

Knapsack Voting

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1. INTRODUCTION

Recently there has been a rise in attempts to directly engage citizens in policy-making with democratic innovations [Aitamurto et al. 2014; Pateman 2012; Smith 2009]. One of these is participatory budgeting [Cabannes 2004], in which a local government asks residents to vote on proposals for how a certain fraction of their total budget should be spent. Participatory budgeting is now gaining popularity in cities like San Francisco, Vallejo, Chicago and New York [PBP]. This development necessitates the need for a more accurate voting mechanism than the traditional “check-the-box” method used in current ballots and motivates the following question: in a scenario where each voter benefits differently from each proposal, and the benefit from the same proposal varies across voters, how should the preferences of voters be aggregated? We address this question by proposing new voting schemes, and discuss their advantages over existing methods both informally in terms of making users take trade-offs into account, and formally by showing that they are partially strategy-proof. Inspired by the classical *Knapsack Problem*, we call this class of schemes *Knapsack Voting*.

2. THE PARTICIPATORY BUDGETING PROBLEM

The participatory budgeting problem addresses the following scenario: the residents of a city, collectively the set of voters \mathcal{V} , are voting on a set \mathcal{P} of proposals that they have identified to be worthwhile, where proposal $j \in \mathcal{P}$ has a cost c_j and there is a fixed total budget of B Dollars. Say the benefit a voter $i \in \mathcal{V}$ gets from proposal j is $v_{i,j}$. Ideally, the government would want to elicit the preferences of each voter to find a feasible allocation among the various proposals that maximizes the average benefit to the voters. Assuming additive utilities, it would want to solve the following problem:

$$\arg \max_{W \subseteq \mathcal{P}} \sum_{j \in W} \frac{1}{|\mathcal{V}|} \sum_{i \in \mathcal{V}} v_{i,j}, \text{ subject to } \sum_{j \in W} c_j \leq B.$$

Given truthful elicitation of the private valuations, this optimization problem reduces to the *Knapsack* problem [Martello and Toth 1990] with \mathcal{P} as the set of items to be fitted into a knapsack of size B . For a proposal j , its average utility ($\frac{1}{|\mathcal{V}|} \sum_{i \in \mathcal{V}} v_{i,j}$), and cost (c_j) correspond to its value, and size, respectively. Assuming fractional solutions are allowed, all we need to do is order the proposals according to the average utility per dollar ($\frac{1}{c_j |\mathcal{V}|} \sum_{i \in \mathcal{V}} v_{i,j}$) that each endows to the voters. It is not clear, however, that voters have a precise knowledge of their values. In addition, there is no established unit in which voters can report their valuations. We also know from social choice theory that elicitation of private preferences runs into celebrated impossibility theorems like those of Arrow and Gibbard-Satterthwaite [Arrow 2012; Satterthwaite 1975]. In light of the above issues, we make each voter directly optimize over the knapsack capacity, or compare proposals according to their benefit per dollar (i.e. the ratios $\frac{v_{i,j}}{c_j}$) in the schemes that we design.

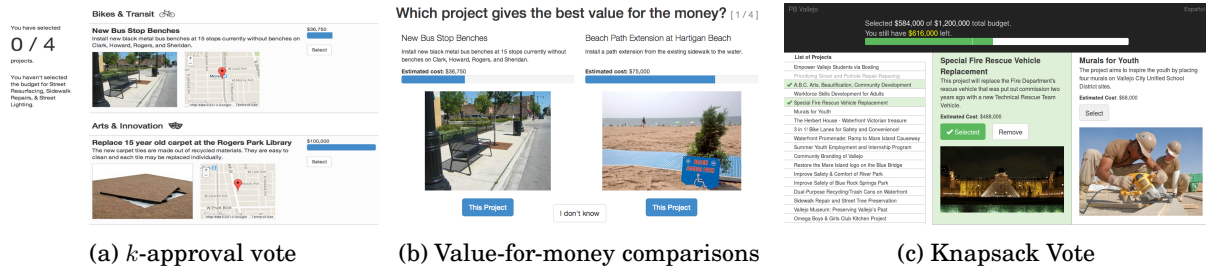


Fig. 1: (a,b) Snapshots of the interface we implemented for Chicago’s 49th Ward, (c) Our proposed interface for knapsack voting.

3. OUR CONTRIBUTIONS

The voting methods used by current participatory budgeting initiatives are ones where each voter chooses all proposals she approves of (*approval voting*), or just her top- k choices (*k-approval voting*, see Fig. 1a). Each proposal is ranked according to the total number of voters that included it, and the higher ranked ones are given a priority. While these methods have a long precedent in social choice [Brams and Fishburn 2007], they don’t require the voter to take the cost of the proposals into account. We now describe two natural approaches to the problem at hand.

3.1 Voting with budget constraints

If asked to propose a valid allocation of the total budget among the various proposals, a voter has to essentially solve a knapsack problem. Hence, we define the first of our voting schemes as follows:

Definition 3.1 (Knapsack Vote). Each voter $i \in \mathcal{V}$ submits a subset $S_i \subseteq \mathcal{P}$, such that it satisfies a budget constraint $\sum_{j \in S_i} c_j \leq B$ (see Fig. 1c).

