13.6 Global structure of the inflationary universe and the anthropic principle

13.6.1 The domain structure of the universe

The process of mini-universe formation and self-reproduction occurs at densities $mM_P^3 \lesssim V(\phi) \lesssim M_P^4$, i.e. at $10^{-5}M_P^4 \lesssim V(\phi) \lesssim M_P^4$ for $m \sim 10^{-5}M_P$. Note that this process may occur at densities many orders of magnitude smaller than M_P^4. Therefore to prove the very existence of the process of self-reproduction of the inflationary universe in our scenario there is no need to appeal to unknown physical processes at densities greater than the Planck density $\rho_P \sim M_P^4$.

On the other hand, it is very important that independently of the origin of the universe in our scenario (whether the universe was created as a whole at $t \sim t_P$, or whether it exists eternally), it now contains an exponentially large (or even infinite) number of causally disconnected mini-universes, and a considerable proportion of these mini-universes were created when the field ϕ was $O(M_P^2/m)$ and its energy density was (almost) as large as M_P^4. (Note that this is true in the chaotic inflationary scenario only, in which inflation may occur even at $V(\phi) \sim M_P^4$.)

The local structure of spacetime inside a new-born mini-universe formed at $V(\phi) \ll M_P^4$ remains unchanged. However, in realistic theories, in which there exist many types of scalar fields Φ with masses $m_\Phi \ll H(\phi)$, large-scale fluctuations of all these fields similar to the fluctuations of the field ϕ are formed. Since the Hubble parameter $H(\phi)$ during mini-universe formation is very large, $H = (8\pi V(\phi)/3M_P^2)^{1/2} \gtrsim 10^{-2}M_P \sim 10^{17}$ GeV, fluctuations of the scalar fields Φ are strong enough to transfer the classical fields Φ in the new-born mini-universe from one local minimum of the effective potential $V(\Phi, \phi)$ to another (Linde, 1983*c*, 1984*c*). This changes the low-energy elementary particle physics inside the new mini-universe. A particular example which is relevant to this effect is the supersymmetric $SU(5)$ model. The effective potential $V(\Phi)$ in this model has several minima of approximately equal depth, and only one of them corresponds to the desirable symmetry breaking $SU(3) \times U(1)$. Even if the universe initially was in one particular vacuum state corresponding to one of these minima of $V(\Phi)$, after inflation it becomes divided into many mini-universes corresponding to *all* possible minima of $V(\Phi)$. A typical time which is necessary for a quantum tunneling from one local minimum of $V(\Phi)$ to another is many orders greater than 10^{10} years, which is the age of the observable part of the universe. In one of these mini-universes the vacuum state corresponds to the $SU(3)\times U(1)$ minimum of $V(\Phi)$. We live in this mini-universe, not for the reason that the whole universe is in the $SU(3)\times U(1)$ symmetric state and not for the reason that the corresponding minimum of $V(\Phi)$ is deeper than other minima, but for the reason that life of our type is impossible in the mini-universes with other types of symmetry breaking. This solves the problem of symmetry breaking in the supersymmetric $SU(5)$ model (Linde, 1983*c*).

