Equiprobable mappings in weighted constraint grammars

Arto Anttila (Stanford), Scott Borgeson (Stanford), and Giorgio Magri (CNRS)

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1.
From phonological equivalences to equiprobability

- This paper is part of a larger project systematically comparing two approaches to **probabilistic** generative phonology:
  - **Stochastic Harmonic Grammar** (SHG)
  - **Maximum Entropy** (ME)

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- In this paper, we illustrate the gist of this argument by focusing on a specific grammatical property: **equivalent mappings**

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  are **equivalent** for categorical vowel harmony
- These phonological equivalences are a key property of phonological systems: so how should this notion of phonological equivalence be extended from the categorical to the probabilistic setting?

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- To illustrate, the fact that words that only differ for length are equivalent for vowel harmony means that the probability of vowel harmony does not depend on the number of syllables

$$\mathbb{P}\big(\text{[maana]} \,\big|\, \text{/maa-nä/}\big) = \mathbb{P}\big(\text{[rakastajana]} \,\big|\, \text{/rakastaja-nä/}\big)$$

2.
ME admits no equiprobable mappings

- Given a winner mapping and a competing loser mapping, we call **difference vector** the vector consisting of the constraint violations of the loser discounted of the violations of the winner

$$\text{difference vector} = \begin{bmatrix} C_1(\text{loser}) - C_1(\text{winner}) \\ C_2(\text{loser}) - C_2(\text{winner}) \\ \vdots \\ C_n(\text{loser}) - C_n(\text{winner}) \end{bmatrix}$$

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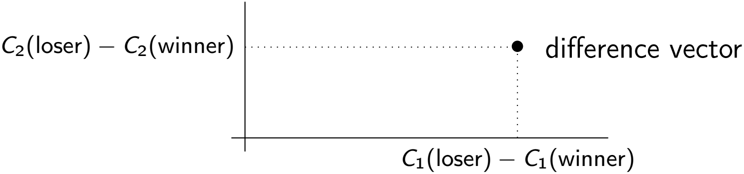
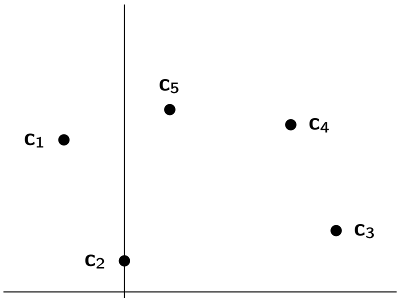
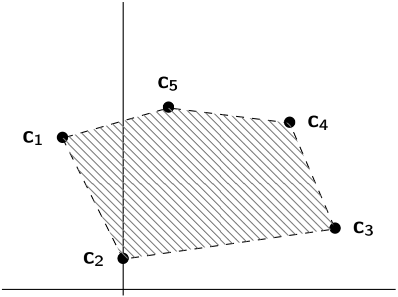
- When there are only $n = 2$ constraints C_1 and C_2, difference vectors can be plotted as points in the Cartesian plane:

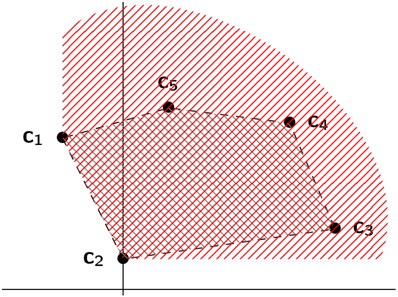




- Suppose the mapping $(/UR/, [SR])$ comes with 5 difference vectors



- Suppose the mapping $(/UR/, [SR])$ comes with 5 difference vectors
- The gray region is the **convex hull** generated by these vectors