Similar to approval voting, we aggregate these votes by giving each proposal a score equal to the number of voters that include it in their votes. The budget is then filled by choosing sets in descending order of their scores. Let S_{-i} denote the cumulative votes of all voters except i , \mathcal{W}_{-i} the set of winners as determined by S_{-i} , and $\mathcal{W}(S_i, S_{-i})$ the set of winners when i ’s vote S_i is added to S_{-i} . A *best response* for i is then defined as a vote S_i^* that satisfies $\sum_{j \in \mathcal{W}(S_i^*, S_{-i})} v_{i,j} = \max_{S_i \in \mathcal{V}} \sum_{j \in \mathcal{W}(S_i, S_{-i})} v_{i,j}$ where $\mathcal{V} \triangleq \{S : S \subseteq \mathcal{P}, \sum_{j \in S} c_j \leq B\}$. The consequence of the budget constraint is that it allows for *partial strategy-proofness* in the best response of a focal voter responding to the votes of all other voters.

THEOREM 3.2 (PARTIAL STRATEGY-PROOFNESS). *Given a best response S_i^* , if $j^* \in S_i^*$, then there is another best response S_i^{**} such that $(\Sigma_{i,j^*} \cap \mathcal{W}_{-i}) \subseteq S_i^{**}$ where $\Sigma_{i,j^*} \triangleq \{j \in \mathcal{P} : \frac{v_{i,j}}{c_j} > \frac{v_{i,j^*}}{c_{j^*}}\}$.*

In other words, when a voter has to vote for a certain proposal j , it is in her best interest to also vote for those that she prefers more than j from among the ones that are winning. This notion is similar in spirit to the ideal of *sincerity* in approval elections given by Brams and Fishburn [2007]. By the following corollary, we show that *knapsack voting* is provably better than *k-approval voting*.

COROLLARY 3.3. *Theorem 3.2 fails to hold when each voter submits a k-approval vote (i.e. $|S_i| = k$), and the winning set is constrained by the budget B .*

3.2 Voting using comparisons

To solve the budgeting problem optimally, we need to rank the proposals according to how much they benefit the electorate per dollar spent. Asking voters to compare different pairs of proposals based on their *value-for-money* is a natural way of doing so.

Definition 3.4 (Value-for-money Vote). For each pair of proposals (j, k) presented to her, voter i is asked to choose a winner $w_i(j, k) = \arg \max_{t \in \{j, k\}} \frac{v_{i,t}}{c_t}$ (see Fig. 1b).

To maintain uniformity across voters and proposals, we choose a uniformly random subset of pairs of a fixed size for each voter. We then aggregate these votes by calculating a strict order which minimizes the number of disagreements with respect to the elicited comparisons. More formally, if $G = (\mathcal{P}, E)$ is a complete directed graph on the set of proposals \mathcal{P} , and the weight $w_{j \rightarrow k}$ of each edge $j \rightarrow k \in E$ is the number of comparisons where j is favored to k , we want to:

PROBLEM 1. Find a strict rank ordering \prec on \mathcal{P} that minimizes $\sum_{j \in \mathcal{P}} \sum_{k \succ j} w_{k \rightarrow j}$.

The above problem is exactly the weighted Minimum Feedback Arc Set problem [Even et al. 1995], and uses an objective similar to the Kemeny-Young method for aggregating complete orders [Young and Levenglick 1978]. While it is NP-hard in general [Hemaspaandra et al. 2005], it turned out to be tractable for our experiment via an LP-relaxation. We now turn to describing our experiment.

4. RESULTS FROM A PRELIMINARY TRIAL

Based off of our earlier work for Chicago’s 49th ward (Figs. 1a,1b), we developed a digital voting system for the participatory budgeting initiative in the City of Vallejo, California, during Sept.-Oct. 2014 (<https://pbstanford.org/vallejo/>). As the ballot had to be available in a paper format too, we could not implement the budget constraint on the votes. For the actual ballot, we implemented a scheme where voters could choose up to 5 of the 25 projects on offer (similar to Fig. 1a). As an addition to the online voting procedure, we asked the voters to participate in our trial to understand the efficacy of using *value-for-money* comparisons in this context. We presented each voter with a uniformly random subset of size 8 chosen from the set of all possible pairs (similar to Fig. 1b). About 20% of the voters used the digital interface, while others used a paper representation of the same ballot.

To aggregate the comparisons, we use the following LP-relaxation [Ali and Meilă 2012; Conitzer et al. 2006] to Problem 1 stated above:

$$\text{minimize } \sum_{j,k \in \mathcal{P}} w_{j \rightarrow k} x_{k \rightarrow j} + w_{k \rightarrow j} x_{j \rightarrow k}; \text{ subject to } \forall j, k \in \mathcal{P}, x_{j \rightarrow k} + x_{k \rightarrow j} = 1, \text{ and } \forall c \in C, \sum_{e \in c} x_e \geq 1.$$

where C is the set of all cycles in G . Quite surprisingly, we observe that we get integer-optimal results on our data implying that we found the optimal aggregate ranking. This hints at some structure in data from *participatory budgeting* problems that deserves further investigation.

Also, we observe qualitative differences between the results from the top-5 procedure and the *value-for-money* comparisons in that some proposals fared very differently. A full exposition of the data is beyond the scope of this article.

5. CONCLUSIONS AND ONGOING WORK

As mentioned before, the Knapsack Problem can be solved optimally using the correct order of items with respect to their relative value-for-money. *Knapsack Voting* schemes present an intuitive way of eliciting this information from the voters. Approval voting with budget constraints admits an interesting notion of strategy-proofness (Theorem 3.2). And *value-for-money* comparisons provide fine-grained information about the voters’ preferences, along with practical tractability of a Kemeny-Young-like method. Furthermore, these schemes are amenable to implementation using interactive digital tools, thereby enhancing the ability of voters to make more informed decisions for *participatory budgeting*. To widen the adoption of our platform, we are currently in conversations with additional partners in New York City, Long Beach (California) and Cambridge (Massachusetts). Further progress along these promising avenues is the aim of our ongoing work.

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