T-orders across categorical and probabilistic constraint-based phonology

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Introduction

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- **Examples:**
  - $CCCV \longrightarrow CCV \longrightarrow CV$
    if a language allows a complex margin cluster, it allows a simpler one
  - $(/cost.us/, [cos.us]) \longrightarrow (/cost.me/, [cos.me])$
    if a dialect of English deletes t/d before V, it also deletes it before C

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  holds provided each grammar in $\mathfrak{T}$ which maps x to y also maps $\widehat{x}$ to $\widehat{y}$
- $\xrightarrow{\mathfrak{T}}$ is a partial order called the **T-order** induced by $\mathfrak{T}$ [Anttila and Andrus 2006]

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- The preceding categorical definition of T-orders is a special case of the probabilistic one

- T-orders are interesting/important because:
  - impose strict limits on categorical and statistical phonological patterns
  - "measure" the amount of typological structure
  - follow from the phonological theory
  - need not be learned, cannot be subverted by learning [Becker et al. 2011]
  - model categorical and gradient phonotactic judgments [Anttila 2008]

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- We develop formal theory of T-orders in constraint-based phonology:
  - two categorical frameworks: categorical OT ($\xrightarrow{\text{OT}}$)
    categorical HG ($\xrightarrow{\text{HG}}$)
  - four probabilistic frameworks: stochastic OT ($\xrightarrow{\text{sOT}}$)
    stochastic (or noisy) HG ($\xrightarrow{\text{sHG}}$)
    partial order OT ($\xrightarrow{\text{poOT}}$)
    Max Ent ($\xrightarrow{\text{ME}}$)

  and we explore its phonological implications

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- **Phonological implications 1**: a surprising equivalence result for OT and HG T-orders under the 2-candidate condition
- **Phonological implications 2**: first ever analytical results on MaxEnt T-orders, showing that they display some counterintuitive properties

Formal results 1

$$\boxed{(x, y) \xrightarrow{\text{ME}} (\widehat{x}, \widehat{y})}$$

$$\boxed{(x, y) \xrightarrow{\text{poOT}} (\widehat{x}, \widehat{y})}$$

$$\boxed{(x, y) \xrightarrow{\text{HG}} (\widehat{x}, \widehat{y})}$$

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- The HG typology is usually at least as large as the OT typology
  hence, a HG T-order entails the OT T-order
  and the vice versa fails in the general case

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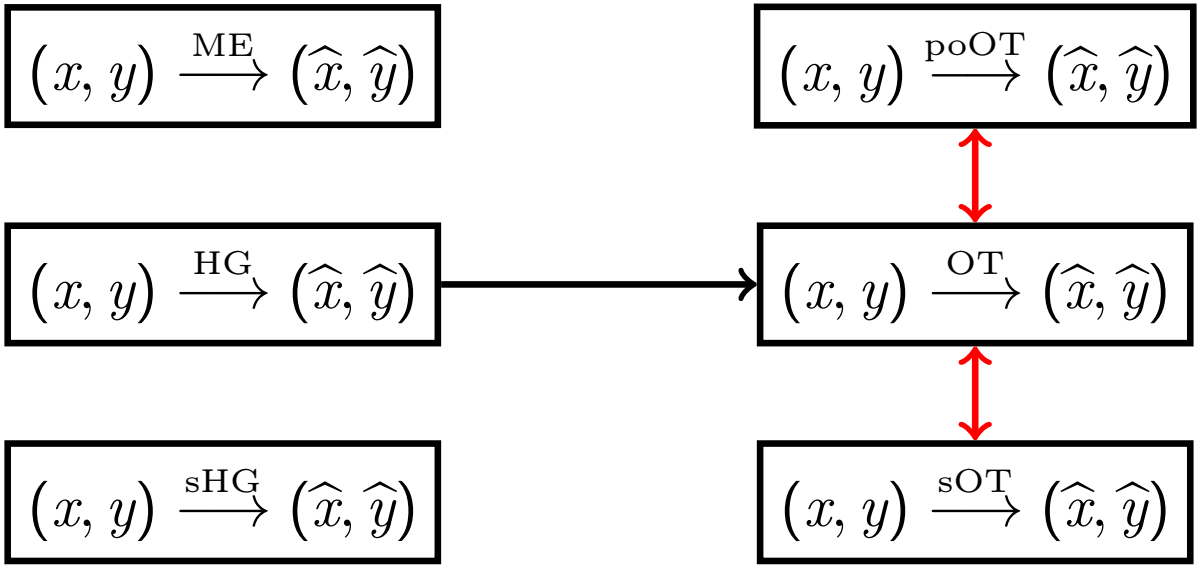
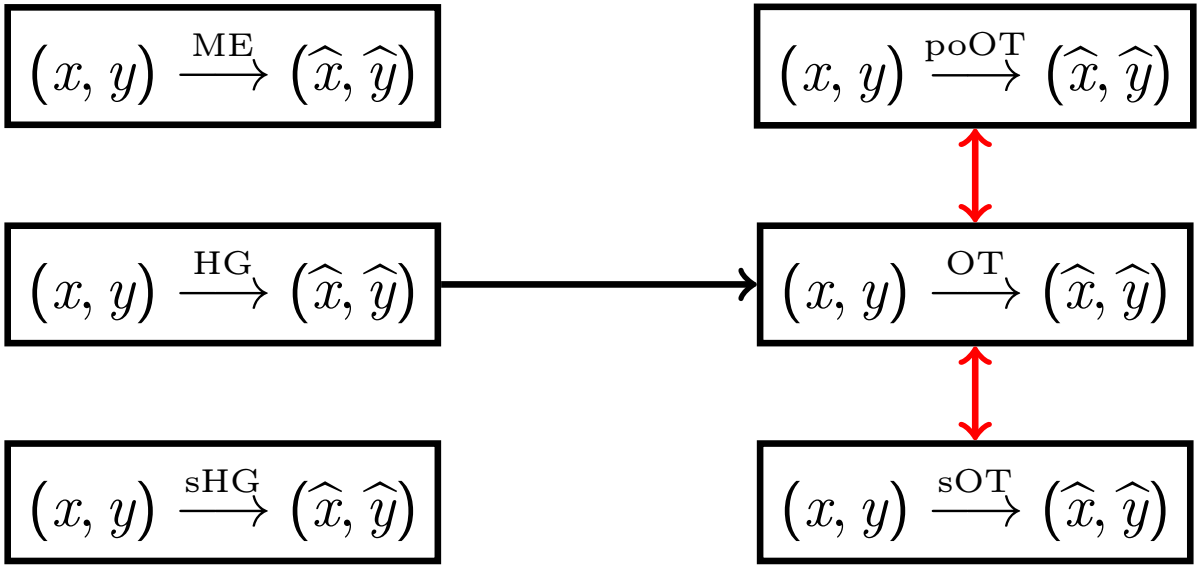
- But see below for more on this!