In order to illustrate new possibilities which appear in the context of the scenario discussed above, let us consider now a toy model which may explain the small value of the vacuum energy density ρ_v (of the cosmological constant), in the observable part of the universe ($|\rho_v|\lesssim 10^{-29}$ g cm^{-3}). This model is probably unrealistic, but nevertheless it may be rather instructive. The model describes an inflaton field ϕ, which drives inflation, and a field Φ with an extremely flat effective potential, $V(\Phi)=\alpha M_P^3(\Phi-C)$, where $\alpha\lesssim 10^{-120}$. It can be shown that during the time interval $t\sim 10^{10}$ years after inflation such a field Φ remains essentially unchanged due to the very small slope of $V(\Phi)$. However, the Brownian motion of this field during inflation is very rapid, and it divides the universe into an exponentially large number of mini-universes containing *all* possible values of the field Φ for which $|V(\phi)+V(\Phi)|\lesssim M_P^4$. After inflation the vacuum energy ρ_v inside these mini-universes is given by $V(\Phi,\phi)=\alpha M_P^3(\Phi-C)+V(\phi_0)$, where ϕ_0 corresponds to the minimum of $V(\phi)$. This quantity in different mini-universes changes continuously from $-M_P^4$ to $+M_P^4$, but life of our type is possible only in those mini-universes in which $|\rho_v|\lesssim 10^{-29}$ g cm^{-3}. Indeed, domains with $V(\Phi,\phi)<0$, $|V(\Phi,\phi)|\gg 10^{-29}$ g cm^{-3} correspond to anti-de Sitter mini-universes with a lifetime $\tau\ll 10^{10}$ years. The domains with $V(\Phi,\phi)\gg 10^{-29}$ g cm^{-3} remain inflationary for $t\gtrsim 10^{10}$ years, and the present density of matter in such domains is negligibly small. For this reason we see ourselves inside a domain with $|\rho_v|\lesssim 10^{-29}$ g cm^{-3}.

To make this model realistic it would be desirable to explain why $V(\Phi)$ is so flat (though similar potentials sometimes appear in realistic models describing unified theories of elementary particles). In any case, the main idea suggested above may be used in other models as well. In our scenario it is not necessary to insist (as is usually done) that the vacuum energy must disappear in a 'true' vacuum state. It is quite sufficient if there exists some relatively stable vacuum-like state with $|\rho_v|\lesssim 10^{-29}$ g cm^{-3}. This requirement is still very restrictive, but it can be satisfied much more easily than the previous one. For a discussion of a similar approach to the cosmological constant problem, see also Sakharov (1984) and (1984*c*).

13.6.2 A scenario of inflationary compactification in the Kaluza–Klein cosmology

In the mini-universes of initial size $O(H^{-1}) \simeq O(M_P^{-1})$, which are formed at $V(\phi) \sim M_P^4$, fluctuations of the metric on a scale $O(M_P^{-1})$ are of the order of unity. Therefore, if a theory of the type of eq. (2.1) can be considered as a part of a Kaluza–Klein (or superstring) theory, then at the moment of mini-universe formation the type of compactification and the number of compactified dimensions inside the new-born mini-universe can be changed (almost) independently of what occurs in causally disconnected regions outside it. (The only possibly constraint on the local changes of spacetime structure stems from topological considerations and may lead, for example, to the formation of a pair of mini-universes with opposite topological numbers.) As a result, our universe becomes divided into an exponentially large number of domains, and all possible types of compactification and all possible (metastable) vacuum states should exist in different domains of the universe. All these domains (mini-universes), in which inflation remains possible after their formation, later become exponentially large (Linde, 1986*a*, *b*, *c*).

For a detailed investigation of this possibility it would be desirable to have a realistic model of inflation in Kaluza–Klein or superstring theories. Unfortunately, the models of Kaluza–Klein inflation that now exist are not completely satisfactory (Maeda and Pollock, 1986; Linde, 1986*c*). On the other hand, we do not know any alternative possibility for solving all the problems that are solved by the inflationary universe scenario. Therefore one may consider the difficulties with the Kaluza–Klein (and superstring) inflation as difficulties of these theories (Maeda and Pollock, 1986). Actually, however, our understanding of these theories is incomplete, and hopefully the corresponding problem will be resolved in future. At present we are just trying to understand some possible features of the Kaluza–Klein inflationary cosmology, which are rather unusual. To illustrate some of the ideas discussed above, let us again consider inflation in the toy model, eq. (2.1).

