Electromagnetic Field Focusing for Short-Range Wireless Power Transmission

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***Abstract*—Wireless power transmission enables remotely-powered implantable devices to reduce the risk of wire snapping, and replacement and corrosion of embedded batteries. However, current autonomous implants remain large in scale due to the operation at very low frequency and the use of unwieldy size of antennas . This paper will first review that the optimal frequency is about 2 orders of magnitude higher than the conventional wisdom; and thereby the power receiving coils can be reduced by more than 100 fold without sacrificing either power efficiency or range. To further maximize the received power, using an array of magnetic current sources increases the power transfer efficiency by an additional 10 dB. The optimal current source distribution is analytically solved and the theoretical upper-bound on the efficiency is investigated. The optimal solution reveals that a finite dimensional source is sufficient to approach the theoretical upper-bound. The efficiency at the low GHz-range is 16-dB higher than that at the low MHz-range for typical depth of the implant.**

I. Introduction

Wireless power transmission finds applications in both consumer and biomedical electronics. In particular, it allows dramatic miniaturization of implantable devices by removing the battery which easily occupies significant space of an implant. Current studies in wireless power transmission into biological tissue tend to operate below 10 MHz because of the common belief that lower operating frequency yields higher power transfer efficiency. Our previous work [1, 2], however, showed that the optimal frequency maximizing the power transfer efficiency is in the low GHz frequency range. The corresponding wavelength inside tissue is a few centimeters which is in the same order as the dimension of the transmit coil. This suggests that we should consider using an array of transmit coils to focus the electromagnetic energy and hence boost up the received power further.

In this paper, we will first review our previous result on choosing the optimal frequency for wireless power transfer over dispersive tissue. Then, we will examine the gain of using an array of transmit coil to deliver power. We formulate the problem as an optimization problem. As the received power is proportional to the square of the induced emf, we optimize the source distributions to maximize the induced emf while minimizing the tissue absorption. In the GHz-range, optimization of the source distribution yields at least an order of magnitude better performance than the non-optimized counterpart for a given transmitter array dimension. Finally, we will address the question: how large should the transmit array be? To answer this question, we first lax the source dimension, and derive the optimal current distribution and hence the theoretical upper-bound on the efficiency for an infinite array size. The optimal current distribution reveals that there is diminishing return on the efficiency when the size of the transmitter array is beyond certain value. This value depends on the frequency of operation and the depth of implants inside the tissue. At the low MHz range, a 2-cm coil with uniform current distribution approaches the efficiency upper-bound. At the low GHz-range, on the other hand, it requires an array size of around 10 cm to approach the upper-bound. The efficiency at the low GHz-range is 16 dB higher than that at the low MHz-range for typical depth of the implant.

II. Optimal Frequency

Let us first review the reasoning behind the popular use of low-frequency carrier in wireless power transmission. As tissue absorption increases with frequency, most analyses assume that lower frequency would yield better transfer efficiency. They therefore omit the displacement current. The propagation of electromagnetic field is then governed by a diffusion equation which is a quasi-static approximation to Maxwell's equations. Solving the diffusion equation reveals that electromagnetic fields decay exponentially inside tissue and the length of diffusion is inversely proportional to the square root of frequency. That is, higher frequency decays faster which agrees with the initial assumption and reinforces the use of low-frequency carrier. However, the diffusion equation is a valid approximation for good conductors, and tissue is better modeled as a low loss dielectric in which displacement current is significant. In this section, we include the displacement current and model the frequency variation of the relative permittivity by the Debye relaxation model [3]:

$$\epsilon_r(\omega) = \epsilon_\infty + \frac{\epsilon_{r0} - \epsilon_\infty}{1 - i\omega\tau} + i\frac{\sigma}{\omega\epsilon_0} \tag{1}$$

where τ is the relaxation time constant, ϵ_{r0} is the static relative permittivity, ϵ_∞ is the relative permittivity at frequencies where $\omega\tau \gg 1$, and σ is the static conductivity. Each type of tissue is characterized by 3 parameters: ϵ_{r0}, ϵ_∞, and τ.