- T-orders in OT, stochastic OT, and partial order OT are identical



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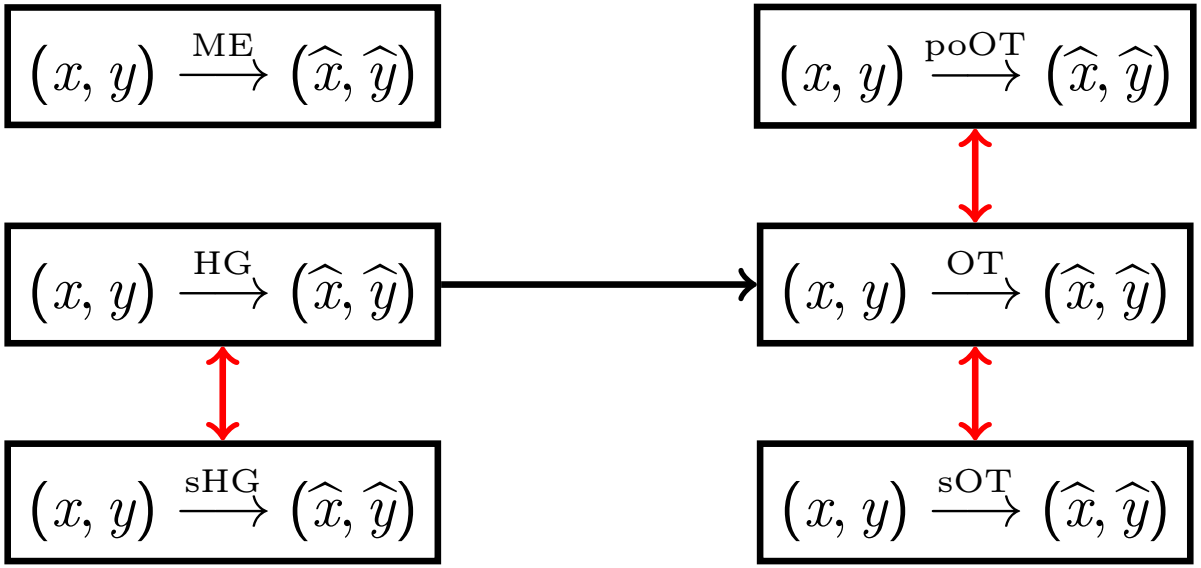

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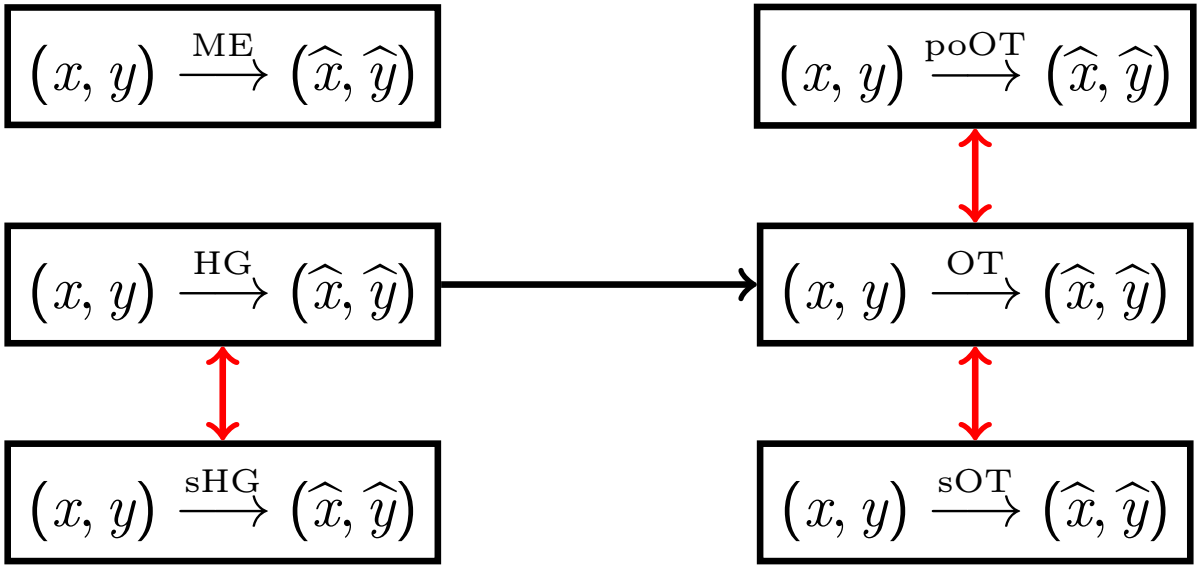

