properties of $\tilde{\Lambda}^{\text{out}}$ on Lagrangian submanifolds Λ' of $T^*X_{c_j}$. But this is exactly what we have proved in Lemma B.3. Thus, we conclude that $\Lambda^{\text{out}}_{j+1}$ is smooth forward-spreading Lagrangian, completing the inductive proof. □

Remark B.6. The outgoing radial set, $R_-(\lambda)\cap {}^{\text{sc}}T^*_{C'_{c_1}} X_{c_1}$ gives another example of a spreading Lagrangian Λ'. (Strictly speaking, $R_-(\lambda)\cap {}^{\text{sc}}T^*_{C'_{c_1}} X_{c_1}$ needs to be 'homogenized' using the $\mathbb{R}^+$ action, i.e. the conic structure, to extend it to a Lagrangian submanifold of $T^*X_{c_1}$.) Namely, the Lagrangian in ${}^{\text{sc}}T^*_{X_{c_1}} X_{c_1}$ corresponding to $R_-(\lambda)$ is a union of the Lagrangians

$$\Lambda = \Lambda_\sigma = \{(w, \sqrt{\lambda - \sigma}\frac{w}{|w|}) : w \in X_c\},\ \sigma \in \Lambda_1, \tag{B.36}$$

so, with the notation of Definition B.1, A is given by

$$A = \frac{\sqrt{\lambda - \sigma}}{|w|^3}(|w|^2 \operatorname{Id} - w \otimes w), \tag{B.37}$$

hence $A \geq 0$. Moreover, $\operatorname{Ker}(A)$ is exactly $\operatorname{span}(w)$. Thus, the proof of Proposition B.4 actually shows that the forward broken bicharacteristic relation maps it to a finite union of smooth Lagrangian submanifolds.

We now indicate the changes required to prove the S-matrix result, Theorem 2.10.

Proof. (Proof of Theorem 2.10.) With the notation of (B.1), if $m = 0$, then γ is an unbroken bicharacteristic, so the endpoints of γ are related by the antipodal relation, which is certainly Lagrangian. So we may assume $m \geq 1$. In addition, due to the assumptions (2.34), the ${}^{\text{sc}}H^a_g$ bicharacteristics will only break upon hitting C_{c_1} with $c_1 \neq a$ (i.e. a-tangential bicharacteristics cannot become normal to a without hitting $C_{a,\text{sing}}$). Similarly, $c_m \neq b$.

The only change from the proof of Proposition B.4 is that we need to replace the first, respectively the last two, Lagrangians in the composition (B.32) by the Lagrangians $\tilde{\Lambda}_\beta \subset T^*C'_b \times T^*X_{c_m}$ and $\tilde{\Lambda}_\alpha \subset T^*X_{c_1} \times T^*C'_a$ given by the (twisted) graphs of $d(-\sqrt{\lambda - \epsilon_\beta}\,\omega \cdot w')$ and $d(-\sqrt{\lambda - \epsilon_\alpha}\,\omega' \cdot w)$ respectively; here we wrote $(\omega, \zeta, w', \xi')$ and $(w, \xi, \omega', \zeta')$ for the coordinates on $T^*C'_b \times T^*X_{c_m}$ and $T^*X_{c_1} \times T^*C'_a$ respectively. Thus, on $\tilde{\Lambda}_\alpha$, $\xi = -\sqrt{\lambda - \epsilon_\alpha}\,\pi_{a,c_1}(\omega')$, so the differential of the projection of $\tilde{\Lambda}_\alpha$ to $T^*X_{c_1}$ is surjective since $a \neq c_1$. In particular, we can choose a Lagrangian subspace V' of $T_{(w,\xi)}T^*X_{c_1}$ which is in the range of the differential of this projection, and which is a graph over X_{c_1} of the form (w, Aw) with A positive definite (e.g. the identity). Then the proof of Proposition B.4 shows that the composite relation

$$\tilde{\Lambda}^{\text{out}}_{a_{m+1},\alpha_{m+1},c_m} \circ \tilde{\Lambda}^{\text{in}}_{c_m,a_m} \circ \tilde{\Lambda}^{\text{out}}_{a_m,\alpha_m,c_{m-1}} \circ \ldots \circ \tilde{\Lambda}^{\text{out}}_{a_2,\alpha_2,c_1} \circ \tilde{\Lambda}^{\text{in}}_{c_1,a_1} \circ \tilde{\Lambda}_\alpha \tag{B.38}$$

is a smooth Lagrangian. Since the differential of the projection of $\tilde{\Lambda}_\beta$ to $T^*X_{c_m}$ is also surjective as $b \neq c_m$, the composition of $\tilde{\Lambda}_\beta$ with (B.38) is transversal, so the result is a smooth Lagrangian submanifold of $T^*C'_b \times T^*C'_a$. In view of Theorem 2.8, this and Remark B.6 complete the proof. □

References

[1] A. Bommier. Propriétés de la matrice de diffusion, 2-amas 2-amas, pour les problèmes à N corps à longue portée. *Ann. Inst. Henri Poincaré*, 59:237–267, 1993.

[2] J. Dereziński. Asymptotic completeness of long-range N-body quantum systems. *Ann. Math.*, 138:427–476, 1993.

[3] J. Dereziński and C. Gérard. *Scattering theory of classical and quantum N-particle systems.* Springer, 1997.