We model the transmitted field from the source by the field emanated from the set of lowest order magnetic multipoles, and assume that the scattered field from the receiver is negligible. Then the electromagnetic fields at a point $\mathbf{r}$ in the medium are:

$$\mathbf{H}(\mathbf{r}) = \frac{iI_tA_t}{4\pi}\left[\alpha_x\boldsymbol{\psi}_x(\mathbf{r}) + \alpha_y\boldsymbol{\psi}_y(\mathbf{r}) + \alpha_z\boldsymbol{\psi}_z(\mathbf{r})\right]$$
$$\mathbf{E}(\mathbf{r}) = -\frac{\omega\mu_0I_tA_t}{4\pi}\left[\alpha_x\boldsymbol{\xi}_x(\mathbf{r}) + \alpha_y\boldsymbol{\xi}_y(\mathbf{r}) + \alpha_z\boldsymbol{\xi}_z(\mathbf{r})\right]$$

where $k = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}$, I_tA_t is the transmit magnetic moment, $\boldsymbol{\psi}_x(\mathbf{r})$ and $\boldsymbol{\xi}_x(\mathbf{r})$ give the respective magnetic and electric fields due to a magnetic dipole pointing in the x direction, and $(\alpha_x, \alpha_y, \alpha_z)$ is the orientation of the transmit dipole. Suppose the receive dipole is at $-\hat{\mathbf{z}}z_f$ and is pointing in the direction $\hat{\mathbf{n}}$, and the tissue region is defined by $z < -d$. Then, we define the power transfer efficiency as the ratio of received power to total tissue absorption:

$$\eta := \frac{\omega^2\mu_0^2A_r^2\left|\mathbf{H}(-\hat{\mathbf{z}}z_f)\cdot\hat{\mathbf{n}}\right|^2}{\omega\epsilon_0\int_{z<-d}\operatorname{Im}\epsilon_r(\omega)\left|\mathbf{E}(\mathbf{r})\right|^2 d\mathbf{r}} \tag{2}$$

where A_r is the area of the receive dipole.

For a given $\hat{\mathbf{n}}$, we derive the frequency that maximizes the efficiency over all possible orientations of the transmit dipole α_m's. The optimal frequency satisfies

$$\omega_{opt} \approx \sqrt{\frac{c\sqrt{\epsilon_{r0}}}{z_f\tau\Delta\epsilon}} \tag{3}$$

Based on the measured data in [4], Table I lists the approximated optimal frequencies for 17 different kinds of tissue assuming $z_f = 1$ cm. All approximated optimal frequencies are in the GHz-range. They are above 1 GHz even for $z_f = 10$ cm. Furthermore, the transfer efficiency at the optimal frequency is approximately proportional to $\frac{1}{z_f^3}$. This implies that the regime for optimal power transmission is in between the far field and the near field.

Table I
APPROXIMATE OPTIMAL FREQUENCY ASSUMING $z_f = 1$ CM.

Type of tissue	f_{opt} (GHz)
Blood	3.54
Bone (cancellous)	3.80
Brain (grey matter)	3.85
Fat (infiltrated)	6.00
Heart	3.75
Kidney	3.81
Lens cortex	3.93
Liver	3.80
Lung	4.90
Muscle	3.93
Skin (wet)	4.01
Spleen	3.79
Tendon	3.17

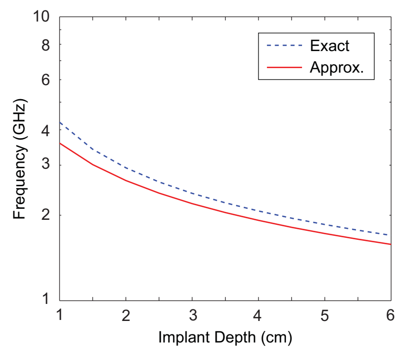

Figure 1. Approximated vs. exact optimal frequency.