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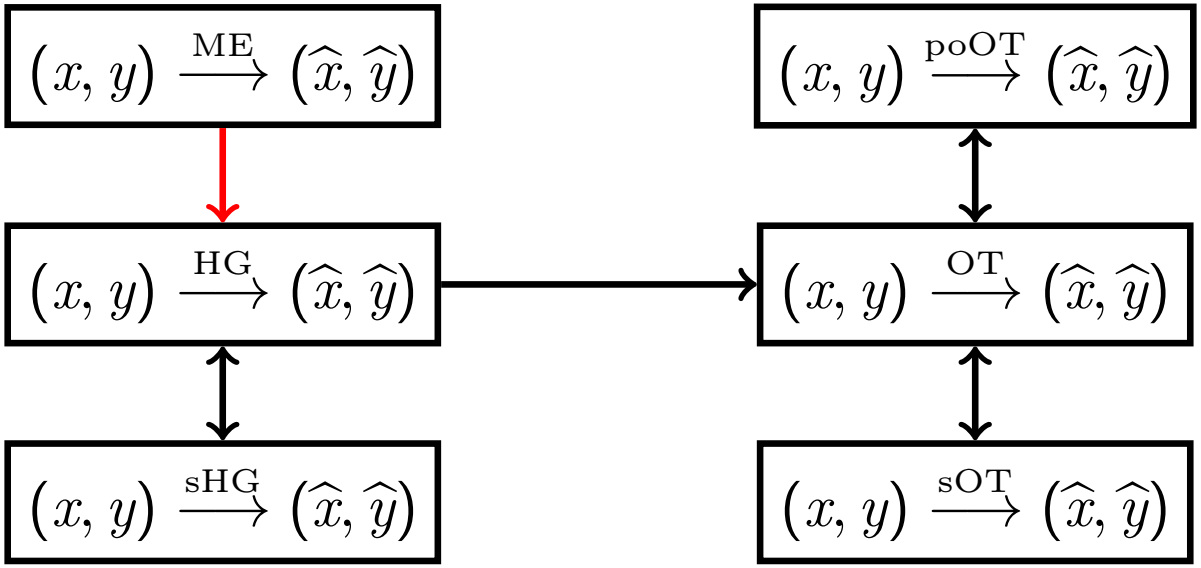
- Thus, stochastic OT/HG are probabilistic variants of categorical OT/HG which don't tamper with the categorical typological structure



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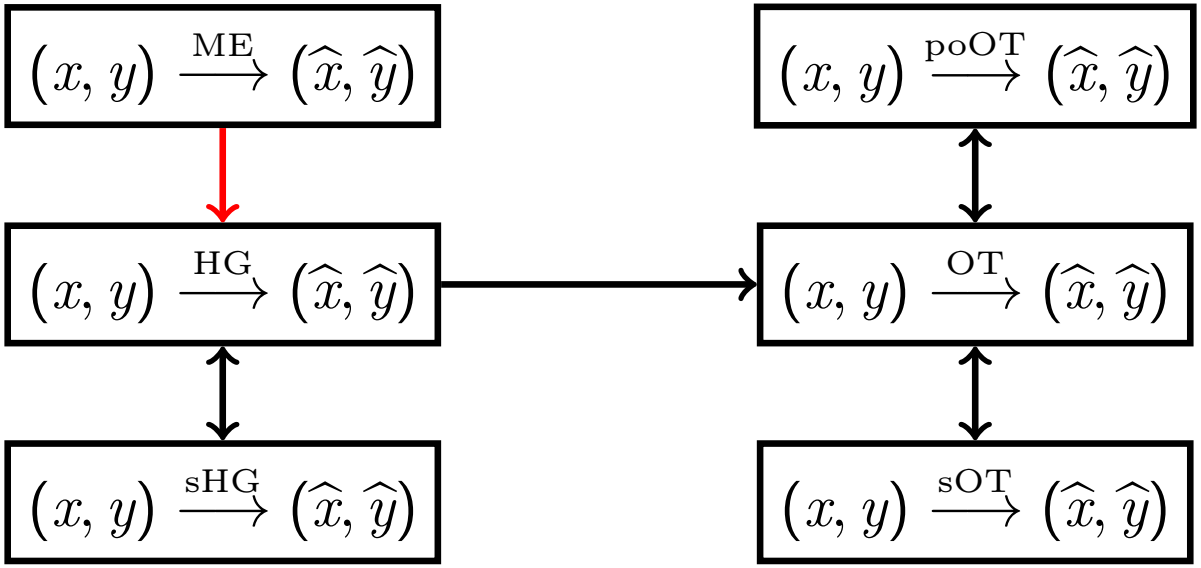