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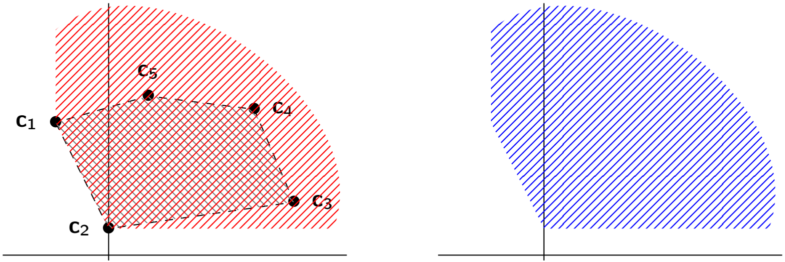
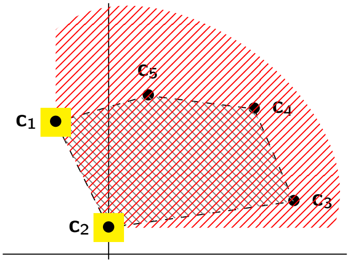
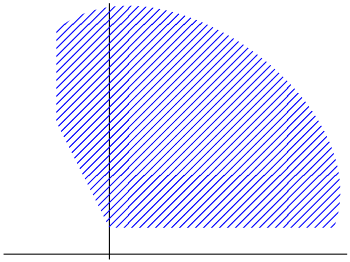


- Two mappings $(/\color{red}{UR}/,[\color{red}{SR}])$ and $(/\color{blue}{\widehat{UR}}/,[\color{blue}{\widehat{SR}}])$ are ME equiprobable if and only if the corresponding red and blue regions coincide





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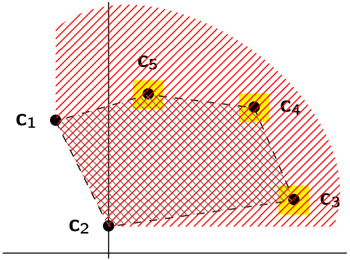

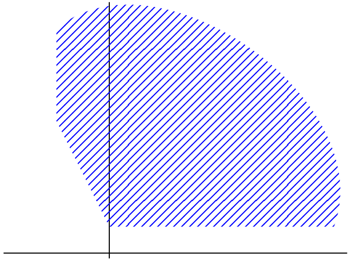

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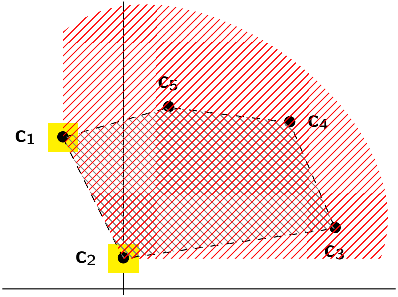
- The difference vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ are **extreme points**: they determine the shape of the region and must be shared by the two mappings
- The difference vectors $\mathbf{c}_3$, $\mathbf{c}_4, \mathbf{c}_5$ are instead **interior points**: they do not contribute to the shape of the region



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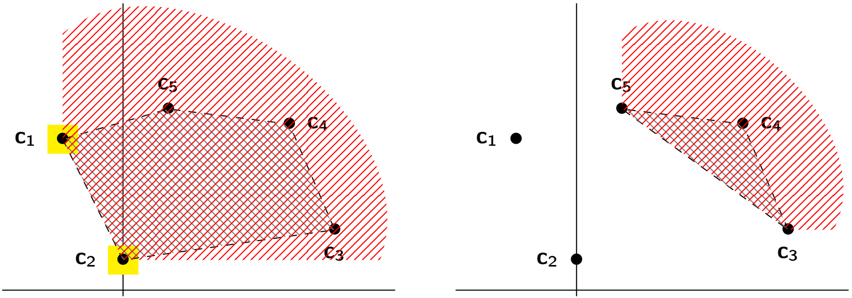

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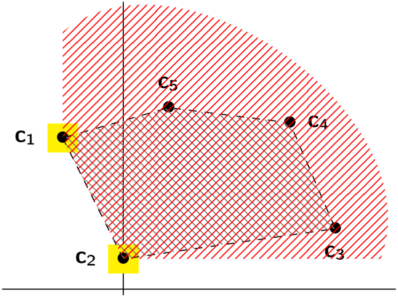