As muscle is the most widely reported tissue, let us take muscle as an example. We consider a receive dipole tilted 45° with respect to the z-axis. The area of the receive dipole is (2 mm)^2. We first compute the efficiency that optimizes the orientation of the transmit dipole for the given receive orientation over the frequency range between 1 MHz and 10 GHz. Then, we find the efficiency that is maximized across this frequency range for different implant depth, $d - d_1$, and compare them with the approximation in (3). Fig. 1 shows these curves. The approximated and exact optimal frequencies are very close.

III. OPTIMAL SOURCE DISTRIBUTION

Next, we replace the single magnetic-dipole source by an array of magnetic dipoles, and the homogeneous medium to an air-muscle half space. The array of magnetic dipoles is abstracted by a magnetic current distribution, $\mathbf{M}(x, y) = \hat{\mathbf{z}}M_z(x, y)$, as illustrated in Fig. 2(a). We first use Weyl identity to decompose the incident electromagnetic fields in

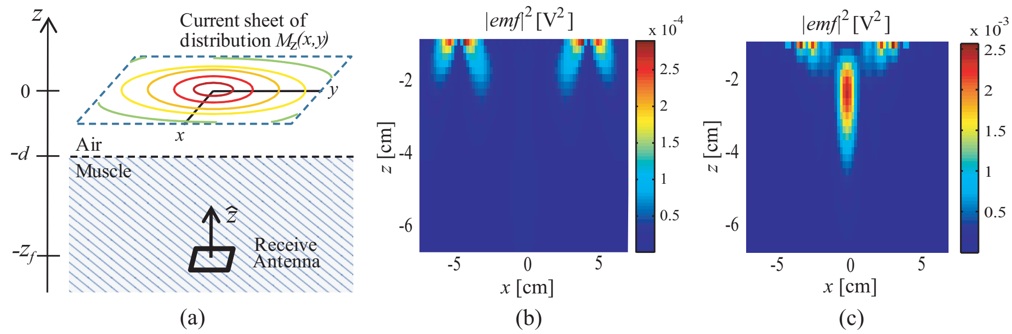

Figure 2. (a) The half-space tissue model and the source. (b) Distribution of induced emf by a uniform source under safety constraint. (c) Corresponding distribution by the optimal source distribution.

the air medium by a continuum of plane waves. As $\mathbf{M}(x, y)$ has the $\hat{\mathbf{z}}$ component only, the electric field are transverse to the air-tissue interface. We will apply the reflection and transmission coefficients for TE waves on each of the incident plane waves. This will yield expressions for the fields in the tissue medium in the spectral domain:

$$\mathcal{H}_{2,z}(k_x, k_y, z) = -\frac{\omega\epsilon_1 e^{i(k_{1z}-k_{2z})d_1 - ik_{2z}z}}{k_{1z}+k_{2z}} \cdot \left(1 - \frac{k_{1z}^2}{k_1^2}\right) \mathcal{M}_z(k_x, k_y) \quad \text{(4a)}$$

$$\boldsymbol{\mathcal{E}}_2(k_x, k_y, z) = \frac{e^{i(k_{1z}-k_{2z})d_1} e^{-ik_{2z}z}}{k_{1z}+k_{2z}} \cdot \begin{bmatrix} k_y \\ -k_x \\ 0 \end{bmatrix} \mathcal{M}_z(k_x, k_y). \quad \text{(4b)}$$

In the expression, $\mathcal{H}_{2,z}(k_x, k_y, z)$, $\boldsymbol{\mathcal{E}}_2(k_x, k_y, z)$, and $\mathcal{M}_z(k_x, k_y)$ are the Fourier transform of $H_{2,z}(x, y, z)$, $\mathbf{E}_2(x, y, z)$, and $M_z(x, y)$ respectively. Index 1 refers to the air medium whereas index 2 refers to the tissue medium. Substituting the above two equations into (2) and using the Parseval's Theorem yield (5).