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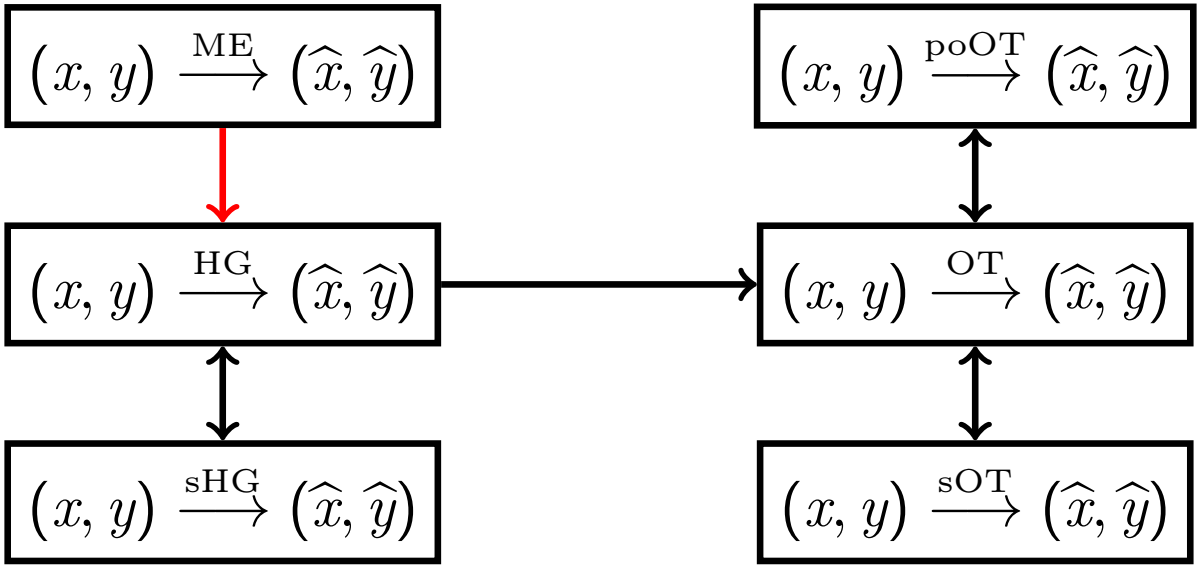

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- The vice versa fails even with 1 constraint and 1 loser!
- Thus, although ME and stochastic HG both look like "innocuous" probabilistic variants of HG, ME actually loses much of the typological structure imposed by HG

Formal results 2

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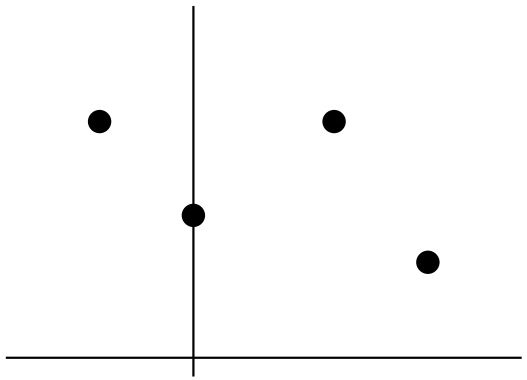
- The **consequent difference vectors** are defined analogously, as pitting the consequent mapping $(\widehat{x}, \widehat{y})$ against one of its losers $(\widehat{x}, \widehat{z})$

- Suppose there are only $n = 2$ constraints:
  - the antecedent difference vectors can be plotted as points in the plane



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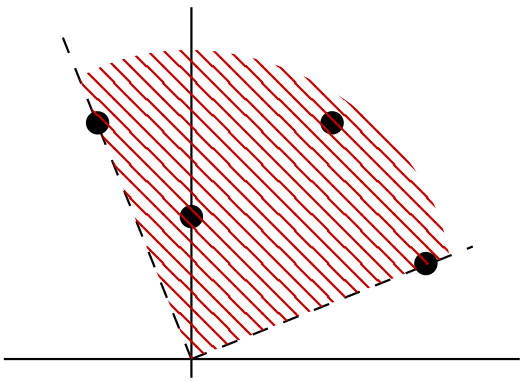

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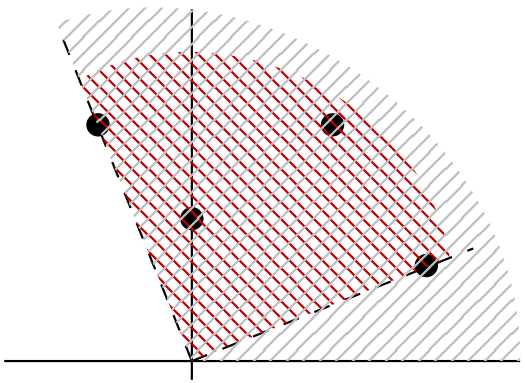

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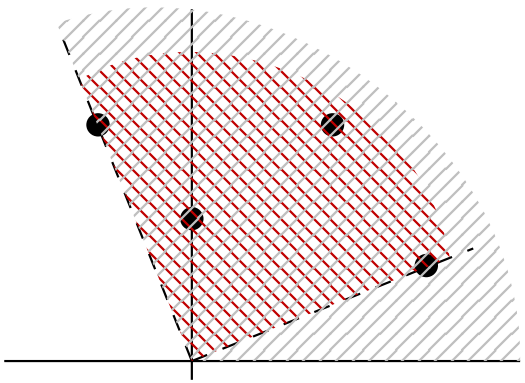