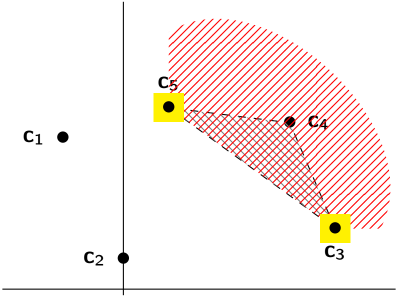

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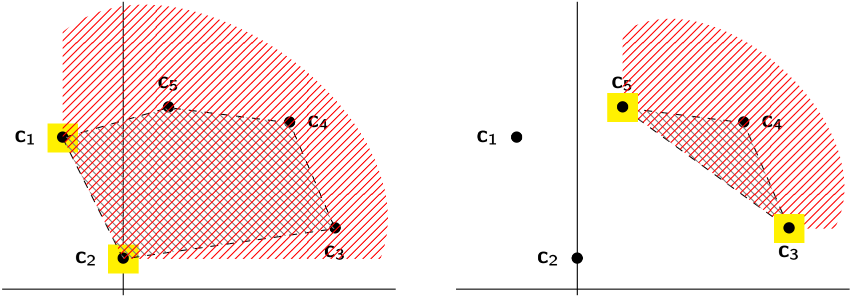

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- And so on

- The conclusion of this reasoning is that the two mappings $(/\textcolor{red}{UR}/, [\textcolor{red}{SR}])$ and $(/\widehat{\textcolor{blue}{UR}}/, [\widehat{\textcolor{blue}{SR}}])$ are equiprobable in ME if and only if the corresponding sets of difference vectors exactly coincide

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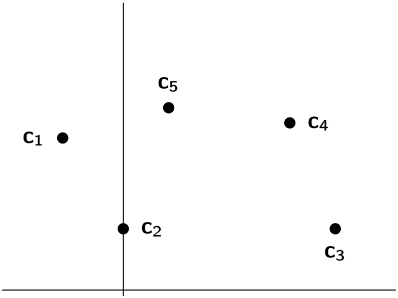
- $\boxed{\text{In conclusion, ME effectively admits no equiprobable mappings}}$

3.
SHG instead does admit equiprobable mappings



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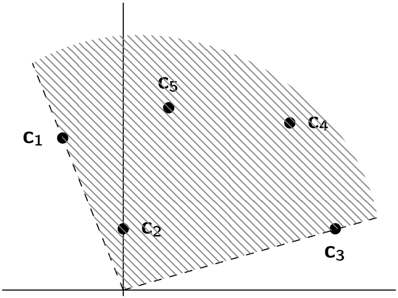

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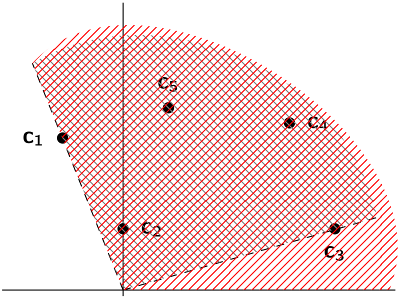

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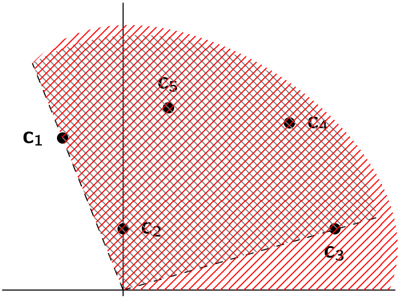

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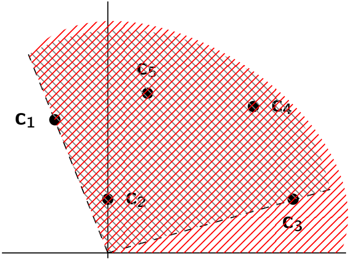
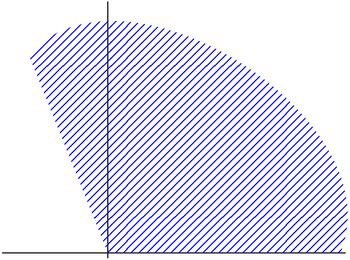
- Same as for ME with cones ( ) rather than convex hulls ( )