In our investigation of inflation in this model we have studied only four-dimensional de Sitter-like solutions dS_4 with metric

$$ds^2 \approx dt^2 - H^{-2}\cosh^2 Ht(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)), \qquad (6.1)$$

where $H^2 = 8\pi V(\phi)/3M_P^2$. However, there exist some other interesting solutions, such as

$$ds^2 = dt^2 - \tfrac{1}{3}H^{-2}((\cosh^2(\sqrt{3}\,Ht)\,d\chi^2 + d\theta^2 + \sin^2\theta\,d\phi^2). \qquad (6.2)$$

This solution describes a universe that is a product of a two-dimensional de Sitter space dS_2 and a compact sphere S_2 with a very small radius $R_2 = (\sqrt{3}\,H)^{-1}$. This is a particular case of a Kantowski–Sachs universe (Kantowski and Sachs, 1966; Kofman *et al.*, 1983; Paul *et al.*, 1986). This solution is unstable; it (locally) transforms into dS_4. However, it is possible to stabilize it, for example by an appropriate addition of R^n terms to the Lagrangian, eq. (2.1) (Deruelle and Madore, 1986; Linde and Zelnikov, 1987).

Recently it was shown that long-wave quantum perturbations of scalar fields ϕ similar to those discussed in Section 13.4 are generated in the $dS_2 \times S_2$ universe (6.2) as well (Kofman and Starobinsky, 1987).† This result is directly related to the main conclusions of the present chapter (Linde, 1986*c*). Namely, if inflation is initially three dimensional as in ordinary de Sitter space dS_4 (6.1), then fluctuations of the scalar field ϕ create some inflationary domains (mini-universe), in which $V(\phi) \sim M_P^4$. Due to large fluctuations of ϕ and of metric $g_{\mu\nu}$ inside domains of a size $O(H^{-1}) \sim M_P^{-1}$, expansion inside some of these domains may become one dimensional (6.2), *independently of what occurs in the nearby domains*. This leads to formation of Kantowski–Sachs 'branches', which spread out of de Sitter space. However, fluctuations in the Kantowski–Sachs mini-universes lead again to creation of domains with $V(\phi) \sim M_P^4$ where de Sitter mini-universes (6.1) can be created. With account taken of this effect, the inflationary universe in our scenario may appear as a system of huge inflationary bubbles dS_4 connected with each other by thin inflationary tubes $dS_2 \times S_2$ of exponentially large length (Fig. 13.2).

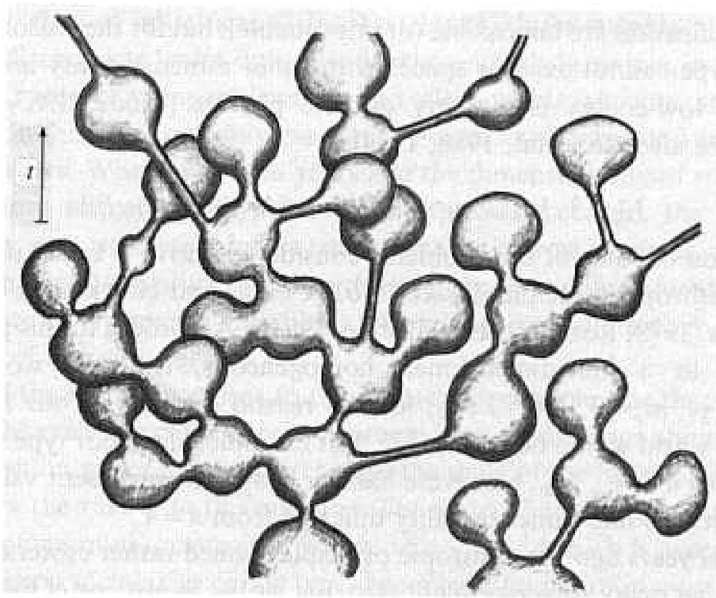
The possibility of existence of such a complicated spacetime structure is directly related to the 'no-hair' theorem for de Sitter space, which is valid also for the one-dimensional exponential expansion (6.2). The processes which occur in a part of a tube (6.2) of initial size $\Delta l \gtrsim H^{-1}$ proceed independently of what occurs in other parts of the universe. Actually, the Kantowski–Sachs universe (6.2) is not the only solution of the Einstein equations with $V(\phi) > 0$ which leads to spontaneous compactification of a

† This is actually a general property of all inflationary models related to the existence of d zero modes of a scalar field ϕ in the Euclidean section S_d of d-dimensional de Sitter space dS_d.

four-dimensional space during inflation. We hope to return to the discussion of this question in a separate publication. Here we would only like to mention that the effect considered above is fairly general. For example, in d-dimensional space ($d>4$) inflation and fluctuations of scalar fields lead to formation of a complicated structure consisting of inflationary mini-universes of all possible types, including mini-universes dS_d, $dS_{d-2}\times S_2$, $dS_{d-3}\times S_3$, etc., connected with each other.