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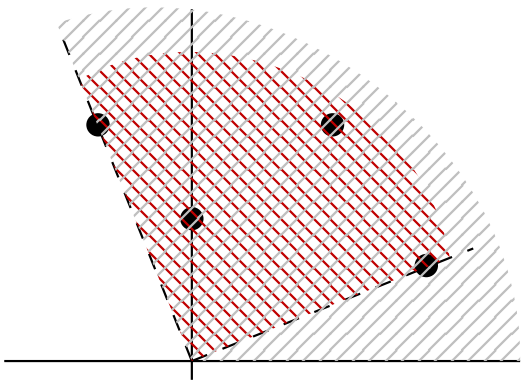

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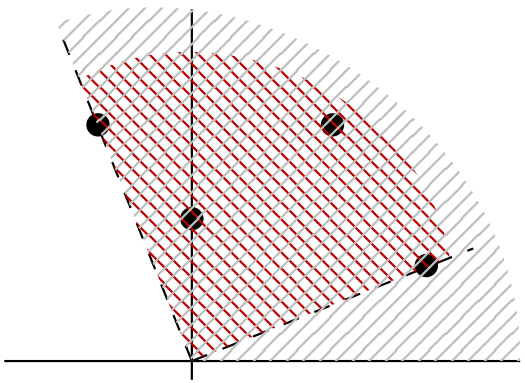

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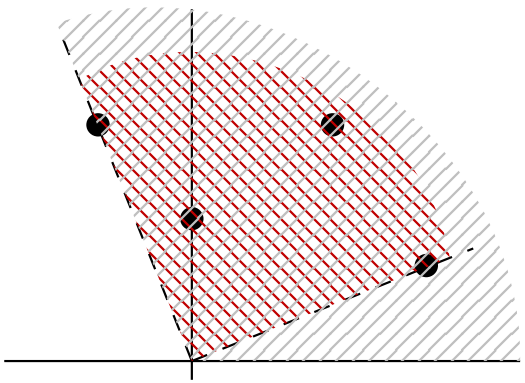

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- While $(x, y) \xrightarrow{\text{HG}} (\widehat{x}, \widehat{y})$ is **expensive** to check directly (universal quantification over weights), the geometric characterization above is **easy** to check (polyhedral feasibility problem)
- We thus advertise new `Python` code to establish HG T-orders

- Suppose again there are only $n = 2$ constraints:
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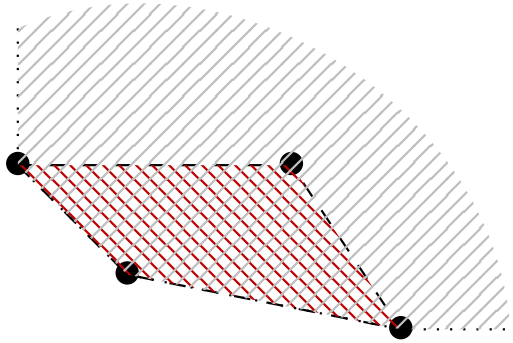

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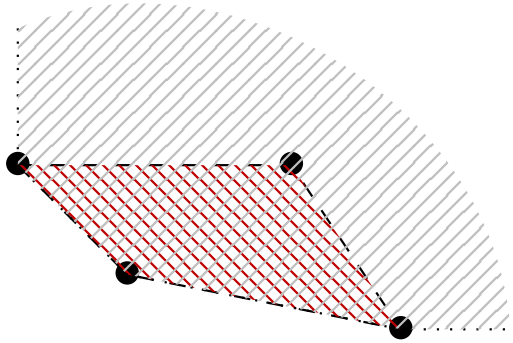

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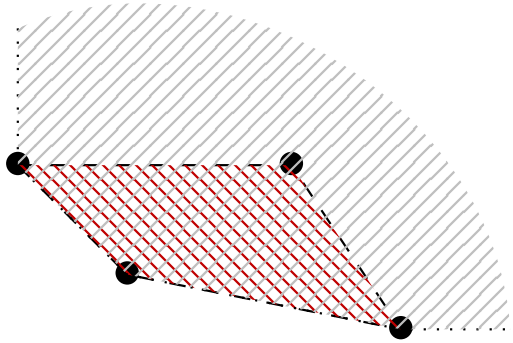

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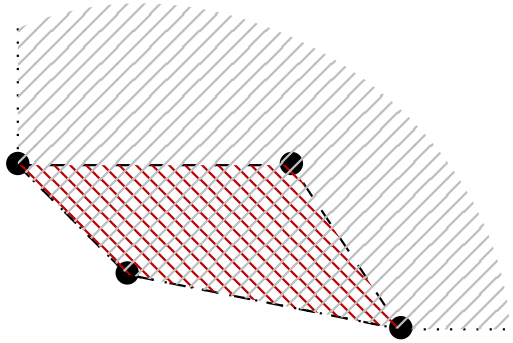

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- C1 is also sufficient when both x and $\widehat{x}$ have at most three candidates
- We derive a more involved sufficient condition which is nonetheless stronger than C1+C2

Phonological applications 1

Geminate devoicing in Japanese

[Kawahara 2006; Pater 2016]

Lyman's Law: One voiced obstruent per word

Recent loanwords violate Lyman's Law:

(a) [bobu] 'Bob' [giga] 'giga'

Voiced geminates are allowed in recent loanwords:

(b) [web:u] 'web' cf. [wip:u] 'whipped (cream)'

Singleton + geminate voiced obstruent yield optional devoicing:

(c) [gud:o] ~ [gut:o] 'good'
[dog:u] ~ [dok:u] 'dog'

Constraints for Japanese (Pater 2016)

IDENT-VOICE	Assign a violation for an output segment that differs from its input correspondent in [voice].
OCP-VOICE	Assign a violation for a two voiced obstruents within the same word.
*VCE-GEM	Assign a violation for a voiced obstruent geminate.