To maximize η, we apply the Cauchy-Schwartz inequality. The $\mathcal{M}_z(k_x, k_y)$ that maximizes the efficiency is

$$\mathcal{M}_z(k_x, k_y) = 2\frac{k_{1z}+k_{2z}}{k_1^2} \operatorname{Im} k_{2z} \cdot e^{ik_{2z}^*(-z_f+d) - ik_{1z}d} \quad (6)$$

which is a circularly symmetrical function. Its inverse Fourier transform yields the source current distribution back to the $\mathbf{r}$-space:

$$M_z(x, y) = \int_0^\infty \frac{k_{1z}+k_{2z}}{\pi k_1^2} \operatorname{Im} k_{2z} k_\rho J_0(k_\rho \rho) \cdot e^{ik_{2z}^*(-z_f+d) - ik_{1z}d_1} \, dk_\rho \quad (7)$$

where $\rho^2 = x^2 + y^2$. The optimal efficiency is

$$\eta_{\text{opt}} = \frac{2A_r^2}{\pi\omega \operatorname{Im} \epsilon_2)} \cdot \int_0^\infty k_\rho^3 \operatorname{Im} k_{2z} e^{2 \operatorname{Im} k_{2z}(-z_f+d)} \, dk_\rho \quad (8)$$

where $k_\rho^2 = k_x^2 + k_y^2$.

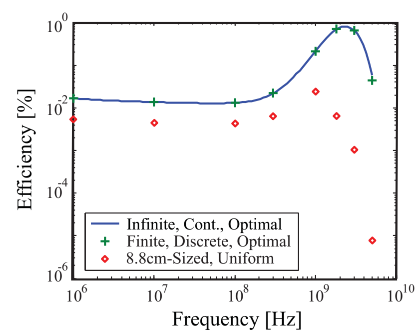

Figure 3. Solid line shows the optimal efficiency achievable by vertical magnetic current source. It is compared with the efficiency of an 8-cm diameter coil with uniform current.

Fig. 3 plots the optimal efficiency versus frequency for $(d, z_f, A_r) = (6\,\text{mm}, 4\,\text{cm}, 4\,\text{mm}^2)$. The optimal efficiency is flat in the low MHz range, increases in the 100's MHz range, peaks at 1.8 GHz, and then decreases with frequency afterwards. The difference in the efficiency is almost two order of magnitude at low MHz versus at low GHz range. In addition, the optimal efficiencies are compared with those from using a single coil of diameter 8 cm with uniform current. There is at least one order of magnitude improvement in the efficiency from optimization at the peak frequency.

To get an idea on how focusing works, Fig. 2(b) and 2(c) show the induced emf at various locations inside the tissue for a uniform source and the optimal source distribution respectively. The

$$\eta = \frac{\left| \frac{\omega\mu A_r}{4\pi^2} \iint \frac{\omega\epsilon_1}{k_{1z}+k_{2z}} \left(\frac{k_x^2+k_y^2}{k_1^2}\right) e^{i(k_{1z}-k_{2z})d+ik_{2z}z_f} \mathcal{M}_z(k_x,k_y)\, dk_x dk_y \right|^2}{\frac{\omega\,\mathrm{Im}\,\epsilon_2}{8\pi^2} \iint \frac{|e^{2ik_{1z}d}|}{2\,\mathrm{Im}\,k_{2z}|k_{1z}+k_{2z}|^2} (k_x^2+k_y^2)|\mathcal{M}_z(k_x,k_y)|^2\, dk_x dk_y} \tag{5}$$

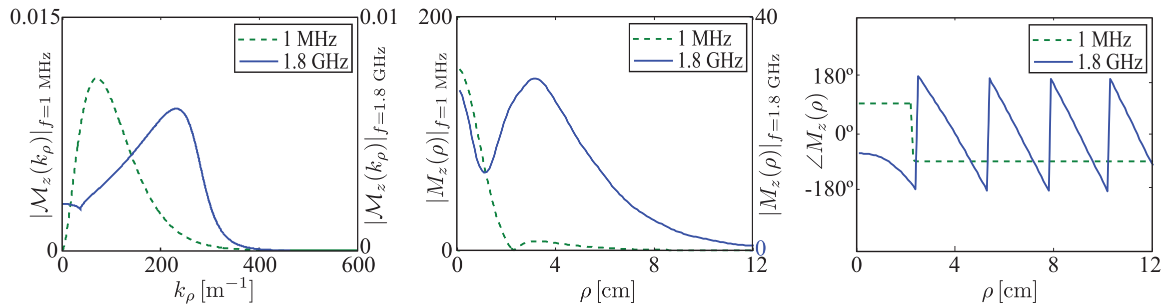

Figure 4. Plot the optimal magnetic current distributions at 1 MHz and 1.8 GHz.