During inflation the radius R_n of each compactified sphere S_n remains of

Fig. 13.2. This picture gives some idea of the global structure of the chaotic inflationary universe. Child; universes created where $V(\phi)\ll M_P^4$ have the same 'genetic code' as their mother universes: they have the same number of dimensions and the same (or almost the same) vacuum structure. However, the universes created where $V(\phi)$ is not much smaller than M_P^4 are 'mutants', which may have different dimensionality and different low-energy elementary particle physics inside them. Each mini-universe may die, but the universe as a whole has no end and may have no beginning. A typical thickness of 'tubes' connecting mini-universes after inflation may become very large. However, during inflation some of the tubes have a very small thickness. For example, some of the de Sitter mini-universes dS_d may be connected by the Kantowski–Sachs tubes $dS_{d-n}\times S_n$, where the radius of the sphere S_n is of the order $O(H^{-1}(\phi))$. The mini-universes compactified during inflation may serve as seeds for the next stage of the process of compactification, which occurs after inflation.



the order $H^{-1}(\phi)$, which initially is as small as M_P^{-1}. The main part of the physical volume of the universes $dS_{d-n} \times S_n$ remains in the state with $V(\phi) \sim M_P^4$ for ever due to the effects connected with the long-wave fluctuations of the field ϕ, see Section 13.4. In those domains of the universe in which the field ϕ slowly decreases, the radius of compactification $R_n(\phi)$ slowly grows as $H^{-1}(\phi)$ unless there exist some dynamical mechanism which fixes the radius of compactification at some value $R_0 \gtrsim M_P^{-1}$. In this sense the domains of the universe compactified during inflation may serve as seeds for different possible types of compactified spaces in the Kaluza–Klein theories.

This provides us with a new scenario of compactification as compared with the scenario of a classical power-law Kasner-like anisotropic expansion discussed by several authors (see, e.g. Freund, 1982; Freund and Oh, 1985). Namely, during inflation at $mM_P^3 \lesssim V(\phi) \lesssim M_P^4$ mini-universes with all possible types of compactification are formed in different causally disconnected regions of the universe, and those mini-universes in which inflation (in the uncompactified directions) remains possible after their formation later become exponentially large. The process of formation of new mini-universes has no end, in the major part of the physical volume of the universe this process occurs even now, and therefore even if the probability of formation of a mini-universe of some particular type is strongly suppressed, many such mini-universes should exist at present. According to this scenario, we live in a four-dimensional spacetime with our type of compactification, not for the reason that other types of compactification are impossible (or improbable), but for the reason that life of *our* type cannot exist in spaces with other dimensionality and with a different low-energy elementary particle physics (Linde, 1983*a*, 1984*c*, 1986*c*; see also Rozental, 1980, 1984).

13.6.3 Inflation and the anthropic principle

Discussion of some of the problems considered above is based on the so-called anthropic principle (Dicke, 1961; Collins and Hawking, 1973; Carr and Rees, 1979; Rozental, 1980; Linde, 1984*c*). According to this principle, we live in a four-dimensional, homogeneous, isotropic world with $e^2/4\pi = \frac{1}{137}$, $m_e = 0.5\,\mathrm{MeV}$, etc., for the reason that life of our type in a different world would be impossible. For example, life of our type would be impossible if e, m_e, M_P, etc., were half (or less) of their present values, or if our spacetime had dimensionality different from $d=4$.