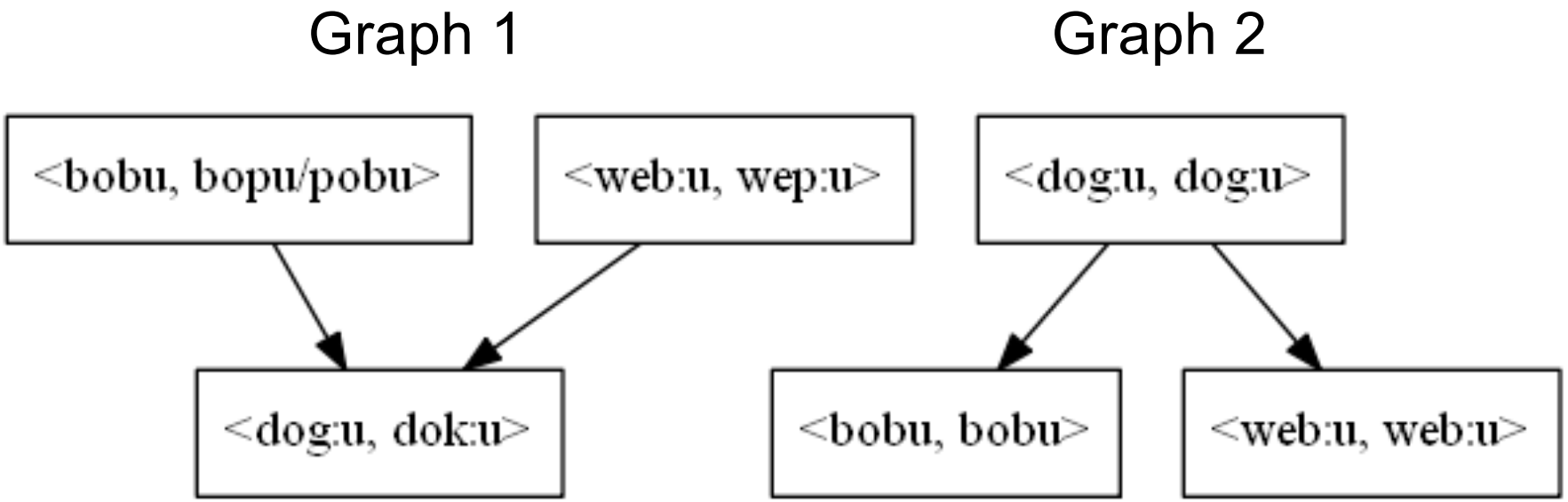
Geminate devoicing in HG (Pater 2016)

	1.5	1	1	Harmony
/bobu/	IDENT-VOICE	OCP-VOICE	*VCE-GEM	
(1a) ☞ bobu		−1		−1
(1b) bopu/pobu	−1			−1.5
/web:u/	IDENT-VOICE	OCP-VOICE	*VCE-GEM	
(2a) ☞ web:u			−1	−1
(2b) wep:u	−1			−1.5
/dog:u/	IDENT-VOICE	OCP-VOICE	*VCE-GEM	
(3a) dog:u		−1	−1	−2
(3b) ☞ dok:u	−1			−1.5

Predicted typology. OT languages are framed.

OT/HG	OT/HG	OT/HG	OT/HG	HG
L1	L2	L3	L4	L5
bobu	bopu/pobu	bobu	bopu/pobu	bobu
web:u	web:u	wep:u	wep:u	web:u
dog:u	dok:u	dok:u	dok:u	dok:u

T-order: OT = HG


Graph 1
Graph 2
<bobu, bopu/pobu>
<web:u, wep:u>
<dog:u, dog:u>
<dog:u, dok:u>
<bobu, bobu>
<web:u, web:u>