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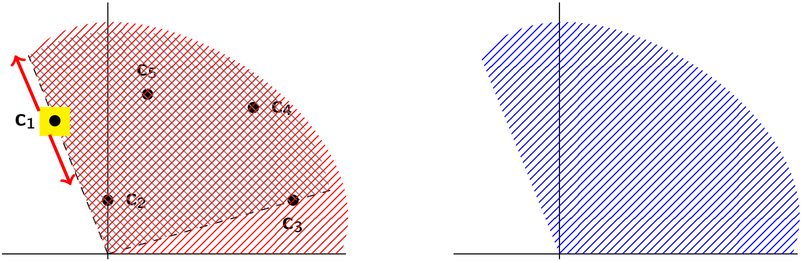

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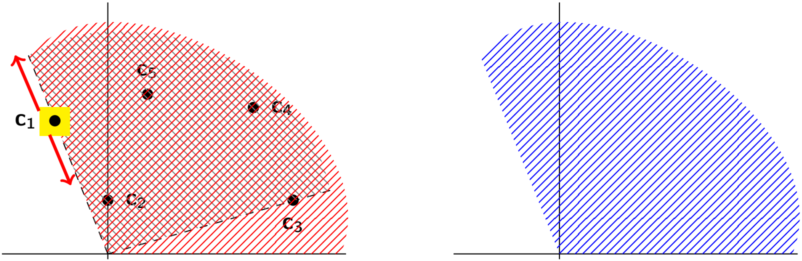

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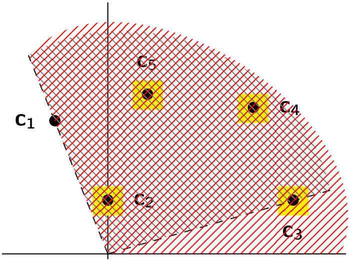

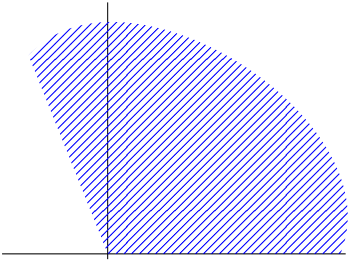

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- And nothing can be said about the interior vectors $\mathbf{c}_2, \ldots, \mathbf{c}_5$

- The conclusion of this reasoning is that two mappings $(/UR/, [SR])$ and $(/\widehat{UR}/, [\widehat{SR}])$ are equiprobable in SHG if and only if each of the **non-interior** difference vectors is a **rescaling** of a non-interior difference vector of the other

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- This SHG condition is way weaker than than the ME condition above:
  - ME condition requires **identity** of difference vectors...
    ...while this SHG condition only requires **rescaling**
  - ME condition applies to **all** difference vectors...
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- $\boxed{\text{In conclusion, SHG does admit equiprobable mappings}}$

4.
A test case: equiprobability in Finnish secondary stress

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- In this final part of the talk, we would like to offer (an admittedly very **preliminary**) argument that the latter might be the case
- We will look at Finnish secondary stress and argue that the equiprobable mappings predicted by SHG are (by and large) attested at least in Finnish while ME over-generates

- Finnish stress is governed by four core rules:
  - Primary stress falls on the initial syllable
  - Secondary stress falls on every other syllable after that...
  - except that a light syllable is skipped if the syllable after that is heavy
  - unless that heavy syllable is final

[Carlson 1978; Hanson and Kiparsky 1996; Elenbaas and Kager 1999; Kiparsky 2003; Karvonen 2005; Karttunen 2006]

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- We focus on the "skipping clause" and note that it leaks in long words: instead of being categorical it exhibits probabilistic variation

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- Skipping depends on vowel quality and preceding syllable weight in a non-categorical/probabilistic manner [Anttila 2012]
  - the "low" vowels */a, ä, o, ö/* attract stress
  - the "high" vowels */e, i, u, y/* repel stress
  - stress is avoided next to a heavy syllable

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- In other words, we have the following correspondences:

		/professori-i-ta/ 'professor-PL-PART'
no skipping $\Longleftrightarrow$ *t*-deletion		(**pró**.fes)(**sò**.re)ja
skipping $\Longleftrightarrow$ *t*-retention		(**pró**.fes.so)(**rèi**.ta)

[Keyser and Kiparsky 1984; Anttila 2012]

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- These phonological forms are evaluated by a set of eight constraints:

WSP	No unstressed heavy syllables
WSP/VV	No unstressed heavy syllables with a long vowel
FTBIN	Feet are disyllabic
PKPROM	No stressed light syllables
ALIGN-L	All feet left
*REV	No trochees with sonority reversal = *(Í.A
*FLAT	No trochees with imperfect sonority = *(Í.I, *(Á.A, *(Í.A
*H.X	No stress next to a heavy syllable

- We computed SHG/ME uniform probability inequalities for this model using $\mathbb{C}$o$\mathbb{G}$e$\mathbb{T}$o: a new suite of $\mathbb{T}$ools for studying categorical and probabilistic constraint-based typologies based on their rich underlying $\mathbb{C}$onvex $\mathbb{G}$eometry [Magri and Anttila 2019: https://cogeto.stanford.edu]

- We computed SHG/ME uniform probability inequalities for this model using CoGeTo: a new suite of Tools for studying categorical and probabilistic constraint-based typologies based on their rich underlying Convex Geometry [Magri and Anttila 2019: https://cogeto.stanford.edu]

- SGH predicts seven blocks of equiprobable mappings ordered through uniform probability inequalities (denoted $\leq$):

(kon.sul)(taa.ti.o)ja (kom.mu)(ni.ke.o)ja (o.pe)(raa.ti.o)ja (al.le)(go.ri.o)ja	$\leq$	(sym.po)(si.u.me)ja (po.ly)(a.mi.de)ja (in.ku)(naa.be.le)ja (lii.rum)(laa.ru.me)ja	$\leq$	(pro.pa)(gan.dis.te)ja (ak.va)(rel.lis.te)ja	$\leq$	(ter.mos)(taat.te)ja (mar.ga)(rii.ne)ja (af.fri)(kaat.to)ja
(pro.pa)(gan.dis)(tei.ta) (ak.va)(rel.lis)(tei.ta)	$\leq$	(po.ly)(a.mi)(dei.ta) (lii.rum)(laa.ru)(mei.ta) (sym.po)(si.u)(mei.ta) (in.ku)(naa.be)(lei.ta)	$\leq$	(al.le)(go.ri)(oi.ta) (kom.mu)(ni.ke)(oi.ta) (kon.sul)(taa.ti)(oi.ta) (o.pe)(raa.ti)(oi.ta)	$\leq$	

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(sym.po)(si.u)(mei.ta)
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(mar.ga)(rii.ne)ja
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- This confirms on a naturalistic example the formal result in the first pat of the talk, that SHG indeed allows for equiprobable mappings

- To start evaluating empirically these predicted equiprobable blocks, we examined Finnish *t*-deletion in a corpus of approximately 9 million nouns (tokens) harvested from Finnish internet sites on April 12, 2005

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- We repeat the equiprobable blocks annotated with token frequency of *t*-retention/deletion for each stem type (type frequencies are similar)

(kon.sul)(taa.ti.o)ja 0.5%
(kom.mu)(ni.ke.o)ja 0.3%
(o.pe)(raa.ti.o)ja 0.0%
(al.le)(go.ri.o)ja 0.0%

$\leq$

(sym.po)(si.u.me)ja 98.6%
(po.ly)(a.mi.de)ja 95.7%
(in.ku)(naa.be.le)ja 9.5%
(lii.rum)(laa.ru.me)ja 18.6%

$\leq$

(pro.pa)(gan.dis.te)ja 100%
(ak.va)(rel.lis.te)ja 100%

$\leq$

(ter.mos)(taat.te)ja 100%
(mar.ga)(rii.ne)ja 100%
(af.fri)(kaat.to)ja 99.7%

$\geq$

(al.le)(go.ri)(oi.ta) 100%
(kom.mu)(ni.ke)(oi.ta) 99.7%
(kon.sul)(taa.ti)(oi.ta) 99.5%
(o.pe)(raa.ti)(oi.ta) 100%