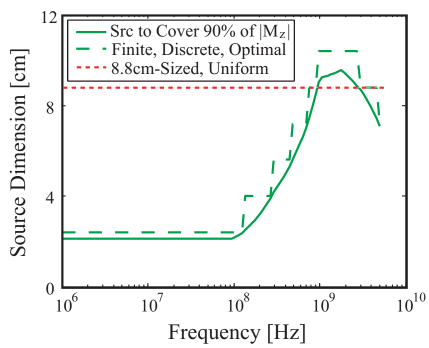

Figure 5. Approximate optimal source dimension.

induced emf shown is normalized to meet the safety constraint where each gram of tissue is not heated up by 1°C.

IV. Optimal Transmit Array Dimension

Finally, we investigate how large the source dimension should be. Fig. 4 plots the optimal source distributions at 1 MHz and 1.8 GHz given by the closed-form expression in (6) and (7). Since the optimal source distributions are circularly symmetric, we plot their distribution in the radial coordinate. The optimal distribution includes more high spatial frequency components at 1.8 GHz than that at 1 MHz. The plot on $|M_z(\rho)|$ reveals that there is diminishing return on using infinite large source. An optimal source dimension therefore exists. From the plot on $\angle M_z(\rho)$, the phase at 1 MHz is almost constant over the range of ρ where $|M_z(\rho)|$ has significant value. In contrast, it varies substantially at 1.8 GHz, implying that coil array should be used.

Fig. 5 shows the source dimension $2\rho_{\text{opt}}$ that satisfies $\int_0^{\rho_{\text{opt}}} |M_z(\rho)|^2 \rho\, d\rho \geq 0.9\|M_z(\rho)\|^2$. We numerically calculate the optimal efficiencies from finite, discrete source distributions following the method in [5]. The approximated efficiency is plotted in Fig 3. Compared with the theoretical upper bound, the finite dimensional source approaches the upper bound. At the low MHz range, a coil of 2 cm in diameter suffices while at the low GHz range, an array of coils of total diameter 10 cm suffices.

V. Conclusion

Wireless power transmission into biological tissue usually operates below 10 MHz, and quasi-static approximation and transformer model are commonly used in the analysis of the power link. In this paper, we perform full-wave analysis and model the power link as a generalized two-port network. The optimal frequency is in the GHz-range which suggests that we should consider using an antenna array to further increase the power transfer efficiency via focusing.

References

[1] A. S. Y. Poon, S. O'Driscoll and T. H. Meng, "Optimal frequency for wireless power transmission over dispersive tissue," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 5, pp.1739–1749, May 2010.

[2] S. O'Driscoll and A. S. Y. Poon and T. H. Meng, "A mm-sized implantable power receiver with adaptive link compensation", *in Proc. IEEE Intl. Solid-State Circuits Conf. (ISSCC)*, Feb. 2009.

[3] A. V. Vorst, A. Rosen, and Y. Kotsuka, *RF/Microwave Interaction with Biological Tissues*, New Jersey, USA: Wiley-IEEE Press, 2006.

[4] S. Gabriel, R. W. Lau, and C. Gabriel, "The dielectric properties of biological tissues: III. parametric models for the dielectric spectrum of tissues," *Phys. Med. Biol.*, Nov. 1996.

[5] S. Kim and A. S. Y. Poon, "Optimizations of source distribution in wireless power transmission for implantable devices," *Antennas and Propagation Society International Symposium (APSURSI), 2010 IEEE*, pp.1-4, July 2010.