Theoretical result

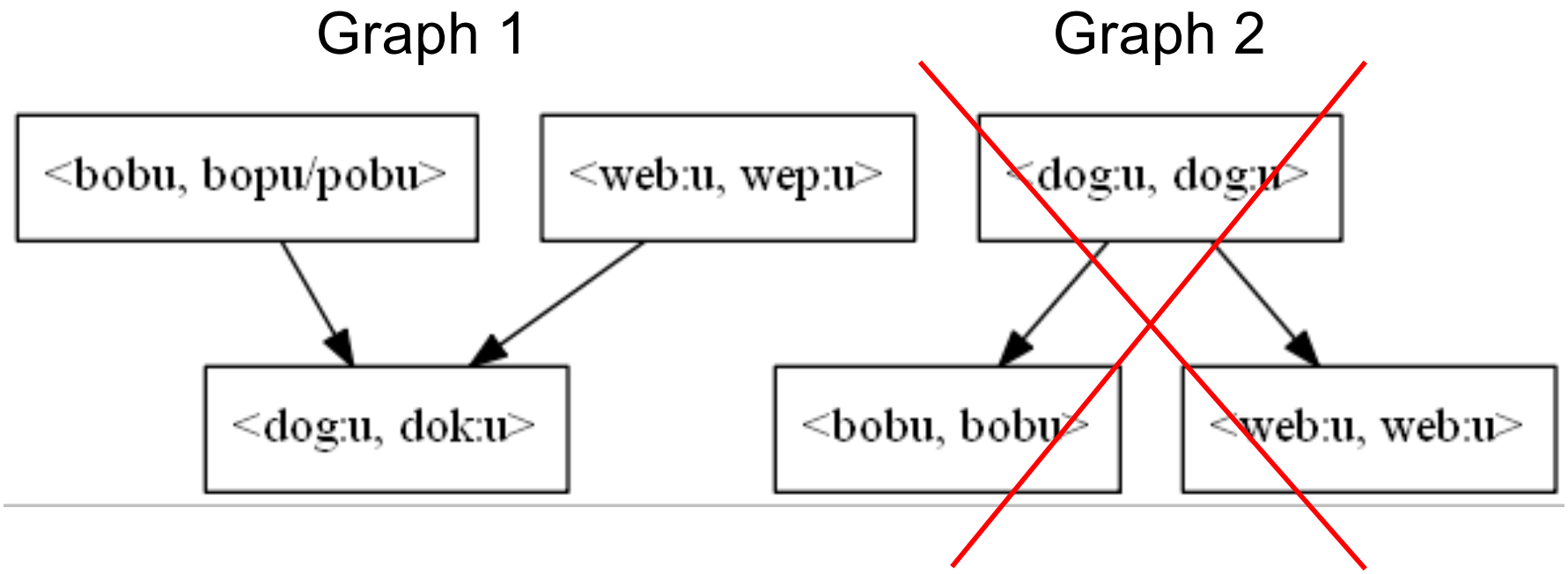
OT and HG T-orders can be identical even when the typologies are different.

- In particular, OT and HG T-orders are identical if there are **only two candidates** (Japanese).
- But this does not exhaust all such cases (e.g., Swedish obstruent voicing, Lombardi 1999, work in progress)

Let's add two harmonically bounded candidates

	1.5	1	1	Harmony
/bobu/	IDENT-VOICE	OCP-VOICE	*VCE-GEM	
(1a) ☞ bobu		−1		−1
(1b) bopu/pobu	−1			−1.5
/web:u/	IDENT-VOICE	OCP-VOICE	*VCE-GEM	
(2a) ☞ web:u			−1	−1
(2b) wep:u	−1			−1.5
/dog:u/	IDENT-VOICE	OCP-VOICE	*VCE-GEM	
(3a) dog:u		−1	−1	−2
(3b) ☞ dok:u	−1			−1.5
(3c) tog:u	−1		−1	−2.5
(3d) tok:u	−2			−3

Maxent T-order



Theoretical result

- In Maxent T-orders, an input with fewer candidates cannot entail one with more candidates.

  Even harmonically bounded candidates count: adding or subtracting them affects the T-order.