Several years ago the anthropic principle seemed rather esoteric, since it implied that many universes might exist, but we live in just one of them, which is sufficiently suitable for us. It was not clear in what sense one could speak of many universes if our universe is unique, and whether such fundamental constants of nature as the dimensionality of spacetime, the vacuum energy, the value of electric charge, etc., can change when one 'travels' from one universe to another.

A possible answer to the first part of this question was suggested by the many-worlds interpretation of quantum mechanics (Everett, 1957). Further investigation of this possibility may be of profound importance. However, it seems doubtful that it is possible to use this interpretation correctly without a proper understanding of the nature of consciousness. Does an observer just observe the universe, or does he 'create' it? (Wheeler, 1979). What is actually split: the universe or consciousness? Can consciousness exist 'by itself' (like spacetime without matter) or is it merely an arena for the manifestation of spacetime and matter? In what sense can consciousness 'choose' a universe to live in?

Thus the development of physics reveals problems which traditionally were beyond the scope of physics. It seems that to go further we must investigate these problems without prejudice, rather than wait until philosophers try to do it for us. However, such a path is not easy to follow, and it seems encouraging that inflation makes it possible to circumvent some of the above-mentioned problems that precluded the justification of the anthropic principle. Namely, from the scenario discussed in this chapter it follows that even if the universe at some time contains only one domain (mini-universe) of initial size $O(H_{(\phi)}^{-1}) \gtrsim O(M_P^{-1})$, later it splits into many causally disconnected mini-universes of exponentially large size, in which all possible types of compactification and all possible vacuum states are realised. In this sense our universe actually consists of many universes of all possible types. Whereas several years ago the dimensionality of spacetime, the vacuum energy density, the value of electric charge, the Yukawa couplings, etc., were regarded as true constants, it now becomes clear that these 'constants' actually depend on the type of compactification and on the mechanism of symmetry breaking, which may be different in different domains of our universe.

One of the main objections to the anthropic principle was the assertion that for the existence of life of our type there is no need for our universe to be so uniform on scales much greater than the scale of the Solar System. We now know the answer to this question. The size of the homogeneous locally Friedmannian mini-universes in which $\delta\rho/\rho \sim 10^{-4}$ (which is necessary for the formation of galaxies of our type) becomes after inflation much greater than the size of the Solar System. For example, in the model (2.1) $\delta\rho/\rho$ on a galactic scale is given by $O(10)(m/M_P)$, which gives $m \sim 10^{-5} M_P$. On the other hand, the size of a uniform part of our universe is greater than $M_P^{-1} \exp[2\pi(M_P/m)] \gtrsim 10^{10^5}$ cm, which is much greater than the observable part of the universe ($\sim 10^{28}$ cm).

This means that actually it is possible to justify some kind of weak anthropic principle in the inflationary cosmology. The line of thought advocated here is an alternative to the old assumption that in a 'true' theory it must be possible to compute unambiguously all masses, coupling constants, etc. From our point of view, it is rather improbable that a 'true' theory must have only one 'true' ground state. In the context of the unified theories of all fundamental interactions, the validity of this assumption becomes unlikely. According to the scenario discussed in this chapter, this assumption is probably incorrect; in any case it is not necessary. The old question of why our universe is the only possible one now is replaced by a new question: in which theories is the existence of mini-universes of our type possible? This question is still very difficult, but it is much easier than the old one.

13.7 Conclusions

The inflationary universe scenario continues to develop rapidly. It appears that the global structure of the universe according to this scenario is determined by quantum effects, that the major part of the physical volume of the universe should even now be in an inflationary state with $V(\phi) \sim M_P^4$, that the universe eternally reproduces itself, and that it consists of exponentially many mini-universes of different types. Some of these results are model-dependent, some are not. We do not know what the fate of the ideas discussed above will be, nor how they will be modified by future developments of elementary particle physics and of the theory of superstrings. In any case, the inflationary universe scenario may serve as a good example, showing how many exciting surprises the theory of gravity developed three hundred years ago can still give us.

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