

Introduction to Optimization

MS&E 111/Engr 62, Fall 2007, Stanford University

Instructor: Ashish Goel

Practice Midterm Exam

1. Write an LP to compute the shortest path distance from s to t in the attached graph. List the basic feasible solutions of your LP.
2. VRM Chapter 3, Problem 10(a) (formulate, but do not solve)
3. VRM Chapter 3, Problem 12(a)
4. Four customers, A, B, C, D are in the market to buy five loaves of bread each. The baker has three types of bread, P, Q, and R. There are 8 loaves of type P, 6 loaves of type Q, and 6 of type R. The baker knows how many dollars each customer is willing to pay for each loaf of bread:

	A	B	C	D
P	1	2	2	1
Q	2	3	2	1
R	3	1	2	1

The baker wants to decide how many of each type of loaf to sell to each customer to maximize her profit. Formulate this as an LP. You may assume that loaves can be split between multiple customers.

Prove that there exists an optimum solution with at most 7 non-zero variables.

(Extra credit challenge) Prove that there exists an optimum solution with at most 6 non-zero variables.

5. You are given N tasks, and M agents. Agent i can execute a fraction f_{ij} of task j in one hour. You have to assign tasks to agents to minimize the time t when all the jobs get completed, i.e. when the latest finishing job completes. Assume that jobs can be done simultaneously.

Formulate this as an LP.

Prove that there exists an optimum solution with at most $N + M - 1$ pairs (i, j) such that agent i executes a non-zero amount of task j .

Provide a numerical example in which an optimal BFS has exactly $N + M - 1$ pairs (i, j) such that agent i executes a non-zero amount of task j .