[4] R. G. Froese and I. Herbst. Exponential bounds and absence of positive eigenvalues of N-body Schrödinger operators. *Commun. Math. Phys.*, 87:429–447, 1982.

[5] R. G. Froese and I. Herbst. A new proof of the Mourre estimate. *Duke Math. J.*, 49:1075–1085, 1982.

[6] C. Gérard, H. Isozaki, and E. Skibsted. *Commutator algebra and resolvent estimates*, volume 23 of *Advanced studies in pure mathematics*, pages 69–82. 1994.

[7] C. Gérard, H. Isozaki, and E. Skibsted. N-body resolvent estimates. *J. Math. Soc. Japan*, 48:135–160, 1996.

[8] G. M. Graf. Asymptotic completeness for N-body short range systems: a new proof. *Commun. Math. Phys.*, 132:73–101, 1990.

[9] A. Hassell. Distorted plane waves for the 3 body Schrödinger operator. *Geom. Funct. Anal.*, to appear.

[10] A. Hassell and A. Vasy. The spectral projections and the resolvent for scattering metrics. *J. d'Analyse Math.*, to appear.

[11] A. Hassell and A. Vasy. Symbolic functional calculus and N-body resolvent estimates. *J. Func. Anal.*, to appear.

[12] L. Hörmander. *The analysis of linear partial differential operators,* vol. 1-4. Springer-Verlag, 1983.

[13] M. Ikawa, editor. *Spectral and scattering theory.* Marcel Dekker, 1994.

[14] H. Isozaki. Structures of S-matrices for three body Schrödinger operators. *Commun. Math. Phys.*, 146:241–258, 1992.

[15] H. Isozaki. A generalization of the radiation condition of Sommerfeld for N-body Schrödinger operators. *Duke Math. J.*, 74:557–584, 1994.

[16] H. Isozaki. *A uniqueness theorem for the N-body Schrödinger equation and its applications.* In Ikawa [13], 1994.

[17] H. Isozaki and J. Kitada. Scattering matrices for two-body schrödinger operators. *Scient. Papers College Arts and Sci., Tokyo University*, 35:81–107, 1985.

[18] A. Jensen. Propagation estimates for Schrödinger-type operators. *Trans. Amer. Math. Soc.*, 291-1:129–144, 1985.

[19] G. Lebeau. Propagation des ondes dans les variétés à coins. *Ann. Scient. Éc. Norm. Sup.*, 30:429–497, 1997.

[20] R. Mazzeo and R. B. Melrose. Pseudodifferential operators on manifolds with fibred boundaries. *Asian Journal of Mathematics*, 2, 1998.

[21] R. B. Melrose. *Differential analysis on manifolds with corners.* In preparation.

[22] R. B. Melrose. Calculus of conormal distributions on manifolds with corners. *International Mathematics Research Notices*, no. 3:51–61, 1992.

[23] R. B. Melrose. *Spectral and scattering theory for the Laplacian on asymptotically Euclidian spaces.* In Ikawa [13], 1994.

[24] R. B. Melrose and J. Sjöstrand. Singularities of boundary value problems. I. *Comm. Pure Appl. Math*, 31:593–617, 1978.

[25] R. B. Melrose and M. Zworski. Scattering metrics and geodesic flow at infinity. *Inventiones Mathematicae*, 124:389–436, 1996.

[26] E. Mourre. Absence of singular continuous spectrum of certain self-adjoint operators. *Commun. Math. Phys.*, 78:391–408, 1981.

[27] P. Perry, I. M. Sigal, and B. Simon. Spectral analysis of N-body Schrödinger operators. *Ann. Math.*, 114:519–567, 1981.

[28] I. M. Sigal and A. Soffer. N-particle scattering problem: asymptotic completeness for short range systems. *Ann. Math.*, 125:35–108, 1987.

[29] I. M. Sigal and A. Soffer. Long-range many-body scattering. *Inventiones Math.*, 99:115–143, 1990.

[30] I. M. Sigal and A. Soffer. Asymptotic completeness of $N \leq 4$-particle systems with the Coulomb-type interactions. *Duke Math. J.*, 71:243–298, 1993.

[31] I. M. Sigal and A. Soffer. Asymptotic completeness of N-particle long-range scattering. *J. Amer. Math. Soc.*, 7:307–334, 1994.

[32] E. Skibsted. Smoothness of N-body scattering amplitudes. *Reviews in Math. Phys.*, 4:619–658, 1992.

[33] A. Vasy. Structure of the resolvent for three-body potentials. *Duke Math. J.*, 90:379–434, 1997.

[34] A. Vasy. Propagation of singularities in many-body scattering in the presence of bound states. *Journées Équations aux Dérivées Partielles, Saint-Jean-de-Monts*, 1999.

[35] A. Vasy. Scattering matrices in many-body scattering. *Commun. Math. Phys.*, 200:105–124, 1999.

[36] A. Vasy. Propagation of singularities in three-body scattering. *Astérisque*, 262, 2000.

[37] A. Vasy. Propagation of singularities in many-body scattering. *Ann. Sci. École Norm. Sup.*, to appear.

[38] X. P. Wang. Microlocal estimates for N-body Schrödinger operators. *J. Fac. Sci. Univ. Tokyo Sect. IA, Math.*, 40:337–385, 1993.

[39] D. Yafaev. Radiation conditions and scattering theory for N-particle Hamiltonians. *Commun. Math. Phys.*, 154:523–554, 1993.