The Atmosphere as a Heat Engine

The Earth's (or any other planet's) atmosphere can be thought of as a giant heat engine, powered by the sun. A simple approximation for the Earth's atmosphere is that it consists of a layer of gas wrapped around the planet, with the gas molecules rising from the surface of the planet, at temperature T_s, to the outer layer of the atmosphere (the stratosphere) at temperature T_s. At the top of the stratosphere, they begin to radiate off energy, and then they cool down, and sink back to the surface, where the process repeats.

Being a bit more specific, we will assume the following. The sun heats up a given gas molecule with heat capacity (at constant pressure) c_P (per molecule) at the surface of the Earth, from T_g to $T_\mathrm{g}+\delta$. This gas molecule then rises adiabatically to the top of the stratosphere. For our purposes, we can treat the atmosphere as adiabatic – i.e., the entropy density of the atmosphere is constant. Our gas molecule will then cool off, at constant pressure, to temperature T_s, and it will then sink down, adiabatically, to the surface of the Earth. The cycle then repeats.

(a) Compute the work done per molecule as it passes through the "atmospheric heat engine" described above, in terms of δ, T_s, T_g, c_P, c_V and/or $\gamma = c_\mathrm{P}/c_\mathrm{V}$.

(b) Show that the efficiency of the engine is given by

$$\eta_\mathrm{atm} = 1 - \frac{T_\mathrm{s}}{T_\mathrm{g}}.$$

(c) Compare the efficiency of the heat engine to the maximal efficiency allowed by thermodynamics. In what limit will the heat engine be most efficient?

Let's estimate how much mechanical power the atmospheric heat engine gives, approximately. The gas will be heated indirectly by the Sun – in particular, the high energy photons from the Sun will hit the surface of the Earth, be absorbed, and then be re-emitted at much longer wavelengths as thermal radiation at temperature T_s.

(d) Using the radius $R \approx 6\times 10^6$ m of the Earth, and the Stefan-Boltzmann law, compute the rate at which the Earth's atmosphere can do work, using that $\delta \approx 1$ K, $T_\mathrm{g} \approx 300$ K and $T_\mathrm{s} \approx 270$ K.[1] Are you surprised by the order of magnitude? Where do you think most of this energy goes?

[1] Not all of this energy will actually be absorbed by gas molecules, but up to at worst a few orders of magnitude this is a decent approximation.