$\geq$

(po.ly)(a.mi)(dei.ta) 4.3%
(lii.rum)(laa.ru)(mei.ta) 81.4%
(sym.po)(si.u)(mei.ta) 1.4%
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$\geq$

(pro.pa)(gan.dis)(tei.ta) 0.0%
(ak.va)(rel.lis)(tei.ta) 0.0%

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(pro.pa)(gan.dis)(tei.ta) 0.0% (ak.va)(rel.lis)(tei.ta) 0.0%	$\leq$	(po.ly)(a.mi)(dei.ta) 4.3% (lii.rum)(laa.ru)(mei.ta) 81.4% (sym.po)(si.u)(mei.ta) 1.4% (in.ku)(naa.be)(lei.ta) 90.5%	$\leq$	(al.le)(go.ri)(oi.ta) 100% (kom.mu)(ni.ke)(oi.ta) 99.7% (kon.sul)(taa.ti)(oi.ta) 99.5% (o.pe)(raa.ti)(oi.ta) 100%	$\leq$	

- **Observation 1**: the SHG equiprobability prediction is consistent with the data in the 5 black blocks (all stems are nearly categorical)

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$\leq$

(ter.mos)(taat.te)ja 100% …

- **Observation 1**: the SHG equiprobability prediction is consistent with the data in the 5 black blocks (all stems are nearly categorical)

- **Observation 2**: the SHG equiprobability prediction is challenged in the 2 red blocks, that we now focus on

- These two problematic red blocks involve four stem types
  *lii.**rum.laa**.ru.mi* *in.**ku.naa**.be.li* *sym.**po.si**.u.mi* *po.**ly.a**.mi.di*
  that differ in their preantepenultimate and antepenultimate syllables (heavy vs. light, high vs. low) and yet SHG predicts they should undergo *t*-deletion/retention at identical rates

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- But the difference between *t*-deletion/retention rates for stems of the *liirumlaarumi*-type and *inkunaabeli*-type is not statistically significant ($\chi^2 = 2.9849$, $df = 1$, $p = 0.08404$)

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- But the difference between *t*-deletion/retention rates for stems of the *liirumlaarumi*-type and *inkunaabeli*-type is not statistically significant ($\chi^2 = 2.9849$, $df = 1$, $p = 0.08404$)
- Furthermore, there are only two stems in the *symposiumi*-type (*symposiumi*, *imperiumi*) and both could be re-analyzed as 4-syllable stems, consistently with high *t*-deletion rate [Anttila and Shapiro 2017]

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- Admittedly, we have no plausible explanation for the unexpectedly high *t*-deletion rate for stem of the *polyamidi*-type ($N = 69$)
- We conclude that the Finnish data largely (although admittedly not completely) support SHG's equiprobability predictions

- Does ME with its ability to make fine-grained distinctions offer a more principled solution to the difficulties just encountered with the two red blocks? This turns out not to be the case

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- In fact, as expected given the formal result in the first part of the talk, ME breaks up these two red equiprobable blocks and orders their stem types through uniform probability inequalities:

(sym.po)(si.u)(mei.ta) 1.4%	$\leq$	(po.ly)(a.mi)(dei.ta) 4.3%	$\leq$	(lii.rum)(laa.ru)(mei.ta) 81.4%	$\leq$	(in.ku)(naa.be)(lei.ta) 90.5%
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- On the **retention side** (top), ME's predictions seem promising: corpus frequencies abide by the predicted probability inequalities
- On the **deletion side** (bottom) though, ME reverses the probabilities, yielding exactly the opposite of what we observe in the data
- We submit that there is simply no way to reconcile ME's counterintuitive probability reversals with the corpus data

Wrapping up:

- We have shown that ME can distinguish between any two different mappings and thus admits no equiprobable mappings

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Grazie!/Thank you!

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