Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2007-2008, Stanford University
Instructor: Ashish Goel
Handout 12: Lab 5. Lab date - 11/16/07

Consider the following problem. Given a bipartite graph, we want to apply non-negative weights, x_i, to each node i such that for every edge (i, j) in the graph, $x_i + x_j \geq 1$. In doing so, we wish to minimize the sum of all the weights, x_i.

a) Represent this problem as a linear program.

b) The problem of finding the smallest set of nodes such that for every edge (i, j) in the graph, either i or j (or both) is in the set is known as the vertex cover problem. To solve this problem with our above LP, what would we have to be able to ensure when we solve the LP?

c) Given the bipartite network with nodes $\{v_1, ..., v_5, w_1, ..., w_5\}$ and the following edges

$$\{(1,1), (1,2), (2,2), (2,4), (3,3), (4,5), (5,5)\}$$

where edge (i, j) represents an edge from v_i to w_j. Set this problem up in Excel and solve it.

d) Does our solution from c) solve the corresponding vertex cover problem?

e) Find the dual of the general form LP from a).

f) With respect to bipartite graphs, what problem does this look like? What is the optimal objective value to the dual of the problem from c)?