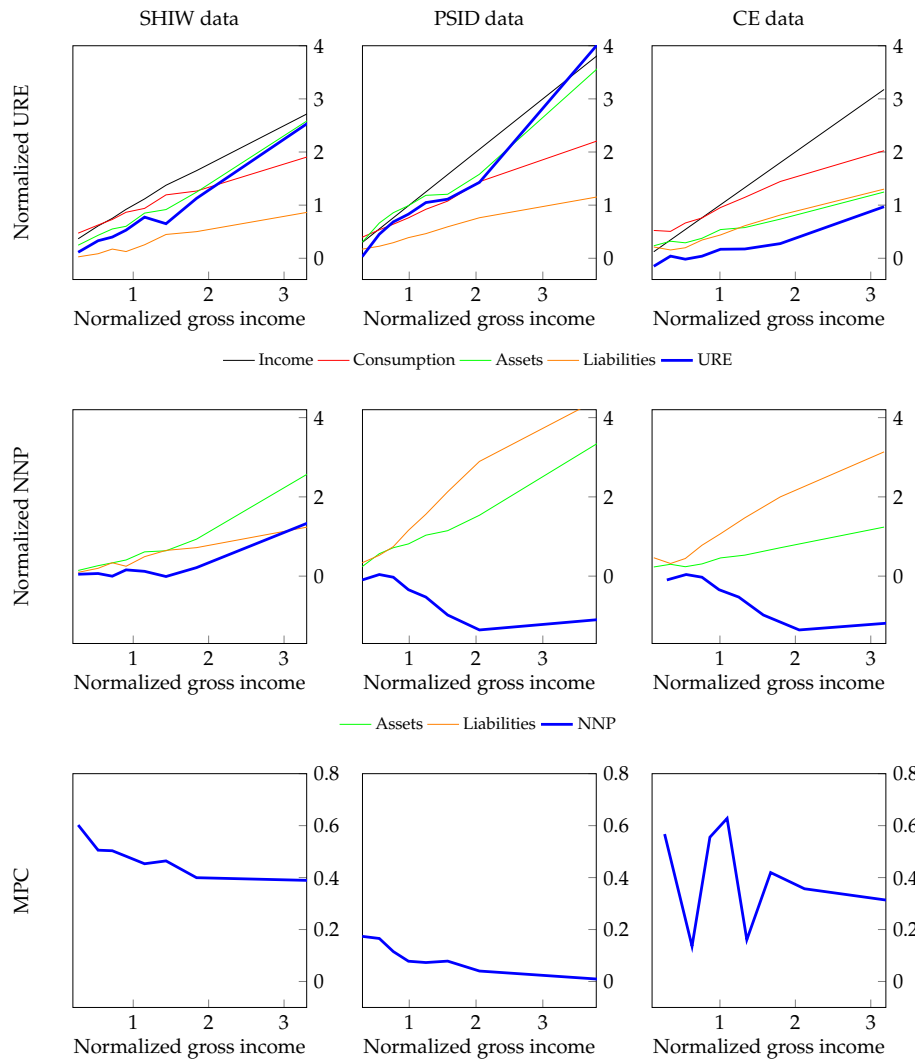
$$\text{Cov}(MPC_i, URE_i) = \text{Cov}(\widehat{MPC}_i, \widehat{URE}_i) + \text{Cov}(\widehat{\epsilon}_i^m, \widehat{\epsilon}_i^u) \quad (\text{C.1})$$

The first term gives the component of explained covariance, and the second the component of unexplained covariance. The explained part of the covariance can be further decomposed as

$$\text{Cov}(\widehat{MPC}_i, \widehat{URE}_i) = \text{Cov}\left(\sum_{j=1}^J \widehat{\beta}_j^m Z_{ij}, \sum_{k=1}^J \widehat{\beta}_k^u Z_{ik}\right) = \sum_{j=1}^J \sum_{k=1}^J \widehat{\beta}_j^m \widehat{\beta}_k^u \text{Cov}(Z_{ij}, Z_{ik}) \quad (\text{C.2})$$

Of course, the 'share of explained covariance' attributed to one particular covariate through this procedure depends on which other covariates are included in  $\mathbf{Z}_i$ .





This figure plots all three exposure measures and estimated MPCs by income in all three surveys. Households are grouped by 8 equally-sized bins of gross income. The  $x$  axis reports the average age in each group, the  $y$  axis reports mean exposure as well as estimated MPC in each bin.

Figure C.3: URE and NNP components by income bins in all three datasets

**Implementation.** Tables C.7–C.9 report the full matrix described by equation (C.2) for each of my three main covariances  $\mathcal{E}_R$ ,  $\mathcal{E}_P$ , and  $\mathcal{E}_Y$  in the SHIW, when all covariates from table 6 are included simultaneously. In the PSID and the CE, this exercise is less interesting since MPCs are only available at the group level, but it is possible to do by using the average value of explanatory variables in each bin. These results can easily be generated using the code provided online.