Phonological applications 2

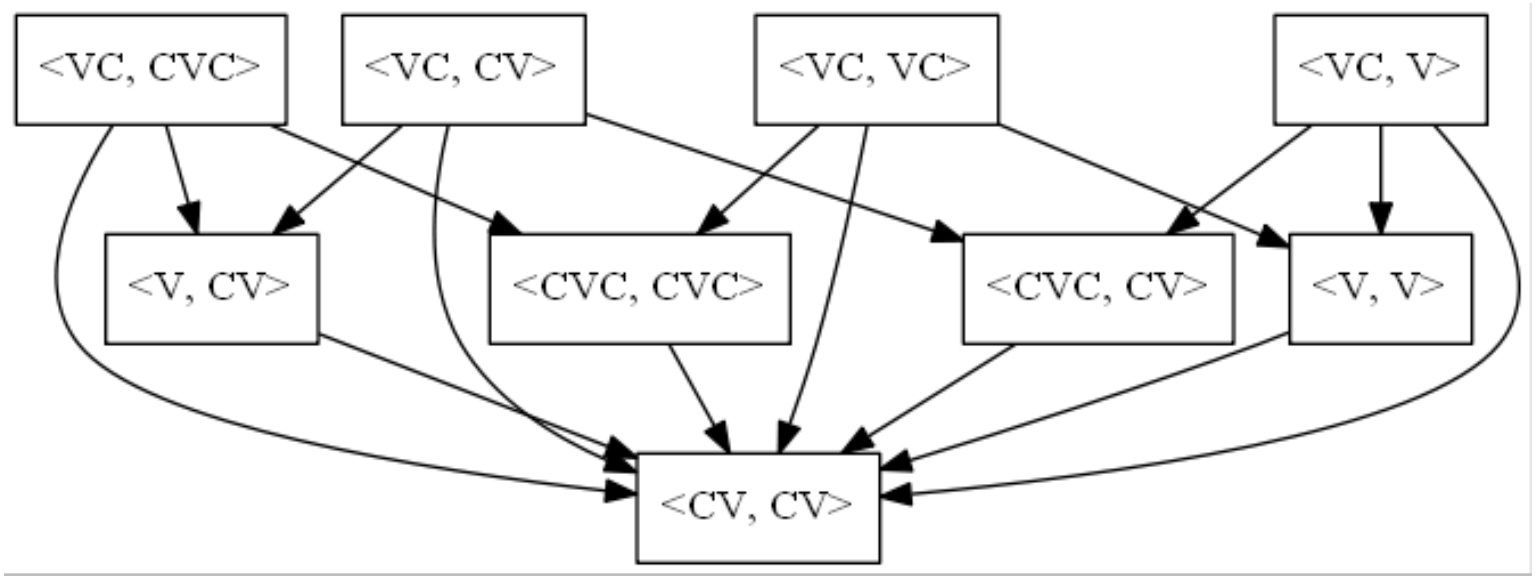
CV syllabification

[Prince and Smolensky 2004]

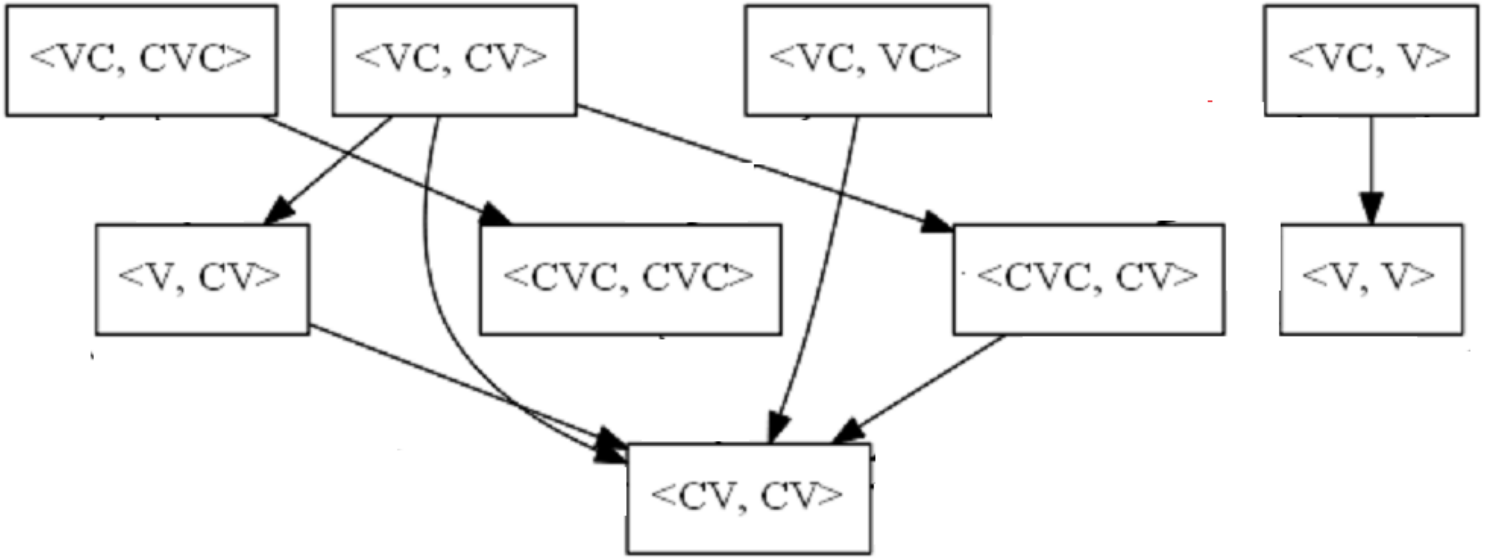
CV syllabification

Input	Output	ONSET	*CODA	MAX	DEP
1. CV	CV				
	CVC		1		1
	VC	1	1	1	1
	V	1		1	
2. CVC	CV			1	
	CVC		1		
	VC	1	1	1	
	V	1		2	
3. VC	CV			1	1
	CVC		1		1
	VC	1	1		
	V	1		1	
4. V	CV				1
	CVC		1		2
	VC	1	1		1
	V	1			

OT/HG T-order



Maxent T-order



What does this mean?

Example: The fact that (CVC, CVC) → (CV, CV) does not hold in Maxent means that there are Maxent grammars where the faithful mapping (CV, CV) has **a smaller probability** than the faithful mapping (CVC, CVC)!

The same holds of the following entailments, which are true in OT/HG, but not in Maxent:

(VC, V) → (CV, CV)
(VC, CVC) → (CV, CV)
(V, V) → (CV, CV)

We now know the following

- OT, HG, and Maxent all have T-orders.
- OT and HG T-orders are identical in certain special cases.
- T-orders can be efficiently computed for all three theories, for HG and Maxent for the first time.
- Maxent T-orders behave in a counterintuitive way: they are sensitive to the **number of candidates**, including harmonically bounded ones.
- Given the standard CV syllabification constraints, Maxent predicts a non-Jakobsonian syllable typology.

Thank you!

Summary: OT vs. HG

Typology	T-orders	Cands	Example
same	same	2	Finnish vowel harmony
same	same	>2	CV syllable typology
same	different	2	CANNOT EXIST
same	different	>2	CANNOT EXIST
different	same	2	Japanese geminate devoicing
different	same	>2	Swedish obstruent devoicing
different	different	2	CANNOT EXIST
different	different	>2	Spanish obstruent lenition/fortition

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