Table C.7: Fraction of  $\mathcal{E}_R$  explained by each pair of SHIW covariates

	Age bins	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	9.83	0.19	0.00	-2.24	-0.07	-0.02	0.01	0.08
Male	0.90	1.69	-0.01	0.24	0.02	0.09	0.01	0.03
Married	-0.29	0.28	-0.02	0.27	0.05	-0.04	0.01	0.01
Years of ed.	-3.30	0.07	-0.00	7.24	0.03	0.52	-0.07	0.00
Family size	2.88	-0.14	0.02	-0.78	-0.22	0.36	-0.01	0.05
Res. South	-0.37	0.32	0.00	5.79	-0.15	11.24	-0.00	0.25
City size	0.64	0.07	-0.00	-2.26	0.01	-0.01	0.96	0.01
Unemployed	1.77	0.13	-0.00	0.02	-0.03	0.33	0.00	0.69

Table C.8: Fraction of  $\mathcal{E}_P$  explained by each pair of SHIW covariates

	Age bins	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	13.29	0.22	-0.06	-2.56	1.52	-0.01	-0.02	-0.15
Male	1.22	1.98	0.28	0.28	-0.35	0.04	-0.01	-0.05
Married	-0.40	0.33	0.49	0.30	-1.06	-0.02	-0.02	-0.01
Years of ed.	-4.47	0.09	0.08	8.27	-0.60	0.22	0.12	-0.00
Family size	3.89	-0.16	-0.43	-0.89	4.48	0.15	0.01	-0.10
Res. South	-0.50	0.37	-0.13	6.62	3.12	4.84	0.00	-0.46
City size	0.87	0.08	0.10	-2.58	-0.14	-0.00	-1.73	-0.01
Unemployed	2.39	0.15	0.03	0.03	0.60	0.14	-0.00	-1.26

Table C.9: Fraction of  $\mathcal{E}_Y$  explained by each pair of SHIW covariates

	Age bins	Male	Married	Years of ed.	Family size	Res. South	City size	Unemployed
Age bins	6.51	0.19	-0.14	-4.82	-2.46	-0.07	-0.02	0.54
Male	0.60	1.71	0.64	0.52	0.56	0.28	-0.01	0.18
Married	-0.19	0.28	1.11	0.57	1.72	-0.11	-0.01	0.04
Years of ed.	-2.19	0.08	0.19	15.56	0.98	1.57	0.09	0.01
Family size	1.91	-0.14	-0.97	-1.67	-7.28	1.09	0.01	0.35
Res. South	-0.24	0.32	-0.30	12.46	-5.06	34.04	0.00	1.65
City size	0.42	0.07	0.22	-4.86	0.23	-0.02	-1.33	0.04
Unemployed	1.17	0.13	0.06	0.05	-0.97	1.00	-0.00	4.52

## C.6 Evaluating the quality of the financial information in U.S. surveys

In order to shed light on the quality of financial data in the PSID and the CE, tables C.10 and C.11 compare the median value of each class of assets and liabilities for households holding these instruments with the comparable number from the Survey of Consumer Finance. All three surveys are analyzed in 2001, the year in which they all overlap. As discussed above, the CE and the PSID group assets and liabilities into coarse categories, making a precise comparison difficult. However, table C.10 illustrates that liabilities in both the CE and the PSID appear to be aligned with numbers from the SCF as far as medians are concerned. This is especially true in the CE. Regarding financial assets, PSID and SCF data are fairly comparable. By contrast, the CE appears to considerably underreport assets, confirming previous findings in the literature.

Table C.10: Median values for financial liabilities — CE v. PSID v. SCF

Liabilities	SCF	CE	PSID	CE/SCF	PSID/SCF
Mortgages on primary residence	72	72.3	73	1.00	1.01
HELOC on primary residence	15	18.9	-	1.26	-
Other residential debt	40	37.9	18	0.95	0.45
Credit cards	1.9	2		1.05	
Vehicle loans	9.2	10.4	6	1.13	0.6
Education loans, personal loans, other	5	1.2		0.24	
Any debt	38.7	40.1	49	1.04	1.26

Units: Thousands of 2001 USD.

Households holding those liabilities in 2001. Medians computed using survey weights.

Table C.11: Median values for financial assets — CE v. PSID v. SCF

Financial Assets	SCF	CE	PSID	CE/SCF	PSID/SCF
Transaction accounts	3.9	1		0.26	
Certificates of deposit	15	3	5	0.2	1
Savings bonds	1	0.8		0.8	
Retirement accounts	29.4	-	31	-	1.12
Stocks	20	25	30	0.64	1.5
Bonds, mutual funds, life insurance, other	20		11		0.65
Any financial asset	28.3	4.5	12	0.16	0.42

Units: Thousands of 2001 USD.

Households holding those assets in 2001. Medians computed using survey weights.

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